Computer Algebra Independent Integration Tests

Summer 2023 edition

3-Logarithms/61-3.2.3-u-log-e-f-a+b-x-^p-c+d-x-^q-^r-^s

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September 5, 2023 Compiled on September 5, 2023 at 11:31am

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CHAPTER 1

INTRODUCTION

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This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [108]. This is test number [61].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	99.07 (107)	0.93 (1)
Rubi	98.15 (106)	1.85 (2)
Maxima	62.96 (68)	37.04 (40)
Maple	41.67 (45)	58.33 (63)
Fricas	37.96 (41)	62.04 (67)
Giac	33.33 (36)	66.67 (72)
Mupad	32.41 (35)	67.59 (73)
Sympy	18.52 (20)	81.48 (88)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

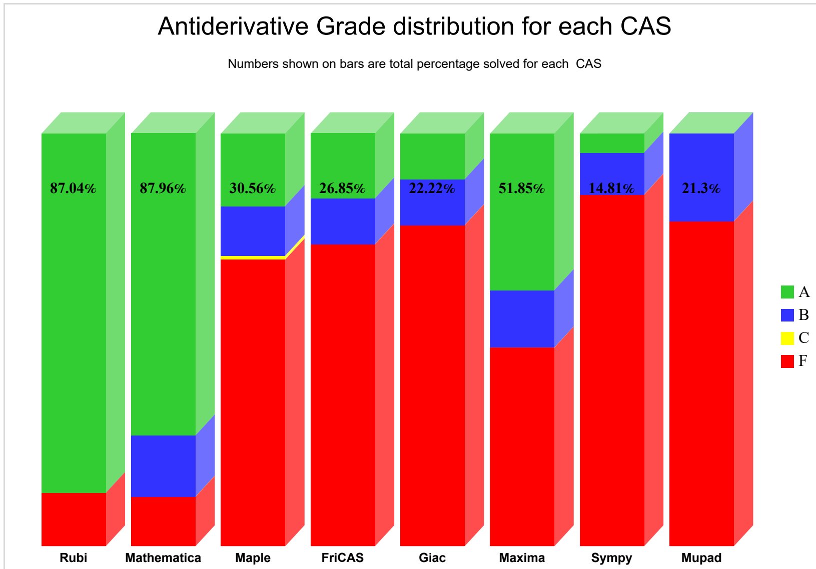
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

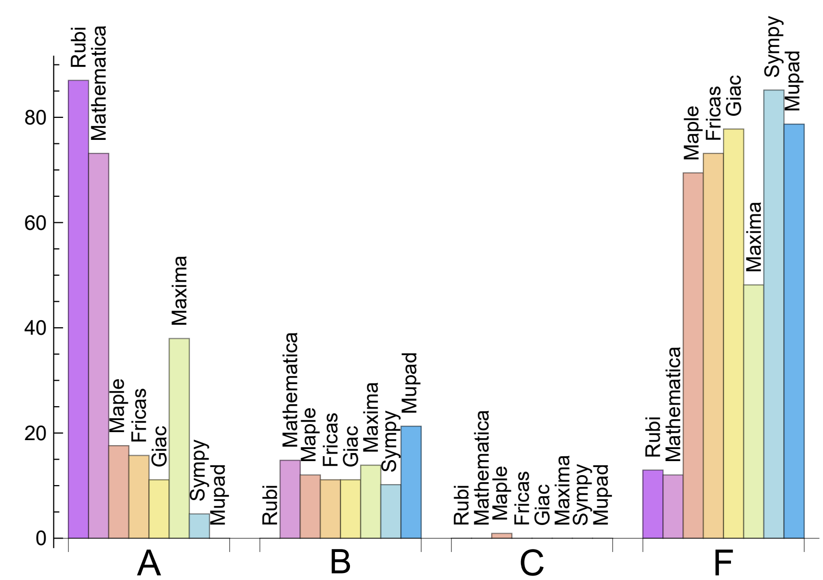
System	% A grade	% B grade	% C grade	% F grade
Rubi	87.037	0.000	0.000	12.963
Mathematica	73.148	14.815	0.000	12.037
Maxima	37.963	13.889	0.000	48.148
Maple	17.593	12.037	0.926	69.444
Fricas	15.741	11.111	0.000	73.148
Giac	11.111	11.111	0.000	77.778
Sympy	4.630	10.185	0.000	85.185
Mupad	0.000	21.296	0.000	78.704

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	1	100.00	0.00	0.00
Rubi	2	100.00	0.00	0.00
Maxima	40	75.00	0.00	25.00
Maple	63	100.00	0.00	0.00
Fricas	67	95.52	4.48	0.00
Giac	72	97.22	2.78	0.00
Mupad	73	0.00	100.00	0.00
Sympy	88	37.50	56.82	5.68

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.28
Maxima	0.45
Mathematica	0.49
Fricas	2.07
Mupad	2.28
Giac	2.45
Sympy	30.14
Maple	36.64

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Sympy	163.75	2.17	65.00	1.98
Fricas	186.63	1.66	59.00	1.29
Maple	240.00	1.88	130.00	1.15
Mupad	275.66	1.69	50.00	1.10
Giac	310.47	2.12	63.50	1.13
Rubi	348.75	1.00	182.50	1.00
Maxima	453.97	2.19	199.00	1.48
Mathematica	1211.81	2.10	164.00	1.00

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

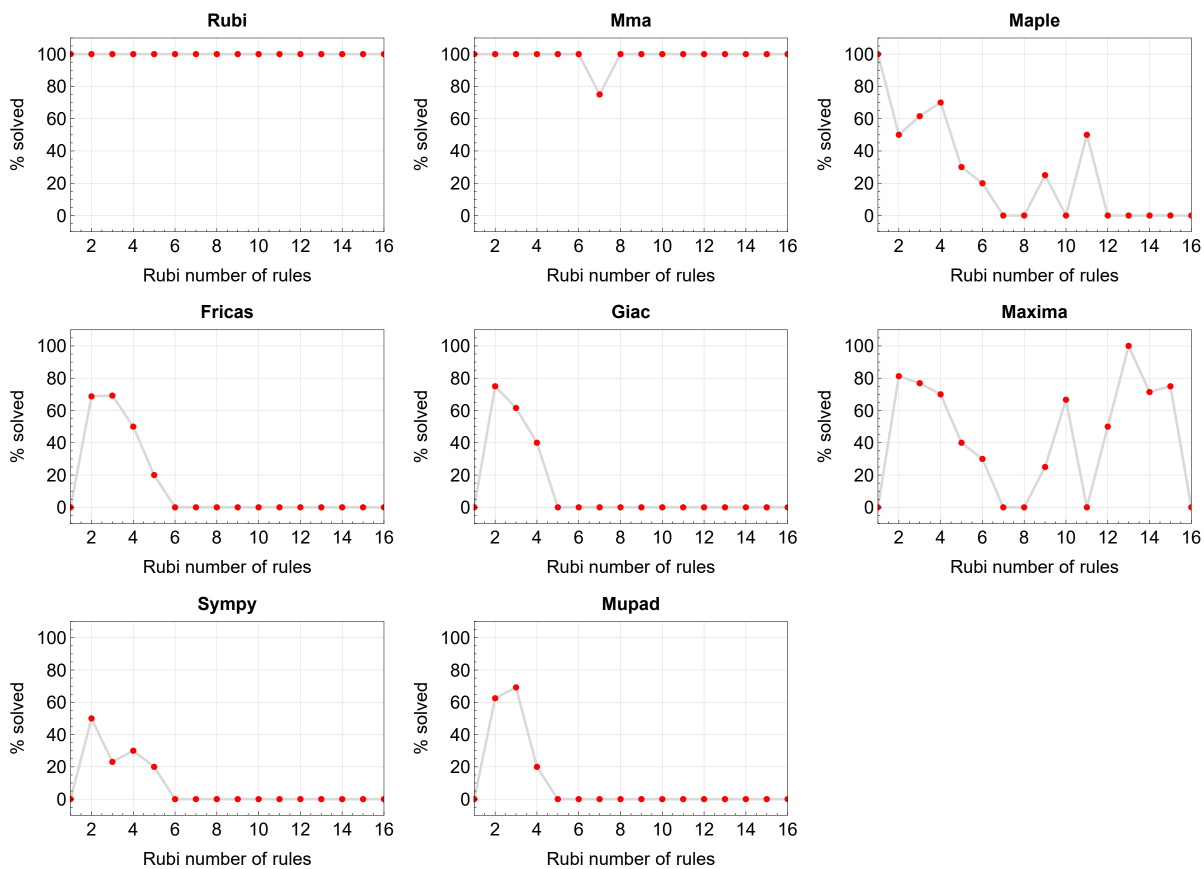


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

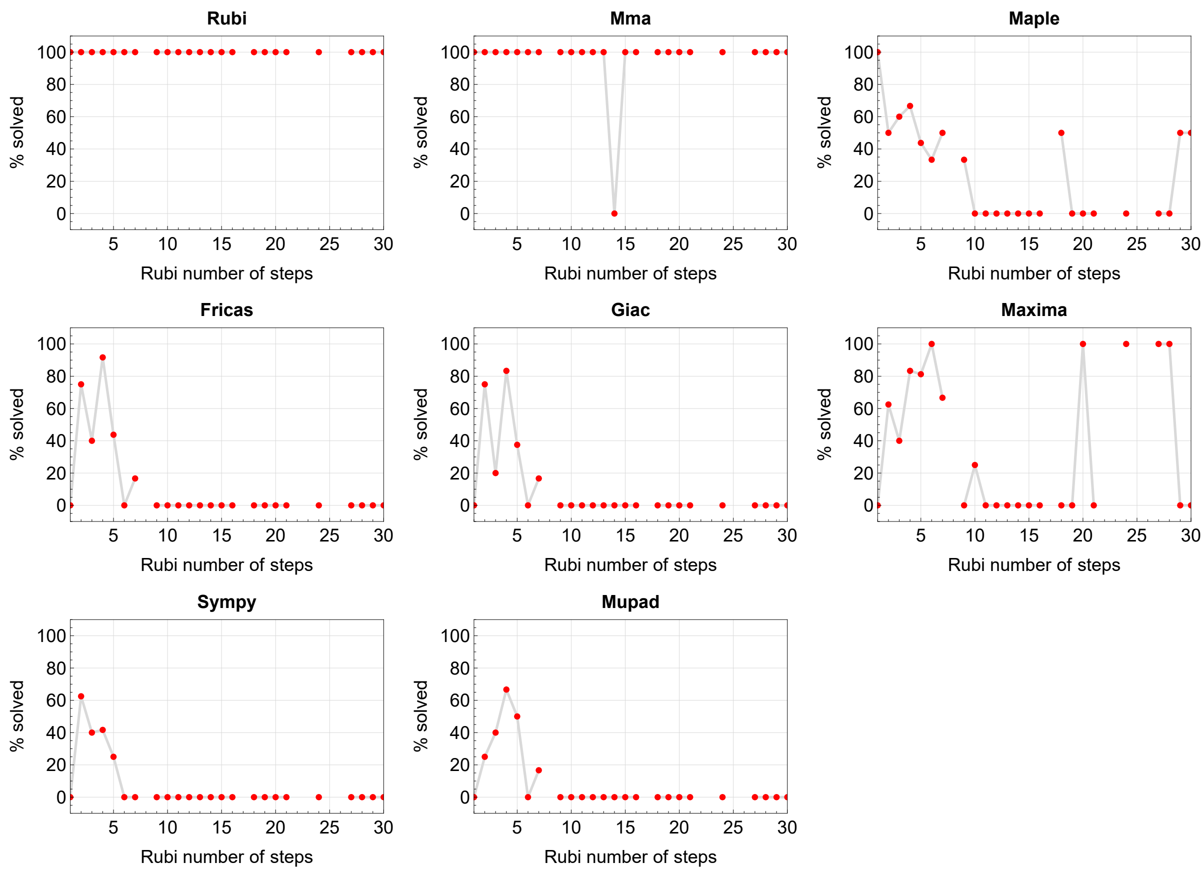


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

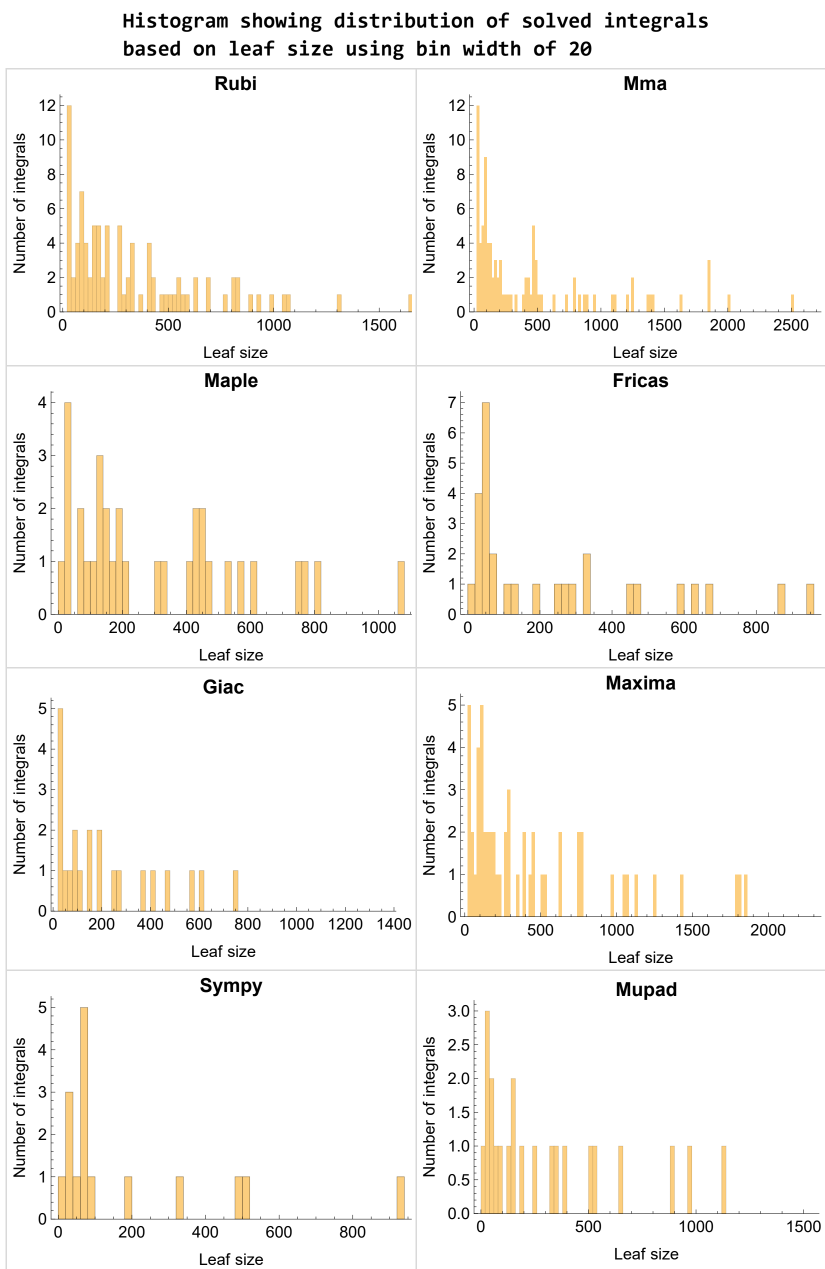


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

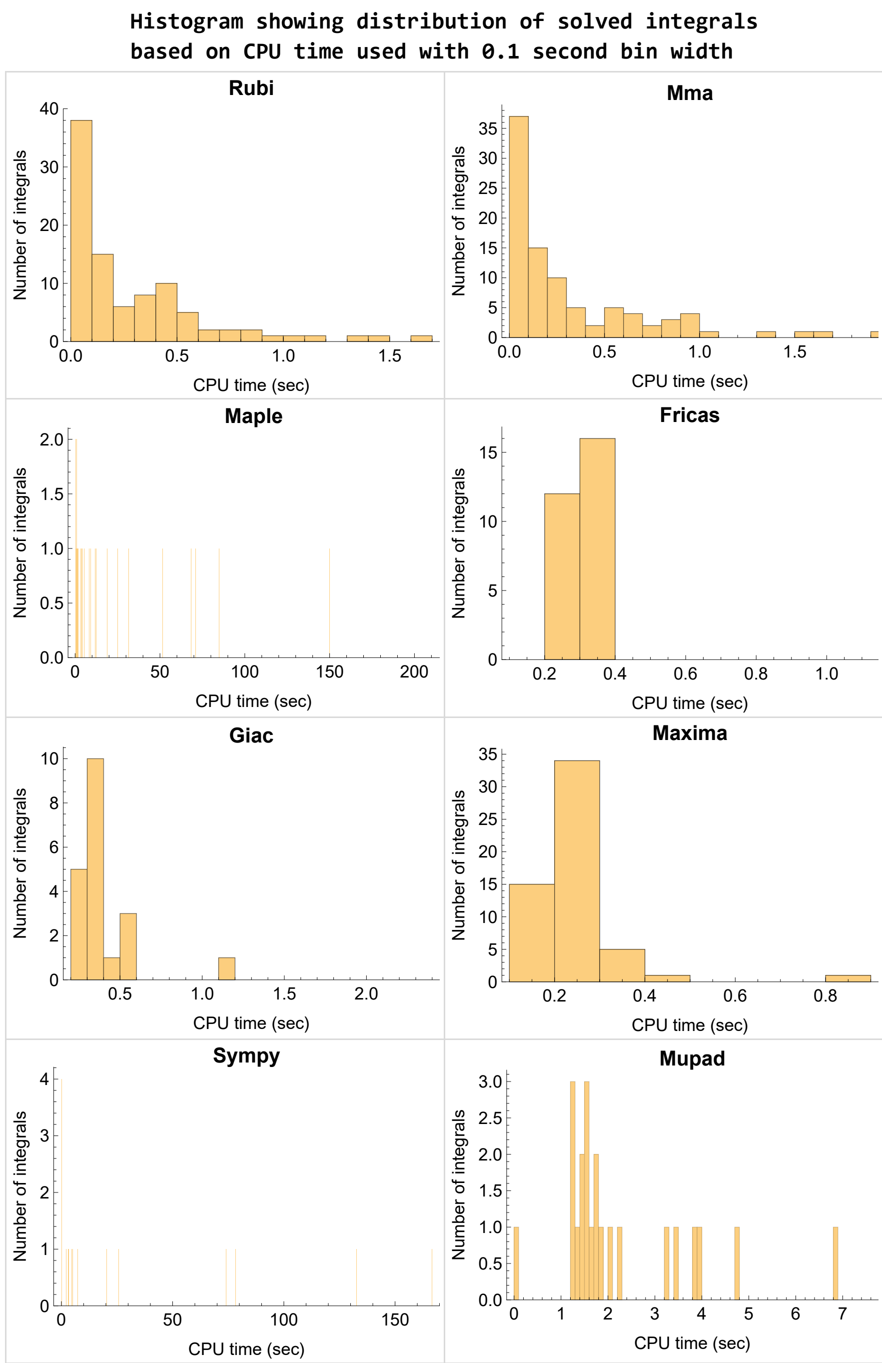


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

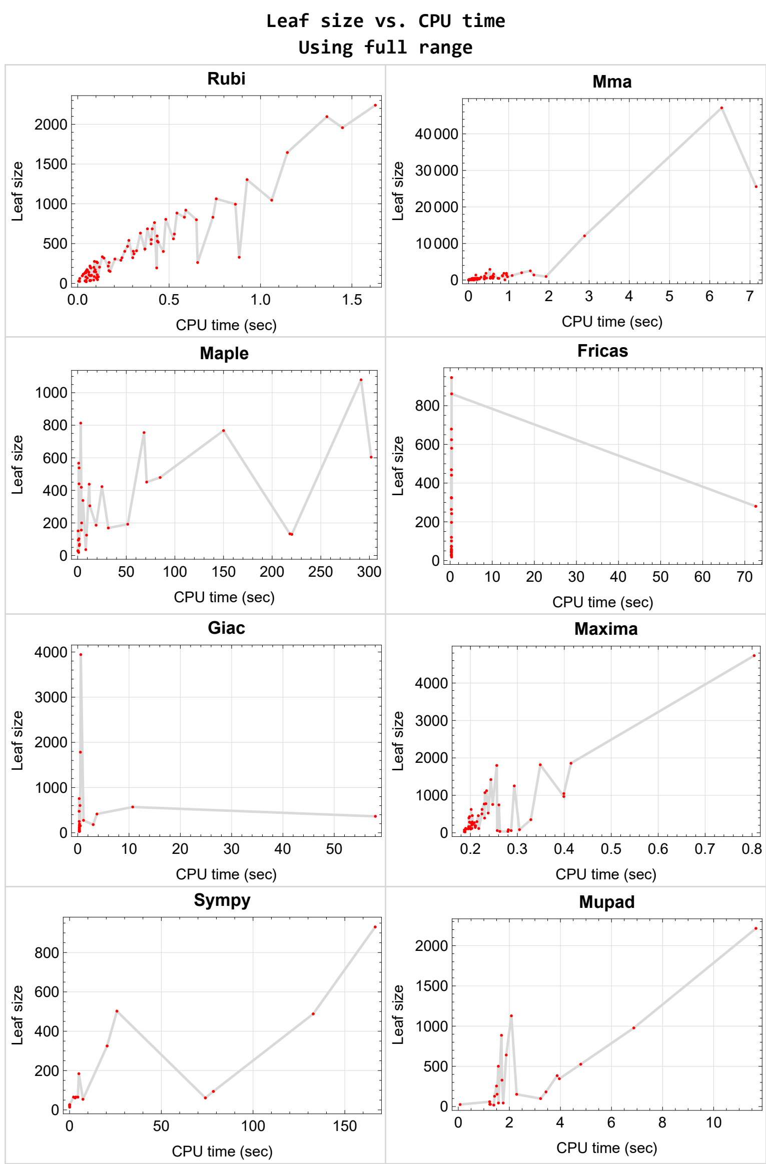


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{54, 55, 60, 61, 62, 63, 65, 66, 70, 71, 72, 73\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

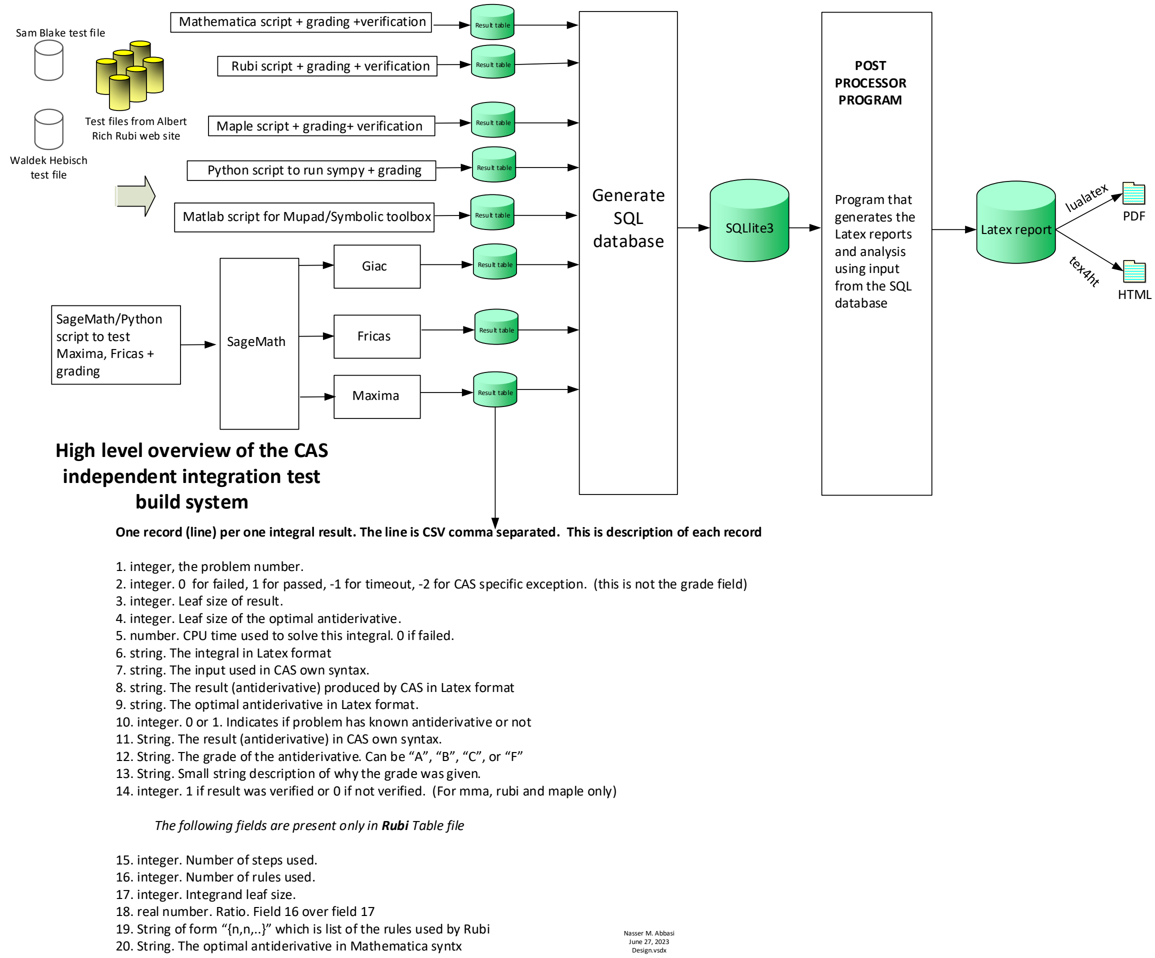
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS 22
2.2 Detailed conclusion table per each integral for all CAS systems 25
2.3 Detailed conclusion table specific for Rubi results 47

2.1 List of integrals sorted by grade for each CAS

Rubi 22
Mma 22
Maple 23
Fricas 23
Maxima 23
Giac 24
Mupad 24
Sympy 24

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 64, 67, 68, 69, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108 }

B grade { }

C grade { }

F normal fail { 74, 75 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 59, 64, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105 }

B grade { 16, 17, 24, 40, 41, 42, 51, 56, 57, 58, 68, 69, 92, 106, 107, 108 }

C grade { }

F normal fail { 67 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 11, 12, 29, 30, 53, 59, 64, 76, 78, 80, 88, 89, 91, 93, 96, 99, 102, 103, 106 }

B grade { 8, 9, 10, 13, 26, 27, 28, 31, 74, 75, 104, 105, 107 }

C grade { 50 }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 51, 52, 56, 57, 58, 67, 68, 69, 77, 79, 81, 82, 83, 84, 85, 86, 87, 90, 92, 94, 95, 97, 98, 100, 101, 108 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 10, 12, 28, 29, 43, 46, 47, 48, 49, 50, 74, 75, 89, 93, 96, 99, 106 }

B grade { 7, 8, 9, 13, 14, 15, 25, 26, 27, 31, 44, 45 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 11, 16, 17, 18, 19, 20, 21, 22, 23, 24, 30, 35, 36, 37, 38, 39, 40, 41, 42, 51, 52, 53, 56, 57, 58, 59, 64, 67, 68, 69, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 94, 95, 97, 98, 100, 101, 102, 103, 104, 105, 107, 108 }

F(-1) timedout fail { 32, 33, 34 }

F(-2) exception fail { }

Maxima

A grade { 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 21, 22, 23, 26, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 40, 41, 47, 48, 53, 59, 74, 75, 79, 93, 94, 96, 97, 99, 100 }

B grade { 7, 15, 24, 25, 34, 42, 44, 45, 46, 49, 50, 77, 81, 88, 89 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 20, 39, 43, 51, 52, 56, 57, 58, 67, 68, 69, 76, 78, 80, 90, 91, 92, 95, 98, 101, 102, 103, 104, 105 }

F(-1) timedout fail { }

F(-2) exception fail { 64, 82, 83, 84, 85, 86, 87, 106, 107, 108 }

Giac

A grade { 10, 12, 13, 27, 28, 29, 31, 43, 47, 48, 96, 99 }

B grade { 7, 8, 9, 14, 15, 32, 33, 34, 46, 49, 50, 93 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 11, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 30, 35, 37, 38, 39, 40, 41, 42, 44, 45, 51, 52, 53, 56, 57, 58, 59, 64, 67, 68, 69, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 94, 95, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108 }

F(-1) timedout fail { 26, 36 }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 7, 8, 9, 10, 12, 13, 14, 15, 25, 26, 27, 28, 29, 31, 32, 33, 34, 74, 75, 89, 93, 96, 99 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 11, 16, 17, 18, 19, 20, 21, 22, 23, 24, 30, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 64, 67, 68, 69, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 94, 95, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108 }

F(-2) exception fail { }

Sympy

A grade { 47, 89, 93, 96, 99 }

B grade { 9, 10, 27, 28, 29, 43, 44, 45, 46, 48, 50 }

C grade { }

F normal fail { 1, 2, 3, 11, 17, 18, 19, 20, 21, 22, 23, 24, 35, 36, 37, 38, 59, 64, 88, 90, 91, 92, 94, 95, 97, 98, 100, 101, 103, 104, 105, 106, 107 }

F(-1) timedout fail { 4, 5, 6, 7, 8, 13, 14, 15, 16, 25, 26, 30, 31, 32, 33, 34, 39, 40, 41, 42, 49, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 67, 68, 69, 70, 71, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 102, 108 }

F(-2) exception fail { 12, 72, 73, 74, 75 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column N.S. means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name Mathematica was abbreviated to MMA.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	404	404	336	0	0	0	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.304	0.275	0.000	0.000	0.000	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	263	263	217	0	0	0	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.171	0.128	0.000	0.000	0.000	0.000	0.000	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	143	143	135	0	0	0	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.099	0.064	0.000	0.000	0.000	0.000	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	217	217	185	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.167	0.097	0.000	0.000	0.000	0.000	0.000	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	322	322	295	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.242	0.272	0.000	0.000	0.000	0.000	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	531	531	470	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.436	0.608	0.000	0.000	0.000	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	201	201	185	0	395	624	0	570	886
N.S.	1	1.00	0.92	0.00	1.97	3.10	0.00	2.84	4.41
time (sec)	N/A	0.070	0.194	0.000	0.197	0.315	0.000	10.717	1.691

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	154	604	285	469	0	415	501
N.S.	1	1.00	0.90	3.51	1.66	2.73	0.00	2.41	2.91
time (sec)	N/A	0.051	0.135	301.793	0.198	0.297	0.000	3.743	1.568

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	127	451	194	325	488	275	255
N.S.	1	1.00	0.89	3.15	1.36	2.27	3.41	1.92	1.78
time (sec)	N/A	0.043	0.092	70.987	0.209	0.287	132.776	1.128	1.497

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	105	305	118	197	325	152	128
N.S.	1	1.00	0.91	2.63	1.02	1.70	2.80	1.31	1.10
time (sec)	N/A	0.030	0.139	12.557	0.190	0.317	20.453	0.503	1.419

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	93	125	164	0	0	0	0
N.S.	1	1.00	0.87	1.17	1.53	0.00	0.00	0.00	0.00
time (sec)	N/A	0.062	0.070	9.167	0.198	0.000	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	89	169	99	120	0	113	99
N.S.	1	1.00	0.94	1.78	1.04	1.26	0.00	1.19	1.04
time (sec)	N/A	0.025	0.041	31.532	0.201	0.302	0.000	0.293	3.222

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	116	767	165	323	0	250	182
N.S.	1	1.00	0.86	5.68	1.22	2.39	0.00	1.85	1.35
time (sec)	N/A	0.046	0.146	150.010	0.196	0.317	0.000	0.292	3.434

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	164	164	141	0	289	580	0	475	346
N.S.	1	1.00	0.86	0.00	1.76	3.54	0.00	2.90	2.11
time (sec)	N/A	0.053	0.221	0.000	0.204	0.344	0.000	0.281	3.958

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	193	193	164	0	459	861	0	756	526
N.S.	1	1.00	0.85	0.00	2.38	4.46	0.00	3.92	2.73
time (sec)	N/A	0.072	0.210	0.000	0.204	0.348	0.000	0.290	4.795

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	920	920	2508	0	1421	0	0	0	0
N.S.	1	1.00	2.73	0.00	1.54	0.00	0.00	0.00	0.00
time (sec)	N/A	0.591	1.538	0.000	0.244	0.000	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	805	805	1853	0	1071	0	0	0	0
N.S.	1	1.00	2.30	0.00	1.33	0.00	0.00	0.00	0.00
time (sec)	N/A	0.482	0.945	0.000	0.231	0.000	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	686	686	1211	0	769	0	0	0	0
N.S.	1	1.00	1.77	0.00	1.12	0.00	0.00	0.00	0.00
time (sec)	N/A	0.382	0.606	0.000	0.229	0.000	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	540	540	781	0	504	0	0	0	0
N.S.	1	1.00	1.45	0.00	0.93	0.00	0.00	0.00	0.00
time (sec)	N/A	0.280	0.398	0.000	0.225	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	431	431	460	0	0	0	0	0	0
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.367	0.118	0.000	0.000	0.000	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	465	465	411	0	392	0	0	0	0
N.S.	1	1.00	0.88	0.00	0.84	0.00	0.00	0.00	0.00
time (sec)	N/A	0.273	0.396	0.000	0.230	0.000	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	632	632	872	0	755	0	0	0	0
N.S.	1	1.00	1.38	0.00	1.19	0.00	0.00	0.00	0.00
time (sec)	N/A	0.343	0.624	0.000	0.247	0.000	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	764	764	1407	0	1252	0	0	0	0
N.S.	1	1.00	1.84	0.00	1.64	0.00	0.00	0.00	0.00
time (sec)	N/A	0.420	0.944	0.000	0.293	0.000	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	884	884	2003	0	1816	0	0	0	0
N.S.	1	1.00	2.27	0.00	2.05	0.00	0.00	0.00	0.00
time (sec)	N/A	0.543	1.322	0.000	0.349	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	334	334	275	0	624	945	0	0	1128
N.S.	1	1.00	0.82	0.00	1.87	2.83	0.00	0.00	3.38
time (sec)	N/A	0.135	0.201	0.000	0.202	0.326	0.000	0.000	2.079

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	231	1079	431	679	0	0	641
N.S.	1	1.00	0.84	3.91	1.56	2.46	0.00	0.00	2.32
time (sec)	N/A	0.091	0.177	291.420	0.198	0.296	0.000	0.000	1.878

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	209	755	269	441	930	363	328
N.S.	1	1.00	0.96	3.46	1.23	2.02	4.27	1.67	1.50
time (sec)	N/A	0.067	0.142	68.290	0.200	0.316	166.570	58.017	1.711

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	120	438	143	242	502	180	153
N.S.	1	1.00	0.75	2.74	0.89	1.51	3.14	1.12	0.96
time (sec)	N/A	0.048	0.132	11.889	0.199	0.345	25.799	3.018	1.520

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	57	61	75	72	184	69	60
N.S.	1	1.00	0.93	1.00	1.23	1.18	3.02	1.13	0.98
time (sec)	N/A	0.011	0.041	1.665	0.187	0.311	5.056	0.314	1.228

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	163	186	186	0	0	0	0
N.S.	1	1.00	1.10	1.26	1.26	0.00	0.00	0.00	0.00
time (sec)	N/A	0.090	0.062	18.984	0.211	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	93	479	123	280	0	191	152
N.S.	1	1.00	0.73	3.74	0.96	2.19	0.00	1.49	1.19
time (sec)	N/A	0.038	0.088	84.840	0.200	72.606	0.000	0.345	2.285

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	202	202	206	0	232	0	0	603	384
N.S.	1	1.00	1.02	0.00	1.15	0.00	0.00	2.99	1.90
time (sec)	N/A	0.089	0.225	0.000	0.205	0.000	0.000	0.448	3.871

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	260	260	254	0	456	0	0	1783	977
N.S.	1	1.00	0.98	0.00	1.75	0.00	0.00	6.86	3.76
time (sec)	N/A	0.108	0.395	0.000	0.217	0.000	0.000	0.519	6.873

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	318	318	480	0	776	0	0	3943	2215
N.S.	1	1.00	1.51	0.00	2.44	0.00	0.00	12.40	6.97
time (sec)	N/A	0.144	0.733	0.000	0.233	0.000	0.000	0.599	11.656

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	2240	2240	1386	0	1799	0	0	0	0
N.S.	1	1.00	0.62	0.00	0.80	0.00	0.00	0.00	0.00
time (sec)	N/A	1.628	1.627	0.000	0.256	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1645	1645	899	0	1123	0	0	0	0
N.S.	1	1.00	0.55	0.00	0.68	0.00	0.00	0.00	0.00
time (sec)	N/A	1.147	0.971	0.000	0.234	0.000	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1063	1063	480	0	623	0	0	0	0
N.S.	1	1.00	0.45	0.00	0.59	0.00	0.00	0.00	0.00
time (sec)	N/A	0.758	0.540	0.000	0.225	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	269	269	389	0	298	0	0	0	0
N.S.	1	1.00	1.45	0.00	1.11	0.00	0.00	0.00	0.00
time (sec)	N/A	0.104	0.124	0.000	0.214	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1471	2096	1370	0	0	0	0	0	0
N.S.	1	1.42	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.363	0.183	0.000	0.000	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	832	832	2930	0	745	0	0	0	0
N.S.	1	1.00	3.52	0.00	0.90	0.00	0.00	0.00	0.00
time (sec)	N/A	0.584	0.533	0.000	0.261	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1304	1304	12086	0	1857	0	0	0	0
N.S.	1	1.00	9.27	0.00	1.42	0.00	0.00	0.00	0.00
time (sec)	N/A	0.926	2.886	0.000	0.414	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1957	1957	47127	0	4732	0	0	0	0
N.S.	1	1.00	24.08	0.00	2.42	0.00	0.00	0.00	0.00
time (sec)	N/A	1.448	6.296	0.000	0.804	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	B	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	42	42	42	0	0	56	94	38	0
N.S.	1	1.00	1.00	0.00	0.00	1.33	2.24	0.90	0.00
time (sec)	N/A	0.088	0.025	0.000	0.000	0.319	78.374	0.298	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	B	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	526	101	65	0	0
N.S.	1	1.00	1.00	0.00	14.22	2.73	1.76	0.00	0.00
time (sec)	N/A	0.070	0.009	0.000	0.238	0.309	4.506	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	B	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	268	74	65	0	0
N.S.	1	1.00	1.00	0.00	7.24	2.00	1.76	0.00	0.00
time (sec)	N/A	0.068	0.010	0.000	0.208	0.295	3.364	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	A	B	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	105	47	61	86	0
N.S.	1	1.00	1.00	0.00	2.84	1.27	1.65	2.32	0.00
time (sec)	N/A	0.045	0.007	0.000	0.195	0.299	3.014	0.317	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	34	0	36	30	54	31	0
N.S.	1	1.00	1.00	0.00	1.06	0.88	1.59	0.91	0.00
time (sec)	N/A	0.066	0.144	0.000	0.263	0.304	7.281	0.328	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	34	0	34	29	61	34	0
N.S.	1	1.00	1.00	0.00	1.00	0.85	1.79	1.00	0.00
time (sec)	N/A	0.067	0.010	0.000	0.280	0.276	74.019	0.349	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	80	59	0	85	0
N.S.	1	1.00	1.00	0.00	2.16	1.59	0.00	2.30	0.00
time (sec)	N/A	0.071	0.009	0.000	0.281	0.280	0.000	0.310	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	151	83	24	65	58	0
N.S.	1	1.00	1.00	5.03	2.77	0.80	2.17	1.93	0.00
time (sec)	N/A	0.040	0.008	0.487	0.305	0.291	2.155	0.306	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	410	410	958	0	0	0	0	0	0
N.S.	1	1.00	2.34	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.322	1.929	0.000	0.000	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	306	306	436	0	0	0	0	0	0
N.S.	1	1.00	1.42	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.203	0.758	0.000	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	166	192	204	0	0	0	0
N.S.	1	1.00	0.97	1.12	1.19	0.00	0.00	0.00	0.00
time (sec)	N/A	0.095	0.087	51.527	0.203	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	48	48	50	48	50	57	0	50	50
N.S.	1	1.00	1.04	1.00	1.04	1.19	0.00	1.04	1.04
time (sec)	N/A	0.037	0.512	0.394	3.443	0.310	0.000	0.326	1.606

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	48	48	50	48	222	91	0	50	50
N.S.	1	1.00	1.04	1.00	4.62	1.90	0.00	1.04	1.04
time (sec)	N/A	0.038	1.505	0.276	2.930	0.300	0.000	0.352	1.753

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	328	328	1241	0	0	0	0	0	0
N.S.	1	1.00	3.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.883	1.086	0.000	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	262	262	839	0	0	0	0	0	0
N.S.	1	1.00	3.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.656	0.551	0.000	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	194	194	451	0	0	0	0	0	0
N.S.	1	1.00	2.32	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.432	0.269	0.000	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	78	103	126	0	0	0	0
N.S.	1	1.00	0.96	1.27	1.56	0.00	0.00	0.00	0.00
time (sec)	N/A	0.045	0.047	1.368	0.196	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	39	39	41	39	41	41	0	41	41
N.S.	1	1.00	1.05	1.00	1.05	1.05	0.00	1.05	1.05
time (sec)	N/A	0.344	0.308	1.010	3.373	0.307	0.000	0.372	1.479

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	39	39	41	39	231	41	0	41	41
N.S.	1	1.00	1.05	1.00	5.92	1.05	0.00	1.05	1.05
time (sec)	N/A	0.293	1.612	1.044	3.123	0.311	0.000	0.358	1.510

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	272	30	71	30	31
N.S.	1	1.00	1.07	1.00	9.71	1.07	2.54	1.07	1.11
time (sec)	N/A	0.017	0.084	1.818	0.274	0.296	4.145	0.585	1.360

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	154	30	66	30	31
N.S.	1	1.00	1.07	1.00	5.50	1.07	2.36	1.07	1.11
time (sec)	N/A	0.016	0.070	0.554	0.268	0.301	6.484	0.415	1.224

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	66	94	0	0	0	0	0
N.S.	1	1.00	0.80	1.15	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.113	0.030	0.694	0.000	0.000	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	30	30	32	30	31
N.S.	1	1.00	1.07	1.00	1.07	1.07	1.14	1.07	1.11
time (sec)	N/A	0.018	0.079	0.444	0.247	0.289	27.260	0.351	1.166

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	83	30	126	30	31
N.S.	1	1.00	1.07	1.00	2.96	1.07	4.50	1.07	1.11
time (sec)	N/A	0.018	0.231	33.559	0.245	0.291	41.297	0.351	1.211

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	620	620	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.529	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	496	496	25557	0	0	0	0	0	0
N.S.	1	1.00	51.53	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.401	7.153	0.000	0.000	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	371	371	1842	0	0	0	0	0	0
N.S.	1	1.00	4.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.308	0.881	0.000	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	45	45	47	45	47	54	0	47	47
N.S.	1	1.00	1.04	1.00	1.04	1.20	0.00	1.04	1.04
time (sec)	N/A	0.379	1.243	0.250	1.719	0.300	0.000	0.413	1.353

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	45	45	47	45	181	54	0	47	47
N.S.	1	1.00	1.04	1.00	4.02	1.20	0.00	1.04	1.04
time (sec)	N/A	0.079	0.798	0.053	0.741	0.304	0.000	0.318	1.302

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	48	48	50	50	154	62	0	50	50
N.S.	1	1.00	1.04	1.04	3.21	1.29	0.00	1.04	1.04
time (sec)	N/A	0.359	0.512	0.936	0.269	0.284	0.000	0.499	1.937

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	48	48	50	50	155	61	0	50	50
N.S.	1	1.00	1.04	1.04	3.23	1.27	0.00	1.04	1.04
time (sec)	N/A	0.344	0.392	0.992	0.267	0.301	0.000	0.545	1.774

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	B	A	A	F(-2)	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	0	44	133	59	50	0	0	44
N.S.	1	0.00	0.98	2.96	1.31	1.11	0.00	0.00	0.98
time (sec)	N/A	0.000	0.905	218.217	0.257	0.298	0.000	0.000	1.758

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	B	A	A	F(-2)	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	0	45	130	58	49	0	0	44
N.S.	1	0.00	1.00	2.89	1.29	1.09	0.00	0.00	0.98
time (sec)	N/A	0.000	0.056	220.254	0.287	0.285	0.000	0.000	1.572

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	560	560	461	538	0	0	0	0	0
N.S.	1	1.00	0.82	0.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.524	0.257	1.516	0.000	0.000	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	550	550	467	0	1047	0	0	0	0
N.S.	1	1.00	0.85	0.00	1.90	0.00	0.00	0.00	0.00
time (sec)	N/A	0.403	0.176	0.000	0.399	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	403	403	413	440	0	0	0	0	0
N.S.	1	1.00	1.02	1.09	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.257	0.091	1.444	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	291	291	421	0	349	0	0	0	0
N.S.	1	1.00	1.45	0.00	1.20	0.00	0.00	0.00	0.00
time (sec)	N/A	0.236	0.082	0.000	0.329	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	518	518	487	567	0	0	0	0	0
N.S.	1	1.00	0.94	1.09	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.440	0.148	1.083	0.000	0.000	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	596	596	479	0	969	0	0	0	0
N.S.	1	1.00	0.80	0.00	1.63	0.00	0.00	0.00	0.00
time (sec)	N/A	0.434	0.230	0.000	0.399	0.000	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1046	1046	1240	0	0	0	0	0	0
N.S.	1	1.00	1.19	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.061	0.842	0.000	0.000	0.000	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	831	831	1105	0	0	0	0	0	0
N.S.	1	1.00	1.33	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.740	0.848	0.000	0.000	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	685	685	539	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.407	0.450	0.000	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	401	401	515	0	0	0	0	0	0
N.S.	1	1.00	1.28	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.468	0.215	0.000	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	800	800	625	0	0	0	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.649	0.569	0.000	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	995	995	721	0	0	0	0	0	0
N.S.	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.863	0.572	0.000	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	84	69	95	0	0	0	0
N.S.	1	1.00	1.83	1.50	2.07	0.00	0.00	0.00	0.00
time (sec)	N/A	0.108	0.014	1.898	0.191	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	141	18	14	0	18
N.S.	1	1.00	1.00	0.95	7.05	0.90	0.70	0.00	0.90
time (sec)	N/A	0.047	0.115	1.076	0.210	0.333	0.058	0.000	1.391

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	82	82	76	0	0	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.108	0.016	0.000	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	110	200	0	0	0	0	0
N.S.	1	1.00	0.73	1.33	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.177	0.031	4.106	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	160	160	785	0	0	0	0	0	0
N.S.	1	1.00	4.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.171	0.312	0.000	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	28	26	25	29	20	153	25
N.S.	1	1.00	1.12	1.04	1.00	1.16	0.80	6.12	1.00
time (sec)	N/A	0.010	0.006	0.397	0.189	0.308	0.048	0.312	0.071

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	67	67	63	0	113	0	0	0	0
N.S.	1	1.00	0.94	0.00	1.69	0.00	0.00	0.00	0.00
time (sec)	N/A	0.052	0.011	0.000	0.218	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	97	97	91	0	0	0	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.073	0.020	0.000	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	28	42	26	29	28
N.S.	1	1.00	1.00	1.04	1.00	1.50	0.93	1.04	1.00
time (sec)	N/A	0.005	0.003	0.227	0.188	0.288	0.057	0.315	1.254

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	67	67	106	0	118	0	0	0	0
N.S.	1	1.00	1.58	0.00	1.76	0.00	0.00	0.00	0.00
time (sec)	N/A	0.102	0.013	0.000	0.200	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	102	102	98	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.075	0.021	0.000	0.000	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	28	41	26	29	28
N.S.	1	1.00	1.00	1.04	1.00	1.46	0.93	1.04	1.00
time (sec)	N/A	0.007	0.004	0.399	0.187	0.303	0.063	0.313	1.233

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	67	67	96	0	118	0	0	0	0
N.S.	1	1.00	1.43	0.00	1.76	0.00	0.00	0.00	0.00
time (sec)	N/A	0.103	0.022	0.000	0.203	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	98	98	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.066	0.022	0.000	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	30	36	0	0	0	0	0
N.S.	1	1.00	0.86	1.03	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.045	0.010	8.383	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	68	156	0	0	0	0	0
N.S.	1	1.00	0.80	1.84	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.090	0.021	3.803	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	135	419	0	0	0	0	0
N.S.	1	1.00	0.96	2.99	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.061	0.032	3.767	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	96	423	0	0	0	0	0
N.S.	1	1.00	0.88	3.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.107	0.022	25.005	0.000	0.000	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	1636	338	0	264	0	0	0
N.S.	1	1.00	8.02	1.66	0.00	1.29	0.00	0.00	0.00
time (sec)	N/A	0.119	0.625	5.451	0.000	0.285	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	1080	813	0	0	0	0	0
N.S.	1	1.00	3.35	2.52	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.301	0.400	3.178	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	433	433	1855	0	0	0	0	0	0
N.S.	1	1.00	4.28	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.368	0.435	0.000	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [95] had the largest ratio of [.5384999999999999979]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	16	10	1.00	32	0.312
2	A	13	8	1.00	32	0.250
3	A	10	6	1.00	30	0.200
4	A	12	7	1.00	32	0.219
5	A	15	9	1.00	32	0.281
6	A	18	11	1.00	32	0.344
7	A	4	3	1.00	29	0.103
8	A	4	3	1.00	29	0.103
9	A	4	3	1.00	29	0.103
10	A	4	2	1.00	27	0.074
11	A	6	6	1.00	29	0.207
12	A	5	4	1.00	29	0.138
13	A	4	3	1.00	29	0.103
14	A	4	3	1.00	29	0.103
15	A	4	3	1.00	29	0.103
16	A	32	14	1.00	31	0.452
17	A	28	14	1.00	31	0.452
18	A	24	14	1.00	31	0.452
19	A	20	13	1.00	29	0.448
20	A	19	15	1.00	31	0.484
21	A	20	12	1.00	31	0.387
22	A	24	13	1.00	31	0.419
23	A	28	13	1.00	31	0.419
24	A	32	13	1.00	31	0.419

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	5	2	1.00	29	0.069
26	A	5	2	1.00	29	0.069
27	A	5	2	1.00	29	0.069
28	A	5	2	1.00	27	0.074
29	A	3	3	1.00	21	0.143
30	A	7	4	1.00	29	0.138
31	A	7	3	1.00	29	0.103
32	A	5	2	1.00	29	0.069
33	A	5	2	1.00	29	0.069
34	A	5	2	1.00	29	0.069
35	A	49	14	1.00	31	0.452
36	A	47	15	1.00	31	0.484
37	A	39	15	1.00	29	0.517
38	A	10	9	1.00	23	0.391
39	A	29	14	1.42	31	0.452
40	A	31	10	1.00	31	0.323
41	A	43	14	1.00	31	0.452
42	A	57	15	1.00	31	0.484
43	A	2	2	1.00	40	0.050
44	A	5	5	1.00	40	0.125
45	A	5	5	1.00	40	0.125
46	A	4	4	1.00	38	0.105
47	A	2	2	1.00	40	0.050
48	A	2	2	1.00	40	0.050
49	A	2	2	1.00	40	0.050
50	A	4	4	1.00	34	0.118
51	A	11	6	1.00	48	0.125
52	A	9	5	1.00	46	0.109
53	A	7	4	1.00	32	0.125
54	N/A	0	0	1.00	48	0.000
55	N/A	0	0	1.00	48	0.000
56	A	13	6	1.00	39	0.154
57	A	11	6	1.00	39	0.154
58	A	9	5	1.00	37	0.135
59	A	5	3	1.00	25	0.120

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	N/A	0	0	1.00	39	0.000
61	N/A	0	0	1.00	39	0.000
62	N/A	0	0	1.00	28	0.000
63	N/A	0	0	1.00	28	0.000
64	A	5	5	1.00	26	0.192
65	N/A	0	0	1.00	28	0.000
66	N/A	0	0	1.00	28	0.000
67	A	14	7	1.00	45	0.156
68	A	12	7	1.00	45	0.156
69	A	10	6	1.00	43	0.140
70	N/A	0	0	1.00	45	0.000
71	N/A	0	0	1.00	45	0.000
72	N/A	0	0	1.00	48	0.000
73	N/A	0	0	1.00	48	0.000
74	F	0	0	N/A	0.000	N/A
75	F	0	0	N/A	0.000	N/A
76	A	30	9	1.00	32	0.281
77	A	27	10	1.00	32	0.312
78	A	18	6	1.00	30	0.200
79	A	7	4	1.00	29	0.138
80	A	29	11	1.00	32	0.344
81	A	31	12	1.00	32	0.375
82	A	37	14	1.00	34	0.412
83	A	30	12	1.00	34	0.353
84	A	21	9	1.00	32	0.281
85	A	7	4	1.00	31	0.129
86	A	31	12	1.00	34	0.353
87	A	40	16	1.00	34	0.471
88	A	5	5	1.00	19	0.263
89	A	3	3	1.00	24	0.125
90	A	3	3	1.00	34	0.088
91	A	3	3	1.00	55	0.055
92	A	3	3	1.00	58	0.052

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
93	A	4	4	1.00	11	0.364
94	A	5	5	1.00	13	0.385
95	A	7	7	1.00	13	0.538
96	A	2	2	1.00	13	0.154
97	A	6	6	1.00	15	0.400
98	A	5	5	1.00	15	0.333
99	A	2	2	1.00	13	0.154
100	A	6	6	1.00	15	0.400
101	A	5	5	1.00	15	0.333
102	A	1	1	1.00	38	0.026
103	A	2	2	1.00	50	0.040
104	A	4	4	1.00	42	0.095
105	A	2	2	1.00	62	0.032
106	A	4	4	1.00	49	0.082
107	A	9	5	1.00	42	0.119
108	A	10	6	1.00	65	0.092

CHAPTER 3

LISTING OF INTEGRALS

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3.1 $\int \left(f+\frac{g}{x}\right)^3 \left(A+B \log \left(e \left(\frac{a+b x}{c+d x}\right)^n\right)\right) \, dx$

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Optimal result

Integrand size = 32, antiderivative size = 404

$$\begin{aligned}
&\int \left(f+\frac{g}{x}\right)^3 \left(A+B \log \left(e \left(\frac{a+b x}{c+d x}\right)^n\right)\right) \, dx \\
&= -\frac{B(bc-ad)g^3n}{2acx}+Af^3x-\frac{1}{2}B\left(\frac{b^2}{a^2}-\frac{d^2}{c^2}\right)g^3n\log(x)+\frac{b^2Bg^3n\log(a+bx)}{2a^2} \\
&\quad -3Bf^2gn\log(x)\log\left(1+\frac{bx}{a}\right)+\frac{Bf^3(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b} \\
&\quad -\frac{g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2x^2}+\frac{3(bc-ad)fg^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{a(c+dx)\left(a-\frac{c(a+bx)}{c+dx}\right)} \\
&\quad +3f^2g\log(x)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \\
&\quad -\frac{B(bc-ad)f^3n\log(c+dx)}{bd}-\frac{Bd^2g^3n\log(c+dx)}{2c^2} \\
&\quad +3Bf^2gn\log(x)\log\left(1+\frac{dx}{c}\right)+\frac{3B(bc-ad)fg^2n\log\left(a-\frac{c(a+bx)}{c+dx}\right)}{ac} \\
&\quad -3Bf^2gn\operatorname{PolyLog}\left(2,-\frac{bx}{a}\right)+3Bf^2gn\operatorname{PolyLog}\left(2,-\frac{dx}{c}\right)
\end{aligned}$$

```
[Out] -1/2*B*(-a*d+b*c)*g^3*n/a/c/x+A*f^3*x-1/2*B*(b^2/a^2-d^2/c^2)*g^3*n*ln(x)+1
/2*b^2*B*g^3*n*ln(b*x+a)/a^2-3*B*f^2*g*n*ln(x)*ln(1+b*x/a)+B*f^3*(b*x+a)*ln
(e*((b*x+a)/(d*x+c))^n)/b-1/2*g^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/x^2+3*(-a
*d+b*c)*f*g^2*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/a/(d*x+c)/(a-c*(b*x+a
)/(d*x+c))+3*f^2*g*ln(x)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))-B*(-a*d+b*c)*f^3*n
*ln(d*x+c)/b/d-1/2*B*d^2*g^3*n*ln(d*x+c)/c^2+3*B*f^2*g*n*ln(x)*ln(1+d*x/c)+
3*B*(-a*d+b*c)*f*g^2*n*ln(a-c*(b*x+a)/(d*x+c))/a/c-3*B*f^2*g*n*polylog(2,-b
*x/a)+3*B*f^2*g*n*polylog(2,-d*x/c)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2608, 2535, 31, 2547, 84, 2553, 2351, 2545, 2354, 2438}

$$\int \left(f+\frac{g}{x}\right)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= -\frac{1}{2}Bg^3n\log(x)\left(\frac{b^2}{a^2}-\frac{d^2}{c^2}\right)+\frac{b^2Bg^3n\log(a+bx)}{2a^2}$$
$$+3f^2g\log(x)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)$$
$$+\frac{3fg^2(a+bx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a(c+dx)\left(a-\frac{c(a+bx)}{c+dx}\right)}-\frac{g^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2x^2}$$
$$+\frac{Bf^3(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}-\frac{Bf^3n(bc-ad)\log(c+dx)}{bd}$$
$$+\frac{3Bfg^2n(bc-ad)\log\left(a-\frac{c(a+bx)}{c+dx}\right)}{ac}-\frac{Bg^3n(bc-ad)}{2acx}-3Bf^2gn\,\text{PolyLog}\left(2,-\frac{bx}{a}\right)$$
$$-3Bf^2gn\log(x)\log\left(\frac{bx}{a}+1\right)+Af^3x-\frac{Bd^2g^3n\log(c+dx)}{2c^2}$$
$$+3Bf^2gn\,\text{PolyLog}\left(2,-\frac{dx}{c}\right)+3Bf^2gn\log(x)\log\left(\frac{dx}{c}+1\right)$$

```
[In] Int[(f + g/x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] -1/2*(B*(b*c - a*d)*g^3*n)/(a*c*x) + A*f^3*x - (B*(b^2/a^2 - d^2/c^2)*g^3*n
*Log[x])/2 + (b^2*B*g^3*n*Log[a + b*x])/(2*a^2) - 3*B*f^2*g*n*Log[x]*Log[1
+ (b*x)/a] + (B*f^3*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b - (g^3*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*x^2) + (3*(b*c - a*d)*f*g^2*(a + b*x
)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*(c + d*x)*(a - (c*(a + b*x))/(
c + d*x))) + 3*f^2*g*Log[x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - (B*(b*
c - a*d)*f^3*n*Log[c + d*x])/(b*d) - (B*d^2*g^3*n*Log[c + d*x])/(2*c^2) + 3
*B*f^2*g*n*Log[x]*Log[1 + (d*x)/c] + (3*B*(b*c - a*d)*f*g^2*n*Log[a - (c*(a
 + b*x))/(c + d*x)])/(a*c) - 3*B*f^2*g*n*PolyLog[2, -((b*x)/a)] + 3*B*f^2*g
*n*PolyLog[2, -((d*x)/c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2535

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rule 2545

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((A + B*Log[e*((
a + b*x)/(c + d*x))^n])/g), x] + (-Dist[b*B*(n/g), Int[Log[f + g*x]/(a + b*
x), x], x] + Dist[B*d*(n/g), Int[Log[f + g*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
```

```
& NeQ[m, -2]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rule 2608

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(f^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+\frac{g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{x^3}\right. \\
&\qquad \left.+\frac{3fg^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{x^2}+\frac{3f^2g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{x}\right)dx \\
&= f^3\int\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx+\left(3f^2g\right)\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x}\,dx \\
&\quad +\left(3fg^2\right)\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2}\,dx+g^3\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^3}\,dx \\
&= Af^3x-\frac{g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2x^2}+3f^2g\log(x)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \\
&\quad +\left(Bf^3\right)\int\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)dx \\
&\quad +\left(3(bc-ad)fg^2\right)\text{Subst}\left(\int\frac{A+B\log\left(ex^n\right)}{(-a+cx)^2}\,dx,x,\frac{a+bx}{c+dx}\right) \\
&\quad -\left(3bBf^2gn\right)\int\frac{\log(x)}{a+bx}\,dx+\left(3Bdf^2gn\right)\int\frac{\log(x)}{c+dx}\,dx \\
&\quad +\frac{1}{2}\left(B(bc-ad)g^3n\right)\int\frac{1}{x^2(a+bx)(c+dx)}\,dx
\end{aligned}$$

$$
\begin{aligned}
&= Af^3x - 3Bf^2gn\log(x)\log\left(1+\frac{bx}{a}\right) + \frac{Bf^3(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b} \\
&\quad - \frac{g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2x^2} + \frac{3(bc-ad)fg^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{a(c+dx)\left(a-\frac{c(a+bx)}{c+dx}\right)} \\
&\quad + 3f^2g\log(x)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) + 3Bf^2gn\log(x)\log\left(1+\frac{dx}{c}\right) \\
&\quad - \frac{(B(bc-ad)f^3n)\int\frac{1}{c+dx}\,dx}{b} + (3Bf^2gn)\int\frac{\log\left(1+\frac{bx}{a}\right)}{x}\,dx \\
&\quad - (3Bf^2gn)\int\frac{\log\left(1+\frac{dx}{c}\right)}{x}\,dx + \frac{(3B(bc-ad)fg^2n)\,\mathrm{Subst}\left(\int\frac{1}{-a+cx}\,dx, x, \frac{a+bx}{c+dx}\right)}{a} \\
&\quad + \frac{1}{2}(B(bc-ad)g^3n)\int\left(\frac{1}{acx^2} + \frac{-bc-ad}{a^2c^2x} - \frac{b^3}{a^2(-bc+ad)(a+bx)}\right. \\
&\qquad\qquad\left. - \frac{d^3}{c^2(bc-ad)(c+dx)}\right)\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{B(bc-ad)g^3n}{2acx} + Af^3x - \frac{B(bc-ad)(bc+ad)g^3n\log(x)}{2a^2c^2} + \frac{b^2Bg^3n\log(a+bx)}{2a^2} \\
&\quad - 3Bf^2gn\log(x)\log\left(1+\frac{bx}{a}\right) + \frac{Bf^3(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b} \\
&\quad - \frac{g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2x^2} + \frac{3(bc-ad)fg^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{a(c+dx)\left(a-\frac{c(a+bx)}{c+dx}\right)} \\
&\quad + 3f^2g\log(x)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) - \frac{B(bc-ad)f^3n\log(c+dx)}{bd} \\
&\quad - \frac{Bd^2g^3n\log(c+dx)}{2c^2} + 3Bf^2gn\log(x)\log\left(1+\frac{dx}{c}\right) \\
&\quad + \frac{3B(bc-ad)fg^2n\log\left(a-\frac{c(a+bx)}{c+dx}\right)}{ac} - 3Bf^2gn\,\mathrm{Li}_2\left(-\frac{bx}{a}\right) + 3Bf^2gn\,\mathrm{Li}_2\left(-\frac{dx}{c}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.83

$$\int \left(f+\frac{g}{x}\right)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = Af^3x+\frac{Bf^3(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b} - \frac{g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2x^2} - \frac{3fg^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{x} + 3f^2g\log(x)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) - \frac{B(bc-ad)f^3n\log(c+dx)}{bd} + \frac{3Bfg^2n((bc-ad)\log(x)-bc\log(a+bx)+ad\log(c+dx))}{ac} + \frac{Bg^3n((-b^2c^2x+a^2d^2x)\log(x)+b^2c^2x\log(a+bx)+a(-bc^2+acd-ad^2x\log(c+dx)))}{2a^2c^2x} - 3Bf^2gn\left(\log(x)\left(\log\left(1+\frac{bx}{a}\right)-\log\left(1+\frac{dx}{c}\right)\right)+\text{PolyLog}\left(2,-\frac{bx}{a}\right) - \text{PolyLog}\left(2,-\frac{dx}{c}\right)\right)$$

```
[In] Integrate[(f + g/x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] A*f^3*x + (B*f^3*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b - (g^3*(A + B*
Log[e*((a + b*x)/(c + d*x))^n]))/(2*x^2) - (3*f*g^2*(A + B*Log[e*((a + b*x)
/(c + d*x))^n]))/x + 3*f^2*g*Log[x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
- (B*(b*c - a*d)*f^3*n*Log[c + d*x])/(b*d) + (3*B*f*g^2*n*((b*c - a*d)*Log[
x] - b*c*Log[a + b*x] + a*d*Log[c + d*x]))/(a*c) + (B*g^3*n*((-(b^2*c^2*x)
+ a^2*d^2*x)*Log[x] + b^2*c^2*x*Log[a + b*x] + a*(-(b*c^2) + a*c*d - a*d^2*
x*Log[c + d*x])))/(2*a^2*c^2*x) - 3*B*f^2*g*n*(Log[x]*(Log[1 + (b*x)/a] - L
og[1 + (d*x)/c]) + PolyLog[2, -((b*x)/a)] - PolyLog[2, -((d*x)/c)])
```

Maple [F]

$$\int \left(f+\frac{g}{x}\right)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)dx$$

```
[In] int((f+g/x)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

```
[Out] int((f+g/x)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Fricas [F]

$$\int \left(f+\frac{g}{x}\right)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)\left(f+\frac{g}{x}\right)^3\,dx$$

```
[In] integrate((f+g/x)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")
[Out] integral((A*f^3*x^3 + 3*A*f^2*g*x^2 + 3*A*f*g^2*x + A*g^3 + (B*f^3*x^3 + 3*
B*f^2*g*x^2 + 3*B*f*g^2*x + B*g^3)*log(e*((b*x + a)/(d*x + c))^n))/x^3, x)
```

Sympy [F]

$$\int \left(f+\frac{g}{x}\right)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\int \frac{\left(A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)\right)(fx+g)^3}{x^3}\,dx$$

```
[In] integrate((f+g/x)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
[Out] Integral((A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n))*(f*x + g)**3/x**3,
 x)
```

Maxima [F]

$$\int \left(f+\frac{g}{x}\right)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)\left(f+\frac{g}{x}\right)^3\,dx$$

```
[In] integrate((f+g/x)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")
[Out] B*f^3*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) - 3*B*f*g^2*n*(b*log(b*x + a)
/a - d*log(d*x + c)/c - (b*c - a*d)*log(x)/(a*c)) + 1/2*B*g^3*n*(b^2*log(b*
x + a)/a^2 - d^2*log(d*x + c)/c^2 - (b*c - a*d)/(a*c*x) - (b^2*c^2 - a^2*d^
2)*log(x)/(a^2*c^2)) + B*f^3*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*f
^3*x - 3*B*f^2*g*integrate(-(log((b*x + a)^n) - log((d*x + c)^n) + log(e))/
x, x) + 3*A*f^2*g*log(x) - 3*B*f*g^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)
/x - 3*A*f*g^2/x - 1/2*B*g^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/x^2 - 1
/2*A*g^3/x^2
```

Giac [F]

$$\int \left(f+\frac{g}{x}\right)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)\left(f+\frac{g}{x}\right)^3\,dx$$

```
[In] integrate((f+g/x)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)*(f + g/x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(f+\frac{g}{x}\right)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\int \left(f+\frac{g}{x}\right)^3\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)\,dx$$

```
[In] int((f + g/x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
[Out] int((f + g/x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)), x)
```

3.2 $\int \left(f+\frac{g}{x}\right)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$

Optimal result 63
Rubi [A] (verified) 64
Mathematica [A] (verified) 67
Maple [F] 67
Fricas [F] 67
Sympy [F] 68
Maxima [F] 68
Giac [F] 68
Mupad [F(-1)] 69

Optimal result

Integrand size = 32, antiderivative size = 263

$$\begin{aligned}
&\int \left(f+\frac{g}{x}\right)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx \\
&= Af^2x-2Bfgn\log(x)\log\left(1+\frac{bx}{a}\right)+\frac{Bf^2(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b} \\
&\quad+\frac{(bc-ad)g^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{a(c+dx)\left(a-\frac{c(a+bx)}{c+dx}\right)} \\
&\quad+2fg\log(x)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-\frac{B(bc-ad)f^2n\log(c+dx)}{bd} \\
&\quad+2Bfgn\log(x)\log\left(1+\frac{dx}{c}\right)+\frac{B(bc-ad)g^2n\log\left(a-\frac{c(a+bx)}{c+dx}\right)}{ac} \\
&\quad-2Bfgn\,\text{PolyLog}\left(2,-\frac{bx}{a}\right)+2Bfgn\,\text{PolyLog}\left(2,-\frac{dx}{c}\right)
\end{aligned}$$

```
[Out] A*f^2*x-2*B*f*g*n*ln(x)*ln(1+b*x/a)+B*f^2*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)
/b+(-a*d+b*c)*g^2*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/a/(d*x+c)/(a-c*(b
*x+a)/(d*x+c))+2*f*g*ln(x)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))-B*(-a*d+b*c)*f^2
*n*ln(d*x+c)/b/d+2*B*f*g*n*ln(x)*ln(1+d*x/c)+B*(-a*d+b*c)*g^2*n*ln(a-c*(b*x
+a)/(d*x+c))/a/c-2*B*f*g*n*polylog(2,-b*x/a)+2*B*f*g*n*polylog(2,-d*x/c)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2608, 2535, 31, 2553, 2351, 2545, 2354, 2438}

$$\begin{aligned}
&\int \left(f+\frac{g}{x}\right)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx \\
&= 2fg\log(x)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right) \\
&\quad+\frac{g^2(a+bx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a(c+dx)\left(a-\frac{c(a+bx)}{c+dx}\right)}+\frac{Bf^2(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b} \\
&\quad-\frac{Bf^2n(bc-ad)\log(c+dx)}{bd}+\frac{Bg^2n(bc-ad)\log\left(a-\frac{c(a+bx)}{c+dx}\right)}{ac} \\
&\quad-2Bfgn\,\text{PolyLog}\left(2,-\frac{bx}{a}\right)-2Bfgn\log(x)\log\left(\frac{bx}{a}+1\right)+Af^2x \\
&\quad+2Bfgn\,\text{PolyLog}\left(2,-\frac{dx}{c}\right)+2Bfgn\log(x)\log\left(\frac{dx}{c}+1\right)
\end{aligned}$$

```
[In] Int[(f + g/x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] A*f^2*x - 2*B*f*g*n*Log[x]*Log[1 + (b*x)/a] + (B*f^2*(a + b*x)*Log[e*((a +
b*x)/(c + d*x))^n])/b + ((b*c - a*d)*g^2*(a + b*x)*(A + B*Log[e*((a + b*x)/
(c + d*x))^n]))/(a*(c + d*x)*(a - (c*(a + b*x))/(c + d*x))) + 2*f*g*Log[x]*
(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - (B*(b*c - a*d)*f^2*n*Log[c + d*x])
/(b*d) + 2*B*f*g*n*Log[x]*Log[1 + (d*x)/c] + (B*(b*c - a*d)*g^2*n*Log[a - (
c*(a + b*x))/(c + d*x)])/(a*c) - 2*B*f*g*n*PolyLog[2, -((b*x)/a)] + 2*B*f*g
*n*PolyLog[2, -((d*x)/c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
```

```
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2535

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rule 2545

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((A + B*Log[e*((
a + b*x)/(c + d*x))^n])/g), x] + (-Dist[b*B*(n/g), Int[Log[f + g*x]/(a + b*
x), x], x] + Dist[B*d*(n/g), Int[Log[f + g*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rule 2608

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(f^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+\frac{g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{x^2}+\frac{2fg\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{x}\right)dx$$

$$= f^2 \int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$$
$$+ (2fg)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x} dx + g^2 \int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2} dx$$
$$= Af^2x + 2fg\log(x)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) + (Bf^2)\int \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) dx$$
$$+ \left((bc-ad)g^2\right)\operatorname{Subst}\left(\int \frac{A+B\log(ex^n)}{(-a+cx)^2} dx, x, \frac{a+bx}{c+dx}\right)$$
$$- (2bBfgn)\int \frac{\log(x)}{a+bx} dx + (2Bdfgn)\int \frac{\log(x)}{c+dx} dx$$
$$= Af^2x - 2Bfgn\log(x)\log\left(1+\frac{bx}{a}\right) + \frac{Bf^2(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}$$
$$+ \frac{(bc-ad)g^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{a(c+dx)\left(a-\frac{c(a+bx)}{c+dx}\right)}$$
$$+ 2fg\log(x)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) + 2Bfgn\log(x)\log\left(1+\frac{dx}{c}\right)$$
$$- \frac{(B(bc-ad)f^2n)\int \frac{1}{c+dx} dx}{b} + (2Bfgn)\int \frac{\log\left(1+\frac{bx}{a}\right)}{x} dx$$
$$- (2Bfgn)\int \frac{\log\left(1+\frac{dx}{c}\right)}{x} dx + \frac{(B(bc-ad)g^2n)\operatorname{Subst}\left(\int \frac{1}{-a+cx} dx, x, \frac{a+bx}{c+dx}\right)}{a}$$
$$= Af^2x - 2Bfgn\log(x)\log\left(1+\frac{bx}{a}\right) + \frac{Bf^2(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}$$
$$+ \frac{(bc-ad)g^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{a(c+dx)\left(a-\frac{c(a+bx)}{c+dx}\right)}$$
$$+ 2fg\log(x)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)$$
$$- \frac{B(bc-ad)f^2n\log(c+dx)}{bd} + 2Bfgn\log(x)\log\left(1+\frac{dx}{c}\right)$$
$$+ \frac{B(bc-ad)g^2n\log\left(a-\frac{c(a+bx)}{c+dx}\right)}{ac} - 2Bfgn\operatorname{Li}_2\left(-\frac{bx}{a}\right) + 2Bfgn\operatorname{Li}_2\left(-\frac{dx}{c}\right)$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.83

$$\int \left(f+\frac{g}{x}\right)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = Af^2x+\frac{Bf^2(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}-\frac{g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{x} + 2fg\log(x)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-\frac{B(bc-ad)f^2n\log(c+dx)}{bd} + \frac{Bg^2n((bc-ad)\log(x)-bc\log(a+bx)+ad\log(c+dx))}{ac} - 2Bfgn\left(\log(x)\left(\log\left(1+\frac{bx}{a}\right)-\log\left(1+\frac{dx}{c}\right)\right)+\operatorname{PolyLog}\left(2,-\frac{bx}{a}\right) - \operatorname{PolyLog}\left(2,-\frac{dx}{c}\right)\right)$$

```
[In] Integrate[(f + g/x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] A*f^2*x + (B*f^2*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b - (g^2*(A + B*
Log[e*((a + b*x)/(c + d*x))^n]))/x + 2*f*g*Log[x]*(A + B*Log[e*((a + b*x)/(
c + d*x))^n]) - (B*(b*c - a*d)*f^2*n*Log[c + d*x])/(b*d) + (B*g^2*n*((b*c -
 a*d)*Log[x] - b*c*Log[a + b*x] + a*d*Log[c + d*x]))/(a*c) - 2*B*f*g*n*(Log
[x]*(Log[1 + (b*x)/a] - Log[1 + (d*x)/c]) + PolyLog[2, -((b*x)/a)] - PolyLo
g[2, -((d*x)/c)])
```

Maple [F]

$$\int \left(f+\frac{g}{x}\right)^2 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)dx$$

```
[In] int((f+g/x)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

```
[Out] int((f+g/x)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Fricas [F]

$$\int \left(f+\frac{g}{x}\right)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)\left(f+\frac{g}{x}\right)^2 dx$$

```
[In] integrate((f+g/x)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")
```

```
[Out] integral((A*f^2*x^2 + 2*A*f*g*x + A*g^2 + (B*f^2*x^2 + 2*B*f*g*x + B*g^2)*l
og(e*((b*x + a)/(d*x + c))^n))/x^2, x)
```

Sympy [F]

$$\int \left(f+\frac{g}{x}\right)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \int \frac{\left(A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)\right)(fx+g)^2}{x^2}\,dx$$

```
[In] integrate((f+g/x)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
[Out] Integral((A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n))*(f*x + g)**2/x**2,
 x)
```

Maxima [F]

$$\int \left(f+\frac{g}{x}\right)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)\left(f+\frac{g}{x}\right)^2\,dx$$

```
[In] integrate((f+g/x)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")
[Out] B*f^2*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) - B*g^2*n*(b*log(b*x + a)/a -
 d*log(d*x + c)/c - (b*c - a*d)*log(x)/(a*c)) + B*f^2*x*log(e*(b*x/(d*x + c
) + a/(d*x + c))^n) + A*f^2*x - 2*B*f*g*integrate(-(log((b*x + a)^n) - log(
(d*x + c)^n) + log(e))/x, x) + 2*A*f*g*log(x) - B*g^2*log(e*(b*x/(d*x + c)
+ a/(d*x + c))^n)/x - A*g^2/x
```

Giac [F]

$$\int \left(f+\frac{g}{x}\right)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)\left(f+\frac{g}{x}\right)^2\,dx$$

```
[In] integrate((f+g/x)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)*(f + g/x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(f+\frac{g}{x}\right)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx \\ = \int \left(f+\frac{g}{x}\right)^2 \left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)\,dx$$

```
[In] int((f + g/x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
[Out] int((f + g/x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)), x)
```

3.3 $\int \left(f+\frac{g}{x}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

Optimal result 70
Rubi [A] (verified) 70
Mathematica [A] (verified) 72
Maple [F] 73
Fricas [F] 73
Sympy [F] 73
Maxima [F] 73
Giac [F] 74
Mupad [F(-1)] 74

Optimal result

Integrand size = 30, antiderivative size = 143

$$\begin{aligned}\int \left(f+\frac{g}{x}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx &= Afx-Bgn\log(x)\log\left(1+\frac{bx}{a}\right)+\frac{Bf(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}\\ &\quad+g\log(x)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-\frac{B(bc-ad)fn\log(c+dx)}{bd}\\ &\quad+Bgn\log(x)\log\left(1+\frac{dx}{c}\right)-Bgn\operatorname{PolyLog}\left(2,-\frac{bx}{a}\right)+Bgn\operatorname{PolyLog}\left(2,-\frac{dx}{c}\right)\end{aligned}$$

```
[Out] A*f*x-B*g*n*ln(x)*ln(1+b*x/a)+B*f*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/b+g*ln(
x)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))-B*(-a*d+b*c)*f*n*ln(d*x+c)/b/d+B*g*n*ln(
x)*ln(1+d*x/c)-B*g*n*polylog(2,-b*x/a)+B*g*n*polylog(2,-d*x/c)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2608, 2535, 31, 2545, 2354, 2438}

$$\begin{aligned}\int \left(f+\frac{g}{x}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx &= g\log(x)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+\frac{Bf(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}\\ &\quad-\frac{Bfn(bc-ad)\log(c+dx)}{bd}-Bgn\operatorname{PolyLog}\left(2,-\frac{bx}{a}\right)-Bgn\log(x)\log\left(\frac{bx}{a}+1\right)\\ &\quad+Afx+Bgn\operatorname{PolyLog}\left(2,-\frac{dx}{c}\right)+Bgn\log(x)\log\left(\frac{dx}{c}+1\right)\end{aligned}$$

```
[In] Int[(f + g/x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] A*f*x - B*g*n*Log[x]*Log[1 + (b*x)/a] + (B*f*(a + b*x)*Log[e*((a + b*x)/(c
+ d*x))^n])/b + g*Log[x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - (B*(b*c -
 a*d)*f*n*Log[c + d*x])/(b*d) + B*g*n*Log[x]*Log[1 + (d*x)/c] - B*g*n*PolyL
og[2, -((b*x)/a)] + B*g*n*PolyLog[2, -((d*x)/c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2535

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rule 2545

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((A + B*Log[e*((
a + b*x)/(c + d*x))^n])/g), x] + (-Dist[b*B*(n/g), Int[Log[f + g*x]/(a + b*
x), x], x] + Dist[B*d*(n/g), Int[Log[f + g*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 2608

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(f\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+\frac{g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{x}\right)dx\\
&= f\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx+g\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x}dx\\
&= Afx+g\log(x)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+(Bf)\int \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)dx\\
&\quad-(bBgn)\int \frac{\log(x)}{a+bx}dx+(Bdgn)\int \frac{\log(x)}{c+dx}dx\\
&= Afx-Bgn\log(x)\log\left(1+\frac{bx}{a}\right)+\frac{Bf(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}\\
&\quad+g\log(x)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+Bgn\log(x)\log\left(1+\frac{dx}{c}\right)\\
&\quad-\frac{(B(bc-ad)fn)\int \frac{1}{c+dx}dx}{b}+(Bgn)\int \frac{\log\left(1+\frac{bx}{a}\right)}{x}dx-(Bgn)\int \frac{\log\left(1+\frac{dx}{c}\right)}{x}dx\\
&= Afx-Bgn\log(x)\log\left(1+\frac{bx}{a}\right)+\frac{Bf(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}\\
&\quad+g\log(x)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-\frac{B(bc-ad)fn\log(c+dx)}{bd}\\
&\quad+Bgn\log(x)\log\left(1+\frac{dx}{c}\right)-Bgn\mathrm{Li}_2\left(-\frac{bx}{a}\right)+Bgn\mathrm{Li}_2\left(-\frac{dx}{c}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.94

$$
\begin{aligned}
&\int \left(f+\frac{g}{x}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx\\
&= Afx+\frac{Bf(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}+g\log(x)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\\
&\quad-\frac{B(bc-ad)fn\log(c+dx)}{bd}-Bgn\left(\log(x)\left(\log\left(1+\frac{bx}{a}\right)-\log\left(1+\frac{dx}{c}\right)\right)\right.\\
&\qquad\left.+\text{PolyLog}\left(2,-\frac{bx}{a}\right)-\text{PolyLog}\left(2,-\frac{dx}{c}\right)\right)
\end{aligned}
$$

```
[In] Integrate[(f + g/x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] A*f*x + (B*f*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b + g*Log[x]*(A + B*
Log[e*((a + b*x)/(c + d*x))^n]) - (B*(b*c - a*d)*f*n*Log[c + d*x])/(b*d) -
B*g*n*(Log[x]*(Log[1 + (b*x)/a] - Log[1 + (d*x)/c]) + PolyLog[2, -((b*x)/a)
] - PolyLog[2, -((d*x)/c)])
```

Maple [F]

$$\int \left(f+\frac{g}{x}\right)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)dx$$

```
[In] int((f+g/x)*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
[Out] int((f+g/x)*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Fricas [F]

$$\int \left(f+\frac{g}{x}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)dx=\int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)\left(f+\frac{g}{x}\right)dx$$

```
[In] integrate((f+g/x)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")
[Out] integral((A*f*x + A*g + (B*f*x + B*g)*log(e*((b*x + a)/(d*x + c))^n))/x, x)
```

Sympy [F]

$$\int \left(f+\frac{g}{x}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)dx=\int \frac{\left(A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)\right)(fx+g)}{x}\,dx$$

```
[In] integrate((f+g/x)*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
[Out] Integral((A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n))*(f*x + g)/x, x)
```

Maxima [F]

$$\int \left(f+\frac{g}{x}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)dx=\int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)\left(f+\frac{g}{x}\right)dx$$

```
[In] integrate((f+g/x)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")
[Out] B*f*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*f*x*log(e*(b*x/(d*x + c) +
a/(d*x + c))^n) + A*f*x - B*g*integrate(-(log((b*x + a)^n) - log((d*x + c)^
n) + log(e))/x, x) + A*g*log(x)
```

Giac [F]

$$\int \left(f+\frac{g}{x}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)\left(f+\frac{g}{x}\right)\,dx$$

```
[In] integrate((f+g/x)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)*(f + g/x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(f+\frac{g}{x}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\int \left(f+\frac{g}{x}\right)\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)\,dx$$

```
[In] int((f + g/x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
[Out] int((f + g/x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)), x)
```

3.4 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+\frac{g}{x}}\,dx$

Optimal result 75
Rubi [A] (verified) 76
Mathematica [A] (verified) 78
Maple [F] 79
Fricas [F] 79
Sympy [F(-1)] 79
Maxima [F] 79
Giac [F] 80
Mupad [F(-1)] 80

Optimal result

Integrand size = 32, antiderivative size = 217

$$\begin{aligned}\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+\frac{g}{x}}\,dx &= \frac{Ax}{f}+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf}-\frac{B(bc-ad)n\log(c+dx)}{bdf}\\ &\quad+\frac{Bgn\log\left(\frac{f(a+bx)}{af-bg}\right)\log(g+fx)}{f^2}\\ &\quad-\frac{g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(g+fx)}{f^2}\\ &\quad-\frac{Bgn\log\left(\frac{f(c+dx)}{cf-dg}\right)\log(g+fx)}{f^2}\\ &\quad+\frac{Bgn\operatorname{PolyLog}\left(2,-\frac{b(g+fx)}{af-bg}\right)}{f^2}-\frac{Bgn\operatorname{PolyLog}\left(2,-\frac{d(g+fx)}{cf-dg}\right)}{f^2}\end{aligned}$$

```
[Out] A*x/f+B*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/b/f-B*(-a*d+b*c)*n*ln(d*x+c)/b/d/
f+B*g*n*ln(f*(b*x+a)/(a*f-b*g))*ln(f*x+g)/f^2-g*(A+B*ln(e*((b*x+a)/(d*x+c))
^n))*ln(f*x+g)/f^2-B*g*n*ln(f*(d*x+c)/(c*f-d*g))*ln(f*x+g)/f^2+B*g*n*polylo
g(2,-b*(f*x+g)/(a*f-b*g))/f^2-B*g*n*polylog(2,-d*(f*x+g)/(c*f-d*g))/f^2
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {2608, 2535, 31, 2545, 2441, 2440, 2438}

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+\frac{g}{x}}\,dx=-\frac{g\log(fx+g)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{f^2}+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf}-\frac{Bn(bc-ad)\log(c+dx)}{bdf}+\frac{Bgn\,\text{PolyLog}\left(2,-\frac{b(g+fx)}{af-bg}\right)}{f^2}+\frac{Bgn\log(fx+g)\log\left(\frac{f(a+bx)}{af-bg}\right)}{f^2}+\frac{Ax}{f}-\frac{Bgn\,\text{PolyLog}\left(2,-\frac{d(g+fx)}{cf-dg}\right)}{f^2}-\frac{Bgn\log(fx+g)\log\left(\frac{f(c+dx)}{cf-dg}\right)}{f^2}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g/x),x]
```

```
[Out] (A*x)/f + (B*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/(b*f) - (B*(b*c - a*
d)*n*Log[c + d*x])/(b*d*f) + (B*g*n*Log[(f*(a + b*x))/(a*f - b*g)]*Log[g +
f*x])/f^2 - (g*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[g + f*x])/f^2 - (
B*g*n*Log[(f*(c + d*x))/(c*f - d*g)]*Log[g + f*x])/f^2 + (B*g*n*PolyLog[2,
-((b*(g + f*x))/(a*f - b*g))])/f^2 - (B*g*n*PolyLog[2, -((d*(g + f*x))/(c*f
 - d*g))])/f^2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
```

```
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2535

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rule 2545

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((A + B*Log[e*((
a + b*x)/(c + d*x))^n])/g), x] + (-Dist[b*B*(n/g), Int[Log[f + g*x]/(a + b*
x), x], x] + Dist[B*d*(n/g), Int[Log[f + g*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 2608

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f} - \frac{g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f(g+fx)}\right) dx$$

$$= \frac{\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx}{f} - \frac{g\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g+fx} dx}{f}$$

$$= \frac{Ax}{f} - \frac{g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(g+fx)}{f^2} + \frac{B\int \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) dx}{f}$$
$$+ \frac{(bBgn)\int \frac{\log(g+fx)}{a+bx} dx}{f^2} - \frac{(Bdgn)\int \frac{\log(g+fx)}{c+dx} dx}{f^2}$$

$$= \frac{Ax}{f} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf} + \frac{Bgn\log\left(\frac{f(a+bx)}{af-bg}\right)\log(g+fx)}{f^2} - \frac{g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(g+fx)}{f^2} - \frac{Bgn\log\left(\frac{f(c+dx)}{cf-dg}\right)\log(g+fx)}{f^2} - \frac{(B(bc-ad)n)\int\frac{1}{c+dx}\,dx}{bf} - \frac{(Bgn)\int\frac{\log\left(\frac{f(a+bx)}{af-bg}\right)}{g+fx}\,dx}{f} + \frac{(Bgn)\int\frac{\log\left(\frac{f(c+dx)}{cf-dg}\right)}{g+fx}\,dx}{f}$$

$$= \frac{Ax}{f} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf} - \frac{B(bc-ad)n\log(c+dx)}{bdf} + \frac{Bgn\log\left(\frac{f(a+bx)}{af-bg}\right)\log(g+fx)}{f^2} - \frac{g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(g+fx)}{f^2} - \frac{Bgn\log\left(\frac{f(c+dx)}{cf-dg}\right)\log(g+fx)}{f^2} - \frac{(Bgn)\text{Subst}\left(\int\frac{\log\left(1+\frac{bx}{af-bg}\right)}{x}\,dx, x, g+fx\right)}{f^2} + \frac{(Bgn)\text{Subst}\left(\int\frac{\log\left(1+\frac{dx}{cf-dg}\right)}{x}\,dx, x, g+fx\right)}{f^2}$$

$$= \frac{Ax}{f} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf} - \frac{B(bc-ad)n\log(c+dx)}{bdf} + \frac{Bgn\log\left(\frac{f(a+bx)}{af-bg}\right)\log(g+fx)}{f^2} - \frac{g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(g+fx)}{f^2} - \frac{Bgn\log\left(\frac{f(c+dx)}{cf-dg}\right)\log(g+fx)}{f^2} + \frac{Bgn\text{Li}_2\left(-\frac{b(g+fx)}{af-bg}\right)}{f^2} - \frac{Bgn\text{Li}_2\left(-\frac{d(g+fx)}{cf-dg}\right)}{f^2}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.85

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+\frac{g}{x}}\,dx$$

$$= \frac{Afx + \frac{Bf(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b} - \frac{B(bc-ad)fn\log(c+dx)}{bd} - g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(g+fx) + Bgn\left(\left(\log\left(\frac{f(a}{af}\right.\right.\right.}{f^2}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g/x),x]
```

```
[Out] (A*f*x + (B*f*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b - (B*(b*c - a*d)*
f*n*Log[c + d*x])/(b*d) - g*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[g +
f*x] + B*g*n*((Log[(f*(a + b*x))/(a*f - b*g)] - Log[(f*(c + d*x))/(c*f - d*
g)])*Log[g + f*x] + PolyLog[2, (b*(g + f*x))/(-(a*f) + b*g)] - PolyLog[2, (
d*(g + f*x))/(-(c*f) + d*g)]))/f^2
```

Maple [F]

$$\int \frac{A + B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{f + \frac{g}{x}}dx$$

[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(f+g/x),x)

[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(f+g/x),x)

Fricas [F]

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f + \frac{g}{x}}\,dx = \int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A}{f + \frac{g}{x}}\,dx$$

[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x),x, algorithm="fricas")

[Out] integral((B*x*log(e*((b*x + a)/(d*x + c))^n) + A*x)/(f*x + g), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f + \frac{g}{x}}\,dx = \text{Timed out}$$

[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(f+g/x),x)

[Out] Timed out

Maxima [F]

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f + \frac{g}{x}}\,dx = \int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A}{f + \frac{g}{x}}\,dx$$

[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x),x, algorithm="maxima")

[Out] A*(x/f - g*log(f*x + g)/f^2) - B*integrate(-(x*log((b*x + a)^n) - x*log((d*x + c)^n) + x*log(e))/(f*x + g), x)

Giac [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+\frac{g}{x}}\,dx = \int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}{f+\frac{g}{x}}\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x),x, algorithm="giac")
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)/(f + g/x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+\frac{g}{x}}\,dx = \int \frac{A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{f+\frac{g}{x}}\,dx$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g/x),x)
[Out] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g/x), x)
```

3.5 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f+\frac{g}{x}\right)^2}\,dx$

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Maple [F] 86
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Sympy [F(-1)] 86
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Giac [F] 87
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Optimal result

Integrand size = 32, antiderivative size = 322

$$\begin{aligned}\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f+\frac{g}{x}\right)^2}\,dx &= \frac{Ax}{f^2}+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf^2}\\
&\quad-\frac{g^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^2(af-bg)(g+fx)}\\
&\quad-\frac{B(bc-ad)n\log(c+dx)}{bdf^2}+\frac{2Bgn\log\left(\frac{f(a+bx)}{af-bg}\right)\log(g+fx)}{f^3}\\
&\quad-\frac{2g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(g+fx)}{f^3}\\
&\quad-\frac{2Bgn\log\left(\frac{f(c+dx)}{cf-dg}\right)\log(g+fx)}{f^3}\\
&\quad+\frac{B(bc-ad)g^2n\log\left(\frac{g+fx}{c+dx}\right)}{f^2(af-bg)(cf-dg)}+\frac{2Bgn\,\text{PolyLog}\left(2,-\frac{b(g+fx)}{af-bg}\right)}{f^3}\\
&\quad-\frac{2Bgn\,\text{PolyLog}\left(2,-\frac{d(g+fx)}{cf-dg}\right)}{f^3}\end{aligned}$$

```
[Out] A*x/f^2+B*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/b/f^2-g^2*(b*x+a)*(A+B*ln(e*((b
*x+a)/(d*x+c))^n))/f^2/(a*f-b*g)/(f*x+g)-B*(-a*d+b*c)*n*ln(d*x+c)/b/d/f^2+2
*B*g*n*ln(f*(b*x+a)/(a*f-b*g))*ln(f*x+g)/f^3-2*g*(A+B*ln(e*((b*x+a)/(d*x+c)
)^n))*ln(f*x+g)/f^3-2*B*g*n*ln(f*(d*x+c)/(c*f-d*g))*ln(f*x+g)/f^3+B*(-a*d+b
*c)*g^2*n*ln((f*x+g)/(d*x+c))/f^2/(a*f-b*g)/(c*f-d*g)+2*B*g*n*polylog(2,-b*
(f*x+g)/(a*f-b*g))/f^3-2*B*g*n*polylog(2,-d*(f*x+g)/(c*f-d*g))/f^3
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {2608, 2535, 31, 2553, 2351, 2545, 2441, 2440, 2438}

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f+\frac{g}{x}\right)^2}\,dx = -\frac{2g\log(fx+g)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{f^3} - \frac{g^2(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{f^2(fx+g)(af-bg)} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf^2} + \frac{Bg^2n(bc-ad)\log\left(\frac{fx+g}{c+dx}\right)}{f^2(af-bg)(cf-dg)} - \frac{Bn(bc-ad)\log(c+dx)}{bdf^2} + \frac{2Bgn\,\text{PolyLog}\left(2,-\frac{b(g+fx)}{af-bg}\right)}{f^3} + \frac{2Bgn\log(fx+g)\log\left(\frac{f(a+bx)}{af-bg}\right)}{f^3} + \frac{Ax}{f^2} - \frac{2Bgn\,\text{PolyLog}\left(2,-\frac{d(g+fx)}{cf-dg}\right)}{f^3} - \frac{2Bgn\log(fx+g)\log\left(\frac{f(c+dx)}{cf-dg}\right)}{f^3}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g/x)^2,x]
```

```
[Out] (A*x)/f^2 + (B*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/(b*f^2) - (g^2*(a
+ b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(f^2*(a*f - b*g)*(g + f*x))
- (B*(b*c - a*d)*n*Log[c + d*x])/(b*d*f^2) + (2*B*g*n*Log[(f*(a + b*x))/(a*
f - b*g)]*Log[g + f*x])/f^3 - (2*g*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*L
og[g + f*x])/f^3 - (2*B*g*n*Log[(f*(c + d*x))/(c*f - d*g)]*Log[g + f*x])/f^
3 + (B*(b*c - a*d)*g^2*n*Log[(g + f*x)/(c + d*x)])/(f^2*(a*f - b*g)*(c*f -
d*g)) + (2*B*g*n*PolyLog[2, -((b*(g + f*x))/(a*f - b*g))])/f^3 - (2*B*g*n*P
olyLog[2, -((d*(g + f*x))/(c*f - d*g))])/f^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
```

```
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2535

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rule 2545

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((A + B*Log[e*((
a + b*x)/(c + d*x))^n])/g), x] + (-Dist[b*B*(n/g), Int[Log[f + g*x]/(a + b*
x), x], x] + Dist[B*d*(n/g), Int[Log[f + g*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rule 2608

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f^2} + \frac{g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^2(g+fx)^2} - \frac{2g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^2(g+fx)}\right) dx$$

$$= \frac{\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx}{f^2} - \frac{(2g)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g+fx}\,dx}{f^2} + \frac{g^2\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(g+fx)^2}\,dx}{f^2}$$

$$= \frac{Ax}{f^2} - \frac{2g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(g+fx)}{f^3} + \frac{B\int \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) dx}{f^2} + \frac{((bc-ad)g^2)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{(-af+bg+(cf-dg)x)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{f^2} + \frac{(2bBgn)\int \frac{\log(g+fx)}{a+bx}\,dx}{f^3} - \frac{(2Bdgn)\int \frac{\log(g+fx)}{c+dx}\,dx}{f^3}$$

$$= \frac{Ax}{f^2} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf^2} - \frac{g^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^2(af-bg)(g+fx)} + \frac{2Bgn\log\left(\frac{f(a+bx)}{af-bg}\right)\log(g+fx)}{f^3} - \frac{2g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(g+fx)}{f^3} - \frac{2Bgn\log\left(\frac{f(c+dx)}{cf-dg}\right)\log(g+fx)}{f^3} - \frac{(B(bc-ad)n)\int \frac{1}{c+dx}\,dx}{bf^2} - \frac{(2Bgn)\int \frac{\log\left(\frac{f(a+bx)}{af-bg}\right)}{g+fx}\,dx}{f^2} + \frac{(2Bgn)\int \frac{\log\left(\frac{f(c+dx)}{cf-dg}\right)}{g+fx}\,dx}{f^2} + \frac{(B(bc-ad)g^2n)\,\text{Subst}\left(\int \frac{1}{-af+bg+(cf-dg)x}\,dx, x, \frac{a+bx}{c+dx}\right)}{f^2(af-bg)}$$

$$= \frac{Ax}{f^2} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf^2} - \frac{g^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^2(af-bg)(g+fx)} - \frac{B(bc-ad)n\log(c+dx)}{bdf^2} + \frac{2Bgn\log\left(\frac{f(a+bx)}{af-bg}\right)\log(g+fx)}{f^3} - \frac{2g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(g+fx)}{f^3} - \frac{2Bgn\log\left(\frac{f(c+dx)}{cf-dg}\right)\log(g+fx)}{f^3} + \frac{B(bc-ad)g^2n\log\left(\frac{g+fx}{c+dx}\right)}{f^2(af-bg)(cf-dg)} - \frac{(2Bgn)\text{Subst}\left(\int \frac{\log\left(1+\frac{bx}{af-bg}\right)}{x}\,dx, x, g+fx\right)}{f^3} + \frac{(2Bgn)\text{Subst}\left(\int \frac{\log\left(1+\frac{dx}{cf-dg}\right)}{x}\,dx, x, g+fx\right)}{f^3}$$

$$= \frac{Ax}{f^2} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf^2} - \frac{g^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^2(af-bg)(g+fx)} - \frac{B(bc-ad)n\log(c+dx)}{bdf^2} + \frac{2Bgn\log\left(\frac{f(a+bx)}{af-bg}\right)\log(g+fx)}{f^3} - \frac{2g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(g+fx)}{f^3} - \frac{2Bgn\log\left(\frac{f(c+dx)}{cf-dg}\right)\log(g+fx)}{f^3} + \frac{B(bc-ad)g^2n\log\left(\frac{g+fx}{c+dx}\right)}{f^2(af-bg)(cf-dg)} + \frac{2Bgn\text{Li}_2\left(-\frac{b(g+fx)}{af-bg}\right)}{f^3} - \frac{2Bgn\text{Li}_2\left(-\frac{d(g+fx)}{cf-dg}\right)}{f^3}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.92

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f+\frac{g}{x}\right)^2}\,dx$$

$$= \frac{Afx + \frac{Bf(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b} - \frac{g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g+fx} - \frac{B(bc-ad)fn\log(c+dx)}{bd} - 2g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(}{}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g/x)^2,x]
```

```
[Out] (A*f*x + (B*f*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b - (g^2*(A + B*Log
[e*((a + b*x)/(c + d*x))^n]))/(g + f*x) - (B*(b*c - a*d)*f*n*Log[c + d*x])/
(b*d) - 2*g*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[g + f*x] + (B*g^2*n*
(b*(-(c*f) + d*g)*Log[a + b*x] + d*(a*f - b*g)*Log[c + d*x] + (b*c - a*d)*f
*Log[g + f*x]))/((a*f - b*g)*(c*f - d*g)) + 2*B*g*n*((Log[(f*(a + b*x))/(a*
f - b*g)] - Log[(f*(c + d*x))/(c*f - d*g)])*Log[g + f*x] + PolyLog[2, (b*(g
 + f*x))/(-(a*f) + b*g)] - PolyLog[2, (d*(g + f*x))/(-(c*f) + d*g)]))/f^3
```

Maple [F]

$$\int \frac{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{\left(f+\frac{g}{x}\right)^2}dx$$

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^2,x)
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^2,x)
```

Fricas [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f+\frac{g}{x}\right)^2}\,dx = \int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}{\left(f+\frac{g}{x}\right)^2}\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^2,x, algorithm="fricas")
[Out] integral((B*x^2*log(e*((b*x + a)/(d*x + c))^n) + A*x^2)/(f^2*x^2 + 2*f*g*x
+ g^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f+\frac{g}{x}\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(f+g/x)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f+\frac{g}{x}\right)^2}\,dx = \int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}{\left(f+\frac{g}{x}\right)^2}\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^2,x, algorithm="maxima")
[Out] -A*(g^2/(f^4*x + f^3*g) - x/f^2 + 2*g*log(f*x + g)/f^3) - B*integrate(-(x^2
*log((b*x + a)^n) - x^2*log((d*x + c)^n) + x^2*log(e))/(f^2*x^2 + 2*f*g*x +
 g^2), x)
```

Giac [F]

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f + \frac{g}{x}\right)^2}\,dx = \int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A}{\left(f + \frac{g}{x}\right)^2}\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^2,x, algorithm="giac")
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)/(f + g/x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f + \frac{g}{x}\right)^2}\,dx = \int \frac{A + B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{\left(f + \frac{g}{x}\right)^2}\,dx$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g/x)^2,x)
[Out] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g/x)^2, x)
```

3.6 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f+\frac{g}{x}\right)^3}\,dx$

Optimal result 88
Rubi [A] (verified) 89
Mathematica [A] (verified) 94
Maple [F] 94
Fricas [F] 94
Sympy [F(-1)] 95
Maxima [F] 95
Giac [F] 95
Mupad [F(-1)] 95

Optimal result

Integrand size = 32, antiderivative size = 531

$$\begin{aligned}\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f+\frac{g}{x}\right)^3}\,dx &= \frac{Ax}{f^3}+\frac{B(bc-ad)g^3n}{2f^3(af-bg)(cf-dg)(g+fx)}-\frac{b^2Bg^3n\log(a+bx)}{2f^4(af-bg)^2}\\
&+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf^3}+\frac{g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2f^4(g+fx)^2}\\
&-\frac{3g^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^3(af-bg)(g+fx)}\\
&-\frac{B(bc-ad)n\log(c+dx)}{bdf^3}+\frac{Bd^2g^3n\log(c+dx)}{2f^4(cf-dg)^2}\\
&+\frac{B(bc-ad)g^3(bcf+adf-2bdg)n\log(g+fx)}{2f^3(af-bg)^2(cf-dg)^2}\\
&+\frac{3Bgn\log\left(\frac{f(a+bx)}{af-bg}\right)\log(g+fx)}{f^4}\\
&-\frac{3g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(g+fx)}{f^4}\\
&-\frac{3Bgn\log\left(\frac{f(c+dx)}{cf-dg}\right)\log(g+fx)}{f^4}\\
&+\frac{3B(bc-ad)g^2n\log\left(\frac{g+fx}{c+dx}\right)}{f^3(af-bg)(cf-dg)}+\frac{3Bgn\operatorname{PolyLog}\left(2,-\frac{b(g+fx)}{af-bg}\right)}{f^4}\\
&-\frac{3Bgn\operatorname{PolyLog}\left(2,-\frac{d(g+fx)}{cf-dg}\right)}{f^4}\end{aligned}$$

```
[Out] A*x/f^3+1/2*B*(-a*d+b*c)*g^3*n/f^3/(a*f-b*g)/(c*f-d*g)/(f*x+g)-1/2*b^2*B*g^
3*n*ln(b*x+a)/f^4/(a*f-b*g)^2+B*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/b/f^3+1/2
*g^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/f^4/(f*x+g)^2-3*g^2*(b*x+a)*(A+B*ln(e*
((b*x+a)/(d*x+c))^n))/f^3/(a*f-b*g)/(f*x+g)-B*(-a*d+b*c)*n*ln(d*x+c)/b/d/f^
3+1/2*B*d^2*g^3*n*ln(d*x+c)/f^4/(c*f-d*g)^2+1/2*B*(-a*d+b*c)*g^3*(a*d*f+b*c
*f-2*b*d*g)*n*ln(f*x+g)/f^3/(a*f-b*g)^2/(c*f-d*g)^2+3*B*g*n*ln(f*(b*x+a)/(a
*f-b*g))*ln(f*x+g)/f^4-3*g*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(f*x+g)/f^4-3*
B*g*n*ln(f*(d*x+c)/(c*f-d*g))*ln(f*x+g)/f^4+3*B*(-a*d+b*c)*g^2*n*ln((f*x+g)
/(d*x+c))/f^3/(a*f-b*g)/(c*f-d*g)+3*B*g*n*polylog(2,-b*(f*x+g)/(a*f-b*g))/f
^4-3*B*g*n*polylog(2,-d*(f*x+g)/(c*f-d*g))/f^4
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 531, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.344, Rules used = {2608, 2535, 31, 2547, 84, 2553, 2351, 2545, 2441, 2440, 2438}

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f+\frac{g}{x}\right)^3}dx = \frac{g^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2f^4(fx+g)^2} - \frac{3g\log(fx+g)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{f^4} - \frac{3g^2(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{f^3(fx+g)(af-bg)} - \frac{b^2Bg^3n\log(a+bx)}{2f^4(af-bg)^2} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf^3} + \frac{Bg^3n(bc-ad)}{2f^3(fx+g)(af-bg)(cf-dg)} + \frac{Bg^3n(bc-ad)\log(fx+g)(adf+bcf-2bdg)}{2f^3(af-bg)^2(cf-dg)^2} + \frac{3Bg^2n(bc-ad)\log\left(\frac{fx+g}{c+dx}\right)}{f^3(af-bg)(cf-dg)} - \frac{Bn(bc-ad)\log(c+dx)}{bdf^3} + \frac{3Bgn\,\text{PolyLog}\left(2,-\frac{b(g+fx)}{af-bg}\right)}{f^4} + \frac{3Bgn\log(fx+g)\log\left(\frac{f(a+bx)}{af-bg}\right)}{f^4} + \frac{Ax}{f^3} + \frac{Bd^2g^3n\log(c+dx)}{2f^4(cf-dg)^2} - \frac{3Bgn\,\text{PolyLog}\left(2,-\frac{d(g+fx)}{cf-dg}\right)}{f^4} - \frac{3Bgn\log(fx+g)\log\left(\frac{f(c+dx)}{cf-dg}\right)}{f^4}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g/x)^3,x]
```

```
[Out] (A*x)/f^3 + (B*(b*c - a*d)*g^3*n)/(2*f^3*(a*f - b*g)*(c*f - d*g)*(g + f*x))
 - (b^2*B*g^3*n*Log[a + b*x])/(2*f^4*(a*f - b*g)^2) + (B*(a + b*x)*Log[e*((
a + b*x)/(c + d*x))^n])/(b*f^3) + (g^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n
]))/(2*f^4*(g + f*x)^2) - (3*g^2*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x
))^n]))/(f^3*(a*f - b*g)*(g + f*x)) - (B*(b*c - a*d)*n*Log[c + d*x])/(b*d*f
^3) + (B*d^2*g^3*n*Log[c + d*x])/(2*f^4*(c*f - d*g)^2) + (B*(b*c - a*d)*g^3
*(b*c*f + a*d*f - 2*b*d*g)*n*Log[g + f*x])/(2*f^3*(a*f - b*g)^2*(c*f - d*g)
^2) + (3*B*g*n*Log[(f*(a + b*x))/(a*f - b*g)]*Log[g + f*x])/f^4 - (3*g*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n])*Log[g + f*x])/f^4 - (3*B*g*n*Log[(f*(c +
 d*x))/(c*f - d*g)]*Log[g + f*x])/f^4 + (3*B*(b*c - a*d)*g^2*n*Log[(g + f*x
)/(c + d*x)])/(f^3*(a*f - b*g)*(c*f - d*g)) + (3*B*g*n*PolyLog[2, -((b*(g +
 f*x))/(a*f - b*g))])/f^4 - (3*B*g*n*PolyLog[2, -((d*(g + f*x))/(c*f - d*g)
)])/f^4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
```

```
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2535

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rule 2545

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((A + B*Log[e*((
a + b*x)/(c + d*x))^n])/g), x] + (-Dist[b*B*(n/g), Int[Log[f + g*x]/(a + b*
x), x], x] + Dist[B*d*(n/g), Int[Log[f + g*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rule 2608

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{A + B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f^3} - \frac{g^3\left(A + B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^3(g+fx)^3} + \frac{3g^2\left(A + B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^3(g+fx)^2} - \frac{3g\left(A + B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^3(g+fx)} \right) dx$$

$$= \frac{\int \left(A + B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx}{f^3} - \frac{(3g)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g+fx} dx}{f^3} + \frac{(3g^2)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(g+fx)^2} dx}{f^3} - \frac{g^3\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(g+fx)^3} dx}{f^3}$$

$$= \frac{Ax}{f^3} + \frac{g^3\left(A + B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2f^4(g+fx)^2} - \frac{3g\left(A + B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(g+fx)}{f^4} + \frac{B\int \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) dx}{f^3} + \frac{(3(bc-ad)g^2)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{(-af+bg+(cf-dg)x)^2} dx, x, \frac{a+bx}{c+dx}\right)}{f^3} + \frac{(3bBgn)\int \frac{\log(g+fx)}{a+bx} dx}{f^4} - \frac{(3Bdgn)\int \frac{\log(g+fx)}{c+dx} dx}{f^4} - \frac{(B(bc-ad)g^3n)\int \frac{1}{(a+bx)(c+dx)(g+fx)^2} dx}{2f^4}$$

$$= \frac{Ax}{f^3} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf^3} + \frac{g^3\left(A + B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2f^4(g+fx)^2} - \frac{3g^2(a+bx)\left(A + B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^3(af-bg)(g+fx)} + \frac{3Bgn\log\left(\frac{f(a+bx)}{af-bg}\right)\log(g+fx)}{f^4} - \frac{3g\left(A + B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(g+fx)}{f^4} - \frac{3Bgn\log\left(\frac{f(c+dx)}{cf-dg}\right)\log(g+fx)}{f^4} - \frac{(B(bc-ad)n)\int \frac{1}{c+dx} dx}{bf^3} - \frac{(3Bgn)\int \frac{\log\left(\frac{f(a+bx)}{af-bg}\right)}{g+fx} dx}{f^3} + \frac{(3Bgn)\int \frac{\log\left(\frac{f(c+dx)}{cf-dg}\right)}{g+fx} dx}{f^3} - \frac{(B(bc-ad)g^3n)\int \left(\frac{b^3}{(bc-ad)(-af+bg)^2(a+bx)} - \frac{d^3}{(bc-ad)(cf-dg)^2(c+dx)} + \frac{f^2}{(af-bg)(cf-dg)(g+fx)^2} - \frac{f^2(bcf+}{(af-bg)^2(cf} \right.}{2f^4} + \frac{(3B(bc-ad)g^2n)\,\text{Subst}\left(\int \frac{1}{-af+bg+(cf-dg)x} dx, x, \frac{a+bx}{c+dx}\right)}{f^3(af-bg)}$$

$$= \frac{Ax}{f^3} + \frac{B(bc-ad)g^3n}{2f^3(af-bg)(cf-dg)(g+fx)} - \frac{b^2Bg^3n\log(a+bx)}{2f^4(af-bg)^2} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf^3} + \frac{g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2f^4(g+fx)^2} - \frac{3g^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^3(af-bg)(g+fx)} - \frac{B(bc-ad)n\log(c+dx)}{bdf^3} + \frac{Bd^2g^3n\log(c+dx)}{2f^4(cf-dg)^2} + \frac{B(bc-ad)g^3(bcf+adf-2bdg)n\log(g+fx)}{2f^3(af-bg)^2(cf-dg)^2} + \frac{3Bgn\log\left(\frac{f(a+bx)}{af-bg}\right)\log(g+fx)}{f^4} - \frac{3g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(g+fx)}{f^4} - \frac{3Bgn\log\left(\frac{f(c+dx)}{cf-dg}\right)\log(g+fx)}{f^4} + \frac{3B(bc-ad)g^2n\log\left(\frac{g+fx}{c+dx}\right)}{f^3(af-bg)(cf-dg)} - \frac{(3Bgn)\text{Subst}\left(\int\frac{\log\left(1+\frac{bx}{af-bg}\right)}{x}\,dx,x,g+fx\right)}{f^4} + \frac{(3Bgn)\text{Subst}\left(\int\frac{\log\left(1+\frac{dx}{cf-dg}\right)}{x}\,dx,x,g+fx\right)}{f^4}$$

$$= \frac{Ax}{f^3} + \frac{B(bc-ad)g^3n}{2f^3(af-bg)(cf-dg)(g+fx)} - \frac{b^2Bg^3n\log(a+bx)}{2f^4(af-bg)^2} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf^3} + \frac{g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2f^4(g+fx)^2} - \frac{3g^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^3(af-bg)(g+fx)} - \frac{B(bc-ad)n\log(c+dx)}{bdf^3} + \frac{Bd^2g^3n\log(c+dx)}{2f^4(cf-dg)^2} + \frac{B(bc-ad)g^3(bcf+adf-2bdg)n\log(g+fx)}{2f^3(af-bg)^2(cf-dg)^2} + \frac{3Bgn\log\left(\frac{f(a+bx)}{af-bg}\right)\log(g+fx)}{f^4} - \frac{3g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(g+fx)}{f^4} - \frac{3Bgn\log\left(\frac{f(c+dx)}{cf-dg}\right)\log(g+fx)}{f^4} + \frac{3B(bc-ad)g^2n\log\left(\frac{g+fx}{c+dx}\right)}{f^3(af-bg)(cf-dg)} + \frac{3Bgn\text{Li}_2\left(-\frac{b(g+fx)}{af-bg}\right)}{f^4} - \frac{3Bgn\text{Li}_2\left(-\frac{d(g+fx)}{cf-dg}\right)}{f^4}$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 470, normalized size of antiderivative = 0.89

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f+\frac{g}{x}\right)^3}\,dx$$

$$= \frac{2Afx + \frac{2Bf(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b} + \frac{g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(g+fx)^2} - \frac{6g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g+fx} - \frac{2B(bc-ad)fn\log(c+dx)}{bd} - 6g(A \cdots}{\cdots}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g/x)^3,x]
```

```
[Out] (2*A*f*x + (2*B*f*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b + (g^3*(A + B
*Log[e*((a + b*x)/(c + d*x))^n]))/(g + f*x)^2 - (6*g^2*(A + B*Log[e*((a + b
*x)/(c + d*x))^n]))/(g + f*x) - (2*B*(b*c - a*d)*f*n*Log[c + d*x])/(b*d) -
6*g*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[g + f*x] + (6*B*g^2*n*(b*(-(
c*f) + d*g)*Log[a + b*x] + d*(a*f - b*g)*Log[c + d*x] + (b*c - a*d)*f*Log[g
 + f*x]))/((a*f - b*g)*(c*f - d*g)) + B*(b*c - a*d)*g^3*n*(-((b^2*Log[a + b
*x])/((b*c - a*d)*(a*f - b*g)^2)) + ((d^2*Log[c + d*x])/(b*c - a*d) + (f*((
(a*f - b*g)*(c*f - d*g))/(g + f*x) + (b*c*f + a*d*f - 2*b*d*g)*Log[g + f*x]
))/(a*f - b*g)^2)/(c*f - d*g)^2) + 6*B*g*n*((Log[(f*(a + b*x))/(a*f - b*g)]
 - Log[(f*(c + d*x))/(c*f - d*g)])*Log[g + f*x] + PolyLog[2, (b*(g + f*x))/
(-(a*f) + b*g)] - PolyLog[2, (d*(g + f*x))/(-(c*f) + d*g)]))/(2*f^4)
```

Maple [F]

$$\int \frac{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{\left(f+\frac{g}{x}\right)^3}dx$$

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^3,x)
```

```
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^3,x)
```

Fricas [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f+\frac{g}{x}\right)^3}\,dx = \int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}{\left(f+\frac{g}{x}\right)^3}\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^3,x, algorithm="fricas")
```

```
[Out] integral((B*x^3*log(e*((b*x + a)/(d*x + c))^n) + A*x^3)/(f^3*x^3 + 3*f^2*g*
x^2 + 3*f*g^2*x + g^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f+\frac{g}{x}\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(f+g/x)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f+\frac{g}{x}\right)^3}\,dx = \int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}{\left(f+\frac{g}{x}\right)^3}\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^3,x, algorithm="maxima")
[Out] -1/2*A*((6*f*g^2*x + 5*g^3)/(f^6*x^2 + 2*f^5*g*x + f^4*g^2) - 2*x/f^3 + 6*g
*log(f*x + g)/f^4) - B*integrate(-(x^3*log((b*x + a)^n) - x^3*log((d*x + c)
^n) + x^3*log(e))/(f^3*x^3 + 3*f^2*g*x^2 + 3*f*g^2*x + g^3), x)
```

Giac [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f+\frac{g}{x}\right)^3}\,dx = \int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}{\left(f+\frac{g}{x}\right)^3}\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^3,x, algorithm="giac")
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)/(f + g/x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f+\frac{g}{x}\right)^3}\,dx = \int \frac{A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{{\left(f+\frac{g}{x}\right)}^3}\,dx$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g/x)^3,x)
[Out] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g/x)^3, x)
```

3.7 $\int (a+bx)^4 \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) \, dx$

Optimal result 96
Rubi [A] (verified) 96
Mathematica [A] (verified) 98
Maple [F] 98
Fricas [B] (verification not implemented) 99
Sympy [F(-1)] 99
Maxima [B] (verification not implemented) 99
Giac [B] (verification not implemented) 100
Mupad [B] (verification not implemented) 102

Optimal result

Integrand size = 29, antiderivative size = 201

$$\int (a+bx)^4 \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) \, dx = -\frac{(bc-ad)^4qrx}{5d^4} + \frac{(bc-ad)^3qr(a+bx)^2}{10bd^3} - \frac{(bc-ad)^2qr(a+bx)^3}{15bd^2} + \frac{(bc-ad)qr(a+bx)^4}{20bd} - \frac{pr(a+bx)^5}{25b} - \frac{qr(a+bx)^5}{25b} + \frac{(bc-ad)^5qr\log(c+dx)}{5bd^5} + \frac{(a+bx)^5\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{5b}$$

```
[Out] -1/5*(-a*d+b*c)^4*q*r*x/d^4+1/10*(-a*d+b*c)^3*q*r*(b*x+a)^2/b/d^3-1/15*(-a*
d+b*c)^2*q*r*(b*x+a)^3/b/d^2+1/20*(-a*d+b*c)*q*r*(b*x+a)^4/b/d-1/25*p*r*(b*
x+a)^5/b-1/25*q*r*(b*x+a)^5/b+1/5*(-a*d+b*c)^5*q*r*ln(d*x+c)/b/d^5+1/5*(b*x
+a)^5*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used

$= \{2581, 32, 45\}$

$$\int (a+bx)^4 \log\left(e(f(a+bx)^p(c+dx)^q)^r\right) dx$$
$$= \frac{qr(bc-ad)^5\log(c+dx)}{5bd^5} - \frac{qrx(bc-ad)^4}{5d^4} + \frac{qr(a+bx)^2(bc-ad)^3}{10bd^3} - \frac{qr(a+bx)^3(bc-ad)^2}{15bd^2} + \frac{(a+bx)^5\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{5b} + \frac{qr(a+bx)^4(bc-ad)}{20bd} - \frac{pr(a+bx)^5}{25b} - \frac{qr(a+bx)^5}{25b}$$

```
[In] Int[(a + b*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] -1/5*((b*c - a*d)^4*q*r*x)/d^4 + ((b*c - a*d)^3*q*r*(a + b*x)^2)/(10*b*d^3)
 - ((b*c - a*d)^2*q*r*(a + b*x)^3)/(15*b*d^2) + ((b*c - a*d)*q*r*(a + b*x)^
4)/(20*b*d) - (p*r*(a + b*x)^5)/(25*b) - (q*r*(a + b*x)^5)/(25*b) + ((b*c -
 a*d)^5*q*r*Log[c + d*x])/(5*b*d^5) + ((a + b*x)^5*Log[e*(f*(a + b*x)^p*(c
+ d*x)^q)^r])/(5*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{(a+bx)^5\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{5b} - \frac{1}{5}(pr)\int (a+bx)^4\,dx - \frac{(dqr)\int \frac{(a+bx)^5}{c+dx}\,dx}{5b}$$

$$
\begin{aligned}
&= -\frac{pr(a+bx)^5}{25b} + \frac{(a+bx)^5\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{5b} \\
&\quad - \frac{(dqr)\int\left(\frac{b(bc-ad)^4}{d^5} - \frac{b(bc-ad)^3(a+bx)}{d^4} + \frac{b(bc-ad)^2(a+bx)^2}{d^3} - \frac{b(bc-ad)(a+bx)^3}{d^2} + \frac{b(a+bx)^4}{d} + \frac{(-bc+ad)^5}{d^5(c+dx)}\right)dx}{5b} \\
&= -\frac{(bc-ad)^4qrx}{5d^4} + \frac{(bc-ad)^3qr(a+bx)^2}{10bd^3} - \frac{(bc-ad)^2qr(a+bx)^3}{15bd^2} \\
&\quad + \frac{(bc-ad)qr(a+bx)^4}{20bd} - \frac{pr(a+bx)^5}{25b} - \frac{qr(a+bx)^5}{25b} \\
&\quad + \frac{(bc-ad)^5qr\log(c+dx)}{5bd^5} + \frac{(a+bx)^5\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{5b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.92

$$
\int(a+bx)^4\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx
$$

$$
= \frac{-\frac{r(60bd(bc-ad)^4(p+5q)x-60b^2(bc-ad)^3(2p+5q)(c+dx)^2+40b^3(bc-ad)^2(3p+5q)(c+dx)^3-15b^4(bc-ad)(4p+5q)(c+dx)^4+12b^5(p+q)(c+dx)^5-6}{60d^5}}{5b}
$$

```
[In] Integrate[(a + b*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] (-1/60*(r*(60*b*d*(b*c - a*d)^4*(p + 5*q)*x - 60*b^2*(b*c - a*d)^3*(2*p + 5
*q)*(c + d*x)^2 + 40*b^3*(b*c - a*d)^2*(3*p + 5*q)*(c + d*x)^3 - 15*b^4*(b*
c - a*d)*(4*p + 5*q)*(c + d*x)^4 + 12*b^5*(p + q)*(c + d*x)^5 - 60*(b*c - a
*d)^5*q*Log[c + d*x]))/d^5 + (a + b*x)^5*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^
r])/(5*b)
```

Maple [F]

$$
\int(bx+a)^4\ln\left(e\left(f(bx+a)^p(dx+c)^q\right)^r\right)dx
$$

```
[In] int((b*x+a)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

```
[Out] int((b*x+a)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(185) = 370$.

Time = 0.32 (sec) , antiderivative size = 624, normalized size of antiderivative = 3.10

$$\int (a+bx)^4 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx =$$
$$-\frac{12\,(b^5d^5p+b^5d^5q)rx^5+15\,(4\,ab^4d^5p-(b^5cd^4-5\,ab^4d^5)q)rx^4+20\,(6\,a^2b^3d^5p+(b^5c^2d^3-5\,ab^4cd^4+1\cdots}{\cdots}$$

```
[In] integrate((b*x+a)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas")
[Out] -1/300*(12*(b^5*d^5*p + b^5*d^5*q)*r*x^5 + 15*(4*a*b^4*d^5*p - (b^5*c*d^4 -
 5*a*b^4*d^5)*q)*r*x^4 + 20*(6*a^2*b^3*d^5*p + (b^5*c^2*d^3 - 5*a*b^4*c*d^4
 + 10*a^2*b^3*d^5)*q)*r*x^3 + 30*(4*a^3*b^2*d^5*p - (b^5*c^3*d^2 - 5*a*b^4*
c^2*d^3 + 10*a^2*b^3*c*d^4 - 10*a^3*b^2*d^5)*q)*r*x^2 + 60*(a^4*b*d^5*p + (
b^5*c^4*d - 5*a*b^4*c^3*d^2 + 10*a^2*b^3*c^2*d^3 - 10*a^3*b^2*c*d^4 + 5*a^4
*b*d^5)*q)*r*x - 60*(b^5*d^5*p*r*x^5 + 5*a*b^4*d^5*p*r*x^4 + 10*a^2*b^3*d^5
*p*r*x^3 + 10*a^3*b^2*d^5*p*r*x^2 + 5*a^4*b*d^5*p*r*x + a^5*d^5*p*r)*log(b*
x + a) - 60*(b^5*d^5*q*r*x^5 + 5*a*b^4*d^5*q*r*x^4 + 10*a^2*b^3*d^5*q*r*x^3
 + 10*a^3*b^2*d^5*q*r*x^2 + 5*a^4*b*d^5*q*r*x + (b^5*c^5 - 5*a*b^4*c^4*d +
10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4)*q*r)*log(d*x + c)
- 60*(b^5*d^5*x^5 + 5*a*b^4*d^5*x^4 + 10*a^2*b^3*d^5*x^3 + 10*a^3*b^2*d^5*x
^2 + 5*a^4*b*d^5*x)*log(e) - 60*(b^5*d^5*r*x^5 + 5*a*b^4*d^5*r*x^4 + 10*a^2
*b^3*d^5*r*x^3 + 10*a^3*b^2*d^5*r*x^2 + 5*a^4*b*d^5*r*x)*log(f))/(b*d^5)
```

Sympy [F(-1)]

Timed out.

$$\int (a+bx)^4 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \text{Timed out}$$

```
[In] integrate((b*x+a)**4*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(185) = 370$.

Time = 0.20 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.97

$$\int (a+bx)^4 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$$
$$= \frac{1}{5}\left(b^4x^5+5\,ab^3x^4+10\,a^2b^2x^3+10\,a^3bx^2+5\,a^4x\right)\log\left(\left(((bx+a)^p(dx+c)^qf\right)^r e\right)$$
$$+\frac{\left(\frac{60\,a^5fp\log(bx+a)}{b}-\frac{12\,b^4d^4f(p+q)x^5+15\,(ab^3d^4f(4\,p+5\,q)-b^4cd^3fq)x^4+20\,(2\,a^2b^2d^4f(3\,p+5\,q)+b^4c^2d^2fq-5\,ab^3cd^3fq)x^3+30\,(2\,a^3bd\cdots}{\cdots}\cdots\right)}{\cdots}$$

```
[In] integrate((b*x+a)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima")
```

```
[Out] 1/5*(b^4*x^5 + 5*a*b^3*x^4 + 10*a^2*b^2*x^3 + 10*a^3*b*x^2 + 5*a^4*x)*log((
(b*x + a)^p*(d*x + c)^q*f)^r*e) + 1/300*(60*a^5*f*p*log(b*x + a)/b - (12*b^
4*d^4*f*(p + q)*x^5 + 15*(a*b^3*d^4*f*(4*p + 5*q) - b^4*c*d^3*f*q)*x^4 + 20
*(2*a^2*b^2*d^4*f*(3*p + 5*q) + b^4*c^2*d^2*f*q - 5*a*b^3*c*d^3*f*q)*x^3 +
30*(2*a^3*b*d^4*f*(2*p + 5*q) - b^4*c^3*d*f*q + 5*a*b^3*c^2*d^2*f*q - 10*a^
2*b^2*c*d^3*f*q)*x^2 + 60*(a^4*d^4*f*(p + 5*q) + b^4*c^4*f*q - 5*a*b^3*c^3*
d*f*q + 10*a^2*b^2*c^2*d^2*f*q - 10*a^3*b*c*d^3*f*q)*x)/d^4 + 60*(b^4*c^5*f
*q - 5*a*b^3*c^4*d*f*q + 10*a^2*b^2*c^3*d^2*f*q - 10*a^3*b*c^2*d^3*f*q + 5*
a^4*c*d^4*f*q)*log(d*x + c)/d^5)*r/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 570 vs. $2(185) = 370$.

Time = 10.72 (sec) , antiderivative size = 570, normalized size of antiderivative = 2.84

$$\begin{aligned}
&\int (a+bx)^4 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx \\
&= \frac{a^5pr\log(bx+a)}{5\,b} - \frac{1}{25}\left(b^4pr + b^4qr - 5\,b^4r\log(f) - 5\,b^4\log(e)\right)x^5 \\
&\quad - \frac{(4\,ab^3dpr - b^4cqr + 5\,ab^3dqr - 20\,ab^3dr\log(f) - 20\,ab^3d\log(e))x^4}{20\,d} \\
&\quad - \frac{(6\,a^2b^2d^2pr + b^4c^2qr - 5\,ab^3cdqr + 10\,a^2b^2d^2qr - 30\,a^2b^2d^2r\log(f) - 30\,a^2b^2d^2\log(e))x^3}{15\,d^2} \\
&\quad + \frac{1}{5}\left(b^4prx^5 + 5\,ab^3prx^4 + 10\,a^2b^2prx^3 + 10\,a^3bprx^2 + 5\,a^4prx\right)\log(bx+a) \\
&\quad + \frac{1}{5}\left(b^4qrx^5 + 5\,ab^3qrx^4 + 10\,a^2b^2qrx^3 + 10\,a^3bqrx^2 + 5\,a^4qrx\right)\log(dx+c) \\
&\quad - \frac{(4\,a^3bd^3pr - b^4c^3qr + 5\,ab^3c^2dqr - 10\,a^2b^2cd^2qr + 10\,a^3bd^3qr - 20\,a^3bd^3r\log(f) - 20\,a^3bd^3\log(e))x^2}{10\,d^3} \\
&\quad - \frac{(a^4d^4pr + b^4c^4qr - 5\,ab^3c^3dqr + 10\,a^2b^2c^2d^2qr - 10\,a^3bcd^3qr + 5\,a^4d^4qr - 5\,a^4d^4r\log(f) - 5\,a^4d^4\log(e)}{5\,d^4} \\
&\quad + \frac{(b^4c^5qr - 5\,ab^3c^4dqr + 10\,a^2b^2c^3d^2qr - 10\,a^3bc^2d^3qr + 5\,a^4cd^4qr)\log(-dx-c)}{5\,d^5}
\end{aligned}$$

```
[In] integrate((b*x+a)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")
```

```
[Out] 1/5*a^5*p*r*log(b*x + a)/b - 1/25*(b^4*p*r + b^4*q*r - 5*b^4*r*log(f) - 5*b
^4*log(e))*x^5 - 1/20*(4*a*b^3*d*p*r - b^4*c*q*r + 5*a*b^3*d*q*r - 20*a*b^3
*d*r*log(f) - 20*a*b^3*d*log(e))*x^4/d - 1/15*(6*a^2*b^2*d^2*p*r + b^4*c^2*
q*r - 5*a*b^3*c*d*q*r + 10*a^2*b^2*d^2*q*r - 30*a^2*b^2*d^2*r*log(f) - 30*a
^2*b^2*d^2*log(e))*x^3/d^2 + 1/5*(b^4*p*r*x^5 + 5*a*b^3*p*r*x^4 + 10*a^2*b^
2*p*r*x^3 + 10*a^3*b*p*r*x^2 + 5*a^4*p*r*x)*log(b*x + a) + 1/5*(b^4*q*r*x^5
 + 5*a*b^3*q*r*x^4 + 10*a^2*b^2*q*r*x^3 + 10*a^3*b*q*r*x^2 + 5*a^4*q*r*x)*l
```

```
og(d*x + c) - 1/10*(4*a^3*b*d^3*p*r - b^4*c^3*q*r + 5*a*b^3*c^2*d*q*r - 10*
a^2*b^2*c*d^2*q*r + 10*a^3*b*d^3*q*r - 20*a^3*b*d^3*r*log(f) - 20*a^3*b*d^3
*log(e))*x^2/d^3 - 1/5*(a^4*d^4*p*r + b^4*c^4*q*r - 5*a*b^3*c^3*d*q*r + 10*
a^2*b^2*c^2*d^2*q*r - 10*a^3*b*c*d^3*q*r + 5*a^4*d^4*q*r - 5*a^4*d^4*r*log(
f) - 5*a^4*d^4*log(e))*x/d^4 + 1/5*(b^4*c^5*q*r - 5*a*b^3*c^4*d*q*r + 10*a^
2*b^2*c^3*d^2*q*r - 10*a^3*b*c^2*d^3*q*r + 5*a^4*c*d^4*q*r)*log(-d*x - c)/d
^5
```

Mupad [B] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 886, normalized size of antiderivative = 4.41

$$\int (a+bx)^4 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$$

$$= \ln\left(e\left(f\left(a+b\,x\right)^p\left(c+d\,x\right)^q\right)^r\right)\left(a^4\,x+2\,a^3\,b\,x^2+2\,a^2\,b^2\,x^3+a\,b^3\,x^4+\frac{b^4\,x^5}{5}\right)$$

$$-x^4\left(\frac{b^3\,r\left(5\,a\,d\,p+b\,c\,p+6\,a\,d\,q\right)}{20\,d}-\frac{b^3\,r\left(p+q\right)\left(5\,a\,d+5\,b\,c\right)}{100\,d}\right)$$

$$+x^3\left(\frac{\left(\frac{b^3\,r\,(5\,a\,d\,p+b\,c\,p+6\,a\,d\,q)}{5\,d}-\frac{b^3\,r\,(p+q)\,(5\,a\,d+5\,b\,c)}{25\,d}\right)\left(5\,a\,d+5\,b\,c\right)}{15\,b\,d}\right.$$

$$\left.-\frac{a\,b^2\,r\left(2\,a\,d\,p+b\,c\,p+3\,a\,d\,q\right)}{3\,d}+\frac{a\,b^3\,c\,r\left(p+q\right)}{15\,d}\right)-x\left(\frac{a^3\,r\left(a\,d\,p+2\,b\,c\,p+3\,a\,d\,q\right)}{d}\right.$$

$$-\frac{\left(5\,a\,d+5\,b\,c\right)\left(\frac{(5\,a\,d+5\,b\,c)\left(\frac{\left(\frac{b^3\,r\,(5\,a\,d\,p+b\,c\,p+6\,a\,d\,q)}{5\,d}-\frac{b^3\,r\,(p+q)\,(5\,a\,d+5\,b\,c)}{25\,d}\right)(5\,a\,d+5\,b\,c)}{5\,b\,d}-\frac{a\,b^2\,r\,(2\,a\,d\,p+b\,c\,p+3\,a\,d\,q)}{d}+\frac{a\,b^3\,c\,r\,(p+q)}{5\,d}\right)}{5\,b\,d}-\cdots\right.}{5\,b\,d}$$

$$\left.+\frac{a\,c\left(\frac{\left(\frac{b^3\,r\,(5\,a\,d\,p+b\,c\,p+6\,a\,d\,q)}{5\,d}-\frac{b^3\,r\,(p+q)\,(5\,a\,d+5\,b\,c)}{25\,d}\right)(5\,a\,d+5\,b\,c)}{5\,b\,d}-\frac{a\,b^2\,r\,(2\,a\,d\,p+b\,c\,p+3\,a\,d\,q)}{d}+\frac{a\,b^3\,c\,r\,(p+q)}{5\,d}\right)}{b\,d}\right)$$

$$-x^2\left(\frac{\left(5\,a\,d+5\,b\,c\right)\left(\frac{\left(\frac{b^3\,r\,(5\,a\,d\,p+b\,c\,p+6\,a\,d\,q)}{5\,d}-\frac{b^3\,r\,(p+q)\,(5\,a\,d+5\,b\,c)}{25\,d}\right)(5\,a\,d+5\,b\,c)}{5\,b\,d}-\frac{a\,b^2\,r\,(2\,a\,d\,p+b\,c\,p+3\,a\,d\,q)}{d}+\frac{a\,b^3\,c\,r\,(p+q)}{5\,d}\right)}{10\,b\,d}\right.$$

$$\left.-\frac{a\,c\left(\frac{b^3\,r\,(5\,a\,d\,p+b\,c\,p+6\,a\,d\,q)}{5\,d}-\frac{b^3\,r\,(p+q)\,(5\,a\,d+5\,b\,c)}{25\,d}\right)}{2\,b\,d}+\frac{a^2\,b\,r\left(a\,d\,p+b\,c\,p+2\,a\,d\,q\right)}{d}\right)$$

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(a + b*x)^4,x)
```

```
[Out] log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(a^4*x + (b^4*x^5)/5 + 2*a^3*b*x^2 + a
*b^3*x^4 + 2*a^2*b^2*x^3) - x^4*((b^3*r*(5*a*d*p + b*c*p + 6*a*d*q))/(20*d)
 - (b^3*r*(p + q)*(5*a*d + 5*b*c))/(100*d)) + x^3*((((b^3*r*(5*a*d*p + b*c*
p + 6*a*d*q))/(5*d) - (b^3*r*(p + q)*(5*a*d + 5*b*c))/(25*d))*(5*a*d + 5*b*
c))/(15*b*d) - (a*b^2*r*(2*a*d*p + b*c*p + 3*a*d*q))/(3*d) + (a*b^3*c*r*(p
+ q))/(15*d)) - x*((a^3*r*(a*d*p + 2*b*c*p + 3*a*d*q))/d - ((5*a*d + 5*b*c)
*(((5*a*d + 5*b*c)*((((b^3*r*(5*a*d*p + b*c*p + 6*a*d*q))/(5*d) - (b^3*r*(p
 + q)*(5*a*d + 5*b*c))/(25*d))*(5*a*d + 5*b*c))/(5*b*d) - (a*b^2*r*(2*a*d*p
 + b*c*p + 3*a*d*q))/d + (a*b^3*c*r*(p + q))/(5*d)))/(5*b*d) - (a*c*((b^3*r
*(5*a*d*p + b*c*p + 6*a*d*q))/(5*d) - (b^3*r*(p + q)*(5*a*d + 5*b*c))/(25*d
)))/(b*d) + (2*a^2*b*r*(a*d*p + b*c*p + 2*a*d*q))/d))/(5*b*d) + (a*c*((((b^
3*r*(5*a*d*p + b*c*p + 6*a*d*q))/(5*d) - (b^3*r*(p + q)*(5*a*d + 5*b*c))/(2
5*d))*(5*a*d + 5*b*c))/(5*b*d) - (a*b^2*r*(2*a*d*p + b*c*p + 3*a*d*q))/d +
(a*b^3*c*r*(p + q))/(5*d)))/(b*d)) - x^2*(((5*a*d + 5*b*c)*((((b^3*r*(5*a*d
*p + b*c*p + 6*a*d*q))/(5*d) - (b^3*r*(p + q)*(5*a*d + 5*b*c))/(25*d))*(5*a
*d + 5*b*c))/(5*b*d) - (a*b^2*r*(2*a*d*p + b*c*p + 3*a*d*q))/d + (a*b^3*c*r
*(p + q))/(5*d)))/(10*b*d) - (a*c*((b^3*r*(5*a*d*p + b*c*p + 6*a*d*q))/(5*d
) - (b^3*r*(p + q)*(5*a*d + 5*b*c))/(25*d)))/(2*b*d) + (a^2*b*r*(a*d*p + b*
c*p + 2*a*d*q))/d) + (log(c + d*x)*((b^4*c^5*q*r)/5 + a^4*c*d^4*q*r + 2*a^2
*b^2*c^3*d^2*q*r - a*b^3*c^4*d*q*r - 2*a^3*b*c^2*d^3*q*r))/d^5 - (b^4*r*x^5
*(p + q))/25 + (a^5*p*r*log(a + b*x))/(5*b)
```

3.8 $\int (a+bx)^3 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx$

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Optimal result

Integrand size = 29, antiderivative size = 172

$$\int (a+bx)^3 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx = \frac{(bc-ad)^3 qrx}{4d^3} - \frac{(bc-ad)^2 qr(a+bx)^2}{8bd^2} + \frac{(bc-ad)qr(a+bx)^3}{12bd} - \frac{pr(a+bx)^4}{16b} - \frac{qr(a+bx)^4}{16b} - \frac{(bc-ad)^4 qr\log(c+dx)}{4bd^4} + \frac{(a+bx)^4 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b}$$

```
[Out] 1/4*(-a*d+b*c)^3*q*r*x/d^3-1/8*(-a*d+b*c)^2*q*r*(b*x+a)^2/b/d^2+1/12*(-a*d+
b*c)*q*r*(b*x+a)^3/b/d-1/16*p*r*(b*x+a)^4/b-1/16*q*r*(b*x+a)^4/b-1/4*(-a*d+
b*c)^4*q*r*ln(d*x+c)/b/d^4+1/4*(b*x+a)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2581, 32, 45}

$$\int (a+bx)^3 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx = -\frac{qr(bc-ad)^4 \log(c+dx)}{4bd^4} + \frac{qrx(bc-ad)^3}{4d^3} - \frac{qr(a+bx)^2(bc-ad)^2}{8bd^2} + \frac{(a+bx)^4 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b} + \frac{qr(a+bx)^3(bc-ad)}{12bd} - \frac{pr(a+bx)^4}{16b} - \frac{qr(a+bx)^4}{16b}$$

```
[In] Int[(a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] ((b*c - a*d)^3*q*r*x)/(4*d^3) - ((b*c - a*d)^2*q*r*(a + b*x)^2)/(8*b*d^2) +
 ((b*c - a*d)*q*r*(a + b*x)^3)/(12*b*d) - (p*r*(a + b*x)^4)/(16*b) - (q*r*(
a + b*x)^4)/(16*b) - ((b*c - a*d)^4*q*r*Log[c + d*x])/(4*b*d^4) + ((a + b*x
)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(4*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(a+bx)^4\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b} - \frac{1}{4}(pr)\int (a+bx)^3\,dx - \frac{(dqr)\int \frac{(a+bx)^4}{c+dx}\,dx}{4b}\\
&= -\frac{pr(a+bx)^4}{16b} + \frac{(a+bx)^4\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b}\\
&\quad - \frac{(dqr)\int\left(-\frac{b(bc-ad)^3}{d^4} + \frac{b(bc-ad)^2(a+bx)}{d^3} - \frac{b(bc-ad)(a+bx)^2}{d^2} + \frac{b(a+bx)^3}{d} + \frac{(-bc+ad)^4}{d^4(c+dx)}\right)dx}{4b}\\
&= \frac{(bc-ad)^3qrx}{4d^3} - \frac{(bc-ad)^2qr(a+bx)^2}{8bd^2} + \frac{(bc-ad)qr(a+bx)^3}{12bd} - \frac{pr(a+bx)^4}{16b}\\
&\quad - \frac{qr(a+bx)^4}{16b} - \frac{(bc-ad)^4qr\log(c+dx)}{4bd^4} + \frac{(a+bx)^4\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.90

$$\int (a+bx)^3 \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) dx$$
$$= \frac{\frac{r\left(12bd(bc-ad)^3(p+4q)x-18b^2(bc-ad)^2(p+2q)(c+dx)^2+4b^3(bc-ad)(3p+4q)(c+dx)^3-3b^4(p+q)(c+dx)^4-12(bc-ad)^4q\log(c+dx)\right)}{12d^4}+(a+bx}{4b}$$

```
[In] Integrate[(a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] ((r*(12*b*d*(b*c - a*d)^3*(p + 4*q)*x - 18*b^2*(b*c - a*d)^2*(p + 2*q)*(c +
 d*x)^2 + 4*b^3*(b*c - a*d)*(3*p + 4*q)*(c + d*x)^3 - 3*b^4*(p + q)*(c + d*
x)^4 - 12*(b*c - a*d)^4*q*Log[c + d*x]))/(12*d^4) + (a + b*x)^4*Log[e*(f*(a
 + b*x)^p*(c + d*x)^q)^r])/(4*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 603 vs. 2(158) = 316.

Time = 301.79 (sec) , antiderivative size = 604, normalized size of antiderivative = 3.51

method	result
parallelrisch	$\frac{12a^4d^4pr+30a^3bc\,d^3pr-48a^2b^2c^2d^2qr+42a\,b^3c^3dqr+24x^2a\,b^3c\,d^3qr+72x\,a^2b^2c\,d^3qr-48xa\,b^3c^2d^2qr+120\ln(bx+a)a^3bc\,d^3pr+}{}$

```
[In] int((b*x+a)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*(12*a^4*d^4*p*r+30*a^3*b*c*d^3*p*r-48*a^2*b^2*c^2*d^2*q*r+42*a*b^3*c^3
*d*q*r+24*x^2*a*b^3*c*d^3*q*r+72*x*a^2*b^2*c*d^3*q*r-48*x*a*b^3*c^2*d^2*q*r
+120*ln(b*x+a)*a^3*b*c*d^3*p*r+168*ln(d*x+c)*a^3*b*c*d^3*q*r-72*ln(d*x+c)*a
^2*b^2*c^2*d^2*q*r+48*ln(d*x+c)*a*b^3*c^3*d*q*r-48*ln(e*(f*(b*x+a)^p*(d*x+c
)^q)^r)*a^4*d^4+48*a^4*d^4*q*r-12*b^4*c^4*q*r+72*x^2*ln(e*(f*(b*x+a)^p*(d*x
+c)^q)^r)*a^2*b^2*d^4+48*x*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*a^3*b*d^4-120*ln
(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*a^3*b*c*d^3+60*ln(b*x+a)*a^4*d^4*p*r+48*ln(d*
x+c)*a^4*d^4*q*r-12*ln(d*x+c)*b^4*c^4*q*r-3*x^4*b^4*d^4*p*r-3*x^4*b^4*d^4*q
*r+48*x^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*a*b^3*d^4+12*x^4*ln(e*(f*(b*x+a)^
p*(d*x+c)^q)^r)*b^4*d^4-12*x^3*a*b^3*d^4*p*r-16*x^3*a*b^3*d^4*q*r+4*x^3*b^4
*c*d^3*q*r-18*x^2*a^2*b^2*d^4*p*r-36*x^2*a^2*b^2*d^4*q*r-6*x^2*b^4*c^2*d^2*
q*r-12*x*a^3*b*d^4*p*r-48*x*a^3*b*d^4*q*r+12*x*b^4*c^3*d*q*r+12*a^3*b*c*d^3
*q*r)/b/d^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 469 vs. $2(158) = 316$.

Time = 0.30 (sec) , antiderivative size = 469, normalized size of antiderivative = 2.73

$$\int (a+bx)^3 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) dx =$$
$$-\frac{3\,(b^4d^4p+b^4d^4q)rx^4+4\,(3\,ab^3d^4p-(b^4cd^3-4\,ab^3d^4)q)rx^3+6\,(3\,a^2b^2d^4p+(b^4c^2d^2-4\,ab^3cd^3+6\,a^2b^2}{}$$

```
[In] integrate((b*x+a)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas")
```

```
[Out] -1/48*(3*(b^4*d^4*p + b^4*d^4*q)*r*x^4 + 4*(3*a*b^3*d^4*p - (b^4*c*d^3 - 4*
a*b^3*d^4)*q)*r*x^3 + 6*(3*a^2*b^2*d^4*p + (b^4*c^2*d^2 - 4*a*b^3*c*d^3 + 6
*a^2*b^2*d^4)*q)*r*x^2 + 12*(a^3*b*d^4*p - (b^4*c^3*d - 4*a*b^3*c^2*d^2 + 6
*a^2*b^2*c*d^3 - 4*a^3*b*d^4)*q)*r*x - 12*(b^4*d^4*p*r*x^4 + 4*a*b^3*d^4*p*
r*x^3 + 6*a^2*b^2*d^4*p*r*x^2 + 4*a^3*b*d^4*p*r*x + a^4*d^4*p*r)*log(b*x +
a) - 12*(b^4*d^4*q*r*x^4 + 4*a*b^3*d^4*q*r*x^3 + 6*a^2*b^2*d^4*q*r*x^2 + 4*
a^3*b*d^4*q*r*x - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*
d^3)*q*r)*log(d*x + c) - 12*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*
x^2 + 4*a^3*b*d^4*x)*log(e) - 12*(b^4*d^4*r*x^4 + 4*a*b^3*d^4*r*x^3 + 6*a^2
*b^2*d^4*r*x^2 + 4*a^3*b*d^4*r*x)*log(f))/(b*d^4)
```

Sympy [F(-1)]

Timed out.

$$\int (a+bx)^3 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) dx = \text{Timed out}$$

```
[In] integrate((b*x+a)**3*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.66

$$\int (a+bx)^3 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) dx$$
$$= \frac{1}{4}\left(b^3x^4+4\,ab^2x^3+6\,a^2bx^2+4\,a^3x\right)\log\left(\left((bx+a)^p(dx+c)^qf\right)^re\right)$$
$$+\frac{\left(\frac{12\,a^4fp\log(bx+a)}{b}-\frac{3\,b^3d^3f(p+q)x^4+4\,(ab^2d^3f(3\,p+4\,q)-b^3cd^2fq)x^3+6\,(3\,a^2bd^3f(p+2\,q)+b^3c^2dfq-4\,ab^2cd^2fq)x^2+12\,(a^3d^3f(p+4\,q)}{d^3}\right.}{48\,f}$$

```
[In] integrate((b*x+a)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima")
```

```
[Out] 1/4*(b^3*x^4 + 4*a*b^2*x^3 + 6*a^2*b*x^2 + 4*a^3*x)*log(((b*x + a)^p*(d*x +
 c)^q*f)^r*e) + 1/48*(12*a^4*f*p*log(b*x + a)/b - (3*b^3*d^3*f*(p + q)*x^4
+ 4*(a*b^2*d^3*f*(3*p + 4*q) - b^3*c*d^2*f*q)*x^3 + 6*(3*a^2*b*d^3*f*(p + 2
*q) + b^3*c^2*d*f*q - 4*a*b^2*c*d^2*f*q)*x^2 + 12*(a^3*d^3*f*(p + 4*q) - b^
3*c^3*f*q + 4*a*b^2*c^2*d*f*q - 6*a^2*b*c*d^2*f*q)*x)/d^3 - 12*(b^3*c^4*f*q
 - 4*a*b^2*c^3*d*f*q + 6*a^2*b*c^2*d^2*f*q - 4*a^3*c*d^3*f*q)*log(d*x + c)/
d^4)*r/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. 2(158) = 316.

Time = 3.74 (sec) , antiderivative size = 415, normalized size of antiderivative = 2.41

$$\int (a+bx)^3 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx$$
$$= \frac{a^4pr\log(bx+a)}{4b} - \frac{1}{16}\left(b^3pr + b^3qr - 4b^3r\log(f) - 4b^3\log(e)\right)x^4$$
$$- \frac{(3ab^2dpr - b^3cqr + 4ab^2dqr - 12ab^2dr\log(f) - 12ab^2d\log(e))x^3}{12d}$$
$$+ \frac{1}{4}\left(b^3prx^4 + 4ab^2prx^3 + 6a^2bprx^2 + 4a^3prx\right)\log(bx+a)$$
$$+ \frac{1}{4}\left(b^3qrx^4 + 4ab^2qrx^3 + 6a^2bqrx^2 + 4a^3qrx\right)\log(dx+c)$$
$$- \frac{(3a^2bd^2pr + b^3c^2qr - 4ab^2cdqr + 6a^2bd^2qr - 12a^2bd^2r\log(f) - 12a^2bd^2\log(e))x^2}{8d^2}$$
$$- \frac{(a^3d^3pr - b^3c^3qr + 4ab^2c^2dqr - 6a^2bcd^2qr + 4a^3d^3qr - 4a^3d^3r\log(f) - 4a^3d^3\log(e))x}{4d^3}$$
$$- \frac{(b^3c^4qr - 4ab^2c^3dqr + 6a^2bc^2d^2qr - 4a^3cd^3qr)\log(-dx-c)}{4d^4}$$

```
[In] integrate((b*x+a)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")
```

```
[Out] 1/4*a^4*p*r*log(b*x + a)/b - 1/16*(b^3*p*r + b^3*q*r - 4*b^3*r*log(f) - 4*b
^3*log(e))*x^4 - 1/12*(3*a*b^2*d*p*r - b^3*c*q*r + 4*a*b^2*d*q*r - 12*a*b^2
*d*r*log(f) - 12*a*b^2*d*log(e))*x^3/d + 1/4*(b^3*p*r*x^4 + 4*a*b^2*p*r*x^3
 + 6*a^2*b*p*r*x^2 + 4*a^3*p*r*x)*log(b*x + a) + 1/4*(b^3*q*r*x^4 + 4*a*b^2
*q*r*x^3 + 6*a^2*b*q*r*x^2 + 4*a^3*q*r*x)*log(d*x + c) - 1/8*(3*a^2*b*d^2*p
*r + b^3*c^2*q*r - 4*a*b^2*c*d*q*r + 6*a^2*b*d^2*q*r - 12*a^2*b*d^2*r*log(f
) - 12*a^2*b*d^2*log(e))*x^2/d^2 - 1/4*(a^3*d^3*p*r - b^3*c^3*q*r + 4*a*b^2
*c^2*d*q*r - 6*a^2*b*c*d^2*q*r + 4*a^3*d^3*q*r - 4*a^3*d^3*r*log(f) - 4*a^3
*d^3*log(e))*x/d^3 - 1/4*(b^3*c^4*q*r - 4*a*b^2*c^3*d*q*r + 6*a^2*b*c^2*d^2
*q*r - 4*a^3*c*d^3*q*r)*log(-d*x - c)/d^4
```

Mupad [B] (verification not implemented)

Time = 1.57 (sec) , antiderivative size = 501, normalized size of antiderivative = 2.91

$$\int (a+bx)^3 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right)\, dx$$
$$= x^2\left(\frac{\left(\frac{b^2 r (4 a d p+b c p+5 a d q)}{4 d}-\frac{b^2 r (p+q) (4 a d+4 b c)}{16 d}\right) (4 a d+4 b c)}{8 b d}-\frac{a b r (3 a d p+2 b c p+5 a d q)}{4 d}+\frac{a b^2 c r (p+q)}{8 d}\right)$$
$$-x^3\left(\frac{b^2 r (4 a d p+b c p+5 a d q)}{12 d}-\frac{b^2 r (p+q) (4 a d+4 b c)}{48 d}\right)$$
$$+\ln\left(e\left(f (a+b x)^p (c+d x)^q\right)^r\right)\left(a^3 x+\frac{3 a^2 b x^2}{2}+a b^2 x^3+\frac{b^3 x^4}{4}\right)$$
$$-x\left(\frac{(4 a d+4 b c)\left(\frac{\left(\frac{b^2 r (4 a d p+b c p+5 a d q)}{4 d}-\frac{b^2 r (p+q) (4 a d+4 b c)}{16 d}\right) (4 a d+4 b c)}{4 b d}-\frac{a b r (3 a d p+2 b c p+5 a d q)}{2 d}+\frac{a b^2 c r (p+q)}{4 d}\right)}{4 b d}\right.$$
$$\left.+\frac{a^2 r (2 a d p+3 b c p+5 a d q)}{2 d}-\frac{a c\left(\frac{b^2 r (4 a d p+b c p+5 a d q)}{4 d}-\frac{b^2 r (p+q) (4 a d+4 b c)}{16 d}\right)}{b d}\right)$$
$$-\frac{\ln (c+d x)\left(-4 q r a^3 c d^3+6 q r a^2 b c^2 d^2-4 q r a b^2 c^3 d+q r b^3 c^4\right)}{4 d^4}$$
$$-\frac{b^3 r x^4 (p+q)}{16}+\frac{a^4 p r \ln (a+b x)}{4 b}$$

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(a + b*x)^3,x)
```

```
[Out] x^2*((((b^2*r*(4*a*d*p + b*c*p + 5*a*d*q))/(4*d) - (b^2*r*(p + q)*(4*a*d +
4*b*c))/(16*d))*(4*a*d + 4*b*c))/(8*b*d) - (a*b*r*(3*a*d*p + 2*b*c*p + 5*a*
d*q))/(4*d) + (a*b^2*c*r*(p + q))/(8*d)) - x^3*((b^2*r*(4*a*d*p + b*c*p + 5
*a*d*q))/(12*d) - (b^2*r*(p + q)*(4*a*d + 4*b*c))/(48*d)) + log(e*(f*(a + b
*x)^p*(c + d*x)^q)^r)*(a^3*x + (b^3*x^4)/4 + (3*a^2*b*x^2)/2 + a*b^2*x^3) -
 x*(((4*a*d + 4*b*c)*((((b^2*r*(4*a*d*p + b*c*p + 5*a*d*q))/(4*d) - (b^2*r*
(p + q)*(4*a*d + 4*b*c))/(16*d))*(4*a*d + 4*b*c))/(4*b*d) - (a*b*r*(3*a*d*p
 + 2*b*c*p + 5*a*d*q))/(2*d) + (a*b^2*c*r*(p + q))/(4*d)))/(4*b*d) + (a^2*r
*(2*a*d*p + 3*b*c*p + 5*a*d*q))/(2*d) - (a*c*((b^2*r*(4*a*d*p + b*c*p + 5*a
*d*q))/(4*d) - (b^2*r*(p + q)*(4*a*d + 4*b*c))/(16*d)))/(b*d)) - (log(c + d
```

```
*x)*(b^3*c^4*q*r - 4*a^3*c*d^3*q*r - 4*a*b^2*c^3*d*q*r + 6*a^2*b*c^2*d^2*q*
r))/(4*d^4) - (b^3*r*x^4*(p + q))/16 + (a^4*p*r*log(a + b*x))/(4*b)
```

3.9 $\int (a+bx)^2 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx$

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Optimal result

Integrand size = 29, antiderivative size = 143

$$\int (a+bx)^2 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx = -\frac{(bc-ad)^2qrx}{3d^2} + \frac{(bc-ad)qr(a+bx)^2}{6bd} - \frac{pr(a+bx)^3}{9b} - \frac{qr(a+bx)^3}{9b} + \frac{(bc-ad)^3qr\log(c+dx)}{3bd^3} + \frac{(a+bx)^3 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b}$$

```
[Out] -1/3*(-a*d+b*c)^2*q*r*x/d^2+1/6*(-a*d+b*c)*q*r*(b*x+a)^2/b/d-1/9*p*r*(b*x+a
)^3/b-1/9*q*r*(b*x+a)^3/b+1/3*(-a*d+b*c)^3*q*r*ln(d*x+c)/b/d^3+1/3*(b*x+a)^
3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2581, 32, 45}

$$\int (a+bx)^2 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx = \frac{qr(bc-ad)^3\log(c+dx)}{3bd^3} - \frac{qrx(bc-ad)^2}{3d^2} + \frac{(a+bx)^3 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b} + \frac{qr(a+bx)^2(bc-ad)}{6bd} - \frac{pr(a+bx)^3}{9b} - \frac{qr(a+bx)^3}{9b}$$

```
[In] Int[(a + b*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] -1/3*((b*c - a*d)^2*q*r*x)/d^2 + ((b*c - a*d)*q*r*(a + b*x)^2)/(6*b*d) - (p
*r*(a + b*x)^3)/(9*b) - (q*r*(a + b*x)^3)/(9*b) + ((b*c - a*d)^3*q*r*Log[c
+ d*x])/(3*b*d^3) + ((a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(3*b
)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(a+bx)^3\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b} - \frac{1}{3}(pr)\int(a+bx)^2\,dx - \frac{(dqr)\int\frac{(a+bx)^3}{c+dx}\,dx}{3b}\\
&= -\frac{pr(a+bx)^3}{9b} + \frac{(a+bx)^3\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b}\\
&\quad - \frac{(dqr)\int\left(\frac{b(bc-ad)^2}{d^3} - \frac{b(bc-ad)(a+bx)}{d^2} + \frac{b(a+bx)^2}{d} + \frac{(-bc+ad)^3}{d^3(c+dx)}\right)dx}{3b}\\
&= -\frac{(bc-ad)^2qrx}{3d^2} + \frac{(bc-ad)qr(a+bx)^2}{6bd} - \frac{pr(a+bx)^3}{9b} - \frac{qr(a+bx)^3}{9b}\\
&\quad + \frac{(bc-ad)^3qr\log(c+dx)}{3bd^3} + \frac{(a+bx)^3\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.89

$$\int (a+bx)^2 \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) dx$$
$$= \frac{-\frac{r\left(6bd(bc-ad)^2(p+3q)x-3b^2(bc-ad)(2p+3q)(c+dx)^2+2b^3(p+q)(c+dx)^3-6(bc-ad)^3q\log(c+dx)\right)}{6d^3} + (a+bx)^3\log\left(e(f(a+bx)^p(c\right.}{3b}$$

```
[In] Integrate[(a + b*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] (-1/6*(r*(6*b*d*(b*c - a*d)^2*(p + 3*q)*x - 3*b^2*(b*c - a*d)*(2*p + 3*q)*(
c + d*x)^2 + 2*b^3*(p + q)*(c + d*x)^3 - 6*(b*c - a*d)^3*q*Log[c + d*x]))/d
^3 + (a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(3*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. 2(131) = 262.

Time = 70.99 (sec) , antiderivative size = 451, normalized size of antiderivative = 3.15

method	result
parallelrisch	$\frac{12a^2bc\,d^2pr+9a^2bc\,d^2qr+6x^3\ln\left(e\left(f(bx+a)^p(dx+c)^q\right)^r\right)b^3d^3-18\ln\left(e\left(f(bx+a)^p(dx+c)^q\right)^r\right)a^3d^3+6c^3qr\,b^3+6a^3d^3pr+18a^3d^3}{}$

```
[In] int((b*x+a)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x,method=_RETURNVERBOSE)
```

```
[Out] 1/18*(12*a^2*b*c*d^2*p*r+9*a^2*b*c*d^2*q*r+6*x^3*ln(e*(f*(b*x+a)^p*(d*x+c)^
q)^r)*b^3*d^3-18*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*a^3*d^3+6*c^3*q*r*b^3+6*a^
3*d^3*p*r+18*a^3*d^3*q*r-15*a*c^2*d*q*r*b^2-2*x^3*b^3*d^3*p*r-2*x^3*b^3*d^3
*q*r+18*x^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*a*b^2*d^3+18*x*ln(e*(f*(b*x+a)^
p*(d*x+c)^q)^r)*a^2*b*d^3-36*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*a^2*b*c*d^2+24
*ln(b*x+a)*a^3*d^3*p*r+18*ln(d*x+c)*a^3*d^3*q*r+6*ln(d*x+c)*b^3*c^3*q*r-6*x
^2*a*b^2*d^3*p*r-9*x^2*a*b^2*d^3*q*r+3*x^2*b^3*c*d^2*q*r-6*x*a^2*b*d^3*p*r-
18*x*a^2*b*d^3*q*r-6*x*b^3*c^2*d*q*r+18*x*a*b^2*c*d^2*q*r+36*ln(b*x+a)*a^2*
b*c*d^2*p*r+54*ln(d*x+c)*a^2*b*c*d^2*q*r-18*ln(d*x+c)*a*b^2*c^2*d*q*r)/b/d^
3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. 2(131) = 262.

Time = 0.29 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.27

$$\int (a+bx)^2 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx = -\frac{2\,(b^3d^3p+b^3d^3q)rx^3+3\,(2\,ab^2d^3p-(b^3cd^2-3\,ab^2d^3)q)rx^2+6\,(a^2bd^3p+(b^3c^2d-3\,ab^2cd^2+3\,a^2bd^3)q)}{}$$

```
[In] integrate((b*x+a)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas")
```

```
[Out] -1/18*(2*(b^3*d^3*p + b^3*d^3*q)*r*x^3 + 3*(2*a*b^2*d^3*p - (b^3*c*d^2 - 3*
a*b^2*d^3)*q)*r*x^2 + 6*(a^2*b*d^3*p + (b^3*c^2*d - 3*a*b^2*c*d^2 + 3*a^2*b
*d^3)*q)*r*x - 6*(b^3*d^3*p*r*x^3 + 3*a*b^2*d^3*p*r*x^2 + 3*a^2*b*d^3*p*r*x
 + a^3*d^3*p*r)*log(b*x + a) - 6*(b^3*d^3*q*r*x^3 + 3*a*b^2*d^3*q*r*x^2 + 3
*a^2*b*d^3*q*r*x + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2)*q*r)*log(d*x +
 c) - 6*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x)*log(e) - 6*(b^3*d^3
*r*x^3 + 3*a*b^2*d^3*r*x^2 + 3*a^2*b*d^3*r*x)*log(f))/(b*d^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 488 vs. 2(124) = 248.

Time = 132.78 (sec) , antiderivative size = 488, normalized size of antiderivative = 3.41

$$\int (a+bx)^2 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx$$

$$= \begin{cases} a^2x\log\left(e(a^pc^qf)^r\right) \\ a^2\left(\frac{c\log\left(e(a^pf(c+dx)^q)^r\right)}{d} - qrx + x\log\left(e(a^pf(c+dx)^q)^r\right)\right) \\ \frac{a^3\log\left(e(c^qf(a+bx)^p)^r\right)}{3b} - \frac{a^2prx}{3} + a^2x\log\left(e(c^qf(a+bx)^p)^r\right) - \frac{abprx^2}{3} + abx^2\log\left(e(c^qf(a+bx)^p)^r\right) - \frac{b^2prx^3}{9} + \cdots \\ -\frac{a^3qr\log\left(\frac{c}{d}+x\right)}{3b} + \frac{a^3\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b} + \frac{a^2cqr\log\left(\frac{c}{d}+x\right)}{d} - \frac{a^2prx}{3} - a^2qrx + a^2x\log\left(e(f(a+bx)^p(c+dx)^q \cdots\right.\right. \end{cases}$$

```
[In] integrate((b*x+a)**2*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r),x)
```

```
[Out] Piecewise((a**2*x*log(e*(a**p*c**q*f)**r), Eq(b, 0) & Eq(d, 0)), (a**2*(c*l
og(e*(a**p*f*(c + d*x)**q)**r)/d - q*r*x + x*log(e*(a**p*f*(c + d*x)**q)**r
)), Eq(b, 0)), (a**3*log(e*(c**q*f*(a + b*x)**p)**r)/(3*b) - a**2*p*r*x/3 +
 a**2*x*log(e*(c**q*f*(a + b*x)**p)**r) - a*b*p*r*x**2/3 + a*b*x**2*log(e*(
c**q*f*(a + b*x)**p)**r) - b**2*p*r*x**3/9 + b**2*x**3*log(e*(c**q*f*(a + b
*x)**p)**r)/3, Eq(d, 0)), (-a**3*q*r*log(c/d + x)/(3*b) + a**3*log(e*(f*(a
+ b*x)**p*(c + d*x)**q)**r)/(3*b) + a**2*c*q*r*log(c/d + x)/d - a**2*p*r*x/
3 - a**2*q*r*x + a**2*x*log(e*(f*(a + b*x)**p*(c + d*x)**q)**r) - a*b*c**2*
```

```
q*r*log(c/d + x)/d**2 + a*b*c*q*r*x/d - a*b*p*r*x**2/3 - a*b*q*r*x**2/2 + a
*b*x**2*log(e*(f*(a + b*x)**p*(c + d*x)**q)**r) + b**2*c**3*q*r*log(c/d + x
)/(3*d**3) - b**2*c**2*q*r*x/(3*d**2) + b**2*c*q*r*x**2/(6*d) - b**2*p*r*x*
*3/9 - b**2*q*r*x**3/9 + b**2*x**3*log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)/
3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.36

$$\int (a+bx)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$$
$$= \frac{1}{3}\left(b^2x^3+3\,abx^2+3\,a^2x\right)\log\left(\left(\left(bx+a\right)^p\left(dx+c\right)^q f\right)^r e\right) + \frac{\left(\frac{6\,a^3fp\log(bx+a)}{b} - \frac{2\,b^2d^2f(p+q)x^3+3\,\left(abd^2f(2\,p+3\,q)-b^2cdfq\right)x^2+6\,\left(a^2d^2f(p+3\,q)+b^2c^2fq-3\,abcdfq\right)x}{d^2} + \frac{6\,\left(b^2c^3fq-3\,abc^2dfq+3\,a^2 \cdots\right.}{d^3}\right.}{18\,f}$$

```
[In] integrate((b*x+a)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*x^3 + 3*a*b*x^2 + 3*a^2*x)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e) +
1/18*(6*a^3*f*p*log(b*x + a)/b - (2*b^2*d^2*f*(p + q)*x^3 + 3*(a*b*d^2*f*(2
*p + 3*q) - b^2*c*d*f*q)*x^2 + 6*(a^2*d^2*f*(p + 3*q) + b^2*c^2*f*q - 3*a*b
*c*d*f*q)*x)/d^2 + 6*(b^2*c^3*f*q - 3*a*b*c^2*d*f*q + 3*a^2*c*d^2*f*q)*log(
d*x + c)/d^3)*r/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. 2(131) = 262.

Time = 1.13 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.92

$$\int (a+bx)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$$
$$= \frac{a^3pr\log\left(bx+a\right)}{3\,b} - \frac{1}{9}\left(b^2pr+b^2qr-3\,b^2r\log\left(f\right)-3\,b^2\log\left(e\right)\right)x^3 - \frac{\left(2\,abdpr-b^2cqr+3\,abdqr-6\,abdr\log\left(f\right)-6\,abd\log\left(e\right)\right)x^2}{6\,d} + \frac{1}{3}\left(b^2prx^3+3\,abprx^2+3\,a^2prx\right)\log\left(bx+a\right) + \frac{1}{3}\left(b^2qrx^3+3\,abqrx^2+3\,a^2qrx\right)\log\left(dx+c\right) - \frac{\left(a^2d^2pr+b^2c^2qr-3\,abcdqr+3\,a^2d^2qr-3\,a^2d^2r\log\left(f\right)-3\,a^2d^2\log\left(e\right)\right)x}{3\,d^2} + \frac{\left(b^2c^3qr-3\,abc^2dqr+3\,a^2cd^2qr\right)\log\left(-dx-c\right)}{3\,d^3}$$

```
[In] integrate((b*x+a)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")
```

```
[Out] 1/3*a^3*p*r*log(b*x + a)/b - 1/9*(b^2*p*r + b^2*q*r - 3*b^2*r*log(f) - 3*b^
2*log(e))*x^3 - 1/6*(2*a*b*d*p*r - b^2*c*q*r + 3*a*b*d*q*r - 6*a*b*d*r*log(
f) - 6*a*b*d*log(e))*x^2/d + 1/3*(b^2*p*r*x^3 + 3*a*b*p*r*x^2 + 3*a^2*p*r*x
)*log(b*x + a) + 1/3*(b^2*q*r*x^3 + 3*a*b*q*r*x^2 + 3*a^2*q*r*x)*log(d*x +
c) - 1/3*(a^2*d^2*p*r + b^2*c^2*q*r - 3*a*b*c*d*q*r + 3*a^2*d^2*q*r - 3*a^2
*d^2*r*log(f) - 3*a^2*d^2*log(e))*x/d^2 + 1/3*(b^2*c^3*q*r - 3*a*b*c^2*d*q*
r + 3*a^2*c*d^2*q*r)*log(-d*x - c)/d^3
```

Mupad [B] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.78

$$\int (a+bx)^2 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx$$
$$= x \left(\frac{\left(\frac{b\,r\,(3\,a\,d\,p+b\,c\,p+4\,a\,d\,q)}{3\,d} - \frac{b\,r\,(p+q)\,(3\,a\,d+3\,b\,c)}{9\,d}\right)\,(3\,a\,d+3\,b\,c)}{3\,b\,d} - \frac{a\,r\,(a\,d\,p+b\,c\,p+2\,a\,d\,q)}{d} + \frac{a\,b\,c\,r\,(p+q)}{3\,d} \right) - x^2 \left(\frac{b\,r\,(3\,a\,d\,p+b\,c\,p+4\,a\,d\,q)}{6\,d} - \frac{b\,r\,(p+q)\,(3\,a\,d+3\,b\,c)}{18\,d} \right)$$
$$+ \ln\left(e\,\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)\,\left(a^2\,x + a\,b\,x^2 + \frac{b^2\,x^3}{3}\right)$$
$$+ \frac{\ln(c+d\,x)\,\left(3\,q\,r\,a^2\,c\,d^2 - 3\,q\,r\,a\,b\,c^2\,d + q\,r\,b^2\,c^3\right)}{3\,d^3} - \frac{b^2\,r\,x^3\,(p+q)}{9} + \frac{a^3\,p\,r\,\ln(a+b\,x)}{3\,b}$$

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(a + b*x)^2,x)
```

```
[Out] x*((((b*r*(3*a*d*p + b*c*p + 4*a*d*q))/(3*d) - (b*r*(p + q)*(3*a*d + 3*b*c)
)/(9*d))*(3*a*d + 3*b*c))/(3*b*d) - (a*r*(a*d*p + b*c*p + 2*a*d*q))/d + (a*
b*c*r*(p + q))/(3*d)) - x^2*((b*r*(3*a*d*p + b*c*p + 4*a*d*q))/(6*d) - (b*r
*(p + q)*(3*a*d + 3*b*c))/(18*d)) + log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(a
^2*x + (b^2*x^3)/3 + a*b*x^2) + (log(c + d*x)*(b^2*c^3*q*r + 3*a^2*c*d^2*q*
r - 3*a*b*c^2*d*q*r))/(3*d^3) - (b^2*r*x^3*(p + q))/9 + (a^3*p*r*log(a + b*
x))/(3*b)
```

3.10 $\int (a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx$

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Optimal result

Integrand size = 27, antiderivative size = 116

$$\int (a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx=-\frac{1}{2}aprx+\frac{(bc-ad)qrx}{2d}-\frac{1}{4}bprx^2-\frac{qr(a+bx)^2}{4b}-\frac{(bc-ad)^2qr\log(c+dx)}{2bd^2}+\frac{(a+bx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b}$$

```
[Out] -1/2*a*p*r*x+1/2*(-a*d+b*c)*q*r*x/d-1/4*b*p*r*x^2-1/4*q*r*(b*x+a)^2/b-1/2*(
-a*d+b*c)^2*q*r*ln(d*x+c)/b/d^2+1/2*(b*x+a)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^
r)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2581, 45}

$$\int (a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx=-\frac{qr(bc-ad)^2\log(c+dx)}{2bd^2}+\frac{(a+bx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b}+\frac{qrx(bc-ad)}{2d}-\frac{qr(a+bx)^2}{4b}-\frac{1}{2}aprx-\frac{1}{4}bprx^2$$

```
[In] Int[(a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] -1/2*(a*p*r*x) + ((b*c - a*d)*q*r*x)/(2*d) - (b*p*r*x^2)/4 - (q*r*(a + b*x)
^2)/(4*b) - ((b*c - a*d)^2*q*r*Log[c + d*x])/(2*b*d^2) + ((a + b*x)^2*Log[e
*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{(a+bx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b} - \frac{1}{2}(pr)\int (a+bx)\,dx - \frac{(dqr)\int \frac{(a+bx)^2}{c+dx}\,dx}{2b}$$

$$= -\frac{1}{2}aprx - \frac{1}{4}bprx^2 + \frac{(a+bx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b} - \frac{(dqr)\int\left(-\frac{b(bc-ad)}{d^2} + \frac{b(a+bx)}{d} + \frac{(-bc+ad)^2}{d^2(c+dx)}\right)dx}{2b}$$

$$= -\frac{1}{2}aprx + \frac{(bc-ad)qrx}{2d} - \frac{1}{4}bprx^2 - \frac{qr(a+bx)^2}{4b} - \frac{(bc-ad)^2qr\log(c+dx)}{2bd^2} + \frac{(a+bx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.91

$$\int (a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)dx = \frac{a^2pr\log(a+bx)}{2b} - \frac{2c(bc-2ad)qr\log(c+dx) + dx(r(-2bcq+2ad(p+2q)+bd(p+q)x) - 2d(2a+bx)\log\left(e(f(a+bx)^p(\right.}{4d^2}$$

```
[In] Integrate[(a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] (a^2*p*r*Log[a + b*x])/(2*b) - (2*c*(b*c - 2*a*d)*q*r*Log[c + d*x] + d*x*(r
*(-2*b*c*q + 2*a*d*(p + 2*q) + b*d*(p + q)*x) - 2*d*(2*a + b*x)*Log[e*(f*(a
 + b*x)^p*(c + d*x)^q)^r]))/(4*d^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(104) = 208.

Time = 12.56 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.63

method	result
parallelrisch	$\frac{-x^2b^2d^2pr-x^2b^2d^2qr+6\ln(bx+a)a^2d^2pr+6\ln(bx+a)abcdpr+4\ln(dx+c)a^2d^2qr+10\ln(dx+c)abcdqr-2\ln(dx+c)b^2c^2qr+2x^2}{}$

```
[In] int((b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-x^2*b^2*d^2*p*r-x^2*b^2*d^2*q*r+6*ln(b*x+a)*a^2*d^2*p*r+6*ln(b*x+a)*a
*b*c*d*p*r+4*ln(d*x+c)*a^2*d^2*q*r+10*ln(d*x+c)*a*b*c*d*q*r-2*ln(d*x+c)*b^2
*c^2*q*r+2*x^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*b^2*d^2-2*x*a*b*d^2*p*r-4*x*
a*b*d^2*q*r+2*x*b^2*c*d*q*r+4*x*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*a*b*d^2+2*a
^2*d^2*p*r+4*a^2*q*r*d^2+3*a*b*c*d*p*r+3*a*b*c*d*q*r-2*b^2*c^2*q*r-4*ln(e*(
f*(b*x+a)^p*(d*x+c)^q)^r)*a^2*d^2-6*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*a*b*c*d
)/b/d^2
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.70

$$\int (a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = -\frac{(b^2d^2p+b^2d^2q)rx^2+2\,(abd^2p-(b^2cd-2\,abd^2)q)rx-2\,(b^2d^2prx^2+2\,abd^2prx+a^2d^2pr)\log\left(bx+a\right)}{}$$

```
[In] integrate((b*x+a)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas")
```

```
[Out] -1/4*((b^2*d^2*p + b^2*d^2*q)*r*x^2 + 2*(a*b*d^2*p - (b^2*c*d - 2*a*b*d^2)*
q)*r*x - 2*(b^2*d^2*p*r*x^2 + 2*a*b*d^2*p*r*x + a^2*d^2*p*r)*log(b*x + a) -
 2*(b^2*d^2*q*r*x^2 + 2*a*b*d^2*q*r*x - (b^2*c^2 - 2*a*b*c*d)*q*r)*log(d*x
+ c) - 2*(b^2*d^2*x^2 + 2*a*b*d^2*x)*log(e) - 2*(b^2*d^2*r*x^2 + 2*a*b*d^2*
r*x)*log(f))/(b*d^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. 2(104) = 208.

Time = 20.45 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.80

$$\int (a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$$
$$=\begin{cases} ax\log\left(e(a^pc^qf)^r\right) \\ a\left(\frac{c\log\left(e(a^pf(c+dx)^q)^r\right)}{d}-qrx+x\log\left(e(a^pf(c+dx)^q)^r\right)\right) \\ \frac{a^2\log\left(e(c^qf(a+bx)^p)^r\right)}{2b}-\frac{aprx}{2}+ax\log\left(e(c^qf(a+bx)^p)^r\right)-\frac{bprx^2}{4}+\frac{bx^2\log\left(e(c^qf(a+bx)^p)^r\right)}{2} \\ -\frac{a^2qr\log\left(\frac{c}{d}+x\right)}{2b}+\frac{a^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b}+\frac{acqr\log\left(\frac{c}{d}+x\right)}{d}-\frac{aprx}{2}-aqrx+ax\log\left(e(f(a+bx)^p(c+dx)^q)^r\right) \end{cases}$$

```
[In] integrate((b*x+a)*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r),x)
```

```
[Out] Piecewise((a*x*log(e*(a**p*c**q*f)**r), Eq(b, 0) & Eq(d, 0)), (a*(c*log(e*(
a**p*f*(c + d*x)**q)**r)/d - q*r*x + x*log(e*(a**p*f*(c + d*x)**q)**r)), Eq
(b, 0)), (a**2*log(e*(c**q*f*(a + b*x)**p)**r)/(2*b) - a*p*r*x/2 + a*x*log(
e*(c**q*f*(a + b*x)**p)**r) - b*p*r*x**2/4 + b*x**2*log(e*(c**q*f*(a + b*x)
**p)**r)/2, Eq(d, 0)), (-a**2*q*r*log(c/d + x)/(2*b) + a**2*log(e*(f*(a + b
*x)**p*(c + d*x)**q)**r)/(2*b) + a*c*q*r*log(c/d + x)/d - a*p*r*x/2 - a*q*r
*x + a*x*log(e*(f*(a + b*x)**p*(c + d*x)**q)**r) - b*c**2*q*r*log(c/d + x)/
(2*d**2) + b*c*q*r*x/(2*d) - b*p*r*x**2/4 - b*q*r*x**2/4 + b*x**2*log(e*(f*
(a + b*x)**p*(c + d*x)**q)**r)/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.02

$$\int (a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \frac{1}{2}\left(bx^2+2\,ax\right)\log\left(\left(\left(bx+a\right)^p\left(dx+c\right)^qf\right)^re\right) + \frac{\left(\frac{2\,a^2fp\log(bx+a)}{b}-\frac{bdf(p+q)x^2+2\,(adf(p+2\,q)-bcfq)x}{d}-\frac{2\,(bc^2fq-2\,acdfq)\log(dx+c)}{d^2}\right)r}{4\,f}$$

```
[In] integrate((b*x+a)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^2 + 2*a*x)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e) + 1/4*(2*a^2*f*p*l
og(b*x + a)/b - (b*d*f*(p + q)*x^2 + 2*(a*d*f*(p + 2*q) - b*c*f*q)*x)/d - 2
*(b*c^2*f*q - 2*a*c*d*f*q)*log(d*x + c)/d^2)*r/f
```

Giac [A] (verification not implemented)

none

Time = 0.50 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.31

$$\int (a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$$
$$= \frac{a^2pr\log(bx+a)}{2\,b} - \frac{1}{4}\,(bpr+bqr-2\,br\log(f)-2\,b\log(e))x^2 + \frac{1}{2}\,\left(bprx^2+2\,aprx\right)\log(bx+a) + \frac{1}{2}\,\left(bqrx^2+2\,aqrx\right)\log(dx+c) - \frac{(adpr-bcqr+2\,adqr-2\,adr\log(f)-2\,ad\log(e))x}{2\,d} - \frac{(bc^2qr-2\,acdqr)\log(-dx-c)}{2\,d^2}$$

```
[In] integrate((b*x+a)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")
```

```
[Out] 1/2*a^2*p*r*log(b*x + a)/b - 1/4*(b*p*r + b*q*r - 2*b*r*log(f) - 2*b*log(e)
)*x^2 + 1/2*(b*p*r*x^2 + 2*a*p*r*x)*log(b*x + a) + 1/2*(b*q*r*x^2 + 2*a*q*r
*x)*log(d*x + c) - 1/2*(a*d*p*r - b*c*q*r + 2*a*d*q*r - 2*a*d*r*log(f) - 2*
a*d*log(e))*x/d - 1/2*(b*c^2*q*r - 2*a*c*d*q*r)*log(-d*x - c)/d^2
```

Mupad [B] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.10

$$\int (a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)\,\left(\frac{b\,x^2}{2}+a\,x\right) - x\,\left(\frac{r\,(2\,a\,d\,p+b\,c\,p+3\,a\,d\,q)}{2\,d} - \frac{r\,(p+q)\,\left(2\,a\,d+2\,b\,c\right)}{4\,d}\right) - \frac{\ln(c+d\,x)\,\left(b\,c^2\,q\,r-2\,a\,c\,d\,q\,r\right)}{2\,d^2} - \frac{b\,r\,x^2\,(p+q)}{4} + \frac{a^2\,p\,r\,\ln(a+b\,x)}{2\,b}$$

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(a + b*x),x)
```

```
[Out] log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(a*x + (b*x^2)/2) - x*((r*(2*a*d*p + b
*c*p + 3*a*d*q))/(2*d) - (r*(p + q)*(2*a*d + 2*b*c))/(4*d)) - (log(c + d*x)
*(b*c^2*q*r - 2*a*c*d*q*r))/(2*d^2) - (b*r*x^2*(p + q))/4 + (a^2*p*r*log(a
+ b*x))/(2*b)
```

3.11 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{a+bx}\,dx$

Optimal result 122
Rubi [A] (verified) 122
Mathematica [A] (verified) 124
Maple [A] (verified) 125
Fricas [F] 125
Sympy [F] 125
Maxima [A] (verification not implemented) 125
Giac [F] 126
Mupad [F(-1)] 126

Optimal result

Integrand size = 29, antiderivative size = 107

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{a+bx}\,dx = -\frac{pr\log^2(a+bx)}{2b} - \frac{qr\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{b} + \frac{\log(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b} - \frac{qr\,\mathrm{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{b}$$

```
[Out] -1/2*p*r*ln(b*x+a)^2/b-q*r*ln(b*x+a)*ln(b*(d*x+c)/(-a*d+b*c))/b+ln(b*x+a)*l
n(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b-q*r*polylog(2,-d*(b*x+a)/(-a*d+b*c))/b
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2580, 2437, 2338, 2441, 2440, 2438}

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{a+bx}\,dx = \frac{\log(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b} - \frac{qr\,\mathrm{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{b} - \frac{qr\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{b} - \frac{pr\log^2(a+bx)}{2b}$$

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x),x]
```

```
[Out] -1/2*(p*r*Log[a + b*x]^2)/b - (q*r*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*
d)])/b + (Log[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/b - (q*r*PolyL
og[2, -((d*(a + b*x))/(b*c - a*d))])/b
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2580

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[Log[g + h*x]*(Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]/h), x] + (-Dist[b*p*(r/h), Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[d*q*(r/h), Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\log(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b} \\
&\quad -(pr)\int \frac{\log(a+bx)}{a+bx}\,dx - \frac{(dqr)\int \frac{\log(a+bx)}{c+dx}\,dx}{b} \\
&= -\frac{qr\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{b} + \frac{\log(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b} \\
&\quad - \frac{(pr)\text{Subst}\left(\int \frac{\log(x)}{x}\,dx, x, a+bx\right)}{b} + (qr)\int \frac{\log\left(\frac{b(c+dx)}{bc-ad}\right)}{a+bx}\,dx \\
&= -\frac{pr\log^2(a+bx)}{2b} - \frac{qr\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{b} \\
&\quad + \frac{\log(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b} \\
&\quad + \frac{(qr)\text{Subst}\left(\int \frac{\log\left(1+\frac{dx}{bc-ad}\right)}{x}\,dx, x, a+bx\right)}{b} \\
&= -\frac{pr\log^2(a+bx)}{2b} - \frac{qr\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{b} \\
&\quad + \frac{\log(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b} - \frac{qr\text{Li}_2\left(-\frac{d(a+bx)}{bc-ad}\right)}{b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.87

$$
\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{a+bx}\,dx =
$$

$$
-\frac{\log(a+bx)\left(pr\log(a+bx)+2qr\log\left(\frac{b(c+dx)}{bc-ad}\right)-2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\right)+2qr\,\text{PolyLog}\left(2,\frac{d(a}{-b}\right.}{2b}
$$

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x),x]
```

```
[Out] -1/2*(Log[a + b*x]*(p*r*Log[a + b*x] + 2*q*r*Log[(b*(c + d*x))/(b*c - a*d)]
 - 2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]) + 2*q*r*PolyLog[2, (d*(a + b*x))
/(-(b*c) + a*d)])/b
```

Maple [A] (verified)

Time = 9.17 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.17

method	result	size
parts	$\frac{\ln(bx+a)\ln\left(e\left(f(bx+a)^{p}(dx+c)^{q}\right)^{r}\right)}{b}-\frac{r\left(\frac{bp\ln(bx+a)^{2}}{2}+bdq\left(\frac{\operatorname{dilog}\left(\frac{-ad+cb+d(bx+a)}{-ad+cb}\right)}{d}+\frac{\ln(bx+a)\ln\left(\frac{-ad+cb+d(bx+a)}{-ad+cb}\right)}{d}\right)\right)}{b^{2}}$	125

[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a),x,method=_RETURNVERBOSE)

[Out] ln(b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b-1/b^2*r*(1/2*b*p*ln(b*x+a)^2+b*d*q*(dilog((-a*d+c*b+d*(b*x+a))/(-a*d+b*c))/d+ln(b*x+a)*ln((-a*d+c*b+d*(b*x+a))/(-a*d+b*c))/d))

Fricas [F]

$$\int \frac{\log\left(e(f(a+bx)^{p}(c+dx)^{q})^{r}\right)}{a+bx}\,dx=\int \frac{\log\left(\left(\left(bx+a\right)^{p}\left(dx+c\right)^{q}f\right)^{r}e\right)}{bx+a}\,dx$$

[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a),x, algorithm="fricas")

[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(b*x + a), x)

Sympy [F]

$$\int \frac{\log\left(e(f(a+bx)^{p}(c+dx)^{q})^{r}\right)}{a+bx}\,dx=\int \frac{\log\left(e\left(f\left(a+bx\right)^{p}\left(c+dx\right)^{q}\right)^{r}\right)}{a+bx}\,dx$$

[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(b*x+a),x)

[Out] Integral(log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)/(a + b*x), x)

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.53

$$\int \frac{\log (e(f(a+bx)^p(c+dx)^q)^r)}{a+bx} dx$$
$$= -\frac{\left(\frac{2\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)fq}{b} - \frac{fp\log(bx+a)^2+2\,fq\log(bx+a)\log(dx+c)}{b}\right)r}{2\,f}$$
$$- \frac{(fp\log(bx+a)+fq\log(dx+c))r\log(bx+a)}{bf}$$
$$+ \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)\log(bx+a)}{b}$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(2*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x +
a*d)/(b*c - a*d)))*f*q/b - (f*p*log(b*x + a)^2 + 2*f*q*log(b*x + a)*log(d*x
 + c))/b)*r/f - (f*p*log(b*x + a) + f*q*log(d*x + c))*r*log(b*x + a)/(b*f)
+ log(((b*x + a)^p*(d*x + c)^q*f)^r*e)*log(b*x + a)/b
```

Giac [F]

$$\int \frac{\log (e(f(a+bx)^p(c+dx)^q)^r)}{a+bx} dx = \int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{bx+a} dx$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log (e(f(a+bx)^p(c+dx)^q)^r)}{a+bx} dx = \int \frac{\ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)}{a+b\,x} dx$$

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(a + b*x),x)
```

```
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(a + b*x), x)
```

3.12 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^2}\,dx$

Optimal result . . . 127
Rubi [A] (verified) . . . 127
Mathematica [A] (verified) . . . 128
Maple [A] (verified) . . . 129
Fricas [A] (verification not implemented) . . . 129
Sympy [F(-2)] . . . 129
Maxima [A] (verification not implemented) . . . 130
Giac [A] (verification not implemented) . . . 130
Mupad [B] (verification not implemented) . . . 130

Optimal result

Integrand size = 29, antiderivative size = 95

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^2}\,dx = -\frac{pr}{b(a+bx)}+\frac{dqr\log(a+bx)}{b(bc-ad)}-\frac{dqr\log(c+dx)}{b(bc-ad)}-\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(a+bx)}$$

```
[Out] -p*r/b/(b*x+a)+d*q*r*ln(b*x+a)/b/(-a*d+b*c)-d*q*r*ln(d*x+c)/b/(-a*d+b*c)-ln
(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/(b*x+a)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2581, 32, 36, 31}

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^2}\,dx = -\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(a+bx)}+\frac{dqr\log(a+bx)}{b(bc-ad)}-\frac{dqr\log(c+dx)}{b(bc-ad)}-\frac{pr}{b(a+bx)}$$

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^2,x]
```

```
[Out] -((p*r)/(b*(a + b*x))) + (d*q*r*Log[a + b*x])/(b*(b*c - a*d)) - (d*q*r*Log[
c + d*x])/(b*(b*c - a*d)) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(b*(a + b*
x))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(a+bx)} + (pr)\int \frac{1}{(a+bx)^2}\,dx + \frac{(dqr)\int \frac{1}{(a+bx)(c+dx)}\,dx}{b} \\
&= -\frac{pr}{b(a+bx)} - \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(a+bx)} + \frac{(dqr)\int \frac{1}{a+bx}\,dx}{bc-ad} - \frac{(d^2qr)\int \frac{1}{c+dx}\,dx}{b(bc-ad)} \\
&= -\frac{pr}{b(a+bx)} + \frac{dqr\log(a+bx)}{b(bc-ad)} - \frac{dqr\log(c+dx)}{b(bc-ad)} - \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(a+bx)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.94

$$
\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^2}\,dx = \frac{r\left(-\frac{p}{a+bx} + \frac{dq\log(a+bx)}{bc-ad} - \frac{dq\log(c+dx)}{bc-ad}\right)}{b} - \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(a+bx)}
$$

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^2,x]
[Out] (r*(-(p/(a + b*x)) + (d*q*Log[a + b*x])/(b*c - a*d) - (d*q*Log[c + d*x])/(b
*c - a*d)))/b - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(b*(a + b*x))
```

Maple [A] (verified)

Time = 31.53 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.78

method	result
parallelrisch	$-\frac{\ln(bx+a)x\,b^{3}d^{2}qr-\ln(dx+c)x\,b^{3}d^{2}qr+\ln(bx+a)a\,b^{2}d^{2}qr-\ln(dx+c)a\,b^{2}d^{2}qr+a\,b^{2}d^{2}pr-b^{3}cdpr+\ln(e(f(bx+a)^{p}(dx+c)^{q})^{r})}{(ad-cb)(bx+a)b^{3}d}$

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(ln(b*x+a)*x*b^3*d^2*q*r-ln(d*x+c)*x*b^3*d^2*q*r+ln(b*x+a)*a*b^2*d^2*q*r-l
n(d*x+c)*a*b^2*d^2*q*r+a*b^2*d^2*p*r-b^3*c*d*p*r+ln(e*(f*(b*x+a)^p*(d*x+c)^
q)^r)*a*b^2*d^2-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*b^3*c*d)/(a*d-b*c)/(b*x+a)/
b^3/d
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.26

$$\int \frac{\log \left(e(f(a+bx)^{p}(c+dx)^{q})^{r}\right)}{(a+bx)^{2}}\,dx = -\frac{(bc-ad)pr+(bc-ad)r\log(f)-(bdqrx+(adq-(bc-ad)p)r)\log(bx+a)+(bdqrx+bcqr)\log(dx}{ab^{2}c-a^{2}bd+(b^{3}c-ab^{2}d)x}$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -((b*c - a*d)*p*r + (b*c - a*d)*r*log(f) - (b*d*q*r*x + (a*d*q - (b*c - a*d
)*p)*r)*log(b*x + a) + (b*d*q*r*x + b*c*q*r)*log(d*x + c) + (b*c - a*d)*log
(e))/(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\log \left(e(f(a+bx)^{p}(c+dx)^{q})^{r}\right)}{(a+bx)^{2}}\,dx = \text{Exception raised: NotImplementedError}$$

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(b*x+a)**2,x)
```

```
[Out] Exception raised: NotImplementedError >> no valid subset found
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.04

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^2}\,dx = \frac{\left(dfq\left(\frac{\log(bx+a)}{bc-ad} - \frac{\log(dx+c)}{bc-ad}\right) - \frac{bfp}{b^2x+ab}\right)r}{bf} - \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{(bx+a)b}$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^2,x, algorithm="maxima")
[Out] (d*f*q*(log(b*x + a)/(b*c - a*d) - log(d*x + c)/(b*c - a*d)) - b*f*p/(b^2*x
 + a*b))*r/(b*f) - log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/((b*x + a)*b)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.19

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^2}\,dx = \frac{dqr\log(bx+a)}{b^2c-abd} - \frac{dqr\log(dx+c)}{b^2c-abd} - \frac{pr\log(bx+a)}{b^2x+ab} - \frac{qr\log(dx+c)}{b^2x+ab} - \frac{pr+r\log(f)+\log(e)}{b^2x+ab}$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^2,x, algorithm="giac")
[Out] d*q*r*log(b*x + a)/(b^2*c - a*b*d) - d*q*r*log(d*x + c)/(b^2*c - a*b*d) - p
*r*log(b*x + a)/(b^2*x + a*b) - q*r*log(d*x + c)/(b^2*x + a*b) - (p*r + r*l
og(f) + log(e))/(b^2*x + a*b)
```

Mupad [B] (verification not implemented)

Time = 3.22 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.04

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^2}\,dx = -\frac{\ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)\,\left(x+\frac{a}{b}\right)}{(a+b\,x)^2} - \frac{p\,r}{x\,b^2+a\,b} + \frac{d\,q\,r\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,2\mathrm{i}}{b\,(a\,d-b\,c)}$$

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(a + b*x)^2,x)
[Out] (d*q*r*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*2i)/(b*(a*d - b*c)) - (p*
r)/(a*b + b^2*x) - (log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(x + a/b))/(a + b*
x)^2
```

3.13 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^3}\,dx$

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Optimal result

Integrand size = 29, antiderivative size = 135

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^3}\,dx = -\frac{pr}{4b(a+bx)^2} - \frac{dqr}{2b(bc-ad)(a+bx)} - \frac{d^2qr\log(a+bx)}{2b(bc-ad)^2} + \frac{d^2qr\log(c+dx)}{2b(bc-ad)^2} - \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b(a+bx)^2}$$

```
[Out] -1/4*p*r/b/(b*x+a)^2-1/2*d*q*r/b/(-a*d+b*c)/(b*x+a)-1/2*d^2*q*r*ln(b*x+a)/b
/(-a*d+b*c)^2+1/2*d^2*q*r*ln(d*x+c)/b/(-a*d+b*c)^2-1/2*ln(e*(f*(b*x+a)^p*(d
*x+c)^q)^r)/b/(b*x+a)^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2581, 32, 46}

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^3}\,dx = -\frac{d^2qr\log(a+bx)}{2b(bc-ad)^2} + \frac{d^2qr\log(c+dx)}{2b(bc-ad)^2} - \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b(a+bx)^2} - \frac{dqr}{2b(a+bx)(bc-ad)} - \frac{pr}{4b(a+bx)^2}$$

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^3,x]
```

```
[Out] -1/4*(p*r)/(b*(a + b*x)^2) - (d*q*r)/(2*b*(b*c - a*d)*(a + b*x)) - (d^2*q*r
*Log[a + b*x])/(2*b*(b*c - a*d)^2) + (d^2*q*r*Log[c + d*x])/(2*b*(b*c - a*d
)^2) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(2*b*(a + b*x)^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b(a+bx)^2}+\frac{1}{2}(pr)\int\frac{1}{(a+bx)^3}\,dx+\frac{(dqr)\int\frac{1}{(a+bx)^2(c+dx)}\,dx}{2b}\\
&= -\frac{pr}{4b(a+bx)^2}-\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b(a+bx)^2}\\
&\quad+\frac{(dqr)\int\left(\frac{b}{(bc-ad)(a+bx)^2}-\frac{bd}{(bc-ad)^2(a+bx)}+\frac{d^2}{(bc-ad)^2(c+dx)}\right)dx}{2b}\\
&= -\frac{pr}{4b(a+bx)^2}-\frac{dqr}{2b(bc-ad)(a+bx)}-\frac{d^2qr\log(a+bx)}{2b(bc-ad)^2}\\
&\quad+\frac{d^2qr\log(c+dx)}{2b(bc-ad)^2}-\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b(a+bx)^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.86

$$\int\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^3}\,dx = \frac{r\left(-\frac{p-\frac{2dq(a+bx)}{-bc+ad}}{2(a+bx)^2}-\frac{d^2q\log(a+bx)}{(bc-ad)^2}+\frac{d^2q\log(c+dx)}{(bc-ad)^2}\right)-\frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^2}}{2b}$$

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^3,x]
```

```
[Out] (r*(-1/2*(p - (2*d*q*(a + b*x))/(-(b*c) + a*d))/(a + b*x)^2 - (d^2*q*Log[a
+ b*x])/(b*c - a*d)^2 + (d^2*q*Log[c + d*x])/(b*c - a*d)^2) - Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]/(a + b*x)^2)/(2*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 766 vs. 2(125) = 250.

Time = 150.01 (sec) , antiderivative size = 767, normalized size of antiderivative = 5.68

method	result
parallelrisch	$-\frac{2\ln(bx+a)x^2a^4b^2cd^2pr+2\ln(bx+a)x^2a^4b^2cd^2qr-4\ln(bx+a)x^2a^3b^3c^2dpr-4\ln(dx+c)x^2a^3b^3c^2dqr+4\ln(bx+a)x\,a^5bcd^2p}{}$

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(2*ln(b*x+a)*x^2*a^4*b^2*c*d^2*p*r+2*ln(b*x+a)*x^2*a^4*b^2*c*d^2*q*r-4
*ln(b*x+a)*x^2*a^3*b^3*c^2*d*p*r-4*ln(d*x+c)*x^2*a^3*b^3*c^2*d*q*r+4*ln(b*x
+a)*x*a^5*b*c*d^2*p*r+4*ln(b*x+a)*x*a^5*b*c*d^2*q*r-8*ln(b*x+a)*x*a^4*b^2*c
^2*d*p*r-8*ln(d*x+c)*x*a^4*b^2*c^2*d*q*r-4*ln(b*x+a)*a^5*b*c^2*d*p*r-4*ln(d
*x+c)*a^5*b*c^2*d*q*r-x^2*a^4*b^2*c*d^2*p*r+2*x^2*a^4*b^2*c*d^2*q*r+2*x^2*a
^3*b^3*c^2*d*p*r-2*x^2*a^3*b^3*c^2*d*q*r-2*x*a^5*b*c*d^2*p*r+2*x*a^5*b*c*d^
2*q*r+4*x*a^4*b^2*c^2*d*p*r-2*x*a^4*b^2*c^2*d*q*r-2*x^2*ln(e*(f*(b*x+a)^p*(
d*x+c)^q)^r)*a^2*b^4*c^3-4*x*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*a^3*b^3*c^3+2*
ln(b*x+a)*x^2*a^2*b^4*c^3*p*r+2*ln(d*x+c)*x^2*a^2*b^4*c^3*q*r+4*ln(b*x+a)*x
*a^3*b^3*c^3*p*r+4*ln(d*x+c)*x*a^3*b^3*c^3*q*r+2*ln(b*x+a)*a^6*c*d^2*p*r+2*
ln(b*x+a)*a^6*c*d^2*q*r+2*ln(b*x+a)*a^4*b^2*c^3*p*r+2*ln(d*x+c)*a^4*b^2*c^3
*q*r-x^2*a^2*b^4*c^3*p*r-2*x^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*a^4*b^2*c*d^
2+4*x^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*a^3*b^3*c^2*d-2*x*a^3*b^3*c^3*p*r-4
*x*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*a^5*b*c*d^2+8*x*ln(e*(f*(b*x+a)^p*(d*x+c
)^q)^r)*a^4*b^2*c^2*d)/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b/(b*x+a)^2/c/a^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(125) = 250.

Time = 0.32 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.39

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^3}\,dx = -\frac{2\,(b^2cd-abd^2)qrx+2\,(b^2c^2-2\,abcd+a^2d^2)r\log(f)+((b^2c^2-2\,abcd+a^2d^2)p+2\,(abcd-a^2d^2)q)r}{4\,(a^2b^3c^2-2}$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(b^2*c*d - a*b*d^2)*q*r*x + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*r*log
(f) + ((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*p + 2*(a*b*c*d - a^2*d^2)*q)*r + 2*(
b^2*d^2*q*r*x^2 + 2*a*b*d^2*q*r*x + (a^2*d^2*q + (b^2*c^2 - 2*a*b*c*d + a^2
*d^2)*p)*r)*log(b*x + a) - 2*(b^2*d^2*q*r*x^2 + 2*a*b*d^2*q*r*x - (b^2*c^2
- 2*a*b*c*d)*q*r)*log(d*x + c) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(e))/
(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3
*d^2)*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^3}\,dx = \text{Timed out}$$

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.22

$$\begin{aligned}\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^3}\,dx &= -\frac{\left(2\,dfq\left(\frac{d\log(bx+a)}{b^2c^2-2\,abcd+a^2d^2}-\frac{d\log(dx+c)}{b^2c^2-2\,abcd+a^2d^2}+\frac{1}{abc-a^2d+(b^2c-abd)x}\right)+\frac{bfp}{b^3x^2+2\,ab^2x+a^2b}\right)r}{4\,bf}\\&\quad-\frac{\log\left(\left((bx+a)^p(dx+c)^qf\right)^re\right)}{2\,(bx+a)^2b}\end{aligned}$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*d*f*q*(d*log(b*x + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - d*log(d*x +
 c)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + 1/(a*b*c - a^2*d + (b^2*c - a*b*d)*x)
) + b*f*p/(b^3*x^2 + 2*a*b^2*x + a^2*b))*r/(b*f) - 1/2*log(((b*x + a)^p*(d*
x + c)^q*f)^r*e)/((b*x + a)^2*b)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.85

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^3}\,dx = -\frac{d^2qr\log\left(bx+a\right)}{2\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)} + \frac{d^2qr\log\left(dx+c\right)}{2\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)} - \frac{pr\log\left(bx+a\right)}{2\left(b^3x^2+2\,ab^2x+a^2b\right)} - \frac{qr\log\left(dx+c\right)}{2\left(b^3x^2+2\,ab^2x+a^2b\right)} - \frac{2\,bdqrx+bcpr-adpr+2\,adqr+2\,bcr\log\left(f\right)-2\,adr\log\left(f\right)+2\,bc\log\left(e\right)-2\,ad\log\left(e\right)}{4\left(b^4cx^2-ab^3dx^2+2\,ab^3cx-2\,a^2b^2dx+a^2b^2c-a^3bd\right)}$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*d^2*q*r*log(b*x + a)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) + 1/2*d^2*q*r
*log(d*x + c)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) - 1/2*p*r*log(b*x + a)/(b
^3*x^2 + 2*a*b^2*x + a^2*b) - 1/2*q*r*log(d*x + c)/(b^3*x^2 + 2*a*b^2*x + a
^2*b) - 1/4*(2*b*d*q*r*x + b*c*p*r - a*d*p*r + 2*a*d*q*r + 2*b*c*r*log(f) -
 2*a*d*r*log(f) + 2*b*c*log(e) - 2*a*d*log(e))/(b^4*c*x^2 - a*b^3*d*x^2 + 2
*a*b^3*c*x - 2*a^2*b^2*d*x + a^2*b^2*c - a^3*b*d)
```

Mupad [B] (verification not implemented)

Time = 3.43 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.35

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^3}\,dx = \frac{\frac{b\,c\,p\,r-a\,d\,p\,r+2\,a\,d\,q\,r}{2\,(a\,d-b\,c)}+\frac{b\,d\,q\,r\,x}{a\,d-b\,c}}{2\,a^2\,b+4\,a\,b^2\,x+2\,b^3\,x^2} - \frac{\ln\left(e\,\left(f\,\left(a+b\,x\right)^p\,\left(c+d\,x\right)^q\right)^r\right)\,\left(\frac{x}{2}+\frac{a}{2\,b}\right)}{{\left(a+b\,x\right)}^3} + \frac{d^2\,q\,r\,\mathrm{atanh}\left(\frac{2\,b^3\,c^2-2\,a^2\,b\,d^2}{2\,b\,{\left(a\,d-b\,c\right)}^2}-\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{b\,{\left(a\,d-b\,c\right)}^2}$$

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(a + b*x)^3,x)
```

```
[Out] ((b*c*p*r - a*d*p*r + 2*a*d*q*r)/(2*(a*d - b*c)) + (b*d*q*r*x)/(a*d - b*c))
/(2*a^2*b + 2*b^3*x^2 + 4*a*b^2*x) - (log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*
(x/2 + a/(2*b)))/(a + b*x)^3 + (d^2*q*r*atanh((2*b^3*c^2 - 2*a^2*b*d^2)/(2*
b*(a*d - b*c)^2) - (2*b*d*x)/(a*d - b*c)))/(b*(a*d - b*c)^2)
```

3.14 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^4} dx$

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Optimal result

Integrand size = 29, antiderivative size = 164

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^4}\,dx = -\frac{pr}{9b(a+bx)^3}-\frac{dqr}{6b(bc-ad)(a+bx)^2}+\frac{d^2qr}{3b(bc-ad)^2(a+bx)}+\frac{d^3qr\log(a+bx)}{3b(bc-ad)^3}-\frac{d^3qr\log(c+dx)}{3b(bc-ad)^3}-\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(a+bx)^3}$$

```
[Out] -1/9*p*r/b/(b*x+a)^3-1/6*d*q*r/b/(-a*d+b*c)/(b*x+a)^2+1/3*d^2*q*r/b/(-a*d+b
*c)^2/(b*x+a)+1/3*d^3*q*r*ln(b*x+a)/b/(-a*d+b*c)^3-1/3*d^3*q*r*ln(d*x+c)/b/
(-a*d+b*c)^3-1/3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/(b*x+a)^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2581, 32, 46}

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^4}\,dx = \frac{d^3qr\log(a+bx)}{3b(bc-ad)^3}-\frac{d^3qr\log(c+dx)}{3b(bc-ad)^3}+\frac{d^2qr}{3b(a+bx)(bc-ad)^2}-\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(a+bx)^3}-\frac{dqr}{6b(a+bx)^2(bc-ad)}-\frac{pr}{9b(a+bx)^3}$$

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^4,x]
```

```
[Out] -1/9*(p*r)/(b*(a + b*x)^3) - (d*q*r)/(6*b*(b*c - a*d)*(a + b*x)^2) + (d^2*q
*r)/(3*b*(b*c - a*d)^2*(a + b*x)) + (d^3*q*r*Log[a + b*x])/(3*b*(b*c - a*d)
^3) - (d^3*q*r*Log[c + d*x])/(3*b*(b*c - a*d)^3) - Log[e*(f*(a + b*x)^p*(c
+ d*x)^q)^r]/(3*b*(a + b*x)^3)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule steps

$$\begin{aligned}
\text{integral} &= -\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(a+bx)^3}+\frac{1}{3}(pr)\int\frac{1}{(a+bx)^4}\,dx+\frac{(dqr)\int\frac{1}{(a+bx)^3(c+dx)}\,dx}{3b}\\
&= -\frac{pr}{9b(a+bx)^3}-\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(a+bx)^3}\\
&\quad+\frac{(dqr)\int\left(\frac{b}{(bc-ad)(a+bx)^3}-\frac{bd}{(bc-ad)^2(a+bx)^2}+\frac{bd^2}{(bc-ad)^3(a+bx)}-\frac{d^3}{(bc-ad)^3(c+dx)}\right)dx}{3b}\\
&= -\frac{pr}{9b(a+bx)^3}-\frac{dqr}{6b(bc-ad)(a+bx)^2}+\frac{d^2qr}{3b(bc-ad)^2(a+bx)}\\
&\quad+\frac{d^3qr\log(a+bx)}{3b(bc-ad)^3}-\frac{d^3qr\log(c+dx)}{3b(bc-ad)^3}-\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(a+bx)^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.86

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^4}\,dx$$
$$=\frac{r\left(\frac{-2p+\frac{3dq(a+bx)}{-bc+ad}+\frac{6d^2q(a+bx)^2}{(bc-ad)^2}}{6(a+bx)^3}+\frac{d^3q\log(a+bx)}{(bc-ad)^3}-\frac{d^3q\log(c+dx)}{(bc-ad)^3}\right)-\frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^3}}{3b}$$

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^4,x]
```

```
[Out] (r*((-2*p + (3*d*q*(a + b*x))/(-(b*c) + a*d) + (6*d^2*q*(a + b*x)^2)/(b*c -
 a*d)^2)/(6*(a + b*x)^3) + (d^3*q*Log[a + b*x])/(b*c - a*d)^3 - (d^3*q*Log[
c + d*x])/(b*c - a*d)^3) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^3
)/(3*b)
```

Maple [F]

$$\int \frac{\ln \left(e\left(f\left(bx+a\right)^p\left(dx+c\right)^q\right)^r\right)}{\left(bx+a\right)^4}dx$$

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^4,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^4,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 580 vs. 2(152) = 304.

Time = 0.34 (sec) , antiderivative size = 580, normalized size of antiderivative = 3.54

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^4}\,dx$$
$$=\frac{6\,(b^3cd^2-ab^2d^3)qrx^2-3\,(b^3c^2d-6\,ab^2cd^2+5\,a^2bd^3)qrx-6\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)r\log{(f)}-}{}$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/18*(6*(b^3*c*d^2 - a*b^2*d^3)*q*r*x^2 - 3*(b^3*c^2*d - 6*a*b^2*c*d^2 + 5*
a^2*b*d^3)*q*r*x - 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*r*
log(f) - (2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*p + 3*(a*b^
2*c^2*d - 4*a^2*b*c*d^2 + 3*a^3*d^3)*q)*r + 6*(b^3*d^3*q*r*x^3 + 3*a*b^2*d^
3*q*r*x^2 + 3*a^2*b*d^3*q*r*x + (a^3*d^3*q - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a
^2*b*c*d^2 - a^3*d^3)*p)*r)*log(b*x + a) - 6*(b^3*d^3*q*r*x^3 + 3*a*b^2*d^3
*q*r*x^2 + 3*a^2*b*d^3*q*r*x + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2)*q*
```

```
r)*log(d*x + c) - 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log
(e))/(a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3 + (b^7*c^
3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*x^3 + 3*(a*b^6*c^3 - 3*a
^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*
b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^4}\,dx = \text{Timed out}$$

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(b*x+a)**4,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.76

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^4}\,dx = \frac{\left(3\left(\frac{2\,d^2\log(bx+a)}{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3} - \frac{2\,d^2\log(dx+c)}{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3} + \frac{2\,bdx-bc+3\,ad}{a^2b^2c^2-2\,a^3bcd+a^4d^2+(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^2+2\,(ab^3c^2-2\,a^2\right.\right.}{18\,bf} - \frac{\log\left(\left((bx+a)^p(dx+c)^qf\right)^re\right)}{3\,(bx+a)^3b}$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^4,x, algorithm="maxima")
[Out] 1/18*(3*(2*d^2*log(b*x + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*
d^3) - 2*d^2*log(d*x + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^
3) + (2*b*d*x - b*c + 3*a*d)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (b^4*c^
2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d
^2)*x))*d*f*q - 2*b*f*p/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b))*r/(b
*f) - 1/3*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/((b*x + a)^3*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 475 vs. 2(152) = 304.

Time = 0.28 (sec) , antiderivative size = 475, normalized size of antiderivative = 2.90

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^4}\,dx = \frac{d^3qr\log(bx+a)}{3\,(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3)} - \frac{d^3qr\log(dx+c)}{3\,(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3)} - \frac{pr\log(bx+a)}{3\,(b^4x^3+3\,ab^3x^2+3\,a^2b^2x+a^3b)} - \frac{qr\log(dx+c)}{3\,(b^4x^3+3\,ab^3x^2+3\,a^2b^2x+a^3b)} + \frac{6\,b^2d^2qrx^2-3\,b^2cdqrx+15\,abd^2qrx-2\,b^2c^2pr+4\,abcdpr-2\,a^2d^2pr-3\,abcdqr+9\,a^2d^2qr-6\,b^2c^2r\log}{18\,(b^6c^2x^3-2\,ab^5cdx^3+a^2b^4d^2x^3+3\,ab^5c^2x^2-6\,a^2b^4cdx^2+3\,a^3b^3d^2x^2+3\,}$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/3*d^3*q*r*log(b*x + a)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b
*d^3) - 1/3*d^3*q*r*log(d*x + c)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2
 - a^3*b*d^3) - 1/3*p*r*log(b*x + a)/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x +
 a^3*b) - 1/3*q*r*log(d*x + c)/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b
) + 1/18*(6*b^2*d^2*q*r*x^2 - 3*b^2*c*d*q*r*x + 15*a*b*d^2*q*r*x - 2*b^2*c^
2*p*r + 4*a*b*c*d*p*r - 2*a^2*d^2*p*r - 3*a*b*c*d*q*r + 9*a^2*d^2*q*r - 6*b
^2*c^2*r*log(f) + 12*a*b*c*d*r*log(f) - 6*a^2*d^2*r*log(f) - 6*b^2*c^2*log(
e) + 12*a*b*c*d*log(e) - 6*a^2*d^2*log(e))/(b^6*c^2*x^3 - 2*a*b^5*c*d*x^3 +
 a^2*b^4*d^2*x^3 + 3*a*b^5*c^2*x^2 - 6*a^2*b^4*c*d*x^2 + 3*a^3*b^3*d^2*x^2
+ 3*a^2*b^4*c^2*x - 6*a^3*b^3*c*d*x + 3*a^4*b^2*d^2*x + a^3*b^3*c^2 - 2*a^4
*b^2*c*d + a^5*b*d^2)
```

Mupad [B] (verification not implemented)

Time = 3.96 (sec) , antiderivative size = 346, normalized size of antiderivative = 2.11

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^4}\,dx = \frac{\frac{x\,(5\,a\,b\,d^2\,q\,r-b^2\,c\,d\,q\,r)}{2\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)} - \frac{2\,a^2\,d^2\,p\,r+2\,b^2\,c^2\,p\,r-9\,a^2\,d^2\,q\,r-4\,a\,b\,c\,d\,p\,r+3\,a\,b\,c\,d\,q\,r}{6\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)} + \frac{b^2\,d^2\,q\,r\,x^2}{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}}{3\,a^3\,b+9\,a^2\,b^2\,x+9\,a\,b^3\,x^2+3\,b^4\,x^3} - \frac{\ln\left(e\,\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)\,\left(\frac{x}{3}+\frac{a}{3\,b}\right)}{(a+b\,x)^4} - \frac{2\,d^3\,q\,r\,\mathrm{atanh}\left(\frac{3\,a^3\,b\,d^3-3\,a^2\,b^2\,c\,d^2-3\,a\,b^3\,c^2\,d+3\,b^4\,c^3}{3\,b\,(a\,d-b\,c)^3} + \frac{2\,b\,d\,x\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{(a\,d-b\,c)^3}\right)}{3\,b\,(a\,d-b\,c)^3}$$

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(a + b*x)^4,x)
```

```
[Out] ((x*(5*a*b*d^2*q*r - b^2*c*d*q*r))/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (2
*a^2*d^2*p*r + 2*b^2*c^2*p*r - 9*a^2*d^2*q*r - 4*a*b*c*d*p*r + 3*a*b*c*d*q*
r)/(6*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (b^2*d^2*q*r*x^2)/(a^2*d^2 + b^2*c
^2 - 2*a*b*c*d))/(3*a^3*b + 3*b^4*x^3 + 9*a^2*b^2*x + 9*a*b^3*x^2) - (log(e
*(f*(a + b*x)^p*(c + d*x)^q)^r)*(x/3 + a/(3*b)))/(a + b*x)^4 - (2*d^3*q*r*a
tanh((3*b^4*c^3 + 3*a^3*b*d^3 - 3*a^2*b^2*c*d^2 - 3*a*b^3*c^2*d)/(3*b*(a*d
- b*c)^3) + (2*b*d*x*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(a*d - b*c)^3))/(3*b*
(a*d - b*c)^3)
```

3.15 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^5} dx$

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Optimal result

Integrand size = 29, antiderivative size = 193

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^5} dx = -\frac{pr}{16b(a+bx)^4} - \frac{dqr}{12b(bc-ad)(a+bx)^3} + \frac{d^2qr}{8b(bc-ad)^2(a+bx)^2} - \frac{d^3qr}{4b(bc-ad)^3(a+bx)} - \frac{d^4qr\log(a+bx)}{4b(bc-ad)^4} + \frac{d^4qr\log(c+dx)}{4b(bc-ad)^4} - \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b(a+bx)^4}$$

```
[Out] -1/16*p*r/b/(b*x+a)^4-1/12*d*q*r/b/(-a*d+b*c)/(b*x+a)^3+1/8*d^2*q*r/b/(-a*d
+b*c)^2/(b*x+a)^2-1/4*d^3*q*r/b/(-a*d+b*c)^3/(b*x+a)-1/4*d^4*q*r*ln(b*x+a)/
b/(-a*d+b*c)^4+1/4*d^4*q*r*ln(d*x+c)/b/(-a*d+b*c)^4-1/4*ln(e*(f*(b*x+a)^p*(
d*x+c)^q)^r)/b/(b*x+a)^4
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used

$= \{2581, 32, 46\}$

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^5}\,dx = -\frac{d^4qr\log(a+bx)}{4b(bc-ad)^4} + \frac{d^4qr\log(c+dx)}{4b(bc-ad)^4} - \frac{d^3qr}{4b(a+bx)(bc-ad)^3} + \frac{d^2qr}{8b(a+bx)^2(bc-ad)^2} - \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b(a+bx)^4} - \frac{dqr}{12b(a+bx)^3(bc-ad)} - \frac{pr}{16b(a+bx)^4}$$

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^5,x]
```

```
[Out] -1/16*(p*r)/(b*(a + b*x)^4) - (d*q*r)/(12*b*(b*c - a*d)*(a + b*x)^3) + (d^2
*q*r)/(8*b*(b*c - a*d)^2*(a + b*x)^2) - (d^3*q*r)/(4*b*(b*c - a*d)^3*(a + b
*x)) - (d^4*q*r*Log[a + b*x])/(4*b*(b*c - a*d)^4) + (d^4*q*r*Log[c + d*x])/
(4*b*(b*c - a*d)^4) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(4*b*(a + b*x)^4
)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 2581

Rubi steps

$$\text{integral} = -\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b(a+bx)^4} + \frac{1}{4}(pr)\int \frac{1}{(a+bx)^5}\,dx + \frac{(dqr)\int \frac{1}{(a+bx)^4(c+dx)}\,dx}{4b}$$

$$= -\frac{pr}{16b(a+bx)^4} - \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b(a+bx)^4} + \frac{(dqr)\int\left(\frac{b}{(bc-ad)(a+bx)^4} - \frac{bd}{(bc-ad)^2(a+bx)^3} + \frac{bd^2}{(bc-ad)^3(a+bx)^2} - \frac{bd^3}{(bc-ad)^4(a+bx)} + \frac{d^4}{(bc-ad)^4(c+dx)}\right)dx}{4b}$$

$$= -\frac{pr}{16b(a+bx)^4} - \frac{dqr}{12b(bc-ad)(a+bx)^3} + \frac{d^2qr}{8b(bc-ad)^2(a+bx)^2} - \frac{d^3qr}{4b(bc-ad)^3(a+bx)} - \frac{d^4qr\log(a+bx)}{4b(bc-ad)^4} + \frac{d^4qr\log(c+dx)}{4b(bc-ad)^4} - \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b(a+bx)^4}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.85

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^5}\,dx = \frac{r\left(\frac{-3p+\frac{4dq(a+bx)}{-bc+ad}+\frac{6d^2q(a+bx)^2}{(bc-ad)^2}-\frac{12d^3q(a+bx)^3}{(bc-ad)^3}}{12(a+bx)^4} - \frac{d^4q\log(a+bx)}{(bc-ad)^4} + \frac{d^4q\log(c+dx)}{(bc-ad)^4}\right) - \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^4}}{4b}$$

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^5,x]
```

```
[Out] (r*((-3*p + (4*d*q*(a + b*x))/(-(b*c) + a*d) + (6*d^2*q*(a + b*x)^2)/(b*c -
 a*d)^2 - (12*d^3*q*(a + b*x)^3)/(b*c - a*d)^3)/(12*(a + b*x)^4) - (d^4*q*L
og[a + b*x])/(b*c - a*d)^4 + (d^4*q*Log[c + d*x])/(b*c - a*d)^4) - Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^4)/(4*b)
```

Maple [F]

$$\int \frac{\ln\left(e\left(f(bx+a)^p(dx+c)^q\right)^r\right)}{(bx+a)^5}dx$$

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^5,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^5,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 861 vs. $2(179) = 358$.

Time = 0.35 (sec) , antiderivative size = 861, normalized size of antiderivative = 4.46

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^5}\,dx = -\frac{12\,(b^4cd^3-ab^3d^4)qrx^3-6\,(b^4c^2d^2-8\,ab^3cd^3+7\,a^2b^2d^4)qrx^2+4\,(b^4c^3d-6\,ab^3c^2d^2+18\,a^2b^2cd^3-13}{}$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/48*(12*(b^4*c*d^3 - a*b^3*d^4)*q*r*x^3 - 6*(b^4*c^2*d^2 - 8*a*b^3*c*d^3
+ 7*a^2*b^2*d^4)*q*r*x^2 + 4*(b^4*c^3*d - 6*a*b^3*c^2*d^2 + 18*a^2*b^2*c*d^
3 - 13*a^3*b*d^4)*q*r*x + 12*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 -
 4*a^3*b*c*d^3 + a^4*d^4)*r*log(f) + (3*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^
2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*p + 2*(2*a*b^3*c^3*d - 9*a^2*b^2*c^2*d
^2 + 18*a^3*b*c*d^3 - 11*a^4*d^4)*q)*r + 12*(b^4*d^4*q*r*x^4 + 4*a*b^3*d^4*
q*r*x^3 + 6*a^2*b^2*d^4*q*r*x^2 + 4*a^3*b*d^4*q*r*x + (a^4*d^4*q + (b^4*c^4
 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*p)*r)*log(b
*x + a) - 12*(b^4*d^4*q*r*x^4 + 4*a*b^3*d^4*q*r*x^3 + 6*a^2*b^2*d^4*q*r*x^2
 + 4*a^3*b*d^4*q*r*x - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3
*b*c*d^3)*q*r)*log(d*x + c) + 12*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d
^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(e))/(a^4*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*a^
6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4 + (b^9*c^4 - 4*a*b^8*c^3*d + 6*
a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*x^4 + 4*(a*b^8*c^4 - 4*a^2
*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*x^3 + 6*(a^
2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^3
*d^4)*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^
3*c*d^3 + a^7*b^2*d^4)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^5}\,dx = \text{Timed out}$$

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(b*x+a)**5,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 459 vs. 2(179) = 358.

Time = 0.20 (sec) , antiderivative size = 459, normalized size of antiderivative = 2.38

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^5}\,dx = -\frac{\left(2\left(\frac{6\,d^3\log(bx+a)}{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}-\frac{6\,d^3\log(dx+c)}{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}+\frac{}{a^3b^3c^3-3\,a^4b^2c^2d+3\,a^5bcd^2-a^6d^3+(b^6}\right.\right.}{} - \frac{\log\left(((bx+a)^p(dx+c)^q f)^r e\right)}{4\,(bx+a)^4 b}$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/48*(2*(6*d^3*log(b*x + a)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 -
 4*a^3*b*c*d^3 + a^4*d^4) - 6*d^3*log(d*x + c)/(b^4*c^4 - 4*a*b^3*c^3*d + 6
*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) + (6*b^2*d^2*x^2 + 2*b^2*c^2 -
7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/(a^3*b^3*c^3 - 3*a^4*b^
2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*
d^2 - a^3*b^3*d^3)*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 -
 a^4*b^2*d^3)*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^
5*b*d^3)*x))*d*f*q + 3*b*f*p/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3
*b^2*x + a^4*b))*r/(b*f) - 1/4*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/((b*x +
 a)^4*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 756 vs. 2(179) = 358.

Time = 0.29 (sec) , antiderivative size = 756, normalized size of antiderivative = 3.92

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^5}\,dx = -\frac{d^4qr\log(bx+a)}{4\,(b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4)} + \frac{d^4qr\log(dx+c)}{4\,(b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4)} - \frac{pr\log(bx+a)}{4\,(b^5x^4+4\,ab^4x^3+6\,a^2b^3x^2+4\,a^3b^2x+a^4b)} - \frac{qr\log(dx+c)}{4\,(b^5x^4+4\,ab^4x^3+6\,a^2b^3x^2+4\,a^3b^2x+a^4b)} - \frac{12\,b^3d^3qrx^3-6\,b^3cd^2qrx^2+42\,ab^2d^3qrx^2+4\,b^3c^2dqrx-20\,ab^2cd^2qrx+52\,a^2bd^3qrx+3\,b^3c^3pr-9\,ab^2c}{48\,(b^8c^3x^4-3\,ab^7c^2dx^4+3\,a^2b^6cd^2x^4-a^3b^5d^3x^4+4\,ab^7c^3x^3-12\,a^2b^6c}$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/4*d^4*q*r*log(b*x + a)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*
a^3*b^2*c*d^3 + a^4*b*d^4) + 1/4*d^4*q*r*log(d*x + c)/(b^5*c^4 - 4*a*b^4*c^
3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) - 1/4*p*r*log(b*x +
a)/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b) - 1/4*q*r*
log(d*x + c)/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b)
- 1/48*(12*b^3*d^3*q*r*x^3 - 6*b^3*c*d^2*q*r*x^2 + 42*a*b^2*d^3*q*r*x^2 + 4
*b^3*c^2*d*q*r*x - 20*a*b^2*c*d^2*q*r*x + 52*a^2*b*d^3*q*r*x + 3*b^3*c^3*p*
r - 9*a*b^2*c^2*d*p*r + 9*a^2*b*c*d^2*p*r - 3*a^3*d^3*p*r + 4*a*b^2*c^2*d*q
*r - 14*a^2*b*c*d^2*q*r + 22*a^3*d^3*q*r + 12*b^3*c^3*r*log(f) - 36*a*b^2*c
^2*d*r*log(f) + 36*a^2*b*c*d^2*r*log(f) - 12*a^3*d^3*r*log(f) + 12*b^3*c^3*
log(e) - 36*a*b^2*c^2*d*log(e) + 36*a^2*b*c*d^2*log(e) - 12*a^3*d^3*log(e))
/(b^8*c^3*x^4 - 3*a*b^7*c^2*d*x^4 + 3*a^2*b^6*c*d^2*x^4 - a^3*b^5*d^3*x^4 +
 4*a*b^7*c^3*x^3 - 12*a^2*b^6*c^2*d*x^3 + 12*a^3*b^5*c*d^2*x^3 - 4*a^4*b^4*
d^3*x^3 + 6*a^2*b^6*c^3*x^2 - 18*a^3*b^5*c^2*d*x^2 + 18*a^4*b^4*c*d^2*x^2 -
 6*a^5*b^3*d^3*x^2 + 4*a^3*b^5*c^3*x - 12*a^4*b^4*c^2*d*x + 12*a^5*b^3*c*d^
2*x - 4*a^6*b^2*d^3*x + a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a
^7*b*d^3)
```

Mupad [B] (verification not implemented)

Time = 4.79 (sec) , antiderivative size = 526, normalized size of antiderivative = 2.73

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^5}\,dx$$
$$= \frac{\frac{3\,b^3\,c^3\,p\,r-3\,a^3\,d^3\,p\,r+22\,a^3\,d^3\,q\,r-9\,a\,b^2\,c^2\,d\,p\,r+9\,a^2\,b\,c\,d^2\,p\,r+4\,a\,b^2\,c^2\,d\,q\,r-14\,a^2\,b\,c\,d^2\,q\,r}{12\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{x\,\left(13\,q\,r\,a^2\,b\,d^3-5\,q\,r\,a\,b^2\,c\,d^2+q\,r\,b^3\,c^2\,d\right)}{3\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+}{4\,a^4\,b+16\,a^3\,b^2\,x+24\,a^2\,b^3\,x^2+16\,a\,b^4\,x^3+4\,b^5\,x}$$
$$-\frac{\ln\left(e\,{\left(f\,{\left(a+b\,x\right)}^p\,{\left(c+d\,x\right)}^q\right)}^r\right)\,\left(\frac{x}{4}+\frac{a}{4\,b}\right)}{{\left(a+b\,x\right)}^5}$$
$$+\frac{d^4\,q\,r\,\mathrm{atanh}\left(\frac{-4\,a^4\,b\,d^4+8\,a^3\,b^2\,c\,d^3-8\,a\,b^4\,c^3\,d+4\,b^5\,c^4}{4\,b\,{\left(a\,d-b\,c\right)}^4}-\frac{2\,b\,d\,x\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{{\left(a\,d-b\,c\right)}^4}\right)}{2\,b\,{\left(a\,d-b\,c\right)}^4}$$

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(a + b*x)^5,x)
```

```
[Out] ((3*b^3*c^3*p*r - 3*a^3*d^3*p*r + 22*a^3*d^3*q*r - 9*a*b^2*c^2*d*p*r + 9*a^
2*b*c*d^2*p*r + 4*a*b^2*c^2*d*q*r - 14*a^2*b*c*d^2*q*r)/(12*(a^3*d^3 - b^3*
c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (x*(13*a^2*b*d^3*q*r + b^3*c^2*d*q*
r - 5*a*b^2*c*d^2*q*r))/(3*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d
^2)) + (d*x^2*(7*a*b^2*d^2*q*r - b^3*c*d*q*r))/(2*(a^3*d^3 - b^3*c^3 + 3*a*
b^2*c^2*d - 3*a^2*b*c*d^2)) + (b^3*d^3*q*r*x^3)/(a^3*d^3 - b^3*c^3 + 3*a*b^
2*c^2*d - 3*a^2*b*c*d^2))/(4*a^4*b + 4*b^5*x^4 + 16*a^3*b^2*x + 16*a*b^4*x^
3 + 24*a^2*b^3*x^2) - (log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(x/4 + a/(4*b))
)/(a + b*x)^5 + (d^4*q*r*atanh((4*b^5*c^4 - 4*a^4*b*d^4 + 8*a^3*b^2*c*d^3 -
 8*a*b^4*c^3*d)/(4*b*(a*d - b*c)^4) - (2*b*d*x*(a^3*d^3 - b^3*c^3 + 3*a*b^2
*c^2*d - 3*a^2*b*c*d^2))/(a*d - b*c)^4))/(2*b*(a*d - b*c)^4)
```

3.16 $\int (a+bx)^4 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) dx$

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Optimal result

Integrand size = 31, antiderivative size = 920

$$\int (a+bx)^4 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx$$
$$= -\frac{a(bc-ad)^3pqr^2x}{5d^3}+\frac{2(bc-ad)^4pqr^2x}{25d^4}+\frac{77(bc-ad)^4q^2r^2x}{150d^4}+\frac{2(bc-ad)^4q(p+q)r^2x}{5d^4}-\frac{b(bc-ad)^3pqr^2x^2}{10d^3}-\frac{(bc-ad)^3pqr^2(a+bx)^2}{25bd^3}-\frac{77(bc-ad)^3q^2r^2(a+bx)^2}{300bd^3}+\frac{16(bc-ad)^2pqr^2(a+bx)^3}{225bd^2}+\frac{47(bc-ad)^2q^2r^2(a+bx)^3}{450bd^2}-\frac{9(bc-ad)pqr^2(a+bx)^4}{200bd}-\frac{9(bc-ad)q^2r^2(a+bx)^4}{200bd}+\frac{2p^2r^2(a+bx)^5}{125b}+\frac{4pqr^2(a+bx)^5}{125b}+\frac{2q^2r^2(a+bx)^5}{125b}-\frac{2(bc-ad)^5pqr^2\log(c+dx)}{25bd^5}-\frac{137(bc-ad)^5q^2r^2\log(c+dx)}{150bd^5}-\frac{2(bc-ad)^5pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{5bd^5}-\frac{(bc-ad)^5q^2r^2\log^2(c+dx)}{5bd^5}-\frac{2(bc-ad)^4qr(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{5bd^4}+\frac{(bc-ad)^3qr(a+bx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{5bd^3}-\frac{2(bc-ad)^2qr(a+bx)^3\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{15bd^2}+\frac{(bc-ad)qr(a+bx)^4\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{10bd}-\frac{2pr(a+bx)^5\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{25b}-\frac{2qr(a+bx)^5\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{25b}+\frac{2(bc-ad)^5qr\log(c+dx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{5bd^5}+\frac{(a+bx)^5\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{5b}-\frac{2(bc-ad)^5pqr^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{5bd^5}$$

```
[Out] -2/25*p*r*(b*x+a)^5*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b-2/25*q*r*(b*x+a)^5*ln
(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b+77/150*(-a*d+b*c)^4*q^2*r^2*x/d^4+4/125*p*q
*r^2*(b*x+a)^5/b-2/5*(-a*d+b*c)^5*p*q*r^2*ln(-d*(b*x+a)/(-a*d+b*c))*ln(d*x+
c)/b/d^5+2/125*p^2*r^2*(b*x+a)^5/b+2/125*q^2*r^2*(b*x+a)^5/b+1/5*(b*x+a)^5*
ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/b-2/25*(-a*d+b*c)^5*p*q*r^2*ln(d*x+c)/b/d
^5-2/5*(-a*d+b*c)^4*q*r*(b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d^4+1/5*(
-a*d+b*c)^3*q*r*(b*x+a)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d^3-2/15*(-a*d+
b*c)^2*q*r*(b*x+a)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d^2+1/10*(-a*d+b*c)*
q*r*(b*x+a)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d+2/5*(-a*d+b*c)^5*q*r*ln(d
```

```
*x+c)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d^5-9/200*(-a*d+b*c)*p*q*r^2*(b*x+a
)^4/b/d-2/5*(-a*d+b*c)^5*p*q*r^2*polylog(2,b*(d*x+c)/(-a*d+b*c))/b/d^5-1/5*
a*(-a*d+b*c)^3*p*q*r^2*x/d^3-1/10*b*(-a*d+b*c)^3*p*q*r^2*x^2/d^3-1/25*(-a*d
+b*c)^3*p*q*r^2*(b*x+a)^2/b/d^3+16/225*(-a*d+b*c)^2*p*q*r^2*(b*x+a)^3/b/d^2
-137/150*(-a*d+b*c)^5*q^2*r^2*ln(d*x+c)/b/d^5-1/5*(-a*d+b*c)^5*q^2*r^2*ln(d
*x+c)^2/b/d^5+2/25*(-a*d+b*c)^4*p*q*r^2*x/d^4+2/5*(-a*d+b*c)^4*q*(p+q)*r^2*
x/d^4-77/300*(-a*d+b*c)^3*q^2*r^2*(b*x+a)^2/b/d^3+47/450*(-a*d+b*c)^2*q^2*r
^2*(b*x+a)^3/b/d^2-9/200*(-a*d+b*c)*q^2*r^2*(b*x+a)^4/b/d
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 920, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.452$, Rules used = {2584, 2581, 32, 45, 2594, 2579, 31, 8, 2580, 2441, 2440, 2438, 2437, 2338}

$$\begin{aligned}
&\int (a+bx)^4 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right) dx \\
&= -\frac{q^2r^2\log^2(c+dx)(bc-ad)^5}{5bd^5} - \frac{137q^2r^2\log(c+dx)(bc-ad)^5}{150bd^5} \\
&\quad - \frac{2pqr^2\log(c+dx)(bc-ad)^5}{25bd^5} - \frac{2pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)(bc-ad)^5}{5bd^5} \\
&\quad + \frac{2qr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)(bc-ad)^5}{5bd^5} \\
&\quad - \frac{2pqr^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)(bc-ad)^5}{5bd^5} + \frac{77q^2r^2x(bc-ad)^4}{150d^4} + \frac{2pqr^2x(bc-ad)^4}{25d^4} \\
&\quad + \frac{2q(p+q)r^2x(bc-ad)^4}{5d^4} - \frac{2qr(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)(bc-ad)^4}{5bd^4} \\
&\quad - \frac{bpqr^2x^2(bc-ad)^3}{10d^3} - \frac{77q^2r^2(a+bx)^2(bc-ad)^3}{300bd^3} - \frac{pqr^2(a+bx)^2(bc-ad)^3}{25bd^3} \\
&\quad - \frac{apqr^2x(bc-ad)^3}{5d^3} + \frac{qr(a+bx)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)(bc-ad)^3}{5bd^3} \\
&\quad + \frac{47q^2r^2(a+bx)^3(bc-ad)^2}{450bd^2} + \frac{16pqr^2(a+bx)^3(bc-ad)^2}{225bd^2} \\
&\quad - \frac{2qr(a+bx)^3\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)(bc-ad)^2}{15bd^2} - \frac{9q^2r^2(a+bx)^4(bc-ad)}{200bd} \\
&\quad - \frac{9pqr^2(a+bx)^4(bc-ad)}{200bd} + \frac{qr(a+bx)^4\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)(bc-ad)}{10bd} \\
&\quad + \frac{2p^2r^2(a+bx)^5}{125b} + \frac{2q^2r^2(a+bx)^5}{125b} + \frac{4pqr^2(a+bx)^5}{125b} \\
&\quad + \frac{(a+bx)^5\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{5b} - \frac{2pr(a+bx)^5\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{25b} \\
&\quad - \frac{2qr(a+bx)^5\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{25b}
\end{aligned}$$

```
[In] Int[(a + b*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] -1/5*(a*(b*c - a*d)^3*p*q*r^2*x)/d^3 + (2*(b*c - a*d)^4*p*q*r^2*x)/(25*d^4)
 + (77*(b*c - a*d)^4*q^2*r^2*x)/(150*d^4) + (2*(b*c - a*d)^4*q*(p + q)*r^2*
x)/(5*d^4) - (b*(b*c - a*d)^3*p*q*r^2*x^2)/(10*d^3) - ((b*c - a*d)^3*p*q*r^
2*(a + b*x)^2)/(25*b*d^3) - (77*(b*c - a*d)^3*q^2*r^2*(a + b*x)^2)/(300*b*d
^3) + (16*(b*c - a*d)^2*p*q*r^2*(a + b*x)^3)/(225*b*d^2) + (47*(b*c - a*d)^
2*q^2*r^2*(a + b*x)^3)/(450*b*d^2) - (9*(b*c - a*d)*p*q*r^2*(a + b*x)^4)/(2
00*b*d) - (9*(b*c - a*d)*q^2*r^2*(a + b*x)^4)/(200*b*d) + (2*p^2*r^2*(a + b
*x)^5)/(125*b) + (4*p*q*r^2*(a + b*x)^5)/(125*b) + (2*q^2*r^2*(a + b*x)^5)/
(125*b) - (2*(b*c - a*d)^5*p*q*r^2*Log[c + d*x])/(25*b*d^5) - (137*(b*c - a
*d)^5*q^2*r^2*Log[c + d*x])/(150*b*d^5) - (2*(b*c - a*d)^5*p*q*r^2*Log[-((d
*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(5*b*d^5) - ((b*c - a*d)^5*q^2*r^2*
Log[c + d*x]^2)/(5*b*d^5) - (2*(b*c - a*d)^4*q*r*(a + b*x)*Log[e*(f*(a + b*
x)^p*(c + d*x)^q)^r])/(5*b*d^4) + ((b*c - a*d)^3*q*r*(a + b*x)^2*Log[e*(f*(
a + b*x)^p*(c + d*x)^q)^r])/(5*b*d^3) - (2*(b*c - a*d)^2*q*r*(a + b*x)^3*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(15*b*d^2) + ((b*c - a*d)*q*r*(a + b*x)
^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(10*b*d) - (2*p*r*(a + b*x)^5*Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(25*b) - (2*q*r*(a + b*x)^5*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/(25*b) + (2*(b*c - a*d)^5*q*r*Log[c + d*x]*Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r])/(5*b*d^5) + ((a + b*x)^5*Log[e*(f*(a + b*x)^
p*(c + d*x)^q)^r]^2)/(5*b) - (2*(b*c - a*d)^5*p*q*r^2*PolyLog[2, (b*(c + d*
x))/(b*c - a*d)])/(5*b*d^5)
```

<u>Rule 8</u>

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

<u>Rule 31</u>

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

<u>Rule 32</u>

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

<u>Rule 45</u>

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

<u>Rule 2338</u>

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2579

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[(a + b*x)*(Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s/b), x] + (Dist[q*r*s*((b*c - a*d)/b), Int[Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r]^(s - 1)/(c + d*x), x], x] - Dist[r*s*(p + q), Int[Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]^(s - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p, q,
r, s}, x] && NeQ[b*c - a*d, 0] && NeQ[p + q, 0] && IGtQ[s, 0] && LtQ[s, 4]
```

Rule 2580

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[Log[g + h*x]*(Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]/h), x] + (-Dist[b*p*(r/h), Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[d*q*(r/h), Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 2584

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1
)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(h*(m + 1))), x] + (-Dist[b*p*r*(
s/(h*(m + 1))), Int[(g + h*x)^(m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1)/(a + b*x)), x], x] - Dist[d*q*r*(s/(h*(m + 1))), Int[(g + h*x)^(m
+ 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(c + d*x)), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2594

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(a+bx)^5 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{5b} \\
&\quad -\frac{1}{5}(2pr)\int (a+bx)^4 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx \\
&\quad -\frac{(2dqr)\int \frac{(a+bx)^5 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{c+dx}\,dx}{5b} \\
&= -\frac{2pr(a+bx)^5 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{25b} \\
&\quad +\frac{(a+bx)^5 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{5b} \\
&\quad -\frac{(2dqr)\int \left(\frac{b(bc-ad)^4 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{d^5} - \frac{b(bc-ad)^3(a+bx) \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{d^4} + \frac{b(bc-ad)^2(a+bx)^2 \log(e}{d^3}\right.}{} \\
&\quad +\frac{1}{25}\left(2p^2r^2\right)\int (a+bx)^4\,dx + \frac{\left(2dpqr^2\right)\int \frac{(a+bx)^5}{c+dx}\,dx}{25b}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{2p^2r^2(a+bx)^5}{125b} - \frac{2pr(a+bx)^5 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{25b} \\
&\quad + \frac{(a+bx)^5 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{5b} \\
&\quad - \frac{1}{5}(2qr)\int (a+bx)^4 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx \\
&\quad + \frac{(2(bc-ad)qr)\int (a+bx)^3 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx}{5d} \\
&\quad - \frac{(2(bc-ad)^2qr)\int (a+bx)^2 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx}{5d^2} \\
&\quad + \frac{(2(bc-ad)^3qr)\int (a+bx) \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx}{5d^3} \\
&\quad - \frac{(2(bc-ad)^4qr)\int \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx}{5d^4} \\
&\quad + \frac{(2(bc-ad)^5qr)\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{c+dx}\,dx}{5bd^4} \\
&\quad + \frac{(2dpqr^2)\int \left(\frac{b(bc-ad)^4}{d^5} - \frac{b(bc-ad)^3(a+bx)}{d^4} + \frac{b(bc-ad)^2(a+bx)^2}{d^3} - \frac{b(bc-ad)(a+bx)^3}{d^2} + \frac{b(a+bx)^4}{d} + \frac{(-bc+ad)^5}{d^5(c+dx)}\right)dx}{25b}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2(bc-ad)^4pqr^2x}{25d^4} - \frac{(bc-ad)^3pqr^2(a+bx)^2}{25bd^3} \\
&+ \frac{2(bc-ad)^2pqr^2(a+bx)^3}{75bd^2} - \frac{(bc-ad)pqr^2(a+bx)^4}{50bd} \\
&+ \frac{2p^2r^2(a+bx)^5}{125b} + \frac{2pqr^2(a+bx)^5}{125b} - \frac{2(bc-ad)^5pqr^2\log(c+dx)}{25bd^5} \\
&- \frac{2(bc-ad)^4qr(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{5bd^4} \\
&+ \frac{(bc-ad)^3qr(a+bx)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{5bd^3} \\
&- \frac{2(bc-ad)^2qr(a+bx)^3\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{15bd^2} \\
&+ \frac{(bc-ad)qr(a+bx)^4\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{10bd} \\
&- \frac{2pr(a+bx)^5\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{25b} \\
&- \frac{2qr(a+bx)^5\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{25b} \\
&+ \frac{2(bc-ad)^5qr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{5bd^5} \\
&+ \frac{(a+bx)^5\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{5b} \\
&+ \frac{1}{25}\left(2pqr^2\right)\int(a+bx)^4\,dx - \frac{\left((bc-ad)pqr^2\right)\int(a+bx)^3\,dx}{10d} \\
&+ \frac{\left(2(bc-ad)^2pqr^2\right)\int(a+bx)^2\,dx}{15d^2} - \frac{\left((bc-ad)^3pqr^2\right)\int(a+bx)\,dx}{5d^3} \\
&- \frac{\left(2(bc-ad)^5pqr^2\right)\int\frac{\log(c+dx)}{a+bx}\,dx}{5d^5} + \frac{\left(2dq^2r^2\right)\int\frac{(a+bx)^5}{c+dx}\,dx}{25b} \\
&- \frac{\left((bc-ad)q^2r^2\right)\int\frac{(a+bx)^4}{c+dx}\,dx}{10b} + \frac{\left(2(bc-ad)^2q^2r^2\right)\int\frac{(a+bx)^3}{c+dx}\,dx}{15bd} \\
&- \frac{\left((bc-ad)^3q^2r^2\right)\int\frac{(a+bx)^2}{c+dx}\,dx}{5bd^2} - \frac{\left(2(bc-ad)^5q^2r^2\right)\int\frac{1}{c+dx}\,dx}{5bd^4} \\
&- \frac{\left(2(bc-ad)^5q^2r^2\right)\int\frac{\log(c+dx)}{c+dx}\,dx}{5bd^4} + \frac{\left(2(bc-ad)^4q(p+q)r^2\right)\int 1\,dx}{5d^4}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{a(bc-ad)^3pqr^2x}{5d^3} + \frac{2(bc-ad)^4pqr^2x}{25d^4} + \frac{2(bc-ad)^4q(p+q)r^2x}{5d^4} \\
&\quad - \frac{b(bc-ad)^3pqr^2x^2}{10d^3} - \frac{(bc-ad)^3pqr^2(a+bx)^2}{25bd^3} \\
&\quad + \frac{16(bc-ad)^2pqr^2(a+bx)^3}{225bd^2} - \frac{9(bc-ad)pqr^2(a+bx)^4}{200bd} \\
&\quad + \frac{2p^2r^2(a+bx)^5}{125b} + \frac{4pqr^2(a+bx)^5}{125b} - \frac{2(bc-ad)^5pqr^2\log(c+dx)}{25bd^5} \\
&\quad - \frac{2(bc-ad)^5q^2r^2\log(c+dx)}{5bd^5} - \frac{2(bc-ad)^5pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{5bd^5} \\
&\quad - \frac{2(bc-ad)^4qr(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{5bd^4} \\
&\quad + \frac{(bc-ad)^3qr(a+bx)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{5bd^3} \\
&\quad - \frac{2(bc-ad)^2qr(a+bx)^3\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{15bd^2} \\
&\quad + \frac{(bc-ad)qr(a+bx)^4\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{10bd} \\
&\quad - \frac{2pr(a+bx)^5\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{25b} \\
&\quad - \frac{2qr(a+bx)^5\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{25b} \\
&\quad + \frac{2(bc-ad)^5qr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{5bd^5} \\
&\quad + \frac{(a+bx)^5\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{5b} + \frac{\left(2(bc-ad)^5pqr^2\right)\int\frac{\log\left(\frac{d(a+bx)}{-bc+ad}\right)}{c+dx}\,dx}{5bd^4} \\
&\quad + \frac{(2dq^2r^2)\int\left(\frac{b(bc-ad)^4}{d^5} - \frac{b(bc-ad)^3(a+bx)}{d^4} + \frac{b(bc-ad)^2(a+bx)^2}{d^3} - \frac{b(bc-ad)(a+bx)^3}{d^2} + \frac{b(a+bx)^4}{d} + \frac{(-bc+ad)^5}{d^5(c+dx)}\right)dx}{25b} \\
&\quad - \frac{((bc-ad)q^2r^2)\int\left(-\frac{b(bc-ad)^3}{d^4} + \frac{b(bc-ad)^2(a+bx)}{d^3} - \frac{b(bc-ad)(a+bx)^2}{d^2} + \frac{b(a+bx)^3}{d} + \frac{(-bc+ad)^4}{d^4(c+dx)}\right)dx}{10b} \\
&\quad + \frac{(2(bc-ad)^2q^2r^2)\int\left(\frac{b(bc-ad)^2}{d^3} - \frac{b(bc-ad)(a+bx)}{d^2} + \frac{b(a+bx)^2}{d} + \frac{(-bc+ad)^3}{d^3(c+dx)}\right)dx}{15bd} \\
&\quad - \frac{((bc-ad)^3q^2r^2)\int\left(-\frac{b(bc-ad)}{d^2} + \frac{b(a+bx)}{d} + \frac{(-bc+ad)^2}{d^2(c+dx)}\right)dx}{5bd^2} \\
&\quad - \frac{(2(bc-ad)^5q^2r^2)\,\mathrm{Subst}\left(\int\frac{\log(x)}{x}\,dx, x, c+dx\right)}{5bd^5}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{a(bc-ad)^3pqr^2x}{5d^3} + \frac{2(bc-ad)^4pqr^2x}{25d^4} + \frac{77(bc-ad)^4q^2r^2x}{150d^4} \\
&+ \frac{2(bc-ad)^4q(p+q)r^2x}{5d^4} - \frac{b(bc-ad)^3pqr^2x^2}{10d^3} - \frac{(bc-ad)^3pqr^2(a+bx)^2}{25bd^3} \\
&- \frac{77(bc-ad)^3q^2r^2(a+bx)^2}{300bd^3} + \frac{16(bc-ad)^2pqr^2(a+bx)^3}{225bd^2} \\
&+ \frac{47(bc-ad)^2q^2r^2(a+bx)^3}{450bd^2} - \frac{9(bc-ad)pqr^2(a+bx)^4}{200bd} \\
&- \frac{9(bc-ad)q^2r^2(a+bx)^4}{200bd} + \frac{2p^2r^2(a+bx)^5}{125b} + \frac{4pqr^2(a+bx)^5}{125b} + \frac{2q^2r^2(a+bx)^5}{125b} \\
&- \frac{2(bc-ad)^5pqr^2\log(c+dx)}{25bd^5} - \frac{137(bc-ad)^5q^2r^2\log(c+dx)}{150bd^5} \\
&- \frac{2(bc-ad)^5pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{5bd^5} - \frac{(bc-ad)^5q^2r^2\log^2(c+dx)}{5bd^5} \\
&- \frac{2(bc-ad)^4qr(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{5bd^4} \\
&+ \frac{(bc-ad)^3qr(a+bx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{5bd^3} \\
&- \frac{2(bc-ad)^2qr(a+bx)^3\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{15bd^2} \\
&+ \frac{(bc-ad)qr(a+bx)^4\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{10bd} \\
&- \frac{2pr(a+bx)^5\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{25b} \\
&- \frac{2qr(a+bx)^5\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{25b} \\
&+ \frac{2(bc-ad)^5qr\log(c+dx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{5bd^5} \\
&+ \frac{(a+bx)^5\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{5b} \\
&+ \frac{(2(bc-ad)^5pqr^2)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{bx}{-bc+ad}\right)}{x}\,dx, x, c+dx\right)}{5bd^5}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{a(bc-ad)^3pqr^2x}{5d^3} + \frac{2(bc-ad)^4pqr^2x}{25d^4} + \frac{77(bc-ad)^4q^2r^2x}{150d^4} \\
&+ \frac{2(bc-ad)^4q(p+q)r^2x}{5d^4} - \frac{b(bc-ad)^3pqr^2x^2}{10d^3} - \frac{(bc-ad)^3pqr^2(a+bx)^2}{25bd^3} \\
&- \frac{77(bc-ad)^3q^2r^2(a+bx)^2}{300bd^3} + \frac{16(bc-ad)^2pqr^2(a+bx)^3}{225bd^2} \\
&+ \frac{47(bc-ad)^2q^2r^2(a+bx)^3}{450bd^2} - \frac{9(bc-ad)pqr^2(a+bx)^4}{200bd} \\
&- \frac{9(bc-ad)q^2r^2(a+bx)^4}{200bd} + \frac{2p^2r^2(a+bx)^5}{125b} + \frac{4pqr^2(a+bx)^5}{125b} + \frac{2q^2r^2(a+bx)^5}{125b} \\
&- \frac{2(bc-ad)^5pqr^2\log(c+dx)}{25bd^5} - \frac{137(bc-ad)^5q^2r^2\log(c+dx)}{150bd^5} \\
&- \frac{2(bc-ad)^5pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{5bd^5} - \frac{(bc-ad)^5q^2r^2\log^2(c+dx)}{5bd^5} \\
&- \frac{2(bc-ad)^4qr(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{5bd^4} \\
&+ \frac{(bc-ad)^3qr(a+bx)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{5bd^3} \\
&- \frac{2(bc-ad)^2qr(a+bx)^3\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{15bd^2} \\
&+ \frac{(bc-ad)qr(a+bx)^4\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{10bd} \\
&- \frac{2pr(a+bx)^5\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{25b} \\
&- \frac{2qr(a+bx)^5\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{25b} \\
&+ \frac{2(bc-ad)^5qr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{5bd^5} \\
&+ \frac{(a+bx)^5\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{5b} - \frac{2(bc-ad)^5pqr^2\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)}{5bd^5}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2508 vs. $2(920) = 1840$.

Time = 1.54 (sec) , antiderivative size = 2508, normalized size of antiderivative = 2.73

$$
\int (a+bx)^4 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \text{Result too large to show}
$$

```
[In] Integrate[(a + b*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] (2*a^5*p*q*r^2)/b + (2*a*b^3*c^4*p*q*r^2)/(5*d^4) - (2*a^2*b^2*c^3*p*q*r^2)
/d^3 + (4*a^3*b*c^2*p*q*r^2)/d^2 - (4*a^4*c*p*q*r^2)/d + (2*a^4*p^2*r^2*x)/
25 + (197*a^4*p*q*r^2*x)/150 + (12*b^4*c^4*p*q*r^2*x)/(25*d^4) - (11*a*b^3*
```

c^3*p*q*r^2*x)/(5*d^3) + (59*a^2*b^2*c^2*p*q*r^2*x)/(15*d^2) - (101*a^3*b*c
*p*q*r^2*x)/(30*d) + 2*a^4*q^2*r^2*x + (137*b^4*c^4*q^2*r^2*x)/(150*d^4) -
(25*a*b^3*c^3*q^2*r^2*x)/(6*d^3) + (22*a^2*b^2*c^2*q^2*r^2*x)/(3*d^2) - (6*
a^3*b*c*q^2*r^2*x)/d + (4*a^3*b*p^2*r^2*x^2)/25 + (283*a^3*b*p*q*r^2*x^2)/3
00 - (7*b^4*c^3*p*q*r^2*x^2)/(50*d^3) + (19*a*b^3*c^2*p*q*r^2*x^2)/(30*d^2)
 - (67*a^2*b^2*c*p*q*r^2*x^2)/(60*d) + a^3*b*q^2*r^2*x^2 - (77*b^4*c^3*q^2*
r^2*x^2)/(300*d^3) + (13*a*b^3*c^2*q^2*r^2*x^2)/(12*d^2) - (5*a^2*b^2*c*q^2
*r^2*x^2)/(3*d) + (4*a^2*b^2*p^2*r^2*x^3)/25 + (257*a^2*b^2*p*q*r^2*x^3)/45
0 + (16*b^4*c^2*p*q*r^2*x^3)/(225*d^2) - (29*a*b^3*c*p*q*r^2*x^3)/(90*d) +
(4*a^2*b^2*q^2*r^2*x^3)/9 + (47*b^4*c^2*q^2*r^2*x^3)/(450*d^2) - (7*a*b^3*c
*q^2*r^2*x^3)/(18*d) + (2*a*b^3*p^2*r^2*x^4)/25 + (41*a*b^3*p*q*r^2*x^4)/20
0 - (9*b^4*c*p*q*r^2*x^4)/(200*d) + (a*b^3*q^2*r^2*x^4)/8 - (9*b^4*c*q^2*r^
2*x^4)/(200*d) + (2*b^4*p^2*r^2*x^5)/125 + (4*b^4*p*q*r^2*x^5)/125 + (2*b^4
*q^2*r^2*x^5)/125 - (a^5*p^2*r^2*Log[a + b*x]^2)/(5*b) + (2*a^5*p*q*r^2*Log
[c + d*x])/b - (2*b^4*c^5*p*q*r^2*Log[c + d*x])/(25*d^5) + (2*a*b^3*c^4*p*q
*r^2*Log[c + d*x])/(5*d^4) - (4*a^2*b^2*c^3*p*q*r^2*Log[c + d*x])/(5*d^3) +
 (4*a^3*b*c^2*p*q*r^2*Log[c + d*x])/(5*d^2) - (2*a^4*c*p*q*r^2*Log[c + d*x]
)/(5*d) - (137*b^4*c^5*q^2*r^2*Log[c + d*x])/(150*d^5) + (25*a*b^3*c^4*q^2*
r^2*Log[c + d*x])/(6*d^4) - (22*a^2*b^2*c^3*q^2*r^2*Log[c + d*x])/(3*d^3) +
 (6*a^3*b*c^2*q^2*r^2*Log[c + d*x])/d^2 - (2*a^4*c*q^2*r^2*Log[c + d*x])/d
- (b^4*c^5*q^2*r^2*Log[c + d*x]^2)/(5*d^5) + (a*b^3*c^4*q^2*r^2*Log[c + d*x
]^2)/d^4 - (2*a^2*b^2*c^3*q^2*r^2*Log[c + d*x]^2)/d^3 + (2*a^3*b*c^2*q^2*r^
2*Log[c + d*x]^2)/d^2 - (a^4*c*q^2*r^2*Log[c + d*x]^2)/d - (2*a^5*p*r*Log[e
(f(a + b*x)^p*(c + d*x)^q)^r])/b - (2*a^4*p*r*x*Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r])/5 - 2*a^4*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - (2*b^4*
c^4*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(5*d^4) + (2*a*b^3*c^3*q*r*
x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d^3 - (4*a^2*b^2*c^2*q*r*x*Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r])/d^2 + (4*a^3*b*c*q*r*x*Log[e*(f*(a + b*x)^p*(
c + d*x)^q)^r])/d - (4*a^3*b*p*r*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/
5 - 2*a^3*b*q*r*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (b^4*c^3*q*r*x^2
Log[e(f*(a + b*x)^p*(c + d*x)^q)^r])/(5*d^3) - (a*b^3*c^2*q*r*x^2*Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r])/d^2 + (2*a^2*b^2*c*q*r*x^2*Log[e*(f*(a + b*x
)^p*(c + d*x)^q)^r])/d - (4*a^2*b^2*p*r*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r])/5 - (4*a^2*b^2*q*r*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/3 - (2*
b^4*c^2*q*r*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(15*d^2) + (2*a*b^3*c
*q*r*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(3*d) - (2*a*b^3*p*r*x^4*Log
[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/5 - (a*b^3*q*r*x^4*Log[e*(f*(a + b*x)^p*
(c + d*x)^q)^r])/2 + (b^4*c*q*r*x^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(
10*d) - (2*b^4*p*r*x^5*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/25 - (2*b^4*q*
r*x^5*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/25 + (2*b^4*c^5*q*r*Log[c + d*x
]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(5*d^5) - (2*a*b^3*c^4*q*r*Log[c +
d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d^4 + (4*a^2*b^2*c^3*q*r*Log[c +
 d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d^3 - (4*a^3*b*c^2*q*r*Log[c +
d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d^2 + (2*a^4*c*q*r*Log[c + d*x]*
Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d + a^4*x*Log[e*(f*(a + b*x)^p*(c + d

```
*x)^q)^r]^2 + 2*a^3*b*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 + 2*a^2*b^
2*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 + a*b^3*x^4*Log[e*(f*(a + b*x)
^p*(c + d*x)^q)^r]^2 + (b^4*x^5*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)/5 +
 (p*r*Log[a + b*x]*(a*d*(a^4*d^4*(288*p - 137*q) - 60*b^4*c^4*q + 270*a*b^3
*c^3*d*q - 470*a^2*b^2*c^2*d^2*q + 385*a^3*b*c*d^3*q)*r - 60*b*c*(b^4*c^4 -
 5*a*b^3*c^3*d + 10*a^2*b^2*c^2*d^2 - 10*a^3*b*c*d^3 + 5*a^4*d^4)*q*r*Log[c
 + d*x] + 60*(b*c - a*d)^5*q*r*Log[(b*(c + d*x))/(b*c - a*d)] + 60*a^5*d^5*
Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(150*b*d^5) + (2*(b*c - a*d)^5*p*q*r
^2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*d^5)
```

Maple [F]

$$\int (bx+a)^4 \ln\left(e(f(bx+a)^p (dx+c)^q)^r\right)^2 dx$$

```
[In] int((b*x+a)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

```
[Out] int((b*x+a)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

Fricas [F]

$$\int (a+bx)^4 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right) dx = \int (bx+a)^4 \log\left(\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2\right) dx$$

```
[In] integrate((b*x+a)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="fricas
")
```

```
[Out] integral((b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4)*log(((b*
x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+bx)^4 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right) dx = \text{Timed out}$$

```
[In] integrate((b*x+a)**4*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 1421, normalized size of antiderivative = 1.54

$$\int (a+bx)^4 \log^2 \left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \text{Too large to display}$$

```
[In] integrate((b*x+a)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="maxima
")
```

```
[Out] 1/5*(b^4*x^5 + 5*a*b^3*x^4 + 10*a^2*b^2*x^3 + 10*a^3*b*x^2 + 5*a^4*x)*log((
(b*x + a)^p*(d*x + c)^q*f)^r*e)^2 + 1/150*(60*a^5*f*p*log(b*x + a)/b - (12*
b^4*d^4*f*(p + q)*x^5 + 15*(a*b^3*d^4*f*(4*p + 5*q) - b^4*c*d^3*f*q)*x^4 +
20*(2*a^2*b^2*d^4*f*(3*p + 5*q) + b^4*c^2*d^2*f*q - 5*a*b^3*c*d^3*f*q)*x^3
+ 30*(2*a^3*b*d^4*f*(2*p + 5*q) - b^4*c^3*d*f*q + 5*a*b^3*c^2*d^2*f*q - 10*
a^2*b^2*c*d^3*f*q)*x^2 + 60*(a^4*d^4*f*(p + 5*q) + b^4*c^4*f*q - 5*a*b^3*c^
3*d*f*q + 10*a^2*b^2*c^2*d^2*f*q - 10*a^3*b*c*d^3*f*q)*x)/d^4 + 60*(b^4*c^5
*f*q - 5*a*b^3*c^4*d*f*q + 10*a^2*b^2*c^3*d^2*f*q - 10*a^3*b*c^2*d^3*f*q +
5*a^4*c*d^4*f*q)*log(d*x + c)/d^5)*r*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/f
 - 1/9000*r^2*(60*((12*p*q + 137*q^2)*b^4*c^5*f^2 - 5*(12*p*q + 125*q^2)*a*
b^3*c^4*d*f^2 + 20*(6*p*q + 55*q^2)*a^2*b^2*c^3*d^2*f^2 - 60*(2*p*q + 15*q^
2)*a^3*b*c^2*d^3*f^2 + 60*(p*q + 5*q^2)*a^4*c*d^4*f^2)*log(d*x + c)/d^5 - 3
600*(b^5*c^5*f^2*p*q - 5*a*b^4*c^4*d*f^2*p*q + 10*a^2*b^3*c^3*d^2*f^2*p*q -
 10*a^3*b^2*c^2*d^3*f^2*p*q + 5*a^4*b*c*d^4*f^2*p*q - a^5*d^5*f^2*p*q)*(log
(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c -
a*d)))/(b*d^5) - (144*(p^2 + 2*p*q + q^2)*b^5*d^5*f^2*x^5 - 1800*a^5*d^5*f^
2*p^2*log(b*x + a)^2 - 45*(9*(p*q + q^2)*b^5*c*d^4*f^2 - (16*p^2 + 41*p*q +
 25*q^2)*a*b^4*d^5*f^2)*x^4 + 20*((32*p*q + 47*q^2)*b^5*c^2*d^3*f^2 - 5*(29
*p*q + 35*q^2)*a*b^4*c*d^4*f^2 + (72*p^2 + 257*p*q + 200*q^2)*a^2*b^3*d^5*f
^2)*x^3 - 30*(7*(6*p*q + 11*q^2)*b^5*c^3*d^2*f^2 - 5*(38*p*q + 65*q^2)*a*b^
4*c^2*d^3*f^2 + 5*(67*p*q + 100*q^2)*a^2*b^3*c*d^4*f^2 - (48*p^2 + 283*p*q
+ 300*q^2)*a^3*b^2*d^5*f^2)*x^2 - 3600*(b^5*c^5*f^2*p*q - 5*a*b^4*c^4*d*f^2
*p*q + 10*a^2*b^3*c^3*d^2*f^2*p*q - 10*a^3*b^2*c^2*d^3*f^2*p*q + 5*a^4*b*c*
d^4*f^2*p*q)*log(b*x + a)*log(d*x + c) - 1800*(b^5*c^5*f^2*q^2 - 5*a*b^4*c^
4*d*f^2*q^2 + 10*a^2*b^3*c^3*d^2*f^2*q^2 - 10*a^3*b^2*c^2*d^3*f^2*q^2 + 5*a
^4*b*c*d^4*f^2*q^2)*log(d*x + c)^2 + 60*((72*p*q + 137*q^2)*b^5*c^4*d*f^2 -
 5*(66*p*q + 125*q^2)*a*b^4*c^3*d^2*f^2 + 10*(59*p*q + 110*q^2)*a^2*b^3*c^2
*d^3*f^2 - 5*(101*p*q + 180*q^2)*a^3*b^2*c*d^4*f^2 + (12*p^2 + 197*p*q + 30
0*q^2)*a^4*b*d^5*f^2)*x - 60*(60*a*b^4*c^4*d*f^2*p*q - 270*a^2*b^3*c^3*d^2*
f^2*p*q + 470*a^3*b^2*c^2*d^3*f^2*p*q - 385*a^4*b*c*d^4*f^2*p*q + (12*p^2 +
 137*p*q)*a^5*d^5*f^2)*log(b*x + a))/(b*d^5))/f^2
```

Giac [F]

$$\int (a+bx)^4 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \int (bx+a)^4 \log\left(\left(((bx+a)^p(dx+c)^q f)^r e\right)^2\right)\,dx$$

```
[In] integrate((b*x+a)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="giac")
[Out] integrate((b*x + a)^4*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^4 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \int \ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)^2 (a+b\,x)^4\,dx$$

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(a + b*x)^4,x)
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(a + b*x)^4, x)
```

3.17 $\int (a+bx)^3 \log^2 \left(e(f(a+bx)^p(c+dx)^q)^r\right) dx$

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Optimal result

Integrand size = 31, antiderivative size = 805

$$\begin{aligned}
&\int (a+bx)^3 \log^2 \left(e(f(a+bx)^p(c+dx)^q)^r\right) dx \\
&= \frac{a(bc-ad)^2pqr^2x}{4d^2} - \frac{(bc-ad)^3pqr^2x}{8d^3} - \frac{13(bc-ad)^3q^2r^2x}{24d^3} - \frac{(bc-ad)^3q(p+q)r^2x}{2d^3} \\
&+ \frac{b(bc-ad)^2pqr^2x^2}{8d^2} + \frac{(bc-ad)^2pqr^2(a+bx)^2}{16bd^2} + \frac{13(bc-ad)^2q^2r^2(a+bx)^2}{48bd^2} \\
&- \frac{7(bc-ad)pqr^2(a+bx)^3}{72bd} - \frac{7(bc-ad)q^2r^2(a+bx)^3}{72bd} + \frac{p^2r^2(a+bx)^4}{32b} \\
&+ \frac{pqr^2(a+bx)^4}{16b} + \frac{q^2r^2(a+bx)^4}{32b} + \frac{(bc-ad)^4pqr^2\log(c+dx)}{8bd^4} \\
&+ \frac{25(bc-ad)^4q^2r^2\log(c+dx)}{24bd^4} + \frac{(bc-ad)^4pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{2bd^4} \\
&+ \frac{(bc-ad)^4q^2r^2\log^2(c+dx)}{4bd^4} + \frac{(bc-ad)^3qr(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2bd^3} \\
&- \frac{(bc-ad)^2qr(a+bx)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4bd^2} \\
&+ \frac{(bc-ad)qr(a+bx)^3\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{6bd} \\
&- \frac{pr(a+bx)^4\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{8b} - \frac{qr(a+bx)^4\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{8b} \\
&- \frac{(bc-ad)^4qr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2bd^4} \\
&+ \frac{(a+bx)^4\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b} + \frac{(bc-ad)^4pqr^2\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{2bd^4}
\end{aligned}$$

```
[Out] 1/4*a*(-a*d+b*c)^2*p*q*r^2*x/d^2-1/8*(-a*d+b*c)^3*p*q*r^2*x/d^3-13/24*(-a*d
+b*c)^3*q^2*r^2*x/d^3-1/2*(-a*d+b*c)^3*q*(p+q)*r^2*x/d^3+1/8*b*(-a*d+b*c)^2
```

```
*p*q*r^2*x^2/d^2+1/16*(-a*d+b*c)^2*p*q*r^2*(b*x+a)^2/b/d^2+13/48*(-a*d+b*c)
^2*q^2*r^2*(b*x+a)^2/b/d^2-7/72*(-a*d+b*c)*p*q*r^2*(b*x+a)^3/b/d-7/72*(-a*d
+b*c)*q^2*r^2*(b*x+a)^3/b/d+1/32*p^2*r^2*(b*x+a)^4/b+1/16*p*q*r^2*(b*x+a)^4
/b+1/32*q^2*r^2*(b*x+a)^4/b+1/8*(-a*d+b*c)^4*p*q*r^2*ln(d*x+c)/b/d^4+25/24*
(-a*d+b*c)^4*q^2*r^2*ln(d*x+c)/b/d^4+1/2*(-a*d+b*c)^4*p*q*r^2*ln(-d*(b*x+a)
/(-a*d+b*c))*ln(d*x+c)/b/d^4+1/4*(-a*d+b*c)^4*q^2*r^2*ln(d*x+c)^2/b/d^4+1/2
*(-a*d+b*c)^3*q*r*(b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d^3-1/4*(-a*d+b
*c)^2*q*r*(b*x+a)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d^2+1/6*(-a*d+b*c)*q*
r*(b*x+a)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d-1/8*p*r*(b*x+a)^4*ln(e*(f*(
b*x+a)^p*(d*x+c)^q)^r)/b-1/8*q*r*(b*x+a)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/
b-1/2*(-a*d+b*c)^4*q*r*ln(d*x+c)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d^4+1/4*
(b*x+a)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/b+1/2*(-a*d+b*c)^4*p*q*r^2*poly
log(2,b*(d*x+c)/(-a*d+b*c))/b/d^4
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 805, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.452, Rules used = {2584, 2581, 32, 45, 2594, 2579, 31, 8, 2580, 2441, 2440, 2438, 2437, 2338}

$$\begin{aligned}
&\int (a+bx)^3 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) \, dx \\
&= \frac{q^2r^2\log^2(c+dx)(bc-ad)^4}{4bd^4}+\frac{25q^2r^2\log(c+dx)(bc-ad)^4}{24bd^4} \\
&+\frac{pqr^2\log(c+dx)(bc-ad)^4}{8bd^4}+\frac{pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)(bc-ad)^4}{2bd^4} \\
&-\frac{qr\log(c+dx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)(bc-ad)^4}{2bd^4} \\
&+\frac{pqr^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)(bc-ad)^4}{2bd^4}-\frac{13q^2r^2x(bc-ad)^3}{24d^3}-\frac{pqr^2x(bc-ad)^3}{8d^3} \\
&-\frac{q(p+q)r^2x(bc-ad)^3}{2d^3}+\frac{qr(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)(bc-ad)^3}{2bd^3} \\
&+\frac{bpqr^2x^2(bc-ad)^2}{8d^2}+\frac{13q^2r^2(a+bx)^2(bc-ad)^2}{48bd^2}+\frac{pqr^2(a+bx)^2(bc-ad)^2}{16bd^2} \\
&+\frac{apqr^2x(bc-ad)^2}{4d^2}-\frac{qr(a+bx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)(bc-ad)^2}{4bd^2} \\
&-\frac{7q^2r^2(a+bx)^3(bc-ad)}{72bd}-\frac{7pqr^2(a+bx)^3(bc-ad)}{72bd} \\
&+\frac{qr(a+bx)^3\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)(bc-ad)}{6bd}+\frac{p^2r^2(a+bx)^4}{32b} \\
&+\frac{q^2r^2(a+bx)^4}{32b}+\frac{pqr^2(a+bx)^4}{16b}+\frac{(a+bx)^4\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4b} \\
&-\frac{pr(a+bx)^4\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{8b}-\frac{qr(a+bx)^4\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{8b}
\end{aligned}$$

```
[In] Int[(a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] (a*(b*c - a*d)^2*p*q*r^2*x)/(4*d^2) - ((b*c - a*d)^3*p*q*r^2*x)/(8*d^3) - (
13*(b*c - a*d)^3*q^2*r^2*x)/(24*d^3) - ((b*c - a*d)^3*q*(p + q)*r^2*x)/(2*d
^3) + (b*(b*c - a*d)^2*p*q*r^2*x^2)/(8*d^2) + ((b*c - a*d)^2*p*q*r^2*(a + b
*x)^2)/(16*b*d^2) + (13*(b*c - a*d)^2*q^2*r^2*(a + b*x)^2)/(48*b*d^2) - (7*
(b*c - a*d)*p*q*r^2*(a + b*x)^3)/(72*b*d) - (7*(b*c - a*d)*q^2*r^2*(a + b*x
)^3)/(72*b*d) + (p^2*r^2*(a + b*x)^4)/(32*b) + (p*q*r^2*(a + b*x)^4)/(16*b)
 + (q^2*r^2*(a + b*x)^4)/(32*b) + ((b*c - a*d)^4*p*q*r^2*Log[c + d*x])/(8*b
*d^4) + (25*(b*c - a*d)^4*q^2*r^2*Log[c + d*x])/(24*b*d^4) + ((b*c - a*d)^4
*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(2*b*d^4) + ((b*c
- a*d)^4*q^2*r^2*Log[c + d*x]^2)/(4*b*d^4) + ((b*c - a*d)^3*q*r*(a + b*x)*L
og[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*b*d^3) - ((b*c - a*d)^2*q*r*(a + b*
x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(4*b*d^2) + ((b*c - a*d)*q*r*(a
+ b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(6*b*d) - (p*r*(a + b*x)^4*L
og[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(8*b) - (q*r*(a + b*x)^4*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/(8*b) - ((b*c - a*d)^4*q*r*Log[c + d*x]*Log[e*(f*(
a + b*x)^p*(c + d*x)^q)^r])/(2*b*d^4) + ((a + b*x)^4*Log[e*(f*(a + b*x)^p*(
c + d*x)^q)^r]^2)/(4*b) + ((b*c - a*d)^4*p*q*r^2*PolyLog[2, (b*(c + d*x))/(
b*c - a*d)])/(2*b*d^4)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2579

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[(a + b*x)*(Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s/b), x] + (Dist[q*r*s*((b*c - a*d)/b), Int[Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r]^(s - 1)/(c + d*x), x], x] - Dist[r*s*(p + q), Int[Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]^(s - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p, q,
r, s}, x] && NeQ[b*c - a*d, 0] && NeQ[p + q, 0] && IGtQ[s, 0] && LtQ[s, 4]
```

Rule 2580

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[Log[g + h*x]*(Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]/h), x] + (-Dist[b*p*(r/h), Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[d*q*(r/h), Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
```

```
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 2584

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1
)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(h*(m + 1))), x] + (-Dist[b*p*r*(
s/(h*(m + 1))), Int[(g + h*x)^(m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1)/(a + b*x)), x], x] - Dist[d*q*r*(s/(h*(m + 1))), Int[(g + h*x)^(m
+ 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(c + d*x)), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2594

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rubi steps

$$\text{integral} = \frac{(a+bx)^4\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b} - \frac{1}{2}(pr)\int(a+bx)^3\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx - \frac{(dqr)\int\frac{(a+bx)^4\log(e(f(a+bx)^p(c+dx)^q)^r)}{c+dx}\,dx}{2b}$$

$$= -\frac{pr(a+bx)^4\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{8b} + \frac{(a+bx)^4\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b} - \frac{(dqr)\int\left(-\frac{b(bc-ad)^3\log(e(f(a+bx)^p(c+dx)^q)^r)}{d^4}+\frac{b(bc-ad)^2(a+bx)\log(e(f(a+bx)^p(c+dx)^q)^r)}{d^3}-\frac{b(bc-ad)(a+bx)^2\log(e}{d^{}}\right.}{2b} + \frac{1}{8}(p^2r^2)\int(a+bx)^3\,dx + \frac{(dpqr^2)\int\frac{(a+bx)^4}{c+dx}\,dx}{8b}$$

$$
\begin{aligned}
&= \frac{p^2r^2(a+bx)^4}{32b} - \frac{pr(a+bx)^4 \log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{8b} \\
&\quad + \frac{(a+bx)^4 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b} \\
&\quad - \frac{1}{2}(qr)\int (a+bx)^3 \log\left(e(f(a+bx)^p(c+dx)^q)^r\right) dx \\
&\quad + \frac{((bc-ad)qr)\int (a+bx)^2 \log\left(e(f(a+bx)^p(c+dx)^q)^r\right) dx}{2d} \\
&\quad - \frac{((bc-ad)^2qr)\int (a+bx) \log\left(e(f(a+bx)^p(c+dx)^q)^r\right) dx}{2d^2} \\
&\quad + \frac{((bc-ad)^3qr)\int \log\left(e(f(a+bx)^p(c+dx)^q)^r\right) dx}{2d^3} \\
&\quad - \frac{((bc-ad)^4qr)\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{c+dx} dx}{2bd^3} \\
&\quad + \frac{(dpqr^2)\int \left(-\frac{b(bc-ad)^3}{d^4} + \frac{b(bc-ad)^2(a+bx)}{d^3} - \frac{b(bc-ad)(a+bx)^2}{d^2} + \frac{b(a+bx)^3}{d} + \frac{(-bc+ad)^4}{d^4(c+dx)}\right) dx}{8b}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{(bc-ad)^3pqr^2x}{8d^3} + \frac{(bc-ad)^2pqr^2(a+bx)^2}{16bd^2} - \frac{(bc-ad)pqr^2(a+bx)^3}{24bd} \\
&+ \frac{p^2r^2(a+bx)^4}{32b} + \frac{pqr^2(a+bx)^4}{32b} + \frac{(bc-ad)^4pqr^2\log(c+dx)}{8bd^4} \\
&+ \frac{(bc-ad)^3qr(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2bd^3} \\
&- \frac{(bc-ad)^2qr(a+bx)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4bd^2} \\
&+ \frac{(bc-ad)qr(a+bx)^3\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{6bd} \\
&- \frac{pr(a+bx)^4\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{8b} \\
&- \frac{qr(a+bx)^4\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{8b} \\
&- \frac{(bc-ad)^4qr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2bd^4} \\
&+ \frac{(a+bx)^4\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b} \\
&+ \frac{1}{8}(pqr^2)\int(a+bx)^3\,dx - \frac{((bc-ad)pqr^2)\int(a+bx)^2\,dx}{6d} \\
&+ \frac{((bc-ad)^2pqr^2)\int(a+bx)\,dx}{4d^2} + \frac{((bc-ad)^4pqr^2)\int\frac{\log(c+dx)}{a+bx}\,dx}{2d^4} \\
&+ \frac{(dq^2r^2)\int\frac{(a+bx)^4}{c+dx}\,dx}{8b} - \frac{((bc-ad)q^2r^2)\int\frac{(a+bx)^3}{c+dx}\,dx}{6b} \\
&+ \frac{((bc-ad)^2q^2r^2)\int\frac{(a+bx)^2}{c+dx}\,dx}{4bd} + \frac{((bc-ad)^4q^2r^2)\int\frac{1}{c+dx}\,dx}{2bd^3} \\
&+ \frac{((bc-ad)^4q^2r^2)\int\frac{\log(c+dx)}{c+dx}\,dx}{2bd^3} - \frac{((bc-ad)^3q(p+q)r^2)\int 1\,dx}{2d^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{a(bc-ad)^2pqr^2x}{4d^2} - \frac{(bc-ad)^3pqr^2x}{8d^3} - \frac{(bc-ad)^3q(p+q)r^2x}{2d^3} \\
&+ \frac{b(bc-ad)^2pqr^2x^2}{8d^2} + \frac{(bc-ad)^2pqr^2(a+bx)^2}{16bd^2} - \frac{7(bc-ad)pqr^2(a+bx)^3}{72bd} \\
&+ \frac{p^2r^2(a+bx)^4}{32b} + \frac{pqr^2(a+bx)^4}{16b} + \frac{(bc-ad)^4pqr^2\log(c+dx)}{8bd^4} \\
&+ \frac{(bc-ad)^4q^2r^2\log(c+dx)}{2bd^4} + \frac{(bc-ad)^4pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{2bd^4} \\
&+ \frac{(bc-ad)^3qr(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2bd^3} \\
&- \frac{(bc-ad)^2qr(a+bx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4bd^2} \\
&+ \frac{(bc-ad)qr(a+bx)^3\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{6bd} \\
&- \frac{pr(a+bx)^4\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{8b} \\
&- \frac{qr(a+bx)^4\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{8b} \\
&- \frac{(bc-ad)^4qr\log(c+dx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2bd^4} \\
&+ \frac{(a+bx)^4\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4b} - \frac{\left((bc-ad)^4pqr^2\right)\int\frac{\log\left(\frac{d(a+bx)}{-bc+ad}\right)}{c+dx}\,dx}{2bd^3} \\
&+ \frac{\left(dq^2r^2\right)\int\left(-\frac{b(bc-ad)^3}{d^4}+\frac{b(bc-ad)^2(a+bx)}{d^3}-\frac{b(bc-ad)(a+bx)^2}{d^2}+\frac{b(a+bx)^3}{d}+\frac{(-bc+ad)^4}{d^4(c+dx)}\right)dx}{8b} \\
&- \frac{\left((bc-ad)q^2r^2\right)\int\left(\frac{b(bc-ad)^2}{d^3}-\frac{b(bc-ad)(a+bx)}{d^2}+\frac{b(a+bx)^2}{d}+\frac{(-bc+ad)^3}{d^3(c+dx)}\right)dx}{6b} \\
&+ \frac{\left((bc-ad)^2q^2r^2\right)\int\left(-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}+\frac{(-bc+ad)^2}{d^2(c+dx)}\right)dx}{4bd} \\
&+ \frac{\left((bc-ad)^4q^2r^2\right)\operatorname{Subst}\left(\int\frac{\log(x)}{x}\,dx,x,c+dx\right)}{2bd^4}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{a(bc-ad)^2pqr^2x}{4d^2} - \frac{(bc-ad)^3pqr^2x}{8d^3} - \frac{13(bc-ad)^3q^2r^2x}{24d^3} - \frac{(bc-ad)^3q(p+q)r^2x}{2d^3} \\
&+ \frac{b(bc-ad)^2pqr^2x^2}{8d^2} + \frac{(bc-ad)^2pqr^2(a+bx)^2}{16bd^2} + \frac{13(bc-ad)^2q^2r^2(a+bx)^2}{48bd^2} \\
&- \frac{7(bc-ad)pqr^2(a+bx)^3}{72bd} - \frac{7(bc-ad)q^2r^2(a+bx)^3}{72bd} + \frac{p^2r^2(a+bx)^4}{32b} \\
&+ \frac{pqr^2(a+bx)^4}{16b} + \frac{q^2r^2(a+bx)^4}{32b} + \frac{(bc-ad)^4pqr^2\log(c+dx)}{8bd^4} \\
&+ \frac{25(bc-ad)^4q^2r^2\log(c+dx)}{24bd^4} + \frac{(bc-ad)^4pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{2bd^4} \\
&+ \frac{(bc-ad)^4q^2r^2\log^2(c+dx)}{4bd^4} + \frac{(bc-ad)^3qr(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2bd^3} \\
&- \frac{(bc-ad)^2qr(a+bx)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4bd^2} \\
&+ \frac{(bc-ad)qr(a+bx)^3\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{6bd} \\
&- \frac{pr(a+bx)^4\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{8b} \\
&- \frac{qr(a+bx)^4\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{8b} \\
&- \frac{(bc-ad)^4qr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2bd^4} \\
&+ \frac{(a+bx)^4\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b} \\
&- \frac{((bc-ad)^4pqr^2)\,\text{Subst}\left(\int \frac{\log\left(1+\frac{bx}{-bc+ad}\right)}{x}\,dx, x, c+dx\right)}{2bd^4}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{a(bc-ad)^2pqr^2x}{4d^2} - \frac{(bc-ad)^3pqr^2x}{8d^3} - \frac{13(bc-ad)^3q^2r^2x}{24d^3} - \frac{(bc-ad)^3q(p+q)r^2x}{2d^3} \\
&+ \frac{b(bc-ad)^2pqr^2x^2}{8d^2} + \frac{(bc-ad)^2pqr^2(a+bx)^2}{16bd^2} + \frac{13(bc-ad)^2q^2r^2(a+bx)^2}{48bd^2} \\
&- \frac{7(bc-ad)pqr^2(a+bx)^3}{72bd} - \frac{7(bc-ad)q^2r^2(a+bx)^3}{72bd} + \frac{p^2r^2(a+bx)^4}{32b} \\
&+ \frac{pqr^2(a+bx)^4}{16b} + \frac{q^2r^2(a+bx)^4}{32b} + \frac{(bc-ad)^4pqr^2\log(c+dx)}{8bd^4} \\
&+ \frac{25(bc-ad)^4q^2r^2\log(c+dx)}{24bd^4} + \frac{(bc-ad)^4pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{2bd^4} \\
&+ \frac{(bc-ad)^4q^2r^2\log^2(c+dx)}{4bd^4} + \frac{(bc-ad)^3qr(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2bd^3} \\
&- \frac{(bc-ad)^2qr(a+bx)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4bd^2} \\
&+ \frac{(bc-ad)qr(a+bx)^3\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{6bd} \\
&- \frac{pr(a+bx)^4\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{8b} \\
&- \frac{qr(a+bx)^4\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{8b} \\
&- \frac{(bc-ad)^4qr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2bd^4} \\
&+ \frac{(a+bx)^4\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b} + \frac{(bc-ad)^4pqr^2\operatorname{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)}{2bd^4}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1853 vs. $2(805) = 1610$.

Time = 0.95 (sec) , antiderivative size = 1853, normalized size of antiderivative = 2.30

$$\int (a+bx)^3 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right) dx$$
$$= \frac{2a^4pqr^2}{b} - \frac{ab^2c^3pqr^2}{2d^3} + \frac{2a^2bc^2pqr^2}{d^2} - \frac{3a^3cpqr^2}{d} + \frac{1}{8}a^3p^2r^2x + \frac{37}{24}a^3pqr^2x - \frac{5b^3c^3pqr^2x}{8d^3}$$
$$+ \frac{9ab^2c^2pqr^2x}{4d^2} - \frac{35a^2bcpqr^2x}{12d} + 2a^3q^2r^2x - \frac{25b^3c^3q^2r^2x}{24d^3} + \frac{11ab^2c^2q^2r^2x}{3d^2} - \frac{9a^2bcq^2r^2x}{2d}$$
$$+ \frac{3}{16}a^2bp^2r^2x^2 + \frac{41}{48}a^2bpqr^2x^2 + \frac{3b^3c^2pqr^2x^2}{16d^2} - \frac{2ab^2cpqr^2x^2}{3d} + \frac{3}{4}a^2bq^2r^2x^2 + \frac{13b^3c^2q^2r^2x^2}{48d^2}$$
$$- \frac{5ab^2cq^2r^2x^2}{6d} + \frac{1}{8}ab^2p^2r^2x^3 + \frac{25}{72}ab^2pqr^2x^3 - \frac{7b^3cpqr^2x^3}{72d} + \frac{2}{9}ab^2q^2r^2x^3 - \frac{7b^3cq^2r^2x^3}{72d}$$
$$+ \frac{1}{32}b^3p^2r^2x^4 + \frac{1}{16}b^3pqr^2x^4 + \frac{1}{32}b^3q^2r^2x^4 - \frac{a^4p^2r^2\log^2(a+bx)}{4b} + \frac{2a^4pqr^2\log(c+dx)}{b}$$
$$+ \frac{b^3c^4pqr^2\log(c+dx)}{8d^4} - \frac{ab^2c^3pqr^2\log(c+dx)}{2d^3} + \frac{3a^2bc^2pqr^2\log(c+dx)}{4d^2}$$
$$- \frac{a^3cpqr^2\log(c+dx)}{2d} + \frac{25b^3c^4q^2r^2\log(c+dx)}{24d^4} - \frac{11ab^2c^3q^2r^2\log(c+dx)}{3d^3}$$
$$+ \frac{9a^2bc^2q^2r^2\log(c+dx)}{2d^2} - \frac{2a^3cq^2r^2\log(c+dx)}{d} + \frac{b^3c^4q^2r^2\log^2(c+dx)}{4d^4}$$
$$- \frac{ab^2c^3q^2r^2\log^2(c+dx)}{d^3} + \frac{3a^2bc^2q^2r^2\log^2(c+dx)}{2d^2} - \frac{a^3cq^2r^2\log^2(c+dx)}{d}$$
$$- \frac{2a^4pr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b} - \frac{1}{2}a^3prx\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)$$
$$- 2a^3qrx\log\left(e(f(a+bx)^p(c+dx)^q)^r\right) + \frac{b^3c^3qrx\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2d^3}$$
$$- \frac{2ab^2c^2qrx\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{d^2} + \frac{3a^2bcqrx\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{d}$$
$$- \frac{3}{4}a^2bprx^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right) - \frac{3}{2}a^2bqrx^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)$$
$$- \frac{b^3c^2qrx^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4d^2} + \frac{ab^2cqrx^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{d}$$
$$- \frac{1}{2}ab^2prx^3\log\left(e(f(a+bx)^p(c+dx)^q)^r\right) - \frac{2}{3}ab^2qrx^3\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)$$
$$+ \frac{b^3cqrx^3\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{6d} - \frac{1}{8}b^3prx^4\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)$$
$$- \frac{1}{8}b^3qrx^4\log\left(e(f(a+bx)^p(c+dx)^q)^r\right) - \frac{b^3c^4qr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2d^4}$$
$$+ \frac{2ab^2c^3qr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{d^3}$$
$$- \frac{3a^2bc^2qr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{d^2}$$
$$+ \frac{2a^3cqr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{d}$$
$$+ a^3x\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right) + \frac{3}{2}a^2bx^2\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)$$
$$+ ab^2x^3\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right) + \frac{1}{4}b^3x^4\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)$$
$$+ \frac{pr\log(a+bx)\Big(ad(5a^3d^3(9p-5q)+12b^3c^3q-42ab^2c^2dq+52a^2bcd^2q)r+12bc(b^3c^3-4ab^2c^2d+6a^2b}{24bd}$$

```
[In] Integrate[(a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]

[Out] (2*a^4*p*q*r^2)/b - (a*b^2*c^3*p*q*r^2)/(2*d^3) + (2*a^2*b*c^2*p*q*r^2)/d^2
 - (3*a^3*c*p*q*r^2)/d + (a^3*p^2*r^2*x)/8 + (37*a^3*p*q*r^2*x)/24 - (5*b^3
*c^3*p*q*r^2*x)/(8*d^3) + (9*a*b^2*c^2*p*q*r^2*x)/(4*d^2) - (35*a^2*b*c*p*q
*r^2*x)/(12*d) + 2*a^3*q^2*r^2*x - (25*b^3*c^3*q^2*r^2*x)/(24*d^3) + (11*a*
b^2*c^2*q^2*r^2*x)/(3*d^2) - (9*a^2*b*c*q^2*r^2*x)/(2*d) + (3*a^2*b*p^2*r^2
*x^2)/16 + (41*a^2*b*p*q*r^2*x^2)/48 + (3*b^3*c^2*p*q*r^2*x^2)/(16*d^2) - (
2*a*b^2*c*p*q*r^2*x^2)/(3*d) + (3*a^2*b*q^2*r^2*x^2)/4 + (13*b^3*c^2*q^2*r^
2*x^2)/(48*d^2) - (5*a*b^2*c*q^2*r^2*x^2)/(6*d) + (a*b^2*p^2*r^2*x^3)/8 + (
25*a*b^2*p*q*r^2*x^3)/72 - (7*b^3*c*p*q*r^2*x^3)/(72*d) + (2*a*b^2*q^2*r^2*
x^3)/9 - (7*b^3*c*q^2*r^2*x^3)/(72*d) + (b^3*p^2*r^2*x^4)/32 + (b^3*p*q*r^2
*x^4)/16 + (b^3*q^2*r^2*x^4)/32 - (a^4*p^2*r^2*Log[a + b*x]^2)/(4*b) + (2*a
^4*p*q*r^2*Log[c + d*x])/b + (b^3*c^4*p*q*r^2*Log[c + d*x])/(8*d^4) - (a*b^
2*c^3*p*q*r^2*Log[c + d*x])/(2*d^3) + (3*a^2*b*c^2*p*q*r^2*Log[c + d*x])/(4
*d^2) - (a^3*c*p*q*r^2*Log[c + d*x])/(2*d) + (25*b^3*c^4*q^2*r^2*Log[c + d*
x])/(24*d^4) - (11*a*b^2*c^3*q^2*r^2*Log[c + d*x])/(3*d^3) + (9*a^2*b*c^2*q
^2*r^2*Log[c + d*x])/(2*d^2) - (2*a^3*c*q^2*r^2*Log[c + d*x])/d + (b^3*c^4*
q^2*r^2*Log[c + d*x]^2)/(4*d^4) - (a*b^2*c^3*q^2*r^2*Log[c + d*x]^2)/d^3 +
(3*a^2*b*c^2*q^2*r^2*Log[c + d*x]^2)/(2*d^2) - (a^3*c*q^2*r^2*Log[c + d*x]^
2)/d - (2*a^4*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/b - (a^3*p*r*x*Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r])/2 - 2*a^3*q*r*x*Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r] + (b^3*c^3*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*d^3)
- (2*a*b^2*c^2*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d^2 + (3*a^2*b*c
*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d - (3*a^2*b*p*r*x^2*Log[e*(f*
(a + b*x)^p*(c + d*x)^q)^r])/4 - (3*a^2*b*q*r*x^2*Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r])/2 - (b^3*c^2*q*r*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(4*
d^2) + (a*b^2*c*q*r*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d - (a*b^2*p*
r*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/2 - (2*a*b^2*q*r*x^3*Log[e*(f*(
a + b*x)^p*(c + d*x)^q)^r])/3 + (b^3*c*q*r*x^3*Log[e*(f*(a + b*x)^p*(c + d*
x)^q)^r])/(6*d) - (b^3*p*r*x^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/8 - (b
^3*q*r*x^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/8 - (b^3*c^4*q*r*Log[c + d
*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*d^4) + (2*a*b^2*c^3*q*r*Log[c
+ d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d^3 - (3*a^2*b*c^2*q*r*Log[c +
 d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d^2 + (2*a^3*c*q*r*Log[c + d*x]
*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d + a^3*x*Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^2 + (3*a^2*b*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)/2 + a*b
^2*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 + (b^3*x^4*Log[e*(f*(a + b*x)
^p*(c + d*x)^q)^r]^2)/4 + (p*r*Log[a + b*x]*(a*d*(5*a^3*d^3*(9*p - 5*q) + 1
2*b^3*c^3*q - 42*a*b^2*c^2*d*q + 52*a^2*b*c*d^2*q)*r + 12*b*c*(b^3*c^3 - 4*
a*b^2*c^2*d + 6*a^2*b*c*d^2 - 4*a^3*d^3)*q*r*Log[c + d*x] - 12*(b*c - a*d)^
4*q*r*Log[(b*(c + d*x))/(b*c - a*d)] + 12*a^4*d^4*Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r]))/(24*b*d^4) - ((b*c - a*d)^4*p*q*r^2*PolyLog[2, (d*(a + b*x))/
(-(b*c) + a*d)])/(2*b*d^4)
```

Maple [F]

$$\int (bx+a)^3 \ln\left(e(f(bx+a)^p (dx+c)^q)^r\right)^2 dx$$

```
[In] int((b*x+a)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
[Out] int((b*x+a)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

Fricas [F]

$$\int (a+bx)^3 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right) dx = \int (bx+a)^3 \log\left(((bx+a)^p(dx+c)^q f)^r e\right)^2 dx$$

```
[In] integrate((b*x+a)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="fricas
")
[Out] integral((b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*log(((b*x + a)^p*(d*x +
c)^q*f)^r*e)^2, x)
```

Sympy [F]

$$\int (a+bx)^3 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right) dx = \int (a+bx)^3 \log\left(e(f(a+bx)^p (c+dx)^q)^r\right)^2 dx$$

```
[In] integrate((b*x+a)**3*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2,x)
[Out] Integral((a + b*x)**3*log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 1071, normalized size of antiderivative = 1.33

$$\int (a+bx)^3 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right) dx = \text{Too large to display}$$

```
[In] integrate((b*x+a)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="maxima
")
[Out] 1/4*(b^3*x^4 + 4*a*b^2*x^3 + 6*a^2*b*x^2 + 4*a^3*x)*log(((b*x + a)^p*(d*x +
 c)^q*f)^r*e)^2 + 1/24*(12*a^4*f*p*log(b*x + a)/b - (3*b^3*d^3*f*(p + q)*x^
4 + 4*(a*b^2*d^3*f*(3*p + 4*q) - b^3*c*d^2*f*q)*x^3 + 6*(3*a^2*b*d^3*f*(p +
 2*q) + b^3*c^2*d*f*q - 4*a*b^2*c*d^2*f*q)*x^2 + 12*(a^3*d^3*f*(p + 4*q) -
```

```
b^3*c^3*f*q + 4*a*b^2*c^2*d*f*q - 6*a^2*b*c*d^2*f*q)*x)/d^3 - 12*(b^3*c^4*f
*q - 4*a*b^2*c^3*d*f*q + 6*a^2*b*c^2*d^2*f*q - 4*a^3*c*d^3*f*q)*log(d*x + c
)/d^4)*r*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/f + 1/288*r^2*(12*((3*p*q + 2
5*q^2)*b^3*c^4*f^2 - 4*(3*p*q + 22*q^2)*a*b^2*c^3*d*f^2 + 18*(p*q + 6*q^2)*
a^2*b*c^2*d^2*f^2 - 12*(p*q + 4*q^2)*a^3*c*d^3*f^2)*log(d*x + c)/d^4 - 144*
(b^4*c^4*f^2*p*q - 4*a*b^3*c^3*d*f^2*p*q + 6*a^2*b^2*c^2*d^2*f^2*p*q - 4*a^
3*b*c*d^3*f^2*p*q + a^4*d^4*f^2*p*q)*(log(b*x + a)*log((b*d*x + a*d)/(b*c -
 a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))/(b*d^4) + (9*(p^2 + 2*p*q +
 q^2)*b^4*d^4*f^2*x^4 - 72*a^4*d^4*f^2*p^2*log(b*x + a)^2 - 4*(7*(p*q + q^2
)*b^4*c*d^3*f^2 - (9*p^2 + 25*p*q + 16*q^2)*a*b^3*d^4*f^2)*x^3 + 6*((9*p*q
+ 13*q^2)*b^4*c^2*d^2*f^2 - 8*(4*p*q + 5*q^2)*a*b^3*c*d^3*f^2 + (9*p^2 + 41
*p*q + 36*q^2)*a^2*b^2*d^4*f^2)*x^2 + 144*(b^4*c^4*f^2*p*q - 4*a*b^3*c^3*d*
f^2*p*q + 6*a^2*b^2*c^2*d^2*f^2*p*q - 4*a^3*b*c*d^3*f^2*p*q)*log(b*x + a)*l
og(d*x + c) + 72*(b^4*c^4*f^2*q^2 - 4*a*b^3*c^3*d*f^2*q^2 + 6*a^2*b^2*c^2*d
^2*f^2*q^2 - 4*a^3*b*c*d^3*f^2*q^2)*log(d*x + c)^2 - 12*(5*(3*p*q + 5*q^2)*
b^4*c^3*d*f^2 - 2*(27*p*q + 44*q^2)*a*b^3*c^2*d^2*f^2 + 2*(35*p*q + 54*q^2)
*a^2*b^2*c*d^3*f^2 - (3*p^2 + 37*p*q + 48*q^2)*a^3*b*d^4*f^2)*x + 12*(12*a*
b^3*c^3*d*f^2*p*q - 42*a^2*b^2*c^2*d^2*f^2*p*q + 52*a^3*b*c*d^3*f^2*p*q - (
3*p^2 + 25*p*q)*a^4*d^4*f^2)*log(b*x + a))/(b*d^4))/f^2
```

Giac [F]

$$\int (a+bx)^3 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \int (bx+a)^3 \log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2\,dx$$

```
[In] integrate((b*x+a)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="giac")
[Out] integrate((b*x + a)^3*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^3 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \int \ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)^2 (a+b\,x)^3\,dx$$

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(a + b*x)^3,x)
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(a + b*x)^3, x)
```

3.18 $\int (a+bx)^2 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$

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Optimal result

Integrand size = 31, antiderivative size = 686

$$
\begin{aligned}
&\int (a+bx)^2 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx \\
&= -\frac{a(bc-ad)pqr^2x}{3d}+\frac{2(bc-ad)^2pqr^2x}{9d^2}+\frac{5(bc-ad)^2q^2r^2x}{9d^2}+\frac{2(bc-ad)^2q(p+q)r^2x}{3d^2} \\
&\quad -\frac{b(bc-ad)pqr^2x^2}{6d}-\frac{(bc-ad)pqr^2(a+bx)^2}{9bd}-\frac{5(bc-ad)q^2r^2(a+bx)^2}{18bd} \\
&\quad +\frac{2p^2r^2(a+bx)^3}{27b}+\frac{4pqr^2(a+bx)^3}{27b}+\frac{2q^2r^2(a+bx)^3}{27b}-\frac{2(bc-ad)^3pqr^2\log(c+dx)}{9bd^3} \\
&\quad -\frac{11(bc-ad)^3q^2r^2\log(c+dx)}{9bd^3}-\frac{2(bc-ad)^3pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{3bd^3} \\
&\quad -\frac{(bc-ad)^3q^2r^2\log^2(c+dx)}{3bd^3}-\frac{2(bc-ad)^2qr(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3bd^2} \\
&\quad +\frac{(bc-ad)qr(a+bx)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3bd} \\
&\quad -\frac{2pr(a+bx)^3\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{9b} \\
&\quad -\frac{2qr(a+bx)^3\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{9b} \\
&\quad +\frac{2(bc-ad)^3qr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3bd^3} \\
&\quad +\frac{(a+bx)^3\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b}-\frac{2(bc-ad)^3pqr^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{3bd^3}
\end{aligned}
$$

```
[Out] -1/3*a*(-a*d+b*c)*p*q*r^2*x/d+2/9*(-a*d+b*c)^2*p*q*r^2*x/d^2+5/9*(-a*d+b*c)
^2*q^2*r^2*x/d^2+2/3*(-a*d+b*c)^2*q*(p+q)*r^2*x/d^2-1/6*b*(-a*d+b*c)*p*q*r^
2*x^2/d-1/9*(-a*d+b*c)*p*q*r^2*(b*x+a)^2/b/d-5/18*(-a*d+b*c)*q^2*r^2*(b*x+a
```

```
)^2/b/d+2/27*p^2*r^2*(b*x+a)^3/b+4/27*p*q*r^2*(b*x+a)^3/b+2/27*q^2*r^2*(b*x
+a)^3/b-2/9*(-a*d+b*c)^3*p*q*r^2*ln(d*x+c)/b/d^3-11/9*(-a*d+b*c)^3*q^2*r^2*
ln(d*x+c)/b/d^3-2/3*(-a*d+b*c)^3*p*q*r^2*ln(-d*(b*x+a)/(-a*d+b*c))*ln(d*x+c
)/b/d^3-1/3*(-a*d+b*c)^3*q^2*r^2*ln(d*x+c)^2/b/d^3-2/3*(-a*d+b*c)^2*q*r*(b*
x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d^2+1/3*(-a*d+b*c)*q*r*(b*x+a)^2*ln(
e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d-2/9*p*r*(b*x+a)^3*ln(e*(f*(b*x+a)^p*(d*x+c
)^q)^r)/b-2/9*q*r*(b*x+a)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b+2/3*(-a*d+b*c
)^3*q*r*ln(d*x+c)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d^3+1/3*(b*x+a)^3*ln(e*
(f*(b*x+a)^p*(d*x+c)^q)^r)^2/b-2/3*(-a*d+b*c)^3*p*q*r^2*polylog(2,b*(d*x+c)
/(-a*d+b*c))/b/d^3
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 686, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.452$, Rules used = {2584, 2581, 32, 45, 2594, 2579, 31, 8, 2580, 2441, 2440, 2438, 2437, 2338}

$$\int (a+bx)^2 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right) dx$$
$$= \frac{2qr(bc-ad)^3\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3bd^3} - \frac{2pqr^2(bc-ad)^3 \text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{3bd^3} - \frac{2pqr^2(bc-ad)^3\log(c+dx)}{9bd^3} - \frac{2pqr^2(bc-ad)^3\log(c+dx)\log\left(-\frac{d(a+bx)}{bc-ad}\right)}{3bd^3} - \frac{q^2r^2(bc-ad)^3\log^2(c+dx)}{3bd^3} - \frac{11q^2r^2(bc-ad)^3\log(c+dx)}{9bd^3} - \frac{2qr(a+bx)(bc-ad)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3bd^2} + \frac{2pqr^2x(bc-ad)^2}{9d^2} + \frac{2qr^2x(p+q)(bc-ad)^2}{3d^2} + \frac{5q^2r^2x(bc-ad)^2}{9d^2} + \frac{(a+bx)^3\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b} + \frac{qr(a+bx)^2(bc-ad)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3bd} - \frac{2pr(a+bx)^3\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{9b} - \frac{2qr(a+bx)^3\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{9b} - \frac{bpqr^2x^2(bc-ad)}{6d} - \frac{pqr^2(a+bx)^2(bc-ad)}{9bd} - \frac{apqr^2x(bc-ad)}{3d} - \frac{5q^2r^2(a+bx)^2(bc-ad)}{18bd} + \frac{2p^2r^2(a+bx)^3}{27b} + \frac{4pqr^2(a+bx)^3}{27b} + \frac{2q^2r^2(a+bx)^3}{27b}$$

```
[In] Int[(a + b*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] -1/3*(a*(b*c - a*d)*p*q*r^2*x)/d + (2*(b*c - a*d)^2*p*q*r^2*x)/(9*d^2) + (5
*(b*c - a*d)^2*q^2*r^2*x)/(9*d^2) + (2*(b*c - a*d)^2*q*(p + q)*r^2*x)/(3*d^
2) - (b*(b*c - a*d)*p*q*r^2*x^2)/(6*d) - ((b*c - a*d)*p*q*r^2*(a + b*x)^2)/
(9*b*d) - (5*(b*c - a*d)*q^2*r^2*(a + b*x)^2)/(18*b*d) + (2*p^2*r^2*(a + b*
x)^3)/(27*b) + (4*p*q*r^2*(a + b*x)^3)/(27*b) + (2*q^2*r^2*(a + b*x)^3)/(27
*b) - (2*(b*c - a*d)^3*p*q*r^2*Log[c + d*x])/(9*b*d^3) - (11*(b*c - a*d)^3*
q^2*r^2*Log[c + d*x])/(9*b*d^3) - (2*(b*c - a*d)^3*p*q*r^2*Log[-((d*(a + b*
x))/(b*c - a*d))]*Log[c + d*x])/(3*b*d^3) - ((b*c - a*d)^3*q^2*r^2*Log[c +
d*x]^2)/(3*b*d^3) - (2*(b*c - a*d)^2*q*r*(a + b*x)*Log[e*(f*(a + b*x)^p*(c
+ d*x)^q)^r])/(3*b*d^2) + ((b*c - a*d)*q*r*(a + b*x)^2*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r])/(3*b*d) - (2*p*r*(a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*
x)^q)^r])/(9*b) - (2*q*r*(a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/
(9*b) + (2*(b*c - a*d)^3*q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)
^r])/(3*b*d^3) + ((a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)/(3*b)
 - (2*(b*c - a*d)^3*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(3*b*d^3
)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
```

```
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2579

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[(a + b*x)*(Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s/b), x] + (Dist[q*r*s*((b*c - a*d)/b), Int[Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r]^(s - 1)/(c + d*x), x], x] - Dist[r*s*(p + q), Int[Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]^(s - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p, q,
r, s}, x] && NeQ[b*c - a*d, 0] && NeQ[p + q, 0] && IGtQ[s, 0] && LtQ[s, 4]
```

Rule 2580

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[Log[g + h*x]*(Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]/h), x] + (-Dist[b*p*(r/h), Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[d*q*(r/h), Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
```

```
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 2584

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1
)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(h*(m + 1))), x] + (-Dist[b*p*r*(
s/(h*(m + 1))), Int[(g + h*x)^(m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1)/(a + b*x)), x], x] - Dist[d*q*r*(s/(h*(m + 1))), Int[(g + h*x)^(m
+ 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(c + d*x)), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2594

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(a+bx)^3\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b} \\
&\quad -\frac{1}{3}(2pr)\int(a+bx)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)dx \\
&\quad -\frac{(2dqr)\int\frac{(a+bx)^3\log(e(f(a+bx)^p(c+dx)^q)^r)}{c+dx}dx}{3b} \\
&= -\frac{2pr(a+bx)^3\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{9b} \\
&\quad +\frac{(a+bx)^3\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b} \\
&\quad -\frac{(2dqr)\int\left(\frac{b(bc-ad)^2\log(e(f(a+bx)^p(c+dx)^q)^r)}{d^3}-\frac{b(bc-ad)(a+bx)\log(e(f(a+bx)^p(c+dx)^q)^r)}{d^2}+\frac{b(a+bx)^2\log(e(f(a+bx)^p}{d}\right.}{3b} \\
&\quad +\frac{1}{9}\left(2p^2r^2\right)\int(a+bx)^2\,dx+\frac{\left(2dpqr^2\right)\int\frac{(a+bx)^3}{c+dx}dx}{9b}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{2p^2r^2(a+bx)^3}{27b} - \frac{2pr(a+bx)^3\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{9b} \\
&\quad + \frac{(a+bx)^3\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b} \\
&\quad - \frac{1}{3}(2qr)\int (a+bx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx \\
&\quad + \frac{(2(bc-ad)qr)\int (a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx}{3d} \\
&\quad - \frac{(2(bc-ad)^2qr)\int \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx}{3d^2} \\
&\quad + \frac{(2(bc-ad)^3qr)\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{c+dx}\,dx}{3bd^2} \\
&\quad + \frac{(2dpqr^2)\int \left(\frac{b(bc-ad)^2}{d^3} - \frac{b(bc-ad)(a+bx)}{d^2} + \frac{b(a+bx)^2}{d} + \frac{(-bc+ad)^3}{d^3(c+dx)}\right)\,dx}{9b} \\
&= \frac{2(bc-ad)^2pqr^2x}{9d^2} - \frac{(bc-ad)pqr^2(a+bx)^2}{9bd} + \frac{2p^2r^2(a+bx)^3}{27b} \\
&\quad + \frac{2pqr^2(a+bx)^3}{27b} - \frac{2(bc-ad)^3pqr^2\log(c+dx)}{9bd^3} \\
&\quad - \frac{2(bc-ad)^2qr(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3bd^2} \\
&\quad + \frac{(bc-ad)qr(a+bx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3bd} \\
&\quad - \frac{2pr(a+bx)^3\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{9b} \\
&\quad - \frac{2qr(a+bx)^3\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{9b} \\
&\quad + \frac{2(bc-ad)^3qr\log(c+dx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3bd^3} \\
&\quad + \frac{(a+bx)^3\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b} + \frac{1}{9}\left(2pqr^2\right)\int (a+bx)^2\,dx \\
&\quad - \frac{((bc-ad)pqr^2)\int (a+bx)\,dx}{3d} - \frac{(2(bc-ad)^3pqr^2)\int \frac{\log(c+dx)}{a+bx}\,dx}{3d^3} \\
&\quad + \frac{(2dq^2r^2)\int \frac{(a+bx)^3}{c+dx}\,dx}{9b} - \frac{((bc-ad)q^2r^2)\int \frac{(a+bx)^2}{c+dx}\,dx}{3b} - \frac{(2(bc-ad)^3q^2r^2)\int \frac{1}{c+dx}\,dx}{3bd^2} \\
&\quad - \frac{(2(bc-ad)^3q^2r^2)\int \frac{\log(c+dx)}{c+dx}\,dx}{3bd^2} + \frac{(2(bc-ad)^2q(p+q)r^2)\int 1\,dx}{3d^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{a(bc-ad)pqr^2x}{3d} + \frac{2(bc-ad)^2pqr^2x}{9d^2} + \frac{2(bc-ad)^2q(p+q)r^2x}{3d^2} \\
&\quad - \frac{b(bc-ad)pqr^2x^2}{6d} - \frac{(bc-ad)pqr^2(a+bx)^2}{9bd} + \frac{2p^2r^2(a+bx)^3}{27b} \\
&\quad + \frac{4pqr^2(a+bx)^3}{27b} - \frac{2(bc-ad)^3pqr^2\log(c+dx)}{9bd^3} \\
&\quad - \frac{2(bc-ad)^3q^2r^2\log(c+dx)}{3bd^3} - \frac{2(bc-ad)^3pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{3bd^3} \\
&\quad - \frac{2(bc-ad)^2qr(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3bd^2} \\
&\quad + \frac{(bc-ad)qr(a+bx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3bd} \\
&\quad - \frac{2pr(a+bx)^3\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{9b} \\
&\quad - \frac{2qr(a+bx)^3\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{9b} \\
&\quad + \frac{2(bc-ad)^3qr\log(c+dx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3bd^3} \\
&\quad + \frac{(a+bx)^3\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b} + \frac{\left(2(bc-ad)^3pqr^2\right)\int\frac{\log\left(\frac{d(a+bx)}{-bc+ad}\right)}{c+dx}\,dx}{3bd^2} \\
&\quad + \frac{\left(2dq^2r^2\right)\int\left(\frac{b(bc-ad)^2}{d^3}-\frac{b(bc-ad)(a+bx)}{d^2}+\frac{b(a+bx)^2}{d}+\frac{(-bc+ad)^3}{d^3(c+dx)}\right)dx}{9b} \\
&\quad - \frac{\left((bc-ad)q^2r^2\right)\int\left(-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}+\frac{(-bc+ad)^2}{d^2(c+dx)}\right)dx}{3b} \\
&\quad - \frac{\left(2(bc-ad)^3q^2r^2\right)\mathrm{Subst}\left(\int\frac{\log(x)}{x}\,dx,x,c+dx\right)}{3bd^3}
\end{aligned}
$$

$$
\begin{aligned}
= & -\frac{a(bc-ad)pqr^2x}{3d}+\frac{2(bc-ad)^2pqr^2x}{9d^2}+\frac{5(bc-ad)^2q^2r^2x}{9d^2} \\
& +\frac{2(bc-ad)^2q(p+q)r^2x}{3d^2}-\frac{b(bc-ad)pqr^2x^2}{6d}-\frac{(bc-ad)pqr^2(a+bx)^2}{9bd} \\
& -\frac{5(bc-ad)q^2r^2(a+bx)^2}{18bd}+\frac{2p^2r^2(a+bx)^3}{27b}+\frac{4pqr^2(a+bx)^3}{27b} \\
& +\frac{2q^2r^2(a+bx)^3}{27b}-\frac{2(bc-ad)^3pqr^2\log(c+dx)}{9bd^3}-\frac{11(bc-ad)^3q^2r^2\log(c+dx)}{9bd^3} \\
& -\frac{2(bc-ad)^3pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{3bd^3}-\frac{(bc-ad)^3q^2r^2\log^2(c+dx)}{3bd^3} \\
& -\frac{2(bc-ad)^2qr(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3bd^2} \\
& +\frac{(bc-ad)qr(a+bx)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3bd} \\
& -\frac{2pr(a+bx)^3\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{9b} \\
& -\frac{2qr(a+bx)^3\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{9b} \\
& +\frac{2(bc-ad)^3qr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3bd^3} \\
& +\frac{(a+bx)^3\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b} \\
& +\frac{(2(bc-ad)^3pqr^2)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{bx}{-bc+ad}\right)}{x}\,dx,x,c+dx\right)}{3bd^3}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{a(bc-ad)pqr^2x}{3d} + \frac{2(bc-ad)^2pqr^2x}{9d^2} + \frac{5(bc-ad)^2q^2r^2x}{9d^2} \\
&\quad + \frac{2(bc-ad)^2q(p+q)r^2x}{3d^2} - \frac{b(bc-ad)pqr^2x^2}{6d} - \frac{(bc-ad)pqr^2(a+bx)^2}{9bd} \\
&\quad - \frac{5(bc-ad)q^2r^2(a+bx)^2}{18bd} + \frac{2p^2r^2(a+bx)^3}{27b} + \frac{4pqr^2(a+bx)^3}{27b} \\
&\quad + \frac{2q^2r^2(a+bx)^3}{27b} - \frac{2(bc-ad)^3pqr^2\log(c+dx)}{9bd^3} - \frac{11(bc-ad)^3q^2r^2\log(c+dx)}{9bd^3} \\
&\quad - \frac{2(bc-ad)^3pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{3bd^3} - \frac{(bc-ad)^3q^2r^2\log^2(c+dx)}{3bd^3} \\
&\quad - \frac{2(bc-ad)^2qr(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3bd^2} \\
&\quad + \frac{(bc-ad)qr(a+bx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3bd} \\
&\quad - \frac{2pr(a+bx)^3\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{9b} \\
&\quad - \frac{2qr(a+bx)^3\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{9b} \\
&\quad + \frac{2(bc-ad)^3qr\log(c+dx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3bd^3} \\
&\quad + \frac{(a+bx)^3\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b} - \frac{2(bc-ad)^3pqr^2\mathrm{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)}{3bd^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 1211, normalized size of antiderivative = 1.77

$$
\begin{aligned}
\int (a+bx)^2\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx = \frac{1}{54}\Bigg(&\frac{108a^3pqr^2}{b}+\frac{36abc^2pqr^2}{d^2}-\frac{108a^2cpqr^2}{d} \\
&+12a^2p^2r^2x+102a^2pqr^2x+\frac{48b^2c^2pqr^2x}{d^2}-\frac{126abcpqr^2x}{d}+108a^2q^2r^2x+\frac{66b^2c^2q^2r^2x}{d^2} \\
&-\frac{162abcq^2r^2x}{d}+12abp^2r^2x^2+39abpqr^2x^2-\frac{15b^2cpqr^2x^2}{d}+27abq^2r^2x^2-\frac{15b^2cq^2r^2x^2}{d} \\
&+4b^2p^2r^2x^3+8b^2pqr^2x^3+4b^2q^2r^2x^3-\frac{18a^3p^2r^2\log^2(a+bx)}{b}+\frac{108a^3pqr^2\log(c+dx)}{b} \\
&-\frac{12b^2c^3pqr^2\log(c+dx)}{d^3}+\frac{36abc^2pqr^2\log(c+dx)}{d^2}-\frac{36a^2cpqr^2\log(c+dx)}{d} \\
&-\frac{66b^2c^3q^2r^2\log(c+dx)}{d^3}+\frac{162abc^2q^2r^2\log(c+dx)}{d^2}-\frac{108a^2cq^2r^2\log(c+dx)}{d} \\
&-\frac{18b^2c^3q^2r^2\log^2(c+dx)}{d^3}+\frac{54abc^2q^2r^2\log^2(c+dx)}{d^2}-\frac{54a^2cq^2r^2\log^2(c+dx)}{d} \\
&-\frac{108a^3pr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b}-36a^2prx\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) \\
&-108a^2qrx\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)-\frac{36b^2c^2qrx\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{d^2} \\
&+\frac{108abcqrx\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{d}-36abprx^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) \\
&-54abqrx^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)+\frac{18b^2cqrx^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{d} \\
&-12b^2prx^3\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)-12b^2qrx^3\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) \\
&+\frac{36b^2c^3qr\log(c+dx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{d^3} \\
&-\frac{108abc^2qr\log(c+dx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{d^2} \\
&+\frac{108a^2cqr\log(c+dx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{d}+54a^2x\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) \\
&+54abx^2\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)+18b^2x^3\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) \\
&+\frac{6pr\log(a+bx)\left(ad(a^2d^2(16p-11q)-6b^2c^2q+15abcdq)\,r-6bc(b^2c^2-3abcd+3a^2d^2)\,qr\log(c+dx)-\right.}{bd^3} \\
&+\frac{36(bc-ad)^3pqr^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{-bc+ad}\right)}{bd^3}\Bigg)
\end{aligned}
$$

```
[In] Integrate[(a + b*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] ((108*a^3*p*q*r^2)/b + (36*a*b*c^2*p*q*r^2)/d^2 - (108*a^2*c*p*q*r^2)/d + 1
2*a^2*p^2*r^2*x + 102*a^2*p*q*r^2*x + (48*b^2*c^2*p*q*r^2*x)/d^2 - (126*a*b
*c*p*q*r^2*x)/d + 108*a^2*q^2*r^2*x + (66*b^2*c^2*q^2*r^2*x)/d^2 - (162*a*b
*c*q^2*r^2*x)/d + 12*a*b*p^2*r^2*x^2 + 39*a*b*p*q*r^2*x^2 - (15*b^2*c*p*q*r
^2*x^2)/d + 27*a*b*q^2*r^2*x^2 - (15*b^2*c*q^2*r^2*x^2)/d + 4*b^2*p^2*r^2*x
^3 + 8*b^2*p*q*r^2*x^3 + 4*b^2*q^2*r^2*x^3 - (18*a^3*p^2*r^2*Log[a + b*x]^2
)/b + (108*a^3*p*q*r^2*Log[c + d*x])/b - (12*b^2*c^3*p*q*r^2*Log[c + d*x])/
d^3 + (36*a*b*c^2*p*q*r^2*Log[c + d*x])/d^2 - (36*a^2*c*p*q*r^2*Log[c + d*x
])/d - (66*b^2*c^3*q^2*r^2*Log[c + d*x])/d^3 + (162*a*b*c^2*q^2*r^2*Log[c +
 d*x])/d^2 - (108*a^2*c*q^2*r^2*Log[c + d*x])/d - (18*b^2*c^3*q^2*r^2*Log[c
 + d*x]^2)/d^3 + (54*a*b*c^2*q^2*r^2*Log[c + d*x]^2)/d^2 - (54*a^2*c*q^2*r^
2*Log[c + d*x]^2)/d - (108*a^3*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/b
- 36*a^2*p*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 108*a^2*q*r*x*Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r] - (36*b^2*c^2*q*r*x*Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r])/d^2 + (108*a*b*c*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d
 - 36*a*b*p*r*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 54*a*b*q*r*x^2*Log
[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (18*b^2*c*q*r*x^2*Log[e*(f*(a + b*x)^p*
(c + d*x)^q)^r])/d - 12*b^2*p*r*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] -
12*b^2*q*r*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (36*b^2*c^3*q*r*Log[c
 + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d^3 - (108*a*b*c^2*q*r*Log[c
+ d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d^2 + (108*a^2*c*q*r*Log[c + d
*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d + 54*a^2*x*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^2 + 54*a*b*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 + 18
*b^2*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 + (6*p*r*Log[a + b*x]*(a*d*
(a^2*d^2*(16*p - 11*q) - 6*b^2*c^2*q + 15*a*b*c*d*q)*r - 6*b*c*(b^2*c^2 - 3
*a*b*c*d + 3*a^2*d^2)*q*r*Log[c + d*x] + 6*(b*c - a*d)^3*q*r*Log[(b*(c + d*
x))/(b*c - a*d)] + 6*a^3*d^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(b*d^3)
 + (36*(b*c - a*d)^3*p*q*r^2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*d
^3))/54
```

Maple [F]

$$\int (bx+a)^2 \ln \left(e(f(bx+a)^p \left(dx+c\right)^q)^r\right)^2 dx$$

```
[In] int((b*x+a)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

```
[Out] int((b*x+a)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

Fricas [F]

$$\int (a+bx)^2 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx = \int (bx+a)^2 \log\left(\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2\,dx$$

```
[In] integrate((b*x+a)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="fricas
")
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2,
x)
```

Sympy [F]

$$\int (a+bx)^2 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx = \int (a+bx)^2 \log\left(e\left(f(a+bx)^p\,(c+dx)^q\right)^r\right)^2 dx$$

```
[In] integrate((b*x+a)**2*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2,x)
[Out] Integral((a + b*x)**2*log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 769, normalized size of antiderivative = 1.12

$$\int (a+bx)^2 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx$$
$$= \frac{1}{3}\left(b^2x^3+3\,abx^2+3\,a^2x\right)\log\left(\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2$$
$$+ \frac{\left(\frac{6\,a^3fp\log(bx+a)}{b} - \frac{2\,b^2d^2f(p+q)x^3+3\left(abd^2f(2\,p+3\,q)-b^2cdfq\right)x^2+6\left(a^2d^2f(p+3\,q)+b^2c^2fq-3\,abcdfq\right)x}{d^2} + \frac{6\left(b^2c^3fq-3\,abc^2dfq+3\,a^2c\cdots\right.}{d^3}\cdots\right.}{9\,f}$$
$$- \frac{r^2\left(\frac{6\left(\left(2\,pq+11\,q^2\right)b^2c^3f^2-3\left(2\,pq+9\,q^2\right)abc^2df^2+6\left(pq+3\,q^2\right)a^2cd^2f^2\right)\log(dx+c)}{d^3} - \frac{36\left(b^3c^3f^2pq-3\,ab^2c^2df^2pq+3\,a^2bcd^2f^2pq-a^3d^3f^2pq\cdots\right.}{bd^3}\cdots\right.}{\cdots}$$

```
[In] integrate((b*x+a)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="maxima
")
[Out] 1/3*(b^2*x^3 + 3*a*b*x^2 + 3*a^2*x)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2
+ 1/9*(6*a^3*f*p*log(b*x + a)/b - (2*b^2*d^2*f*(p + q)*x^3 + 3*(a*b*d^2*f*(
2*p + 3*q) - b^2*c*d*f*q)*x^2 + 6*(a^2*d^2*f*(p + 3*q) + b^2*c^2*f*q - 3*a*
b*c*d*f*q)*x)/d^2 + 6*(b^2*c^3*f*q - 3*a*b*c^2*d*f*q + 3*a^2*c*d^2*f*q)*log
```

```
(d*x + c)/d^3)*r*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/f - 1/54*r^2*(6*((2*p
*q + 11*q^2)*b^2*c^3*f^2 - 3*(2*p*q + 9*q^2)*a*b*c^2*d*f^2 + 6*(p*q + 3*q^2
)*a^2*c*d^2*f^2)*log(d*x + c)/d^3 - 36*(b^3*c^3*f^2*p*q - 3*a*b^2*c^2*d*f^2
*p*q + 3*a^2*b*c*d^2*f^2*p*q - a^3*d^3*f^2*p*q)*(log(b*x + a)*log((b*d*x +
a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))/(b*d^3) - (4*(p^
2 + 2*p*q + q^2)*b^3*d^3*f^2*x^3 - 18*a^3*d^3*f^2*p^2*log(b*x + a)^2 - 3*(5
*(p*q + q^2)*b^3*c*d^2*f^2 - (4*p^2 + 13*p*q + 9*q^2)*a*b^2*d^3*f^2)*x^2 -
36*(b^3*c^3*f^2*p*q - 3*a*b^2*c^2*d*f^2*p*q + 3*a^2*b*c*d^2*f^2*p*q)*log(b*
x + a)*log(d*x + c) - 18*(b^3*c^3*f^2*q^2 - 3*a*b^2*c^2*d*f^2*q^2 + 3*a^2*b
*c*d^2*f^2*q^2)*log(d*x + c)^2 + 6*((8*p*q + 11*q^2)*b^3*c^2*d*f^2 - 3*(7*p
*q + 9*q^2)*a*b^2*c*d^2*f^2 + (2*p^2 + 17*p*q + 18*q^2)*a^2*b*d^3*f^2)*x -
6*(6*a*b^2*c^2*d*f^2*p*q - 15*a^2*b*c*d^2*f^2*p*q + (2*p^2 + 11*p*q)*a^3*d^
3*f^2)*log(b*x + a))/(b*d^3))/f^2
```

Giac [F]

$$\int (a+bx)^2 \log^2 \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx = \int (bx+a)^2 \log \left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2 \, dx$$

```
[In] integrate((b*x+a)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="giac")
[Out] integrate((b*x + a)^2*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^2 \log^2 \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx = \int \ln \left(e \left(f \left(a+b\,x\right)^p \left(c+d\,x\right)^q\right)^r\right)^2 \left(a+b\,x\right)^2 \,dx$$

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(a + b*x)^2,x)
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(a + b*x)^2, x)
```

3.19 $\int (a+bx)\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx$

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Optimal result

Integrand size = 29, antiderivative size = 540

$$\begin{aligned}
&\int (a+bx)\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx \\
&= \frac{1}{2}ap^2r^2x+\frac{1}{2}apqr^2x-\frac{(bc-ad)pqr^2x}{2d}-\frac{(bc-ad)q^2r^2x}{2d}-\frac{(bc-ad)q(p+q)r^2x}{d} \\
&\quad+\frac{1}{4}bp^2r^2x^2+\frac{1}{4}bpqr^2x^2+\frac{pqr^2(a+bx)^2}{4b}+\frac{q^2r^2(a+bx)^2}{4b}+\frac{(bc-ad)^2pqr^2\log(c+dx)}{2bd^2} \\
&\quad+\frac{3(bc-ad)^2q^2r^2\log(c+dx)}{2bd^2}+\frac{(bc-ad)^2pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{bd^2} \\
&\quad+\frac{(bc-ad)^2q^2r^2\log^2(c+dx)}{2bd^2}+\frac{(bc-ad)qr(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{bd} \\
&\quad-\frac{pr(a+bx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b}-\frac{qr(a+bx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b} \\
&\quad-\frac{(bc-ad)^2qr\log(c+dx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{bd^2} \\
&\quad+\frac{(a+bx)^2\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b}+\frac{(bc-ad)^2pqr^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{bd^2}
\end{aligned}$$

```
[Out] 1/2*a*p^2*r^2*x+1/2*a*p*q*r^2*x-1/2*(-a*d+b*c)*p*q*r^2*x/d-1/2*(-a*d+b*c)*q
^2*r^2*x/d-(-a*d+b*c)*q*(p+q)*r^2*x/d+1/4*b*p^2*r^2*x^2+1/4*b*p*q*r^2*x^2+1
/4*p*q*r^2*(b*x+a)^2/b+1/4*q^2*r^2*(b*x+a)^2/b+1/2*(-a*d+b*c)^2*p*q*r^2*ln(
d*x+c)/b/d^2+3/2*(-a*d+b*c)^2*q^2*r^2*ln(d*x+c)/b/d^2+(-a*d+b*c)^2*p*q*r^2*
ln(-d*(b*x+a)/(-a*d+b*c))*ln(d*x+c)/b/d^2+1/2*(-a*d+b*c)^2*q^2*r^2*ln(d*x+c
)^2/b/d^2+(-a*d+b*c)*q*r*(b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d-1/2*p*
r*(b*x+a)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b-1/2*q*r*(b*x+a)^2*ln(e*(f*(b*
x+a)^p*(d*x+c)^q)^r)/b-(-a*d+b*c)^2*q*r*ln(d*x+c)*ln(e*(f*(b*x+a)^p*(d*x+c)
^q)^r)/b/d^2+1/2*(b*x+a)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/b+(-a*d+b*c)^2
*p*q*r^2*polylog(2,b*(d*x+c)/(-a*d+b*c))/b/d^2
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 540, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.448$, Rules used = {2584, 2581, 45, 2594, 2579, 31, 8, 2580, 2441, 2440, 2438, 2437, 2338}

$$\begin{aligned}
&\int (a+bx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx \\
&= -\frac{qr(bc-ad)^2\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{bd^2} \\
&\quad+\frac{pqr^2(bc-ad)^2\,\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{bd^2}+\frac{pqr^2(bc-ad)^2\log(c+dx)}{2bd^2} \\
&\quad+\frac{pqr^2(bc-ad)^2\log(c+dx)\log\left(-\frac{d(a+bx)}{bc-ad}\right)}{bd^2}+\frac{q^2r^2(bc-ad)^2\log^2(c+dx)}{2bd^2} \\
&\quad+\frac{3q^2r^2(bc-ad)^2\log(c+dx)}{2bd^2}+\frac{(a+bx)^2\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b} \\
&\quad+\frac{qr(a+bx)(bc-ad)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{bd} \\
&\quad-\frac{pr(a+bx)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b}-\frac{qr(a+bx)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b} \\
&\quad-\frac{pqr^2x(bc-ad)}{2d}-\frac{qr^2x(p+q)(bc-ad)}{d}-\frac{q^2r^2x(bc-ad)}{2d}+\frac{pqr^2(a+bx)^2}{4b} \\
&\quad+\frac{q^2r^2(a+bx)^2}{4b}+\frac{1}{2}ap^2r^2x+\frac{1}{2}apqr^2x+\frac{1}{4}bp^2r^2x^2+\frac{1}{4}bpqr^2x^2
\end{aligned}$$

```
[In] Int[(a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] (a*p^2*r^2*x)/2 + (a*p*q*r^2*x)/2 - ((b*c - a*d)*p*q*r^2*x)/(2*d) - ((b*c -
 a*d)*q^2*r^2*x)/(2*d) - ((b*c - a*d)*q*(p + q)*r^2*x)/d + (b*p^2*r^2*x^2)/
4 + (b*p*q*r^2*x^2)/4 + (p*q*r^2*(a + b*x)^2)/(4*b) + (q^2*r^2*(a + b*x)^2)
/(4*b) + ((b*c - a*d)^2*p*q*r^2*Log[c + d*x])/(2*b*d^2) + (3*(b*c - a*d)^2*
q^2*r^2*Log[c + d*x])/(2*b*d^2) + ((b*c - a*d)^2*p*q*r^2*Log[-((d*(a + b*x)
)/(b*c - a*d))]*Log[c + d*x])/(b*d^2) + ((b*c - a*d)^2*q^2*r^2*Log[c + d*x]
^2)/(2*b*d^2) + ((b*c - a*d)*q*r*(a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q
)^r])/(b*d) - (p*r*(a + b*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*b)
- (q*r*(a + b*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*b) - ((b*c - a*
d)^2*q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(b*d^2) + ((a +
 b*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)/(2*b) + ((b*c - a*d)^2*p*q*
r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(b*d^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2579

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[(a + b*x)*(Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s/b), x] + (Dist[q*r*s*((b*c - a*d)/b), Int[Log[e*(f*(a + b*x)^p*(c +
```

```
 d*x)^q)^r]^(s - 1)/(c + d*x), x], x] - Dist[r*s*(p + q), Int[Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]^(s - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p, q,
r, s}, x] && NeQ[b*c - a*d, 0] && NeQ[p + q, 0] && IGtQ[s, 0] && LtQ[s, 4]
```

Rule 2580

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[Log[g + h*x]*(Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]/h), x] + (-Dist[b*p*(r/h), Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[d*q*(r/h), Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 2584

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1
)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(h*(m + 1))), x] + (-Dist[b*p*r*(
s/(h*(m + 1))), Int[(g + h*x)^(m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1)/(a + b*x)), x], x] - Dist[d*q*r*(s/(h*(m + 1))), Int[(g + h*x)^(m
+ 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(c + d*x)), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2594

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rubi steps

$$\text{integral} = \frac{(a+bx)^2 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b}$$
$$-(pr)\int (a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx$$
$$-\frac{(dqr)\int \frac{(a+bx)^2\log(e(f(a+bx)^p(c+dx)^q)^r)}{c+dx}\,dx}{b}$$
$$= -\frac{pr(a+bx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b}$$
$$+\frac{(a+bx)^2\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b}$$
$$-\frac{(dqr)\int \left(-\frac{b(bc-ad)\log(e(f(a+bx)^p(c+dx)^q)^r)}{d^2}+\frac{b(a+bx)\log(e(f(a+bx)^p(c+dx)^q)^r)}{d}+\frac{(-bc+ad)^2\log(e(f(a+bx)^p(c+dx)^q}{d^2(c+dx)}\right.}{b}$$
$$+\frac{1}{2}\left(p^2r^2\right)\int (a+bx)\,dx+\frac{(dpqr^2)\int \frac{(a+bx)^2}{c+dx}\,dx}{2b}$$
$$= \frac{1}{2}ap^2r^2x+\frac{1}{4}bp^2r^2x^2-\frac{pr(a+bx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b}$$
$$+\frac{(a+bx)^2\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b}$$
$$-(qr)\int (a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx$$
$$+\frac{((bc-ad)qr)\int \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx}{d}$$
$$-\frac{((bc-ad)^2qr)\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{c+dx}\,dx}{bd}$$
$$+\frac{(dpqr^2)\int \left(-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}+\frac{(-bc+ad)^2}{d^2(c+dx)}\right)\,dx}{2b}$$

$$
\begin{aligned}
&= \frac{1}{2}ap^2r^2x - \frac{(bc-ad)pqr^2x}{2d} + \frac{1}{4}bp^2r^2x^2 + \frac{pqr^2(a+bx)^2}{4b} \\
&\quad + \frac{(bc-ad)^2pqr^2\log(c+dx)}{2bd^2} + \frac{(bc-ad)qr(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{bd} \\
&\quad - \frac{pr(a+bx)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b} \\
&\quad - \frac{qr(a+bx)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b} \\
&\quad - \frac{(bc-ad)^2qr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{bd^2} \\
&\quad + \frac{(a+bx)^2\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b} + \frac{1}{2}(pqr^2)\int(a+bx)\,dx \\
&\quad + \frac{((bc-ad)^2pqr^2)\int\frac{\log(c+dx)}{a+bx}\,dx}{d^2} + \frac{(dq^2r^2)\int\frac{(a+bx)^2}{c+dx}\,dx}{2b} + \frac{((bc-ad)^2q^2r^2)\int\frac{1}{c+dx}\,dx}{bd} \\
&\quad + \frac{((bc-ad)^2q^2r^2)\int\frac{\log(c+dx)}{c+dx}\,dx}{bd} - \frac{((bc-ad)q(p+q)r^2)\int 1\,dx}{d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{2}ap^2r^2x + \frac{1}{2}apqr^2x - \frac{(bc-ad)pqr^2x}{2d} - \frac{(bc-ad)q(p+q)r^2x}{d} \\
&\quad + \frac{1}{4}bp^2r^2x^2 + \frac{1}{4}bpqr^2x^2 + \frac{pqr^2(a+bx)^2}{4b} + \frac{(bc-ad)^2pqr^2\log(c+dx)}{2bd^2} \\
&\quad + \frac{(bc-ad)^2q^2r^2\log(c+dx)}{bd^2} + \frac{(bc-ad)^2pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{bd^2} \\
&\quad + \frac{(bc-ad)qr(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{bd} \\
&\quad - \frac{pr(a+bx)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b} \\
&\quad - \frac{qr(a+bx)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b} \\
&\quad - \frac{(bc-ad)^2qr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{bd^2} \\
&\quad + \frac{(a+bx)^2\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b} - \frac{((bc-ad)^2pqr^2)\int\frac{\log\left(\frac{d(a+bx)}{-bc+ad}\right)}{c+dx}\,dx}{bd} \\
&\quad + \frac{(dq^2r^2)\int\left(-\frac{b(bc-ad)}{d^2} + \frac{b(a+bx)}{d} + \frac{(-bc+ad)^2}{d^2(c+dx)}\right)dx}{2b} \\
&\quad + \frac{((bc-ad)^2q^2r^2)\,\mathrm{Subst}\left(\int\frac{\log(x)}{x}\,dx, x, c+dx\right)}{bd^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{2}ap^2r^2x + \frac{1}{2}apqr^2x - \frac{(bc-ad)pqr^2x}{2d} - \frac{(bc-ad)q^2r^2x}{2d} \\
&\quad - \frac{(bc-ad)q(p+q)r^2x}{d} + \frac{1}{4}bp^2r^2x^2 + \frac{1}{4}bpqr^2x^2 + \frac{pqr^2(a+bx)^2}{4b} \\
&\quad + \frac{q^2r^2(a+bx)^2}{4b} + \frac{(bc-ad)^2pqr^2\log(c+dx)}{2bd^2} + \frac{3(bc-ad)^2q^2r^2\log(c+dx)}{2bd^2} \\
&\quad + \frac{(bc-ad)^2pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{bd^2} + \frac{(bc-ad)^2q^2r^2\log^2(c+dx)}{2bd^2} \\
&\quad + \frac{(bc-ad)qr(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{bd} \\
&\quad - \frac{pr(a+bx)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b} \\
&\quad - \frac{qr(a+bx)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b} \\
&\quad - \frac{(bc-ad)^2qr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{bd^2} \\
&\quad + \frac{(a+bx)^2\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b} \\
&\quad - \frac{((bc-ad)^2pqr^2)\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{bx}{-bc+ad}\right)}{x}\,dx, x, c+dx\right)}{bd^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{2}ap^2r^2x + \frac{1}{2}apqr^2x - \frac{(bc-ad)pqr^2x}{2d} - \frac{(bc-ad)q^2r^2x}{2d} - \frac{(bc-ad)q(p+q)r^2x}{d} \\
&\quad + \frac{1}{4}bp^2r^2x^2 + \frac{1}{4}bpqr^2x^2 + \frac{pqr^2(a+bx)^2}{4b} + \frac{q^2r^2(a+bx)^2}{4b} \\
&\quad + \frac{(bc-ad)^2pqr^2\log(c+dx)}{2bd^2} + \frac{3(bc-ad)^2q^2r^2\log(c+dx)}{2bd^2} \\
&\quad + \frac{(bc-ad)^2pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{bd^2} + \frac{(bc-ad)^2q^2r^2\log^2(c+dx)}{2bd^2} \\
&\quad + \frac{(bc-ad)qr(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{bd} \\
&\quad - \frac{pr(a+bx)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b} \\
&\quad - \frac{qr(a+bx)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b} \\
&\quad - \frac{(bc-ad)^2qr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{bd^2} \\
&\quad + \frac{(a+bx)^2\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b} + \frac{(bc-ad)^2pqr^2\operatorname{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)}{bd^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 781, normalized size of antiderivative = 1.45

$$\int (a+bx)\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx$$
$$=\frac{-4abcdpqr^2+8a^2d^2pqr^2+2abd^2p^2r^2x-6b^2cdpqr^2x+10abd^2pqr^2x-6b^2cdq^2r^2x+8abd^2q^2r^2x+b^2d^2p^2}{}$$

```
[In] Integrate[(a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] (-4*a*b*c*d*p*q*r^2 + 8*a^2*d^2*p*q*r^2 + 2*a*b*d^2*p^2*r^2*x - 6*b^2*c*d*p
*q*r^2*x + 10*a*b*d^2*p*q*r^2*x - 6*b^2*c*d*q^2*r^2*x + 8*a*b*d^2*q^2*r^2*x
 + b^2*d^2*p^2*r^2*x^2 + 2*b^2*d^2*p*q*r^2*x^2 + b^2*d^2*q^2*r^2*x^2 - 2*a^
2*d^2*p^2*r^2*Log[a + b*x]^2 + 2*b^2*c^2*p*q*r^2*Log[c + d*x] - 4*a*b*c*d*p
*q*r^2*Log[c + d*x] + 8*a^2*d^2*p*q*r^2*Log[c + d*x] + 6*b^2*c^2*q^2*r^2*Lo
g[c + d*x] - 8*a*b*c*d*q^2*r^2*Log[c + d*x] + 2*b^2*c^2*q^2*r^2*Log[c + d*x
]^2 - 4*a*b*c*d*q^2*r^2*Log[c + d*x]^2 - 8*a^2*d^2*p*r*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r] - 4*a*b*d^2*p*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 4
*b^2*c*d*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 8*a*b*d^2*q*r*x*Log[e
*(f*(a + b*x)^p*(c + d*x)^q)^r] - 2*b^2*d^2*p*r*x^2*Log[e*(f*(a + b*x)^p*(c
 + d*x)^q)^r] - 2*b^2*d^2*q*r*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 4*
b^2*c^2*q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 8*a*b*c*d*q
*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 4*a*b*d^2*x*Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]^2 + 2*b^2*d^2*x^2*Log[e*(f*(a + b*x)^p*(c + d*
x)^q)^r]^2 + 2*p*r*Log[a + b*x]*(2*b*c*(b*c - 2*a*d)*q*r*Log[c + d*x] - 2*(
b*c - a*d)^2*q*r*Log[(b*(c + d*x))/(b*c - a*d)] + a*d*(3*a*d*(p - q)*r + 2*
b*c*q*r + 2*a*d*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])) - 4*(b*c - a*d)^2*p*
q*r^2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/(4*b*d^2)
```

Maple [F]

$$\int (bx+a)\ln \left(e(f(bx+a)^{p} (dx+c)^{q})^{r}\right)^{2}dx$$

```
[In] int((b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
[Out] int((b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

Fricas [F]

$$\int (a+bx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \int (bx+a)\log\left(\left(((bx+a)^p(dx+c)^q f)^r\, e\right)^2\, dx\right.$$

```
[In] integrate((b*x+a)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="fricas")
[Out] integral((b*x + a)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

Sympy [F]

$$\int (a+bx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \int (a+bx)\log\left(e(f(a+bx)^p\,(c+dx)^q)^r\right)^2 dx$$

```
[In] integrate((b*x+a)*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2,x)
[Out] Integral((a + b*x)*log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 504, normalized size of antiderivative = 0.93

$$\int (a+bx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \frac{1}{2}\left(bx^2+2\,ax\right)\log\left(\left(((bx+a)^p(dx+c)^q f)^r\, e\right)^2\right.$$
$$+\frac{\left(\frac{2\,a^2 fp\log(bx+a)}{b}-\frac{bdf(p+q)x^2+2\,(adf(p+2\,q)-bcfq)x}{d}-\frac{2\,(bc^2fq-2\,acdfq)\log(dx+c)}{d^2}\right)r\log\left(((bx+a)^p(dx+c)^q f)^r\, e\right)}{2\,f}$$
$$+\frac{r^2\left(\frac{2\,((pq+3\,q^2)bc^2f^2-2\,(pq+2\,q^2)acdf^2)\log(dx+c)}{d^2}-\frac{4\,(b^2c^2f^2pq-2\,abcdf^2pq+a^2d^2f^2pq)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)}{bd^2}\right.}{}$$

```
[In] integrate((b*x+a)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^2 + 2*a*x)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2 + 1/2*(2*a^2*f*p
*log(b*x + a)/b - (b*d*f*(p + q)*x^2 + 2*(a*d*f*(p + 2*q) - b*c*f*q)*x)/d -
 2*(b*c^2*f*q - 2*a*c*d*f*q)*log(d*x + c)/d^2)*r*log(((b*x + a)^p*(d*x + c)
^q*f)^r*e)/f + 1/4*r^2*(2*((p*q + 3*q^2)*b*c^2*f^2 - 2*(p*q + 2*q^2)*a*c*d*
f^2)*log(d*x + c)/d^2 - 4*(b^2*c^2*f^2*p*q - 2*a*b*c*d*f^2*p*q + a^2*d^2*f^
2*p*q)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a
*d)/(b*c - a*d)))/(b*d^2) - (2*a^2*d^2*f^2*p^2*log(b*x + a)^2 - (p^2 + 2*p*
q + q^2)*b^2*d^2*f^2*x^2 - 4*(b^2*c^2*f^2*p*q - 2*a*b*c*d*f^2*p*q)*log(b*x
+ a)*log(d*x + c) - 2*(b^2*c^2*f^2*q^2 - 2*a*b*c*d*f^2*q^2)*log(d*x + c)^2
+ 2*(3*(p*q + q^2)*b^2*c*d*f^2 - (p^2 + 5*p*q + 4*q^2)*a*b*d^2*f^2)*x - 2*(
2*a*b*c*d*f^2*p*q - (p^2 + 3*p*q)*a^2*d^2*f^2)*log(b*x + a))/(b*d^2))/f^2
```

Giac [F]

$$\int (a+bx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx=\int (bx+a)\log\left(\left(\left((bx+a)^p(dx+c)^qf\right)^re\right)^2\,dx\right.$$

```
[In] integrate((b*x+a)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="giac")
[Out] integrate((b*x + a)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx=\int \ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)^2(a+b\,x)\,dx$$

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(a + b*x),x)
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(a + b*x), x)
```

3.20 $\int \frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{a+bx}\,dx$

Optimal result 200
Rubi [A] (verified) 201
Mathematica [A] (verified) 208
Maple [F] 208
Fricas [F] 208
Sympy [F] 209
Maxima [F] 209
Giac [F] 209
Mupad [F(-1)] 209

Optimal result

Integrand size = 31, antiderivative size = 431

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{a+bx}\,dx = \frac{\log^3\left((a+bx)^{pr}\right)}{3bpr} - \frac{q\log^2\left((a+bx)^{pr}\right)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{bp} + \frac{\log^2\left((a+bx)^{pr}\right)\log\left((c+dx)^{qr}\right)}{bpr} + \frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log^2\left((c+dx)^{qr}\right)}{b} - \frac{2qr\log\left((a+bx)^{pr}\right)\operatorname{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{b} + \frac{2qr\log\left((c+dx)^{qr}\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{b} - \frac{1}{4}\left(\log\left((a+bx)^{pr}\right)+\log\left((c+dx)^{qr}\right) - \log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)\left(\frac{\left(\log\left((a+bx)^{pr}\right)-\log\left((c+dx)^{qr}\right)+\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)^2}{bpr} + 8\left(\frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log\left((c+dx)^{qr}\right)}{b} + \frac{qr\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{b}\right)\right) + \frac{2pqr^2\operatorname{PolyLog}\left(3,-\frac{d(a+bx)}{bc-ad}\right)}{b} - \frac{2q^2r^2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{bc-ad}\right)}{b}$$

```
[Out] 1/3*ln((b*x+a)^(p*r))^3/b/p/r-q*ln((b*x+a)^(p*r))^2*ln(b*(d*x+c)/(-a*d+b*c)
)/b/p+ln((b*x+a)^(p*r))^2*ln((d*x+c)^(q*r))/b/p/r+ln(-d*(b*x+a)/(-a*d+b*c))
*ln((d*x+c)^(q*r))^2/b-2*q*r*ln((b*x+a)^(p*r))*polylog(2,-d*(b*x+a)/(-a*d+b
*c))/b+2*q*r*ln((d*x+c)^(q*r))*polylog(2,b*(d*x+c)/(-a*d+b*c))/b-1/4*(ln((b
*x+a)^(p*r))+ln((d*x+c)^(q*r))-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))*((ln((b*x+a
)^(p*r))-ln((d*x+c)^(q*r))+ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))^2/b/p/r+8*ln(-d
*(b*x+a)/(-a*d+b*c))*ln((d*x+c)^(q*r))/b+8*q*r*polylog(2,b*(d*x+c)/(-a*d+b*
```

```
c))/b)+2*p*q*r^2*polylog(3,-d*(b*x+a)/(-a*d+b*c))/b-2*q^2*r^2*polylog(3,b*(
d*x+c)/(-a*d+b*c))/b
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.484$, Rules used = {2582, 6874, 2437, 2339, 30, 2481, 2422, 2354, 2421, 6724, 2443, 2441, 2440, 2438, 6818}

$$\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{a+bx}\,dx$$

$$= -\frac{1}{4}\left(-\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)+\log\left((a+bx)^{pr}\right)+\log\left((c+dx)^{qr}\right)\right)\left(\frac{\left(\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)+\log\left((a+bx)^{pr}\right)-\log\left((c+dx)^{qr}\right)\right)^2}{bpr}+8\left(\frac{qr\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{b}+\frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log\left((c+dx)^{qr}\right)}{b}\right)\right)+\frac{2pqr^2\operatorname{PolyLog}\left(3,-\frac{d(a+bx)}{bc-ad}\right)}{b}-\frac{2qr\operatorname{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)\log\left((a+bx)^{pr}\right)}{b}-\frac{q\log\left(\frac{b(c+dx)}{bc-ad}\right)\log^2\left((a+bx)^{pr}\right)}{bp}+\frac{\log^2\left((a+bx)^{pr}\right)\log\left((c+dx)^{qr}\right)}{bpr}-\frac{2q^2r^2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{bc-ad}\right)}{b}+\frac{2qr\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)\log\left((c+dx)^{qr}\right)}{b}+\frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log^2\left((c+dx)^{qr}\right)}{b}+\frac{\log^3\left((a+bx)^{pr}\right)}{3bpr}$$

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x),x]
```

```
[Out] Log[(a + b*x)^(p*r)]^3/(3*b*p*r) - (q*Log[(a + b*x)^(p*r)]^2*Log[(b*(c + d*
x))/(b*c - a*d)])/(b*p) + (Log[(a + b*x)^(p*r)]^2*Log[(c + d*x)^(q*r)])/(b*
p*r) + (Log[-((d*(a + b*x))/(b*c - a*d))]*Log[(c + d*x)^(q*r)]^2)/b - (2*q*
r*Log[(a + b*x)^(p*r)]*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/b + (2*q*r
*Log[(c + d*x)^(q*r)]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/b - ((Log[(a +
 b*x)^(p*r)] + Log[(c + d*x)^(q*r)] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])
*((Log[(a + b*x)^(p*r)] - Log[(c + d*x)^(q*r)] + Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r])^2/(b*p*r) + 8*((Log[-((d*(a + b*x))/(b*c - a*d))]*Log[(c + d*x)
^(q*r)])/b + (q*r*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/b)))/4 + (2*p*q*r^
2*PolyLog[3, -((d*(a + b*x))/(b*c - a*d))])/b - (2*q^2*r^2*PolyLog[3, (b*(c
 + d*x))/(b*c - a*d)])/b
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2422

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[f*m*(r/(b*n*(p + 1))), Int[x^(m -
 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2582

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^2/((g_.) + (h_.)*(x_)), x_Symbol] :> Int[(Log[(a + b*x)^(p*r)] + Lo
g[(c + d*x)^(q*r)])^2/(g + h*x), x] + Simp[(Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r] - Log[(a + b*x)^(p*r)] - Log[(c + d*x)^(q*r)])*(2*Int[Log[(c + d*x)^(
q*r)]/(g + h*x), x] + Int[(Log[(a + b*x)^(p*r)] - Log[(c + d*x)^(q*r)] + Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(g + h*x), x]), x] /; FreeQ[{a, b, c, d
, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0] && EqQ[b*g - a*h, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6818

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \int \frac{(\log((a+bx)^{pr}) + \log((c+dx)^{qr}))^2}{a+bx}\,dx - (\log((a+bx)^{pr}) + \log((c+dx)^{qr}) - \log(e(f(a+bx)^p(c+dx)^q)^r))\left(2\int \frac{\log((c+dx)^{qr})}{a+bx}\,dx + \int \frac{\log((a+bx)^{pr}) - \log((c+dx)^{qr}) + \log(e(f(a+bx)^p(c+dx)^q)^r)}{a+bx}\,dx\right)$$

$$= -\Bigg((\log((a+bx)^{pr}) + \log((c+dx)^{qr}) - \log(e(f(a+bx)^p(c+dx)^q)^r))\Bigg(\frac{(\log((a+bx)^{pr}) - \log((c+dx)^{qr}) + \log(e(f(a+bx)^p(c+dx)^q)}{4bpr} + 2\left(\frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log((c+dx)^{qr})}{b} - \frac{(dqr)\int \frac{\log\left(\frac{d(a+bx)}{-bc+ad}\right)}{c+dx}\,dx}{b}\right)\Bigg)\Bigg) + \int \left(\frac{\log^2((a+bx)^{pr})}{a+bx} + \frac{2\log((a+bx)^{pr})\log((c+dx)^{qr})}{a+bx} + \frac{\log^2((c+dx)^{qr})}{a+bx}\right)dx$$

$$= 2\int \frac{\log\left((a+bx)^{pr}\right)\log\left((c+dx)^{qr}\right)}{a+bx}\,dx - \left(\log\left((a+bx)^{pr}\right) + \log\left((c+dx)^{qr}\right)\right.$$

$$\left. - \log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)\left(\frac{\left(\log\left((a+bx)^{pr}\right) - \log\left((c+dx)^{qr}\right) + \log\left(e(f(a+bx)^p(c+dx)^q\right.\right.}{4bpr}\right.$$

$$+ 2\left(\frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log\left((c+dx)^{qr}\right)}{b}\right.$$

$$\left.\left.\left. - \frac{(qr)\text{Subst}\left(\int \frac{\log\left(1+\frac{bx}{-bc+ad}\right)}{x}\,dx, x, c+dx\right)}{b}\right)\right)\right)$$

$$+ \int \frac{\log^2\left((a+bx)^{pr}\right)}{a+bx}\,dx + \int \frac{\log^2\left((c+dx)^{qr}\right)}{a+bx}\,dx$$

$$= \frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log^2\left((c+dx)^{qr}\right)}{b} - \left(\log\left((a+bx)^{pr}\right) + \log\left((c+dx)^{qr}\right)\right.$$

$$\left. - \log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)\left(\frac{\left(\log\left((a+bx)^{pr}\right) - \log\left((c+dx)^{qr}\right) + \log\left(e(f(a+bx)^p(c+dx)^q\right.\right.}{4bpr}\right.$$

$$\left.\left. + 2\left(\frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log\left((c+dx)^{qr}\right)}{b} + \frac{qr\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)}{b}\right)\right)\right)$$

$$+ \frac{\text{Subst}\left(\int \frac{\log^2(x^{pr})}{x}\,dx, x, a+bx\right)}{b}$$

$$+ \frac{2\text{Subst}\left(\int \frac{\log(x^{pr})\log\left(\left(\frac{bc-ad}{b}+\frac{dx}{b}\right)^{qr}\right)}{x}\,dx, x, a+bx\right)}{b}$$

$$- \frac{(2dqr)\int \frac{\log\left(\frac{d(a+bx)}{-bc+ad}\right)\log\left((c+dx)^{qr}\right)}{c+dx}\,dx}{b}$$

$$= \frac{\log^2((a+bx)^{pr})\log((c+dx)^{qr})}{bpr} + \frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log^2((c+dx)^{qr})}{b}$$
$$-(\log((a+bx)^{pr}) + \log((c+dx)^{qr})$$
$$-\log(e(f(a+bx)^p(c+dx)^q)^r))\left(\frac{(\log((a+bx)^{pr}) - \log((c+dx)^{qr}) + \log(e(f(a+bx)^p(c+dx)^q)^r}{4bpr}\right.$$
$$\left.+2\left(\frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log((c+dx)^{qr})}{b} + \frac{qr\mathrm{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)}{b}\right)\right)$$
$$-\frac{(dq)\mathrm{Subst}\left(\int\frac{\log^2(x^{pr})}{\frac{bc-ad}{b}+\frac{dx}{b}}\,dx, x, a+bx\right)}{b^2p} + \frac{\mathrm{Subst}\left(\int x^2\,dx, x, \log((a+bx)^{pr})\right)}{bpr}$$
$$-\frac{(2qr)\mathrm{Subst}\left(\int\frac{\log(x^{qr})\log\left(\frac{d\left(\frac{-bc+ad}{d}+\frac{bx}{d}\right)}{-bc+ad}\right)}{x}\,dx, x, c+dx\right)}{b}$$

$$= \frac{\log^3((a+bx)^{pr})}{3bpr} - \frac{q\log^2((a+bx)^{pr})\log\left(\frac{b(c+dx)}{bc-ad}\right)}{bp}$$
$$+\frac{\log^2((a+bx)^{pr})\log((c+dx)^{qr})}{bpr} + \frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log^2((c+dx)^{qr})}{b}$$
$$+\frac{2qr\log((c+dx)^{qr})\mathrm{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)}{b} - (\log((a+bx)^{pr}) + \log((c+dx)^{qr})$$
$$-\log(e(f(a+bx)^p(c+dx)^q)^r))\left(\frac{(\log((a+bx)^{pr}) - \log((c+dx)^{qr}) + \log(e(f(a+bx)^p(c+dx)^q)^r}{4bpr}\right.$$
$$\left.+2\left(\frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log((c+dx)^{qr})}{b} + \frac{qr\mathrm{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)}{b}\right)\right)$$
$$+\frac{(2qr)\mathrm{Subst}\left(\int\frac{\log(x^{pr})\log\left(1+\frac{dx}{bc-ad}\right)}{x}\,dx, x, a+bx\right)}{b}$$
$$-\frac{(2q^2r^2)\mathrm{Subst}\left(\int\frac{\mathrm{Li}_2\left(-\frac{bx}{-bc+ad}\right)}{x}\,dx, x, c+dx\right)}{b}$$

$$
\begin{aligned}
&= \frac{\log^3\left((a+bx)^{pr}\right)}{3bpr} - \frac{q\log^2\left((a+bx)^{pr}\right)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{bp} \\
&+ \frac{\log^2\left((a+bx)^{pr}\right)\log\left((c+dx)^{qr}\right)}{bpr} + \frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log^2\left((c+dx)^{qr}\right)}{b} \\
&- \frac{2qr\log\left((a+bx)^{pr}\right)\operatorname{Li}_2\left(-\frac{d(a+bx)}{bc-ad}\right)}{b} + \frac{2qr\log\left((c+dx)^{qr}\right)\operatorname{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)}{b} \\
&- \left(\log\left((a+bx)^{pr}\right) + \log\left((c+dx)^{qr}\right)\right. \\
&\left.-\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\right)\left(\frac{\left(\log\left((a+bx)^{pr}\right) - \log\left((c+dx)^{qr}\right) + \log\left(e\left(f(a+bx)^p(c+dx)^q\right.\right.\right.}{4bpr}\right. \\
&\qquad\left. + 2\left(\frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log\left((c+dx)^{qr}\right)}{b} + \frac{qr\operatorname{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)}{b}\right)\right) \\
&- \frac{2q^2r^2\operatorname{Li}_3\left(\frac{b(c+dx)}{bc-ad}\right)}{b} + \frac{(2pqr^2)\operatorname{Subst}\left(\int \frac{\operatorname{Li}_2\left(-\frac{dx}{bc-ad}\right)}{x}\,dx, x, a+bx\right)}{b}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\log^3\left((a+bx)^{pr}\right)}{3bpr} - \frac{q\log^2\left((a+bx)^{pr}\right)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{bp} \\
&+ \frac{\log^2\left((a+bx)^{pr}\right)\log\left((c+dx)^{qr}\right)}{bpr} + \frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log^2\left((c+dx)^{qr}\right)}{b} \\
&- \frac{2qr\log\left((a+bx)^{pr}\right)\operatorname{Li}_2\left(-\frac{d(a+bx)}{bc-ad}\right)}{b} + \frac{2qr\log\left((c+dx)^{qr}\right)\operatorname{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)}{b} \\
&- \left(\log\left((a+bx)^{pr}\right) + \log\left((c+dx)^{qr}\right)\right. \\
&\left.-\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\right)\left(\frac{\left(\log\left((a+bx)^{pr}\right) - \log\left((c+dx)^{qr}\right) + \log\left(e\left(f(a+bx)^p(c+dx)^q\right.\right.\right.}{4bpr}\right. \\
&\qquad\left. + 2\left(\frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log\left((c+dx)^{qr}\right)}{b} + \frac{qr\operatorname{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)}{b}\right)\right) \\
&+ \frac{2pqr^2\operatorname{Li}_3\left(-\frac{d(a+bx)}{bc-ad}\right)}{b} - \frac{2q^2r^2\operatorname{Li}_3\left(\frac{b(c+dx)}{bc-ad}\right)}{b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.07

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{a+bx}\,dx$$

$$=\frac{p^2r^2\log^3(a+bx)+6pqr^2\log^2(a+bx)\log(c+dx)-6pqr^2\log(a+bx)\log\left(\frac{d(a+bx)}{-bc+ad}\right)\log(c+dx)+3q^2r^2\log}{}$$

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x),x]
```

```
[Out] (p^2*r^2*Log[a + b*x]^3 + 6*p*q*r^2*Log[a + b*x]^2*Log[c + d*x] - 6*p*q*r^2
*Log[a + b*x]*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] + 3*q^2*r^2*Lo
g[a + b*x]*Log[c + d*x]^2 - 3*q^2*r^2*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log
[c + d*x]^2 - 3*p*q*r^2*Log[a + b*x]^2*Log[(b*(c + d*x))/(b*c - a*d)] - 3*p
*r*Log[a + b*x]^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 6*q*r*Log[a + b*x]
*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 6*q*r*Log[(d*(a + b*x)
)/(-(b*c) + a*d)]*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 3*Log
[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 - 6*p*q*r^2*Log[a + b*x]*P
olyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 6*q*r*(-(p*r*Log[a + b*x]) + Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r])*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] + 6
*p*q*r^2*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)] - 6*q^2*r^2*PolyLog[3, (b
*(c + d*x))/(b*c - a*d)])/(3*b)
```

Maple [F]

$$\int \frac{\ln\left(e\left(f(bx+a)^p(dx+c)^q\right)^r\right)^2}{bx+a}dx$$

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a),x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a),x)
```

Fricas [F]

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{a+bx}\,dx=\int \frac{\log\left(\left((bx+a)^p(dx+c)^qf\right)^re\right)^2}{bx+a}\,dx$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b*x + a), x)
```

Sympy [F]

$$\int \frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{a+bx}\,dx = \int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)^2}{a+bx}\,dx$$

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(b*x+a),x)
[Out] Integral(log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)**2/(a + b*x), x)
```

Maxima [F]

$$\int \frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{a+bx}\,dx = \int \frac{\log\left(((bx+a)^p(dx+c)^qf)^re\right)^2}{bx+a}\,dx$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a),x, algorithm="maxima")
[Out] log(b*x + a)*log(((d*x + c)^q)^r)^2/b + integrate(((r^2*log(f)^2 + 2*r*log(
e)*log(f) + log(e)^2)*b*d*x + (r^2*log(f)^2 + 2*r*log(e)*log(f) + log(e)^2)
*b*c + (b*d*x + b*c)*log(((b*x + a)^p)^r)^2 + 2*((r*log(f) + log(e))*b*d*x
+ (r*log(f) + log(e))*b*c)*log(((b*x + a)^p)^r) + 2*((r*log(f) + log(e))*b*
d*x + (r*log(f) + log(e))*b*c - (b*d*q*r*x + a*d*q*r)*log(b*x + a) + (b*d*x
 + b*c)*log(((b*x + a)^p)^r))*log(((d*x + c)^q)^r))/(b^2*d*x^2 + a*b*c + (b
^2*c + a*b*d)*x), x)
```

Giac [F]

$$\int \frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{a+bx}\,dx = \int \frac{\log\left(((bx+a)^p(dx+c)^qf)^re\right)^2}{bx+a}\,dx$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a),x, algorithm="giac")
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{a+bx}\,dx = \int \frac{\ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)^2}{a+b\,x}\,dx$$

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(a + b*x),x)
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(a + b*x), x)
```

3.21 $\int \frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^2} dx$

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Optimal result

Integrand size = 31, antiderivative size = 465

$$\begin{aligned}\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^2}\,dx =& -\frac{2p^2r^2}{b(a+bx)}+\frac{2dpqr^2\log(a+bx)}{b(bc-ad)} \\ &-\frac{dpqr^2\log^2(a+bx)}{b(bc-ad)}-\frac{2dpqr^2\log(c+dx)}{b(bc-ad)} \\ &+\frac{2dpqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{b(bc-ad)} \\ &+\frac{dq^2r^2\log^2(c+dx)}{b(bc-ad)} \\ &-\frac{2dq^2r^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{b(bc-ad)} \\ &-\frac{2pr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(a+bx)} \\ &+\frac{2dqr\log(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(bc-ad)} \\ &-\frac{2dqr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(bc-ad)} \\ &-\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(a+bx)} \\ &-\frac{2dq^2r^2\operatorname{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{b(bc-ad)} \\ &+\frac{2dpqr^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{b(bc-ad)}\end{aligned}$$

```
[Out] -2*p^2*r^2/b/(b*x+a)+2*d*p*q*r^2*ln(b*x+a)/b/(-a*d+b*c)-d*p*q*r^2*ln(b*x+a)
^2/b/(-a*d+b*c)-2*d*p*q*r^2*ln(d*x+c)/b/(-a*d+b*c)+2*d*p*q*r^2*ln(-d*(b*x+a
)/(-a*d+b*c))*ln(d*x+c)/b/(-a*d+b*c)+d*q^2*r^2*ln(d*x+c)^2/b/(-a*d+b*c)-2*d
*q^2*r^2*ln(b*x+a)*ln(b*(d*x+c)/(-a*d+b*c))/b/(-a*d+b*c)-2*p*r*ln(e*(f*(b*x
+a)^p*(d*x+c)^q)^r)/b/(b*x+a)+2*d*q*r*ln(b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q
)^r)/b/(-a*d+b*c)-2*d*q*r*ln(d*x+c)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/(-a*d
+b*c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/b/(b*x+a)-2*d*q^2*r^2*polylog(2,-d*
(b*x+a)/(-a*d+b*c))/b/(-a*d+b*c)+2*d*p*q*r^2*polylog(2,b*(d*x+c)/(-a*d+b*c)
)/b/(-a*d+b*c)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 465, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {2584, 2581, 32, 36, 31, 2594, 2580, 2437, 2338, 2441, 2440, 2438}

$$\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^2}\,dx = -\frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(a+bx)} + \frac{2dqr\log(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(bc-ad)} - \frac{2dqr\log(c+dx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(bc-ad)} - \frac{2pr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(a+bx)} + \frac{2dpqr^2\,\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{b(bc-ad)} - \frac{dpqr^2\log^2(a+bx)}{b(bc-ad)} + \frac{2dpqr^2\log(a+bx)}{b(bc-ad)} - \frac{2dpqr^2\log(c+dx)}{b(bc-ad)} + \frac{2dpqr^2\log(c+dx)\log\left(-\frac{d(a+bx)}{bc-ad}\right)}{b(bc-ad)} - \frac{2dq^2r^2\,\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{b(bc-ad)} + \frac{dq^2r^2\log^2(c+dx)}{b(bc-ad)} - \frac{2dq^2r^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{b(bc-ad)} - \frac{2p^2r^2}{b(a+bx)}$$

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x)^2,x]
```

```
[Out] (-2*p^2*r^2)/(b*(a + b*x)) + (2*d*p*q*r^2*Log[a + b*x])/(b*(b*c - a*d)) - (
d*p*q*r^2*Log[a + b*x]^2)/(b*(b*c - a*d)) - (2*d*p*q*r^2*Log[c + d*x])/(b*(
b*c - a*d)) + (2*d*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/
(b*(b*c - a*d)) + (d*q^2*r^2*Log[c + d*x]^2)/(b*(b*c - a*d)) - (2*d*q^2*r^2
```

```
*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(b*(b*c - a*d)) - (2*p*r*Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(b*(a + b*x)) + (2*d*q*r*Log[a + b*x]*Log
[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(b*(b*c - a*d)) - (2*d*q*r*Log[c + d*x]*
Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(b*(b*c - a*d)) - Log[e*(f*(a + b*x)^
p*(c + d*x)^q)^r]^2/(b*(a + b*x)) - (2*d*q^2*r^2*PolyLog[2, -((d*(a + b*x))
/(b*c - a*d))])/(b*(b*c - a*d)) + (2*d*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*
c - a*d)])/(b*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2580

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[Log[g + h*x]*(Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]/h), x] + (-Dist[b*p*(r/h), Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[d*q*(r/h), Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 2584

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1
)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(h*(m + 1))), x] + (-Dist[b*p*r*(
s/(h*(m + 1))), Int[(g + h*x)^(m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1)/(a + b*x)), x], x] - Dist[d*q*r*(s/(h*(m + 1))), Int[(g + h*x)^(m
+ 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(c + d*x)), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2594

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(a+bx)} + (2pr)\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^2}\,dx \\
&\quad + \frac{(2dqr)\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)(c+dx)}\,dx}{b} \\
&= -\frac{2pr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(a+bx)} - \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(a+bx)} \\
&\quad + \frac{(2dqr)\int \left(\frac{b\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(bc-ad)(a+bx)} - \frac{d\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(bc-ad)(c+dx)}\right)dx}{b} \\
&\quad + \left(2p^2r^2\right)\int \frac{1}{(a+bx)^2}\,dx + \frac{\left(2dpqr^2\right)\int \frac{1}{(a+bx)(c+dx)}\,dx}{b} \\
&= -\frac{2p^2r^2}{b(a+bx)} - \frac{2pr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(a+bx)} \\
&\quad - \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(a+bx)} + \frac{(2dqr)\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{a+bx}\,dx}{bc-ad} \\
&\quad - \frac{\left(2d^2qr\right)\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{c+dx}\,dx}{b(bc-ad)} + \frac{\left(2dpqr^2\right)\int \frac{1}{a+bx}\,dx}{bc-ad} - \frac{\left(2d^2pqr^2\right)\int \frac{1}{c+dx}\,dx}{b(bc-ad)} \\
&= -\frac{2p^2r^2}{b(a+bx)} + \frac{2dpqr^2\log(a+bx)}{b(bc-ad)} - \frac{2dpqr^2\log(c+dx)}{b(bc-ad)} \\
&\quad - \frac{2pr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(a+bx)} \\
&\quad + \frac{2dqr\log(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(bc-ad)} \\
&\quad - \frac{2dqr\log(c+dx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(bc-ad)} \\
&\quad - \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(a+bx)} - \frac{\left(2dpqr^2\right)\int \frac{\log(a+bx)}{a+bx}\,dx}{bc-ad} \\
&\quad + \frac{\left(2dpqr^2\right)\int \frac{\log(c+dx)}{a+bx}\,dx}{bc-ad} - \frac{\left(2d^2q^2r^2\right)\int \frac{\log(a+bx)}{c+dx}\,dx}{b(bc-ad)} + \frac{\left(2d^2q^2r^2\right)\int \frac{\log(c+dx)}{c+dx}\,dx}{b(bc-ad)}
\end{aligned}
$$

$$= -\frac{2p^2r^2}{b(a+bx)} + \frac{2dpqr^2\log(a+bx)}{b(bc-ad)} - \frac{2dpqr^2\log(c+dx)}{b(bc-ad)} + \frac{2dpqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{b(bc-ad)} - \frac{2dq^2r^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{b(bc-ad)} - \frac{2pr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(a+bx)} + \frac{2dqr\log(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(bc-ad)} - \frac{2dqr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(bc-ad)} - \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(a+bx)} - \frac{(2dpqr^2)\,\text{Subst}\left(\int\frac{\log(x)}{x}\,dx,x,a+bx\right)}{b(bc-ad)} - \frac{(2d^2pqr^2)\int\frac{\log\left(\frac{d(a+bx)}{-bc+ad}\right)}{c+dx}\,dx}{b(bc-ad)} + \frac{(2dq^2r^2)\int\frac{\log\left(\frac{b(c+dx)}{bc-ad}\right)}{a+bx}\,dx}{bc-ad} + \frac{(2dq^2r^2)\,\text{Subst}\left(\int\frac{\log(x)}{x}\,dx,x,c+dx\right)}{b(bc-ad)}$$

$$= -\frac{2p^2r^2}{b(a+bx)} + \frac{2dpqr^2\log(a+bx)}{b(bc-ad)} - \frac{dpqr^2\log^2(a+bx)}{b(bc-ad)} - \frac{2dpqr^2\log(c+dx)}{b(bc-ad)} + \frac{2dpqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{b(bc-ad)} + \frac{dq^2r^2\log^2(c+dx)}{b(bc-ad)} - \frac{2dq^2r^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{b(bc-ad)} - \frac{2pr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(a+bx)} + \frac{2dqr\log(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(bc-ad)} - \frac{2dqr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(bc-ad)} - \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(a+bx)} - \frac{(2dpqr^2)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{bx}{-bc+ad}\right)}{x}\,dx,x,c+dx\right)}{b(bc-ad)} + \frac{(2dq^2r^2)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{dx}{bc-ad}\right)}{x}\,dx,x,a+bx\right)}{b(bc-ad)}$$

$$= -\frac{2p^2r^2}{b(a+bx)} + \frac{2dpqr^2\log(a+bx)}{b(bc-ad)} - \frac{dpqr^2\log^2(a+bx)}{b(bc-ad)} - \frac{2dpqr^2\log(c+dx)}{b(bc-ad)} + \frac{2dpqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{b(bc-ad)} + \frac{dq^2r^2\log^2(c+dx)}{b(bc-ad)} - \frac{2dq^2r^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{b(bc-ad)} - \frac{2pr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(a+bx)} + \frac{2dqr\log(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(bc-ad)} - \frac{2dqr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(bc-ad)} - \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(a+bx)} - \frac{2dq^2r^2\text{Li}_2\left(-\frac{d(a+bx)}{bc-ad}\right)}{b(bc-ad)} + \frac{2dpqr^2\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)}{b(bc-ad)}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 411, normalized size of antiderivative = 0.88

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^2}\,dx = \frac{-2bcp^2r^2 + 2adp^2r^2 - dpqr^2(a+bx)\log^2(a+bx) - 2adpqr^2\log(c+dx) - 2bdpqr^2x\log(c+dx) + adq^2r^2 \ldots}{}$$

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x)^2,x]
```

```
[Out] (-2*b*c*p^2*r^2 + 2*a*d*p^2*r^2 - d*p*q*r^2*(a + b*x)*Log[a + b*x]^2 - 2*a*
d*p*q*r^2*Log[c + d*x] - 2*b*d*p*q*r^2*x*Log[c + d*x] + a*d*q^2*r^2*Log[c +
 d*x]^2 + b*d*q^2*r^2*x*Log[c + d*x]^2 - 2*b*c*p*r*Log[e*(f*(a + b*x)^p*(c
+ d*x)^q)^r] + 2*a*d*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 2*a*d*q*r*L
og[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 2*b*d*q*r*x*Log[c + d*x]
*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - b*c*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^2 + a*d*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 + 2*d*q*r*(a + b*x)*Lo
g[a + b*x]*(p*r + p*r*Log[c + d*x] - (p + q)*r*Log[(b*(c + d*x))/(b*c - a*d
)] + Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]) - 2*d*q*(p + q)*r^2*(a + b*x)*Po
lyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(b*c - a*d)*(a + b*x))
```

Maple [F]

$$\int \frac{\ln \left(e(f(bx+a)^p\,(dx+c)^q)^r\right)^2}{(bx+a)^2}\,dx$$

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^2,x)
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^2,x)
```

Fricas [F]

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^2}\,dx = \int \frac{\log\left(\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2\right)}{(bx+a)^2}\,dx$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^2,x, algorithm="fricas
")
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b^2*x^2 + 2*a*b*x + a^2),
x)
```

Sympy [F]

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^2}\,dx = \int \frac{\log\left(e\left(f(a+bx)^p\,(c+dx)^q\right)^r\right)^2}{(a+bx)^2}\,dx$$

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(b*x+a)**2,x)
[Out] Integral(log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)**2/(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 392, normalized size of antiderivative = 0.84

$$\begin{aligned}
&\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^2}\,dx \\
&= \frac{2\left(\frac{dfq\log(bx+a)}{bc-ad} - \frac{dfq\log(dx+c)}{bc-ad} - \frac{fp}{bx+a}\right)r\log\left(\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)\right)}{bf} \\
&\quad - \frac{\left(\frac{2\,df^2pq\log(dx+c)}{bc-ad} + \frac{2\left(pq+q^2\right)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)df^2}{bc-ad} + \frac{2\,bcf^2p^2-2\,adf^2p^2+\left(bdf^2pqx+adf^2pq\right)\log(bx+a)^2 \cdots}{\cdots}\right.}{bf^2} \\
&\quad - \frac{\log\left(\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2\right)}{(bx+a)b}
\end{aligned}$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^2,x, algorithm="maxima
")
```

```
[Out] 2*(d*f*q*log(b*x + a)/(b*c - a*d) - d*f*q*log(d*x + c)/(b*c - a*d) - f*p/(b
*x + a))*r*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(b*f) - (2*d*f^2*p*q*log(d*
x + c)/(b*c - a*d) + 2*(p*q + q^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a
*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*d*f^2/(b*c - a*d) + (2*b*c*f^
2*p^2 - 2*a*d*f^2*p^2 + (b*d*f^2*p*q*x + a*d*f^2*p*q)*log(b*x + a)^2 - 2*(b
*d*f^2*p*q*x + a*d*f^2*p*q)*log(b*x + a)*log(d*x + c) - (b*d*f^2*q^2*x + a*
d*f^2*q^2)*log(d*x + c)^2 - 2*(b*d*f^2*p*q*x + a*d*f^2*p*q)*log(b*x + a))/(
a*b*c - a^2*d + (b^2*c - a*b*d)*x))*r^2/(b*f^2) - log(((b*x + a)^p*(d*x + c
)^q*f)^r*e)^2/((b*x + a)*b)
```

Giac [F]

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^2}\,dx = \int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{(bx+a)^2}\,dx$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^2}\,dx = \int \frac{\ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)^2}{(a+b\,x)^2}\,dx$$

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(a + b*x)^2,x)
```

```
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(a + b*x)^2, x)
```

3.22 $\int \frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^3}\,dx$

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Optimal result

Integrand size = 31, antiderivative size = 632

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^3}\,dx = -\frac{p^2r^2}{4b(a+bx)^2} - \frac{3dpqr^2}{2b(bc-ad)(a+bx)} - \frac{d^2pqr^2\log(a+bx)}{2b(bc-ad)^2} + \frac{d^2q^2r^2\log(a+bx)}{b(bc-ad)^2} + \frac{d^2pqr^2\log^2(a+bx)}{2b(bc-ad)^2} + \frac{d^2pqr^2\log(c+dx)}{2b(bc-ad)^2} - \frac{d^2q^2r^2\log(c+dx)}{b(bc-ad)^2} - \frac{d^2pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{b(bc-ad)^2} - \frac{d^2q^2r^2\log^2(c+dx)}{2b(bc-ad)^2} + \frac{d^2q^2r^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{b(bc-ad)^2} - \frac{pr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b(a+bx)^2} - \frac{dqr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(bc-ad)(a+bx)} - \frac{d^2qr\log(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(bc-ad)^2} + \frac{d^2qr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(bc-ad)^2} - \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b(a+bx)^2} + \frac{d^2q^2r^2\,\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{b(bc-ad)^2} - \frac{d^2pqr^2\,\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{b(bc-ad)^2}$$

```
[Out] -1/4*p^2*r^2/b/(b*x+a)^2-3/2*d*p*q*r^2/b/(-a*d+b*c)/(b*x+a)-1/2*d^2*p*q*r^2
*ln(b*x+a)/b/(-a*d+b*c)^2+d^2*q^2*r^2*ln(b*x+a)/b/(-a*d+b*c)^2+1/2*d^2*p*q*
r^2*ln(b*x+a)^2/b/(-a*d+b*c)^2+1/2*d^2*p*q*r^2*ln(d*x+c)/b/(-a*d+b*c)^2-d^2
*q^2*r^2*ln(d*x+c)/b/(-a*d+b*c)^2-d^2*p*q*r^2*ln(-d*(b*x+a)/(-a*d+b*c))*ln(
d*x+c)/b/(-a*d+b*c)^2-1/2*d^2*q^2*r^2*ln(d*x+c)^2/b/(-a*d+b*c)^2+d^2*q^2*r^
2*ln(b*x+a)*ln(b*(d*x+c)/(-a*d+b*c))/b/(-a*d+b*c)^2-1/2*p*r*ln(e*(f*(b*x+a)
^p*(d*x+c)^q)^r)/b/(b*x+a)^2-d*q*r*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/(-a*d+
```

```
b*c)/(b*x+a)-d^2*q*r*ln(b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/(-a*d+b*c)
^2+d^2*q*r*ln(d*x+c)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/(-a*d+b*c)^2-1/2*ln(
e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/b/(b*x+a)^2+d^2*q^2*r^2*polylog(2,-d*(b*x+a)
/(-a*d+b*c))/b/(-a*d+b*c)^2-d^2*p*q*r^2*polylog(2,b*(d*x+c)/(-a*d+b*c))/b/(
-a*d+b*c)^2
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 632, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {2584, 2581, 32, 46, 2594, 36, 31, 2580, 2437, 2338, 2441, 2440, 2438}

$$\begin{aligned}\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^3}\,dx = &-\frac{d^2qr\log(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(bc-ad)^2}\\&+\frac{d^2qr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(bc-ad)^2}\\&-\frac{d^2pqr^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{b(bc-ad)^2}+\frac{d^2pqr^2\log^2(a+bx)}{2b(bc-ad)^2}\\&-\frac{d^2pqr^2\log(a+bx)}{2b(bc-ad)^2}+\frac{d^2pqr^2\log(c+dx)}{2b(bc-ad)^2}\\&-\frac{d^2pqr^2\log(c+dx)\log\left(-\frac{d(a+bx)}{bc-ad}\right)}{b(bc-ad)^2}\\&+\frac{d^2q^2r^2\operatorname{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{b(bc-ad)^2}\\&-\frac{d^2q^2r^2\log^2(c+dx)}{2b(bc-ad)^2}\\&+\frac{d^2q^2r^2\log(a+bx)}{b(bc-ad)^2}-\frac{d^2q^2r^2\log(c+dx)}{b(bc-ad)^2}\\&+\frac{d^2q^2r^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{b(bc-ad)^2}\\&-\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b(a+bx)^2}\\&-\frac{dqr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(a+bx)(bc-ad)}\\&-\frac{pr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b(a+bx)^2}\\&-\frac{3dpqr^2}{2b(a+bx)(bc-ad)}-\frac{p^2r^2}{4b(a+bx)^2}\end{aligned}$$

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x)^3,x]
```

```
[Out] -1/4*(p^2*r^2)/(b*(a + b*x)^2) - (3*d*p*q*r^2)/(2*b*(b*c - a*d)*(a + b*x))
- (d^2*p*q*r^2*Log[a + b*x])/(2*b*(b*c - a*d)^2) + (d^2*q^2*r^2*Log[a + b*x
])/(b*(b*c - a*d)^2) + (d^2*p*q*r^2*Log[a + b*x]^2)/(2*b*(b*c - a*d)^2) + (
d^2*p*q*r^2*Log[c + d*x])/(2*b*(b*c - a*d)^2) - (d^2*q^2*r^2*Log[c + d*x])/
(b*(b*c - a*d)^2) - (d^2*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c +
d*x])/(b*(b*c - a*d)^2) - (d^2*q^2*r^2*Log[c + d*x]^2)/(2*b*(b*c - a*d)^2)
+ (d^2*q^2*r^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(b*(b*c - a*d)^
2) - (p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*b*(a + b*x)^2) - (d*q*r*
Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(b*(b*c - a*d)*(a + b*x)) - (d^2*q*r*
Log[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(b*(b*c - a*d)^2) + (d^2
*q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(b*(b*c - a*d)^2) -
 Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(2*b*(a + b*x)^2) + (d^2*q^2*r^2*Po
lyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(b*(b*c - a*d)^2) - (d^2*p*q*r^2*Po
lyLog[2, (b*(c + d*x))/(b*c - a*d)])/(b*(b*c - a*d)^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
```

```
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2580

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[Log[g + h*x]*(Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]/h), x] + (-Dist[b*p*(r/h), Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[d*q*(r/h), Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 2584

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1
)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(h*(m + 1))), x] + (-Dist[b*p*r*(
s/(h*(m + 1))), Int[(g + h*x)^(m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1)/(a + b*x)), x], x] - Dist[d*q*r*(s/(h*(m + 1))), Int[(g + h*x)^(m
```

```
+ 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(c + d*x)), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2594

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2} + (pr)\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^3}\,dx \\
&\quad + \frac{(dqr)\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^2(c+dx)}\,dx}{b} \\
&= -\frac{pr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2} - \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2} \\
&\quad + \frac{(dqr)\int \left(\frac{b\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(bc-ad)(a+bx)^2} - \frac{bd\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(bc-ad)^2(a+bx)} + \frac{d^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(bc-ad)^2(c+dx)}\right)dx}{b} \\
&\quad + \frac{1}{2}\left(p^2r^2\right)\int \frac{1}{(a+bx)^3}\,dx + \frac{\left(dpqr^2\right)\int \frac{1}{(a+bx)^2(c+dx)}\,dx}{2b} \\
&= -\frac{p^2r^2}{4b(a+bx)^2} - \frac{pr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2} \\
&\quad - \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2} - \frac{\left(d^2qr\right)\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{a+bx}\,dx}{(bc-ad)^2} \\
&\quad + \frac{\left(d^3qr\right)\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{c+dx}\,dx}{b(bc-ad)^2} + \frac{(dqr)\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^2}\,dx}{bc-ad} \\
&\quad + \frac{\left(dpqr^2\right)\int \left(\frac{b}{(bc-ad)(a+bx)^2} - \frac{bd}{(bc-ad)^2(a+bx)} + \frac{d^2}{(bc-ad)^2(c+dx)}\right)dx}{2b}
\end{aligned}
$$

$$= -\frac{p^2r^2}{4b(a+bx)^2} - \frac{dpqr^2}{2b(bc-ad)(a+bx)} - \frac{d^2pqr^2\log(a+bx)}{2b(bc-ad)^2} + \frac{d^2pqr^2\log(c+dx)}{2b(bc-ad)^2}$$
$$- \frac{pr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2} - \frac{dqr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(bc-ad)(a+bx)}$$
$$- \frac{d^2qr\log(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(bc-ad)^2}$$
$$+ \frac{d^2qr\log(c+dx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(bc-ad)^2} - \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2}$$
$$+ \frac{(d^2pqr^2)\int\frac{\log(a+bx)}{a+bx}\,dx}{(bc-ad)^2} - \frac{(d^2pqr^2)\int\frac{\log(c+dx)}{a+bx}\,dx}{(bc-ad)^2} + \frac{(dpqr^2)\int\frac{1}{(a+bx)^2}\,dx}{bc-ad}$$
$$+ \frac{(d^3q^2r^2)\int\frac{\log(a+bx)}{c+dx}\,dx}{b(bc-ad)^2} - \frac{(d^3q^2r^2)\int\frac{\log(c+dx)}{c+dx}\,dx}{b(bc-ad)^2} + \frac{(d^2q^2r^2)\int\frac{1}{(a+bx)(c+dx)}\,dx}{b(bc-ad)}$$

$$= -\frac{p^2r^2}{4b(a+bx)^2} - \frac{3dpqr^2}{2b(bc-ad)(a+bx)} - \frac{d^2pqr^2\log(a+bx)}{2b(bc-ad)^2} + \frac{d^2pqr^2\log(c+dx)}{2b(bc-ad)^2}$$
$$- \frac{d^2pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{b(bc-ad)^2} + \frac{d^2q^2r^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{b(bc-ad)^2}$$
$$- \frac{pr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2} - \frac{dqr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(bc-ad)(a+bx)}$$
$$- \frac{d^2qr\log(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(bc-ad)^2}$$
$$+ \frac{d^2qr\log(c+dx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(bc-ad)^2}$$
$$- \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2} + \frac{(d^2pqr^2)\,\mathrm{Subst}\left(\int\frac{\log(x)}{x}\,dx, x, a+bx\right)}{b(bc-ad)^2}$$
$$+ \frac{(d^3pqr^2)\int\frac{\log\left(\frac{d(a+bx)}{-bc+ad}\right)}{c+dx}\,dx}{b(bc-ad)^2} + \frac{(d^2q^2r^2)\int\frac{1}{a+bx}\,dx}{(bc-ad)^2} - \frac{(d^2q^2r^2)\int\frac{\log\left(\frac{b(c+dx)}{bc-ad}\right)}{a+bx}\,dx}{(bc-ad)^2}$$
$$- \frac{(d^2q^2r^2)\,\mathrm{Subst}\left(\int\frac{\log(x)}{x}\,dx, x, c+dx\right)}{b(bc-ad)^2} - \frac{(d^3q^2r^2)\int\frac{1}{c+dx}\,dx}{b(bc-ad)^2}$$

$$= -\frac{p^2r^2}{4b(a+bx)^2} - \frac{3dpqr^2}{2b(bc-ad)(a+bx)} - \frac{d^2pqr^2\log(a+bx)}{2b(bc-ad)^2} + \frac{d^2q^2r^2\log(a+bx)}{b(bc-ad)^2} + \frac{d^2pqr^2\log^2(a+bx)}{2b(bc-ad)^2} + \frac{d^2pqr^2\log(c+dx)}{2b(bc-ad)^2} - \frac{d^2q^2r^2\log(c+dx)}{b(bc-ad)^2} - \frac{d^2pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{b(bc-ad)^2} - \frac{d^2q^2r^2\log^2(c+dx)}{2b(bc-ad)^2} + \frac{d^2q^2r^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{b(bc-ad)^2} - \frac{pr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b(a+bx)^2} - \frac{dqr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(bc-ad)(a+bx)} - \frac{d^2qr\log(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(bc-ad)^2} + \frac{d^2qr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(bc-ad)^2} - \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b(a+bx)^2} + \frac{(d^2pqr^2)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{bx}{-bc+ad}\right)}{x}\,dx,x,c+dx\right)}{b(bc-ad)^2} - \frac{(d^2q^2r^2)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{dx}{bc-ad}\right)}{x}\,dx,x,a+bx\right)}{b(bc-ad)^2}$$

$$= -\frac{p^2r^2}{4b(a+bx)^2} - \frac{3dpqr^2}{2b(bc-ad)(a+bx)} - \frac{d^2pqr^2\log(a+bx)}{2b(bc-ad)^2} + \frac{d^2q^2r^2\log(a+bx)}{b(bc-ad)^2} + \frac{d^2pqr^2\log^2(a+bx)}{2b(bc-ad)^2} + \frac{d^2pqr^2\log(c+dx)}{2b(bc-ad)^2} - \frac{d^2q^2r^2\log(c+dx)}{b(bc-ad)^2} - \frac{d^2pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{b(bc-ad)^2} - \frac{d^2q^2r^2\log^2(c+dx)}{2b(bc-ad)^2} + \frac{d^2q^2r^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{b(bc-ad)^2} - \frac{pr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b(a+bx)^2} - \frac{dqr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(bc-ad)(a+bx)} - \frac{d^2qr\log(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(bc-ad)^2} + \frac{d^2qr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(bc-ad)^2} - \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b(a+bx)^2} + \frac{d^2q^2r^2\text{Li}_2\left(-\frac{d(a+bx)}{bc-ad}\right)}{b(bc-ad)^2} - \frac{d^2pqr^2\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)}{b(bc-ad)^2}$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 872, normalized size of antiderivative = 1.38

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^3}\,dx =$$

$$-\frac{b^2c^2p^2r^2 - 2abcdp^2r^2 + a^2d^2p^2r^2 + 6abcdpqr^2 - 6a^2d^2pqr^2 + 6b^2cdpqr^2x - 6abd^2pqr^2x - 2d^2pqr^2(a+}{}$$

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x)^3,x]
```

```
[Out] -1/4*(b^2*c^2*p^2*r^2 - 2*a*b*c*d*p^2*r^2 + a^2*d^2*p^2*r^2 + 6*a*b*c*d*p*q
*r^2 - 6*a^2*d^2*p*q*r^2 + 6*b^2*c*d*p*q*r^2*x - 6*a*b*d^2*p*q*r^2*x - 2*d^
2*p*q*r^2*(a + b*x)^2*Log[a + b*x]^2 - 2*a^2*d^2*p*q*r^2*Log[c + d*x] + 4*a
^2*d^2*q^2*r^2*Log[c + d*x] - 4*a*b*d^2*p*q*r^2*x*Log[c + d*x] + 8*a*b*d^2*
q^2*r^2*x*Log[c + d*x] - 2*b^2*d^2*p*q*r^2*x^2*Log[c + d*x] + 4*b^2*d^2*q^2
*r^2*x^2*Log[c + d*x] + 2*a^2*d^2*q^2*r^2*Log[c + d*x]^2 + 4*a*b*d^2*q^2*r^
2*x*Log[c + d*x]^2 + 2*b^2*d^2*q^2*r^2*x^2*Log[c + d*x]^2 - 2*d^2*q*r*(a +
b*x)^2*Log[a + b*x]*(-(p*r) + 2*q*r - 2*p*r*Log[c + d*x] + 2*(p + q)*r*Log[
(b*(c + d*x))/(b*c - a*d)] - 2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]) + 2*b^
2*c^2*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 4*a*b*c*d*p*r*Log[e*(f*(a
+ b*x)^p*(c + d*x)^q)^r] + 2*a^2*d^2*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^
r] + 4*a*b*c*d*q*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 4*a^2*d^2*q*r*Log
[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 4*b^2*c*d*q*r*x*Log[e*(f*(a + b*x)^p*(c
 + d*x)^q)^r] - 4*a*b*d^2*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 4*a^
2*d^2*q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 8*a*b*d^2*q*r
*x*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 4*b^2*d^2*q*r*x^2*Lo
g[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 2*b^2*c^2*Log[e*(f*(a + b
*x)^p*(c + d*x)^q)^r]^2 - 4*a*b*c*d*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2
+ 2*a^2*d^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 - 4*d^2*q*(p + q)*r^2*(a
 + b*x)^2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(b*c - a*d)^2*(a + b
*x)^2)
```

Maple [F]

$$\int \frac{\ln\left(e\left(f(bx+a)^p(dx+c)^q\right)^r\right)^2}{(bx+a)^3}dx$$

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^3,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^3,x)
```

Fricas [F]

$$\int \frac{\log^2 \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^3}\,dx = \int \frac{\log \left(\left(\left(bx+a\right)^p\left(dx+c\right)^q f\right)^r e\right)^2}{\left(bx+a\right)^3}\,dx$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^3,x, algorithm="fricas
")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b^3*x^3 + 3*a*b^2*x^2 + 3*
a^2*b*x + a^3), x)
```

Sympy [F]

$$\int \frac{\log^2 \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^3}\,dx = \int \frac{\log \left(e\left(f\left(a+bx\right)^p\left(c+dx\right)^q\right)^r\right)^2}{\left(a+bx\right)^3}\,dx$$

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(b*x+a)**3,x)
```

```
[Out] Integral(log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)**2/(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 755, normalized size of antiderivative = 1.19

$$\int \frac{\log^2 \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^3}\,dx = -\frac{\left(\frac{2\,d^2 fq\log(bx+a)}{b^2c^2-2\,abcd+a^2d^2} - \frac{2\,d^2 fq\log(dx+c)}{b^2c^2-2\,abcd+a^2d^2} + \frac{2\,bdfqx-adf(p-2\,q)+bcfp}{a^2bc-a^3d+(b^3c-ab^2d)x^2+2\,(ab^2c-a^2bd)x}\right) r\log\left(\left(\left(bx+a\right)^p\left(dx+c\right)^q f\right)^r e\right)}{2\,bf} + \frac{\left(\frac{4\,(pq+q^2)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)d^2f^2}{b^2c^2-2\,abcd+a^2d^2} + \frac{2\,(pq-2\,q^2)d^2f^2\log(dx+c)}{b^2c^2-2\,abcd+a^2d^2} - \frac{b^2c^2f^2p^2-2\,(p^2-3\,pq)abcdf^2+(p^2-6\,pq)\cdots}{\cdots}\cdots}{\cdots} - \frac{\log\left(\left(\left(bx+a\right)^p\left(dx+c\right)^q f\right)^r e\right)^2}{2\,\left(bx+a\right)^2 b}$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^3,x, algorithm="maxima
")
```

```
[Out] -1/2*(2*d^2*f*q*log(b*x + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - 2*d^2*f*q*lo
g(d*x + c)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + (2*b*d*f*q*x - a*d*f*(p - 2*q)
```

```
 + b*c*f*p)/(a^2*b*c - a^3*d + (b^3*c - a*b^2*d)*x^2 + 2*(a*b^2*c - a^2*b*d
)*x))*r*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(b*f) + 1/4*(4*(p*q + q^2)*(lo
g(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c -
 a*d)))*d^2*f^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + 2*(p*q - 2*q^2)*d^2*f^2*l
og(d*x + c)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - (b^2*c^2*f^2*p^2 - 2*(p^2 - 3
*p*q)*a*b*c*d*f^2 + (p^2 - 6*p*q)*a^2*d^2*f^2 - 2*(b^2*d^2*f^2*p*q*x^2 + 2*
a*b*d^2*f^2*p*q*x + a^2*d^2*f^2*p*q)*log(b*x + a)^2 + 4*(b^2*d^2*f^2*p*q*x^
2 + 2*a*b*d^2*f^2*p*q*x + a^2*d^2*f^2*p*q)*log(b*x + a)*log(d*x + c) + 2*(b
^2*d^2*f^2*q^2*x^2 + 2*a*b*d^2*f^2*q^2*x + a^2*d^2*f^2*q^2)*log(d*x + c)^2
+ 6*(b^2*c*d*f^2*p*q - a*b*d^2*f^2*p*q)*x + 2*((p*q - 2*q^2)*b^2*d^2*f^2*x^
2 + 2*(p*q - 2*q^2)*a*b*d^2*f^2*x + (p*q - 2*q^2)*a^2*d^2*f^2)*log(b*x + a)
)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d
^2)*x^2 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x))*r^2/(b*f^2) - 1/2*l
og(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/((b*x + a)^2*b)
```

Giac [F]

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^3}\,dx=\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{(bx+a)^3}\,dx$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^3,x, algorithm="giac")
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^3}\,dx=\int \frac{\ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)^2}{(a+b\,x)^3}\,dx$$

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(a + b*x)^3,x)
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(a + b*x)^3, x)
```

3.23 $\int \frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^4} \, dx$

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Optimal result

Integrand size = 31, antiderivative size = 764

$$\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^4}\,dx = -\frac{2p^2r^2}{27b(a+bx)^3} - \frac{5dpqr^2}{18b(bc-ad)(a+bx)^2} + \frac{8d^2pqr^2}{9b(bc-ad)^2(a+bx)} - \frac{d^2q^2r^2}{3b(bc-ad)^2(a+bx)} + \frac{2d^3pqr^2\log(a+bx)}{9b(bc-ad)^3} - \frac{d^3q^2r^2\log(a+bx)}{b(bc-ad)^3} - \frac{d^3pqr^2\log^2(a+bx)}{3b(bc-ad)^3} - \frac{2d^3pqr^2\log(c+dx)}{9b(bc-ad)^3} + \frac{d^3q^2r^2\log(c+dx)}{b(bc-ad)^3} + \frac{2d^3pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{3b(bc-ad)^3} + \frac{d^3q^2r^2\log^2(c+dx)}{3b(bc-ad)^3} - \frac{2d^3q^2r^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{3b(bc-ad)^3} - \frac{2pr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{9b(a+bx)^3} - \frac{dqr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b(bc-ad)(a+bx)^2} + \frac{2d^2qr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b(bc-ad)^2(a+bx)} + \frac{2d^3qr\log(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b(bc-ad)^3} - \frac{2d^3qr\log(c+dx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b(bc-ad)^3} - \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b(a+bx)^3} - \frac{2d^3q^2r^2\,\mathrm{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{3b(bc-ad)^3} + \frac{2d^3pqr^2\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{3b(bc-ad)^3}$$

```
[Out] -2/27*p^2*r^2/b/(b*x+a)^3-5/18*d*p*q*r^2/b/(-a*d+b*c)/(b*x+a)^2+8/9*d^2*p*q
*r^2/b/(-a*d+b*c)^2/(b*x+a)-1/3*d^2*q^2*r^2/b/(-a*d+b*c)^2/(b*x+a)+2/9*d^3*
```

```
p*q*r^2*ln(b*x+a)/b/(-a*d+b*c)^3-d^3*q^2*r^2*ln(b*x+a)/b/(-a*d+b*c)^3-1/3*d
^3*p*q*r^2*ln(b*x+a)^2/b/(-a*d+b*c)^3-2/9*d^3*p*q*r^2*ln(d*x+c)/b/(-a*d+b*c
)^3+d^3*q^2*r^2*ln(d*x+c)/b/(-a*d+b*c)^3+2/3*d^3*p*q*r^2*ln(-d*(b*x+a)/(-a*
d+b*c))*ln(d*x+c)/b/(-a*d+b*c)^3+1/3*d^3*q^2*r^2*ln(d*x+c)^2/b/(-a*d+b*c)^3
-2/3*d^3*q^2*r^2*ln(b*x+a)*ln(b*(d*x+c)/(-a*d+b*c))/b/(-a*d+b*c)^3-2/9*p*r*
ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/(b*x+a)^3-1/3*d*q*r*ln(e*(f*(b*x+a)^p*(d*
x+c)^q)^r)/b/(-a*d+b*c)/(b*x+a)^2+2/3*d^2*q*r*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^
r)/b/(-a*d+b*c)^2/(b*x+a)+2/3*d^3*q*r*ln(b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q
)^r)/b/(-a*d+b*c)^3-2/3*d^3*q*r*ln(d*x+c)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b
/(-a*d+b*c)^3-1/3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/b/(b*x+a)^3-2/3*d^3*q^2
*r^2*polylog(2,-d*(b*x+a)/(-a*d+b*c))/b/(-a*d+b*c)^3+2/3*d^3*p*q*r^2*polylo
g(2,b*(d*x+c)/(-a*d+b*c))/b/(-a*d+b*c)^3
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 764, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules

used = {2584, 2581, 32, 46, 2594, 36, 31, 2580, 2437, 2338, 2441, 2440, 2438}

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^4}\,dx = \frac{2d^3qr\log(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(bc-ad)^3} - \frac{2d^3qr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(bc-ad)^3} + \frac{2d^3pqr^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{3b(bc-ad)^3} - \frac{d^3pqr^2\log^2(a+bx)}{3b(bc-ad)^3} + \frac{2d^3pqr^2\log(a+bx)}{9b(bc-ad)^3} - \frac{2d^3pqr^2\log(c+dx)}{9b(bc-ad)^3} + \frac{2d^3pqr^2\log(c+dx)\log\left(-\frac{d(a+bx)}{bc-ad}\right)}{3b(bc-ad)^3} - \frac{2d^3q^2r^2\operatorname{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{3b(bc-ad)^3} + \frac{d^3q^2r^2\log^2(c+dx)}{3b(bc-ad)^3} - \frac{d^3q^2r^2\log(a+bx)}{b(bc-ad)^3} + \frac{d^3q^2r^2\log(c+dx)}{b(bc-ad)^3} - \frac{2d^3q^2r^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{3b(bc-ad)^3} + \frac{2d^2qr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(a+bx)(bc-ad)^2} + \frac{8d^2pqr^2}{9b(a+bx)(bc-ad)^2} - \frac{d^2q^2r^2}{3b(a+bx)(bc-ad)^2} - \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(a+bx)^3} - \frac{dqr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(a+bx)^2(bc-ad)} - \frac{2pr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{9b(a+bx)^3} - \frac{5dpqr^2}{18b(a+bx)^2(bc-ad)} - \frac{2p^2r^2}{27b(a+bx)^3}$$

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x)^4,x]
```

```
[Out] (-2*p^2*r^2)/(27*b*(a + b*x)^3) - (5*d*p*q*r^2)/(18*b*(b*c - a*d)*(a + b*x)
^2) + (8*d^2*p*q*r^2)/(9*b*(b*c - a*d)^2*(a + b*x)) - (d^2*q^2*r^2)/(3*b*(b
*c - a*d)^2*(a + b*x)) + (2*d^3*p*q*r^2*Log[a + b*x])/(9*b*(b*c - a*d)^3) -
 (d^3*q^2*r^2*Log[a + b*x])/(b*(b*c - a*d)^3) - (d^3*p*q*r^2*Log[a + b*x]^2
)/(3*b*(b*c - a*d)^3) - (2*d^3*p*q*r^2*Log[c + d*x])/(9*b*(b*c - a*d)^3) +
```

```
(d^3*q^2*r^2*Log[c + d*x])/(b*(b*c - a*d)^3) + (2*d^3*p*q*r^2*Log[-((d*(a +
 b*x))/(b*c - a*d))]*Log[c + d*x])/(3*b*(b*c - a*d)^3) + (d^3*q^2*r^2*Log[c
 + d*x]^2)/(3*b*(b*c - a*d)^3) - (2*d^3*q^2*r^2*Log[a + b*x]*Log[(b*(c + d*
x))/(b*c - a*d)])/(3*b*(b*c - a*d)^3) - (2*p*r*Log[e*(f*(a + b*x)^p*(c + d*
x)^q)^r])/(9*b*(a + b*x)^3) - (d*q*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/
(3*b*(b*c - a*d)*(a + b*x)^2) + (2*d^2*q*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q
)^r])/(3*b*(b*c - a*d)^2*(a + b*x)) + (2*d^3*q*r*Log[a + b*x]*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/(3*b*(b*c - a*d)^3) - (2*d^3*q*r*Log[c + d*x]*Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(3*b*(b*c - a*d)^3) - Log[e*(f*(a + b*x)^
p*(c + d*x)^q)^r]^2/(3*b*(a + b*x)^3) - (2*d^3*q^2*r^2*PolyLog[2, -((d*(a +
 b*x))/(b*c - a*d))])/(3*b*(b*c - a*d)^3) + (2*d^3*p*q*r^2*PolyLog[2, (b*(c
 + d*x))/(b*c - a*d)])/(3*b*(b*c - a*d)^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2580

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[Log[g + h*x]*(Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]/h), x] + (-Dist[b*p*(r/h), Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[d*q*(r/h), Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 2584

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1
)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(h*(m + 1))), x] + (-Dist[b*p*r*(
s/(h*(m + 1))), Int[(g + h*x)^(m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1)/(a + b*x)), x], x] - Dist[d*q*r*(s/(h*(m + 1))), Int[(g + h*x)^(m
+ 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(c + d*x)), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2594

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rubi steps

$$\text{integral} = -\frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b(a+bx)^3} + \frac{1}{3}(2pr)\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^4}\,dx + \frac{(2dqr)\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^3(c+dx)}\,dx}{3b}$$

$$= -\frac{2pr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{9b(a+bx)^3} - \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b(a+bx)^3} + \frac{(2dqr)\int \left(\frac{b\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(bc-ad)(a+bx)^3} - \frac{bd\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(bc-ad)^2(a+bx)^2} + \frac{bd^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(bc-ad)^3(a+bx)} - \frac{d^3\log\left(e\left(f(a+b\cdots\right.\right.}{(bc-ad)\cdots}\right.}{3b} + \frac{1}{9}\left(2p^2r^2\right)\int \frac{1}{(a+bx)^4}\,dx + \frac{\left(2dpqr^2\right)\int \frac{1}{(a+bx)^3(c+dx)}\,dx}{9b}$$

$$= -\frac{2p^2r^2}{27b(a+bx)^3} - \frac{2pr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{9b(a+bx)^3} - \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b(a+bx)^3} + \frac{\left(2d^3qr\right)\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{a+bx}\,dx}{3(bc-ad)^3} - \frac{\left(2d^4qr\right)\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{c+dx}\,dx}{3b(bc-ad)^3} - \frac{\left(2d^2qr\right)\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^2}\,dx}{3(bc-ad)^2} + \frac{(2dqr)\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^3}\,dx}{3(bc-ad)} + \frac{\left(2dpqr^2\right)\int \left(\frac{b}{(bc-ad)(a+bx)^3} - \frac{bd}{(bc-ad)^2(a+bx)^2} + \frac{bd^2}{(bc-ad)^3(a+bx)} - \frac{d^3}{(bc-ad)^3(c+dx)}\right)dx}{9b}$$

$$= -\frac{2p^2r^2}{27b(a+bx)^3} - \frac{dpqr^2}{9b(bc-ad)(a+bx)^2} + \frac{2d^2pqr^2}{9b(bc-ad)^2(a+bx)} + \frac{2d^3pqr^2\log(a+bx)}{9b(bc-ad)^3} - \frac{2d^3pqr^2\log(c+dx)}{9b(bc-ad)^3} - \frac{2pr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{9b(a+bx)^3} - \frac{dqr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(bc-ad)(a+bx)^2} + \frac{2d^2qr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(bc-ad)^2(a+bx)} + \frac{2d^3qr\log(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(bc-ad)^3} - \frac{2d^3qr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(bc-ad)^3} - \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(a+bx)^3} - \frac{(2d^3pqr^2)\int\frac{\log(a+bx)}{a+bx}\,dx}{3(bc-ad)^3} + \frac{(2d^3pqr^2)\int\frac{\log(c+dx)}{a+bx}\,dx}{3(bc-ad)^3} - \frac{(2d^2pqr^2)\int\frac{1}{(a+bx)^2}\,dx}{3(bc-ad)^2} + \frac{(dpqr^2)\int\frac{1}{(a+bx)^3}\,dx}{3(bc-ad)} - \frac{(2d^4q^2r^2)\int\frac{\log(a+bx)}{c+dx}\,dx}{3b(bc-ad)^3} + \frac{(2d^4q^2r^2)\int\frac{\log(c+dx)}{c+dx}\,dx}{3b(bc-ad)^3} - \frac{(2d^3q^2r^2)\int\frac{1}{(a+bx)(c+dx)}\,dx}{3b(bc-ad)^2} + \frac{(d^2q^2r^2)\int\frac{1}{(a+bx)^2(c+dx)}\,dx}{3b(bc-ad)}$$

$$= -\frac{2p^2r^2}{27b(a+bx)^3} - \frac{5dpqr^2}{18b(bc-ad)(a+bx)^2} + \frac{8d^2pqr^2}{9b(bc-ad)^2(a+bx)} + \frac{2d^3pqr^2\log(a+bx)}{9b(bc-ad)^3} - \frac{2d^3pqr^2\log(c+dx)}{9b(bc-ad)^3} + \frac{2d^3pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{3b(bc-ad)^3} - \frac{2d^3q^2r^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{3b(bc-ad)^3} - \frac{2pr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{9b(a+bx)^3} - \frac{dqr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(bc-ad)(a+bx)^2} + \frac{2d^2qr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(bc-ad)^2(a+bx)} + \frac{2d^3qr\log(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(bc-ad)^3} - \frac{2d^3qr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(bc-ad)^3} - \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(a+bx)^3} - \frac{(2d^3pqr^2)\,\text{Subst}\left(\int\frac{\log(x)}{x}\,dx, x, a+bx\right)}{3b(bc-ad)^3} - \frac{(2d^4pqr^2)\int\frac{\log\left(\frac{d(a+bx)}{-bc+ad}\right)}{c+dx}\,dx}{3b(bc-ad)^3} - \frac{(2d^3q^2r^2)\int\frac{1}{a+bx}\,dx}{3(bc-ad)^3} + \frac{(2d^3q^2r^2)\int\frac{\log\left(\frac{b(c+dx)}{bc-ad}\right)}{a+bx}\,dx}{3(bc-ad)^3} + \frac{(2d^3q^2r^2)\,\text{Subst}\left(\int\frac{\log(x)}{x}\,dx, x, c+dx\right)}{3b(bc-ad)^3} + \frac{(2d^4q^2r^2)\int\frac{1}{c+dx}\,dx}{3b(bc-ad)^3} + \frac{(d^2q^2r^2)\int\left(\frac{b}{(bc-ad)(a+bx)^2} - \frac{bd}{(bc-ad)^2(a+bx)} + \frac{d^2}{(bc-ad)^2(c+dx)}\right)dx}{3b(bc-ad)}$$

$$= -\frac{2p^2r^2}{27b(a+bx)^3} - \frac{5dpqr^2}{18b(bc-ad)(a+bx)^2} + \frac{8d^2pqr^2}{9b(bc-ad)^2(a+bx)}$$
$$- \frac{d^2q^2r^2}{3b(bc-ad)^2(a+bx)} + \frac{2d^3pqr^2\log(a+bx)}{9b(bc-ad)^3} - \frac{d^3q^2r^2\log(a+bx)}{b(bc-ad)^3}$$
$$- \frac{d^3pqr^2\log^2(a+bx)}{3b(bc-ad)^3} - \frac{2d^3pqr^2\log(c+dx)}{9b(bc-ad)^3} + \frac{d^3q^2r^2\log(c+dx)}{b(bc-ad)^3}$$
$$+ \frac{2d^3pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{3b(bc-ad)^3} + \frac{d^3q^2r^2\log^2(c+dx)}{3b(bc-ad)^3}$$
$$- \frac{2d^3q^2r^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{3b(bc-ad)^3} - \frac{2pr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{9b(a+bx)^3}$$
$$- \frac{dqr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(bc-ad)(a+bx)^2} + \frac{2d^2qr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(bc-ad)^2(a+bx)}$$
$$+ \frac{2d^3qr\log(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(bc-ad)^3}$$
$$- \frac{2d^3qr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(bc-ad)^3}$$
$$- \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(a+bx)^3} - \frac{(2d^3pqr^2)\,\mathrm{Subst}\left(\int\frac{\log\left(1+\frac{bx}{-bc+ad}\right)}{x}\,dx, x, c+dx\right)}{3b(bc-ad)^3}$$
$$+ \frac{(2d^3q^2r^2)\,\mathrm{Subst}\left(\int\frac{\log\left(1+\frac{dx}{bc-ad}\right)}{x}\,dx, x, a+bx\right)}{3b(bc-ad)^3}$$

$$
\begin{aligned}
&= -\frac{2p^2r^2}{27b(a+bx)^3} - \frac{5dpqr^2}{18b(bc-ad)(a+bx)^2} + \frac{8d^2pqr^2}{9b(bc-ad)^2(a+bx)} \\
&\quad - \frac{d^2q^2r^2}{3b(bc-ad)^2(a+bx)} + \frac{2d^3pqr^2\log(a+bx)}{9b(bc-ad)^3} - \frac{d^3q^2r^2\log(a+bx)}{b(bc-ad)^3} \\
&\quad - \frac{d^3pqr^2\log^2(a+bx)}{3b(bc-ad)^3} - \frac{2d^3pqr^2\log(c+dx)}{9b(bc-ad)^3} + \frac{d^3q^2r^2\log(c+dx)}{b(bc-ad)^3} \\
&\quad + \frac{2d^3pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{3b(bc-ad)^3} + \frac{d^3q^2r^2\log^2(c+dx)}{3b(bc-ad)^3} \\
&\quad - \frac{2d^3q^2r^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{3b(bc-ad)^3} - \frac{2pr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{9b(a+bx)^3} \\
&\quad - \frac{dqr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(bc-ad)(a+bx)^2} + \frac{2d^2qr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(bc-ad)^2(a+bx)} \\
&\quad + \frac{2d^3qr\log(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(bc-ad)^3} \\
&\quad - \frac{2d^3qr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(bc-ad)^3} \\
&\quad - \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(a+bx)^3} - \frac{2d^3q^2r^2\operatorname{Li}_2\left(-\frac{d(a+bx)}{bc-ad}\right)}{3b(bc-ad)^3} + \frac{2d^3pqr^2\operatorname{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)}{3b(bc-ad)^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 1407, normalized size of antiderivative = 1.84

$$
\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^4}\,dx =
$$

$$
-\frac{4b^3c^3p^2r^2 - 12ab^2c^2dp^2r^2 + 12a^2bcd^2p^2r^2 - 4a^3d^3p^2r^2 + 15ab^2c^2dpqr^2 - 78a^2bcd^2pqr^2 + 63a^3d^3pqr^2 +}{}
$$

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x)^4,x]
```

```
[Out] -1/54*(4*b^3*c^3*p^2*r^2 - 12*a*b^2*c^2*d*p^2*r^2 + 12*a^2*b*c*d^2*p^2*r^2
- 4*a^3*d^3*p^2*r^2 + 15*a*b^2*c^2*d*p*q*r^2 - 78*a^2*b*c*d^2*p*q*r^2 + 63*
a^3*d^3*p*q*r^2 + 18*a^2*b*c*d^2*q^2*r^2 - 18*a^3*d^3*q^2*r^2 + 15*b^3*c^2*
d*p*q*r^2*x - 126*a*b^2*c*d^2*p*q*r^2*x + 111*a^2*b*d^3*p*q*r^2*x + 36*a*b^
2*c*d^2*q^2*r^2*x - 36*a^2*b*d^3*q^2*r^2*x - 48*b^3*c*d^2*p*q*r^2*x^2 + 48*
a*b^2*d^3*p*q*r^2*x^2 + 18*b^3*c*d^2*q^2*r^2*x^2 - 18*a*b^2*d^3*q^2*r^2*x^2
 + 18*d^3*p*q*r^2*(a + b*x)^3*Log[a + b*x]^2 + 12*a^3*d^3*p*q*r^2*Log[c + d
*x] - 54*a^3*d^3*q^2*r^2*Log[c + d*x] + 36*a^2*b*d^3*p*q*r^2*x*Log[c + d*x]
 - 162*a^2*b*d^3*q^2*r^2*x*Log[c + d*x] + 36*a*b^2*d^3*p*q*r^2*x^2*Log[c +
d*x] - 162*a*b^2*d^3*q^2*r^2*x^2*Log[c + d*x] + 12*b^3*d^3*p*q*r^2*x^3*Log[
c + d*x] - 54*b^3*d^3*q^2*r^2*x^3*Log[c + d*x] - 18*a^3*d^3*q^2*r^2*Log[c +
```

```
d*x]^2 - 54*a^2*b*d^3*q^2*r^2*x*Log[c + d*x]^2 - 54*a*b^2*d^3*q^2*r^2*x^2*
Log[c + d*x]^2 - 18*b^3*d^3*q^2*r^2*x^3*Log[c + d*x]^2 + 12*b^3*c^3*p*r*Log
[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 36*a*b^2*c^2*d*p*r*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r] + 36*a^2*b*c*d^2*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
- 12*a^3*d^3*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 18*a*b^2*c^2*d*q*r*
Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 72*a^2*b*c*d^2*q*r*Log[e*(f*(a + b*x
)^p*(c + d*x)^q)^r] + 54*a^3*d^3*q*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] +
 18*b^3*c^2*d*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 108*a*b^2*c*d^2*
q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 90*a^2*b*d^3*q*r*x*Log[e*(f*(a
 + b*x)^p*(c + d*x)^q)^r] - 36*b^3*c*d^2*q*r*x^2*Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r] + 36*a*b^2*d^3*q*r*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 36
*a^3*d^3*q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 108*a^2*b*
d^3*q*r*x*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 108*a*b^2*d^3
*q*r*x^2*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 36*b^3*d^3*q*r
*x^3*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 18*b^3*c^3*Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r]^2 - 54*a*b^2*c^2*d*Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^2 + 54*a^2*b*c*d^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 - 18*a
^3*d^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 - 6*d^3*q*r*(a + b*x)^3*Log[a
 + b*x]*(2*p*r - 9*q*r + 6*p*r*Log[c + d*x] - 6*(p + q)*r*Log[(b*(c + d*x))
/(b*c - a*d)] + 6*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]) + 36*d^3*q*(p + q)*
r^2*(a + b*x)^3*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(b*c - a*d)^3*
(a + b*x)^3)
```

Maple [F]

$$\int \frac{\ln \left(e(f(bx+a)^p\,(dx+c)^q)^r\right)^2}{(bx+a)^4}dx$$

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^4,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^4,x)
```

Fricas [F]

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^4}\,dx = \int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{(bx+a)^4}\,dx$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^4,x, algorithm="fricas
")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b^4*x^4 + 4*a*b^3*x^3 + 6*
a^2*b^2*x^2 + 4*a^3*b*x + a^4), x)
```

Sympy [F]

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^4}\,dx = \int \frac{\log\left(e\left(f\left(a+bx\right)^p\left(c+dx\right)^q\right)^r\right)^2}{\left(a+bx\right)^4}\,dx$$

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(b*x+a)**4,x)
[Out] Integral(log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)**2/(a + b*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 1252, normalized size of antiderivative = 1.64

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^4}\,dx = \text{Too large to display}$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^4,x, algorithm="maxima
")
[Out] 1/9*(6*d^3*f*q*log(b*x + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*
d^3) - 6*d^3*f*q*log(d*x + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3) + (6*b^2*d^2*f*q*x^2 + a*b*c*d*f*(4*p - 3*q) - a^2*d^2*f*(2*p - 9*q)
 - 2*b^2*c^2*f*p - 3*(b^2*c*d*f*q - 5*a*b*d^2*f*q)*x)/(a^3*b^2*c^2 - 2*a^4*
b*c*d + a^5*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^3 + 3*(a*b^4*c^2
- 2*a^2*b^3*c*d + a^3*b^2*d^2)*x^2 + 3*(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b
*d^2)*x))*r*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(b*f) - 1/54*(36*(p*q + q^
2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/
(b*c - a*d)))*d^3*f^2/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) +
 6*(2*p*q - 9*q^2)*d^3*f^2*log(d*x + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*
c*d^2 - a^3*d^3) + (4*b^3*c^3*f^2*p^2 - 3*(4*p^2 - 5*p*q)*a*b^2*c^2*d*f^2 +
 6*(2*p^2 - 13*p*q + 3*q^2)*a^2*b*c*d^2*f^2 - (4*p^2 - 63*p*q + 18*q^2)*a^3
*d^3*f^2 - 6*((8*p*q - 3*q^2)*b^3*c*d^2*f^2 - (8*p*q - 3*q^2)*a*b^2*d^3*f^2
)*x^2 + 18*(b^3*d^3*f^2*p*q*x^3 + 3*a*b^2*d^3*f^2*p*q*x^2 + 3*a^2*b*d^3*f^2
*p*q*x + a^3*d^3*f^2*p*q)*log(b*x + a)^2 - 36*(b^3*d^3*f^2*p*q*x^3 + 3*a*b^
2*d^3*f^2*p*q*x^2 + 3*a^2*b*d^3*f^2*p*q*x + a^3*d^3*f^2*p*q)*log(b*x + a)*l
og(d*x + c) - 18*(b^3*d^3*f^2*q^2*x^3 + 3*a*b^2*d^3*f^2*q^2*x^2 + 3*a^2*b*d
^3*f^2*q^2*x + a^3*d^3*f^2*q^2)*log(d*x + c)^2 + 3*(5*b^3*c^2*d*f^2*p*q - 6
*(7*p*q - 2*q^2)*a*b^2*c*d^2*f^2 + (37*p*q - 12*q^2)*a^2*b*d^3*f^2)*x - 6*(
(2*p*q - 9*q^2)*b^3*d^3*f^2*x^3 + 3*(2*p*q - 9*q^2)*a*b^2*d^3*f^2*x^2 + 3*(
2*p*q - 9*q^2)*a^2*b*d^3*f^2*x + (2*p*q - 9*q^2)*a^3*d^3*f^2)*log(b*x + a))
/(a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*
b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c
^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2*d^3)*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*
d + 3*a^4*b^2*c*d^2 - a^5*b*d^3)*x))*r^2/(b*f^2) - 1/3*log(((b*x + a)^p*(d*
x + c)^q*f)^r*e)^2/((b*x + a)^3*b)
```

Giac [F]

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^4}\,dx=\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{(bx+a)^4}\,dx$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^4,x, algorithm="giac")
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b*x + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^4}\,dx=\int \frac{\ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)^2}{(a+b\,x)^4}\,dx$$

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(a + b*x)^4,x)
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(a + b*x)^4, x)
```

3.24 $\int \frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^5} \, dx$

Optimal result 244
Rubi [A] (verified) 245
Mathematica [B] (verified) 253
Maple [F] 255
Fricas [F] 255
Sympy [F] 255
Maxima [B] (verification not implemented) 255
Giac [F] 256
Mupad [F(-1)] 257

Optimal result

Integrand size = 31, antiderivative size = 884

$$\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^5}\,dx = -\frac{p^2r^2}{32b(a+bx)^4} - \frac{7dpqr^2}{72b(bc-ad)(a+bx)^3} + \frac{3d^2pqr^2}{16b(bc-ad)^2(a+bx)^2} - \frac{d^2q^2r^2}{12b(bc-ad)^2(a+bx)^2} - \frac{5d^3pqr^2}{8b(bc-ad)^3(a+bx)} + \frac{5d^3q^2r^2}{12b(bc-ad)^3(a+bx)} - \frac{d^4pqr^2\log(a+bx)}{8b(bc-ad)^4} + \frac{11d^4q^2r^2\log(a+bx)}{12b(bc-ad)^4} + \frac{d^4pqr^2\log^2(a+bx)}{4b(bc-ad)^4} + \frac{d^4pqr^2\log(c+dx)}{8b(bc-ad)^4} - \frac{11d^4q^2r^2\log(c+dx)}{12b(bc-ad)^4} - \frac{d^4pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{2b(bc-ad)^4} - \frac{d^4q^2r^2\log^2(c+dx)}{4b(bc-ad)^4} + \frac{d^4q^2r^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{2b(bc-ad)^4} - \frac{pr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{8b(a+bx)^4} - \frac{dqr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{6b(bc-ad)(a+bx)^3} + \frac{d^2qr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4b(bc-ad)^2(a+bx)^2} - \frac{d^3qr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(bc-ad)^3(a+bx)} - \frac{d^4qr\log(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(bc-ad)^4} + \frac{d^4qr\log(c+dx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(bc-ad)^4} - \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4b(a+bx)^4} + \frac{d^4q^2r^2\operatorname{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{2b(bc-ad)^4} - \frac{d^4pqr^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{2b(bc-ad)^4}$$

```
[Out] -1/32*p^2*r^2/b/(b*x+a)^4-7/72*d*p*q*r^2/b/(-a*d+b*c)/(b*x+a)^3+3/16*d^2*p*
q*r^2/b/(-a*d+b*c)^2/(b*x+a)^2-1/12*d^2*q^2*r^2/b/(-a*d+b*c)^2/(b*x+a)^2-5/
8*d^3*p*q*r^2/b/(-a*d+b*c)^3/(b*x+a)+5/12*d^3*q^2*r^2/b/(-a*d+b*c)^3/(b*x+a
)-1/8*d^4*p*q*r^2*ln(b*x+a)/b/(-a*d+b*c)^4+11/12*d^4*q^2*r^2*ln(b*x+a)/b/(-
a*d+b*c)^4+1/4*d^4*p*q*r^2*ln(b*x+a)^2/b/(-a*d+b*c)^4+1/8*d^4*p*q*r^2*ln(d*
x+c)/b/(-a*d+b*c)^4-11/12*d^4*q^2*r^2*ln(d*x+c)/b/(-a*d+b*c)^4-1/2*d^4*p*q*
r^2*ln(-d*(b*x+a)/(-a*d+b*c))*ln(d*x+c)/b/(-a*d+b*c)^4-1/4*d^4*q^2*r^2*ln(d
*x+c)^2/b/(-a*d+b*c)^4+1/2*d^4*q^2*r^2*ln(b*x+a)*ln(b*(d*x+c)/(-a*d+b*c))/b
/(-a*d+b*c)^4-1/8*p*r*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/(b*x+a)^4-1/6*d*q*r
*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/(-a*d+b*c)/(b*x+a)^3+1/4*d^2*q*r*ln(e*(f
*(b*x+a)^p*(d*x+c)^q)^r)/b/(-a*d+b*c)^2/(b*x+a)^2-1/2*d^3*q*r*ln(e*(f*(b*x+
a)^p*(d*x+c)^q)^r)/b/(-a*d+b*c)^3/(b*x+a)-1/2*d^4*q*r*ln(b*x+a)*ln(e*(f*(b*
x+a)^p*(d*x+c)^q)^r)/b/(-a*d+b*c)^4+1/2*d^4*q*r*ln(d*x+c)*ln(e*(f*(b*x+a)^p
*(d*x+c)^q)^r)/b/(-a*d+b*c)^4-1/4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/b/(b*x+
a)^4+1/2*d^4*q^2*r^2*polylog(2,-d*(b*x+a)/(-a*d+b*c))/b/(-a*d+b*c)^4-1/2*d^
4*p*q*r^2*polylog(2,b*(d*x+c)/(-a*d+b*c))/b/(-a*d+b*c)^4
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 884, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.419, Rules

used = {2584, 2581, 32, 46, 2594, 36, 31, 2580, 2437, 2338, 2441, 2440, 2438}

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^5}\,dx = \frac{pqr^2\log^2(a+bx)d^4}{4b(bc-ad)^4} - \frac{q^2r^2\log^2(c+dx)d^4}{4b(bc-ad)^4} + \frac{11q^2r^2\log(a+bx)d^4}{12b(bc-ad)^4} - \frac{pqr^2\log(a+bx)d^4}{8b(bc-ad)^4} - \frac{11q^2r^2\log(c+dx)d^4}{12b(bc-ad)^4} + \frac{pqr^2\log(c+dx)d^4}{8b(bc-ad)^4} - \frac{pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)d^4}{2b(bc-ad)^4} + \frac{q^2r^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)d^4}{2b(bc-ad)^4} - \frac{qr\log(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)d^4}{2b(bc-ad)^4} + \frac{qr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)d^4}{2b(bc-ad)^4} + \frac{q^2r^2\,\mathrm{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)d^4}{2b(bc-ad)^4} - \frac{pqr^2\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)d^4}{2b(bc-ad)^4} - \frac{qr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)d^3}{2b(bc-ad)^3(a+bx)} + \frac{5q^2r^2d^3}{12b(bc-ad)^3(a+bx)} - \frac{5pqr^2d^3}{8b(bc-ad)^3(a+bx)} + \frac{qr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)d^2}{4b(bc-ad)^2(a+bx)^2} - \frac{q^2r^2d^2}{12b(bc-ad)^2(a+bx)^2} + \frac{3pqr^2d^2}{16b(bc-ad)^2(a+bx)^2} - \frac{qr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)d}{6b(bc-ad)(a+bx)^3} - \frac{7pqr^2d}{72b(bc-ad)(a+bx)^3} - \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b(a+bx)^4} - \frac{pr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{8b(a+bx)^4} - \frac{p^2r^2}{32b(a+bx)^4}$$

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x)^5,x]
```

```
[Out] -1/32*(p^2*r^2)/(b*(a + b*x)^4) - (7*d*p*q*r^2)/(72*b*(b*c - a*d)*(a + b*x)
^3) + (3*d^2*p*q*r^2)/(16*b*(b*c - a*d)^2*(a + b*x)^2) - (d^2*q^2*r^2)/(12*
b*(b*c - a*d)^2*(a + b*x)^2) - (5*d^3*p*q*r^2)/(8*b*(b*c - a*d)^3*(a + b*x)
) + (5*d^3*q^2*r^2)/(12*b*(b*c - a*d)^3*(a + b*x)) - (d^4*p*q*r^2*Log[a + b
*x])/(8*b*(b*c - a*d)^4) + (11*d^4*q^2*r^2*Log[a + b*x])/(12*b*(b*c - a*d)^
4) + (d^4*p*q*r^2*Log[a + b*x]^2)/(4*b*(b*c - a*d)^4) + (d^4*p*q*r^2*Log[c
+ d*x])/(8*b*(b*c - a*d)^4) - (11*d^4*q^2*r^2*Log[c + d*x])/(12*b*(b*c - a*
d)^4) - (d^4*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(2*b*(
b*c - a*d)^4) - (d^4*q^2*r^2*Log[c + d*x]^2)/(4*b*(b*c - a*d)^4) + (d^4*q^2
*r^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(2*b*(b*c - a*d)^4) - (p*
r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(8*b*(a + b*x)^4) - (d*q*r*Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r])/(6*b*(b*c - a*d)*(a + b*x)^3) + (d^2*q*r*Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(4*b*(b*c - a*d)^2*(a + b*x)^2) - (d^3*q*
r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*b*(b*c - a*d)^3*(a + b*x)) - (d^
4*q*r*Log[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*b*(b*c - a*d)^4
) + (d^4*q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*b*(b*c -
 a*d)^4) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(4*b*(a + b*x)^4) + (d^4*
q^2*r^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(2*b*(b*c - a*d)^4) - (d^
4*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(2*b*(b*c - a*d)^4)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2580

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[Log[g + h*x]*(Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]/h), x] + (-Dist[b*p*(r/h), Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[d*q*(r/h), Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 2584

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1
```

```
)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(h*(m + 1))), x] + (-Dist[b*p*r*(
s/(h*(m + 1))), Int[(g + h*x)^(m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1)/(a + b*x)), x], x] - Dist[d*q*r*(s/(h*(m + 1))), Int[(g + h*x)^(m
+ 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(c + d*x)), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2594

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4b(a+bx)^4}+\frac{1}{2}(pr)\int\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^5}\,dx \\
&\quad+\frac{(dqr)\int\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^4(c+dx)}\,dx}{2b} \\
&= -\frac{pr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{8b(a+bx)^4}-\frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4b(a+bx)^4} \\
&\quad+\frac{(dqr)\int\left(\frac{b\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(bc-ad)(a+bx)^4}-\frac{bd\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(bc-ad)^2(a+bx)^3}+\frac{bd^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(bc-ad)^3(a+bx)^2}-\frac{bd^3\log(e(f(a+}{(bc-ad}\right.}{2b} \\
&\quad+\frac{1}{8}\left(p^2r^2\right)\int\frac{1}{(a+bx)^5}\,dx+\frac{\left(dpqr^2\right)\int\frac{1}{(a+bx)^4(c+dx)}\,dx}{8b} \\
&= -\frac{p^2r^2}{32b(a+bx)^4}-\frac{pr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{8b(a+bx)^4} \\
&\quad-\frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4b(a+bx)^4}-\frac{\left(d^4qr\right)\int\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{a+bx}\,dx}{2(bc-ad)^4} \\
&\quad+\frac{\left(d^5qr\right)\int\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{c+dx}\,dx}{2b(bc-ad)^4}+\frac{\left(d^3qr\right)\int\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^2}\,dx}{2(bc-ad)^3} \\
&\quad-\frac{\left(d^2qr\right)\int\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^3}\,dx}{2(bc-ad)^2}+\frac{(dqr)\int\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^4}\,dx}{2(bc-ad)} \\
&\quad+\frac{\left(dpqr^2\right)\int\left(\frac{b}{(bc-ad)(a+bx)^4}-\frac{bd}{(bc-ad)^2(a+bx)^3}+\frac{bd^2}{(bc-ad)^3(a+bx)^2}-\frac{bd^3}{(bc-ad)^4(a+bx)}+\frac{d^4}{(bc-ad)^4(c+dx)}\right)dx}{8b}
\end{aligned}$$

$$
\begin{aligned}
= &-\frac{p^2r^2}{32b(a+bx)^4}-\frac{dpqr^2}{24b(bc-ad)(a+bx)^3}+\frac{d^2pqr^2}{16b(bc-ad)^2(a+bx)^2}\\
&-\frac{d^3pqr^2}{8b(bc-ad)^3(a+bx)}-\frac{d^4pqr^2\log(a+bx)}{8b(bc-ad)^4}+\frac{d^4pqr^2\log(c+dx)}{8b(bc-ad)^4}\\
&-\frac{pr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{8b(a+bx)^4}-\frac{dqr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{6b(bc-ad)(a+bx)^3}\\
&+\frac{d^2qr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b(bc-ad)^2(a+bx)^2}-\frac{d^3qr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b(bc-ad)^3(a+bx)}\\
&-\frac{d^4qr\log(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b(bc-ad)^4}\\
&+\frac{d^4qr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b(bc-ad)^4}-\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b(a+bx)^4}\\
&+\frac{(d^4pqr^2)\int\frac{\log(a+bx)}{a+bx}\,dx}{2(bc-ad)^4}-\frac{(d^4pqr^2)\int\frac{\log(c+dx)}{a+bx}\,dx}{2(bc-ad)^4}+\frac{(d^3pqr^2)\int\frac{1}{(a+bx)^2}\,dx}{2(bc-ad)^3}\\
&-\frac{(d^2pqr^2)\int\frac{1}{(a+bx)^3}\,dx}{4(bc-ad)^2}+\frac{(dpqr^2)\int\frac{1}{(a+bx)^4}\,dx}{6(bc-ad)}+\frac{(d^5q^2r^2)\int\frac{\log(a+bx)}{c+dx}\,dx}{2b(bc-ad)^4}\\
&-\frac{(d^5q^2r^2)\int\frac{\log(c+dx)}{c+dx}\,dx}{2b(bc-ad)^4}+\frac{(d^4q^2r^2)\int\frac{1}{(a+bx)(c+dx)}\,dx}{2b(bc-ad)^3}\\
&-\frac{(d^3q^2r^2)\int\frac{1}{(a+bx)^2(c+dx)}\,dx}{4b(bc-ad)^2}+\frac{(d^2q^2r^2)\int\frac{1}{(a+bx)^3(c+dx)}\,dx}{6b(bc-ad)}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{p^2r^2}{32b(a+bx)^4} - \frac{7dpqr^2}{72b(bc-ad)(a+bx)^3} + \frac{3d^2pqr^2}{16b(bc-ad)^2(a+bx)^2} \\
&\quad - \frac{5d^3pqr^2}{8b(bc-ad)^3(a+bx)} - \frac{d^4pqr^2\log(a+bx)}{8b(bc-ad)^4} + \frac{d^4pqr^2\log(c+dx)}{8b(bc-ad)^4} \\
&\quad - \frac{d^4pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{2b(bc-ad)^4} + \frac{d^4q^2r^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{2b(bc-ad)^4} \\
&\quad - \frac{pr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{8b(a+bx)^4} - \frac{dqr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{6b(bc-ad)(a+bx)^3} \\
&\quad + \frac{d^2qr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4b(bc-ad)^2(a+bx)^2} - \frac{d^3qr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(bc-ad)^3(a+bx)} \\
&\quad - \frac{d^4qr\log(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(bc-ad)^4} \\
&\quad + \frac{d^4qr\log(c+dx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(bc-ad)^4} \\
&\quad - \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4b(a+bx)^4} + \frac{(d^4pqr^2)\,\text{Subst}\left(\int \frac{\log(x)}{x}\,dx, x, a+bx\right)}{2b(bc-ad)^4} \\
&\quad + \frac{(d^5pqr^2)\int \frac{\log\left(\frac{d(a+bx)}{-bc+ad}\right)}{c+dx}\,dx}{2b(bc-ad)^4} + \frac{(d^4q^2r^2)\int \frac{1}{a+bx}\,dx}{2(bc-ad)^4} - \frac{(d^4q^2r^2)\int \frac{\log\left(\frac{b(c+dx)}{bc-ad}\right)}{a+bx}\,dx}{2(bc-ad)^4} \\
&\quad - \frac{(d^4q^2r^2)\,\text{Subst}\left(\int \frac{\log(x)}{x}\,dx, x, c+dx\right)}{2b(bc-ad)^4} - \frac{(d^5q^2r^2)\int \frac{1}{c+dx}\,dx}{2b(bc-ad)^4} \\
&\quad - \frac{(d^3q^2r^2)\int \left(\frac{b}{(bc-ad)(a+bx)^2} - \frac{bd}{(bc-ad)^2(a+bx)} + \frac{d^2}{(bc-ad)^2(c+dx)}\right)dx}{4b(bc-ad)^2} \\
&\quad + \frac{(d^2q^2r^2)\int \left(\frac{b}{(bc-ad)(a+bx)^3} - \frac{bd}{(bc-ad)^2(a+bx)^2} + \frac{bd^2}{(bc-ad)^3(a+bx)} - \frac{d^3}{(bc-ad)^3(c+dx)}\right)dx}{6b(bc-ad)}
\end{aligned}
$$

$$= -\frac{p^2r^2}{32b(a+bx)^4} - \frac{7dpqr^2}{72b(bc-ad)(a+bx)^3} + \frac{3d^2pqr^2}{16b(bc-ad)^2(a+bx)^2} - \frac{d^2q^2r^2}{12b(bc-ad)^2(a+bx)^2} - \frac{5d^3pqr^2}{8b(bc-ad)^3(a+bx)} + \frac{5d^3q^2r^2}{12b(bc-ad)^3(a+bx)} - \frac{d^4pqr^2\log(a+bx)}{8b(bc-ad)^4} + \frac{11d^4q^2r^2\log(a+bx)}{12b(bc-ad)^4} + \frac{d^4pqr^2\log^2(a+bx)}{4b(bc-ad)^4} + \frac{d^4pqr^2\log(c+dx)}{8b(bc-ad)^4} - \frac{11d^4q^2r^2\log(c+dx)}{12b(bc-ad)^4} - \frac{d^4pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{2b(bc-ad)^4} - \frac{d^4q^2r^2\log^2(c+dx)}{4b(bc-ad)^4} + \frac{d^4q^2r^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{2b(bc-ad)^4} - \frac{pr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{8b(a+bx)^4} - \frac{dqr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{6b(bc-ad)(a+bx)^3} + \frac{d^2qr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b(bc-ad)^2(a+bx)^2} - \frac{d^3qr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b(bc-ad)^3(a+bx)} - \frac{d^4qr\log(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b(bc-ad)^4} + \frac{d^4qr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b(bc-ad)^4} - \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b(a+bx)^4} + \frac{(d^4pqr^2)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{bx}{-bc+ad}\right)}{x}\,dx,x,c+dx\right)}{2b(bc-ad)^4} - \frac{(d^4q^2r^2)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{dx}{bc-ad}\right)}{x}\,dx,x,a+bx\right)}{2b(bc-ad)^4}$$

$$
\begin{aligned}
&= -\frac{p^2r^2}{32b(a+bx)^4} - \frac{7dpqr^2}{72b(bc-ad)(a+bx)^3} + \frac{3d^2pqr^2}{16b(bc-ad)^2(a+bx)^2} \\
&- \frac{d^2q^2r^2}{12b(bc-ad)^2(a+bx)^2} - \frac{5d^3pqr^2}{8b(bc-ad)^3(a+bx)} + \frac{5d^3q^2r^2}{12b(bc-ad)^3(a+bx)} \\
&- \frac{d^4pqr^2\log(a+bx)}{8b(bc-ad)^4} + \frac{11d^4q^2r^2\log(a+bx)}{12b(bc-ad)^4} + \frac{d^4pqr^2\log^2(a+bx)}{4b(bc-ad)^4} \\
&+ \frac{d^4pqr^2\log(c+dx)}{8b(bc-ad)^4} - \frac{11d^4q^2r^2\log(c+dx)}{12b(bc-ad)^4} - \frac{d^4pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{2b(bc-ad)^4} \\
&- \frac{d^4q^2r^2\log^2(c+dx)}{4b(bc-ad)^4} + \frac{d^4q^2r^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{2b(bc-ad)^4} \\
&- \frac{pr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{8b(a+bx)^4} - \frac{dqr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{6b(bc-ad)(a+bx)^3} \\
&+ \frac{d^2qr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b(bc-ad)^2(a+bx)^2} - \frac{d^3qr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b(bc-ad)^3(a+bx)} \\
&- \frac{d^4qr\log(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b(bc-ad)^4} \\
&+ \frac{d^4qr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b(bc-ad)^4} \\
&- \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b(a+bx)^4} + \frac{d^4q^2r^2\text{Li}_2\left(-\frac{d(a+bx)}{bc-ad}\right)}{2b(bc-ad)^4} - \frac{d^4pqr^2\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)}{2b(bc-ad)^4}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2003 vs. $2(884) = 1768$.

Time = 1.32 (sec) , antiderivative size = 2003, normalized size of antiderivative = 2.27

$$
\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^5}\,dx = \text{Result too large to show}
$$

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x)^5,x]
```

```
[Out] (-9*b^4*c^4*p^2*r^2 + 36*a*b^3*c^3*d*p^2*r^2 - 54*a^2*b^2*c^2*d^2*p^2*r^2 +
 36*a^3*b*c*d^3*p^2*r^2 - 9*a^4*d^4*p^2*r^2 - 28*a*b^3*c^3*d*p*q*r^2 + 138*
a^2*b^2*c^2*d^2*p*q*r^2 - 372*a^3*b*c*d^3*p*q*r^2 + 262*a^4*d^4*p*q*r^2 - 2
4*a^2*b^2*c^2*d^2*q^2*r^2 + 168*a^3*b*c*d^3*q^2*r^2 - 144*a^4*d^4*q^2*r^2 -
 28*b^4*c^3*d*p*q*r^2*x + 192*a*b^3*c^2*d^2*p*q*r^2*x - 840*a^2*b^2*c*d^3*p
*q*r^2*x + 676*a^3*b*d^4*p*q*r^2*x - 48*a*b^3*c^2*d^2*q^2*r^2*x + 456*a^2*b
^2*c*d^3*q^2*r^2*x - 408*a^3*b*d^4*q^2*r^2*x + 54*b^4*c^2*d^2*p*q*r^2*x^2 -
 648*a*b^3*c*d^3*p*q*r^2*x^2 + 594*a^2*b^2*d^4*p*q*r^2*x^2 - 24*b^4*c^2*d^2
*q^2*r^2*x^2 + 408*a*b^3*c*d^3*q^2*r^2*x^2 - 384*a^2*b^2*d^4*q^2*r^2*x^2 -
180*b^4*c*d^3*p*q*r^2*x^3 + 180*a*b^3*d^4*p*q*r^2*x^3 + 120*b^4*c*d^3*q^2*r
```

^2*x^3 - 120*a*b^3*d^4*q^2*r^2*x^3 + 72*d^4*p*q*r^2*(a + b*x)^4*Log[a + b*x
]^2 + 36*a^4*d^4*p*q*r^2*Log[c + d*x] - 264*a^4*d^4*q^2*r^2*Log[c + d*x] +
144*a^3*b*d^4*p*q*r^2*x*Log[c + d*x] - 1056*a^3*b*d^4*q^2*r^2*x*Log[c + d*x
] + 216*a^2*b^2*d^4*p*q*r^2*x^2*Log[c + d*x] - 1584*a^2*b^2*d^4*q^2*r^2*x^2
*Log[c + d*x] + 144*a*b^3*d^4*p*q*r^2*x^3*Log[c + d*x] - 1056*a*b^3*d^4*q^2
*r^2*x^3*Log[c + d*x] + 36*b^4*d^4*p*q*r^2*x^4*Log[c + d*x] - 264*b^4*d^4*q
^2*r^2*x^4*Log[c + d*x] - 72*a^4*d^4*q^2*r^2*Log[c + d*x]^2 - 288*a^3*b*d^4
*q^2*r^2*x*Log[c + d*x]^2 - 432*a^2*b^2*d^4*q^2*r^2*x^2*Log[c + d*x]^2 - 28
8*a*b^3*d^4*q^2*r^2*x^3*Log[c + d*x]^2 - 72*b^4*d^4*q^2*r^2*x^4*Log[c + d*x
]^2 + 12*d^4*q*r*(a + b*x)^4*Log[a + b*x]*(-3*p*r + 22*q*r - 12*p*r*Log[c +
 d*x] + 12*(p + q)*r*Log[(b*(c + d*x))/(b*c - a*d)] - 12*Log[e*(f*(a + b*x)
^p*(c + d*x)^q)^r]) - 36*b^4*c^4*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] +
 144*a*b^3*c^3*d*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 216*a^2*b^2*c^2
*d^2*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 144*a^3*b*c*d^3*p*r*Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r] - 36*a^4*d^4*p*r*Log[e*(f*(a + b*x)^p*(c + d*
x)^q)^r] - 48*a*b^3*c^3*d*q*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 216*a^
2*b^2*c^2*d^2*q*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 432*a^3*b*c*d^3*q*
r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 264*a^4*d^4*q*r*Log[e*(f*(a + b*x)
^p*(c + d*x)^q)^r] - 48*b^4*c^3*d*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r
] + 288*a*b^3*c^2*d^2*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 864*a^2*
b^2*c*d^3*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 624*a^3*b*d^4*q*r*x*
Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 72*b^4*c^2*d^2*q*r*x^2*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r] - 576*a*b^3*c*d^3*q*r*x^2*Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r] + 504*a^2*b^2*d^4*q*r*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
- 144*b^4*c*d^3*q*r*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 144*a*b^3*d^
4*q*r*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 144*a^4*d^4*q*r*Log[c + d*
x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 576*a^3*b*d^4*q*r*x*Log[c + d*x]*
Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 864*a^2*b^2*d^4*q*r*x^2*Log[c + d*x]
Log[e(f*(a + b*x)^p*(c + d*x)^q)^r] + 576*a*b^3*d^4*q*r*x^3*Log[c + d*x]*
Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 144*b^4*d^4*q*r*x^4*Log[c + d*x]*Log
[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 72*b^4*c^4*Log[e*(f*(a + b*x)^p*(c + d*
x)^q)^r]^2 + 288*a*b^3*c^3*d*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 - 432*a
^2*b^2*c^2*d^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 + 288*a^3*b*c*d^3*Log
[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 - 72*a^4*d^4*Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^2 + 144*d^4*q*(p + q)*r^2*(a + b*x)^4*PolyLog[2, (d*(a + b*x))/(
-(b*c) + a*d)])/(288*b*(b*c - a*d)^4*(a + b*x)^4)

Maple [F]

$$\int \frac{\ln \left(e(f(bx+a)^p (dx+c)^q)^r\right)^2}{(bx+a)^5}dx$$

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^5,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^5,x)
```

Fricas [F]

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^5}\,dx = \int \frac{\log\left(\left((bx+a)^p(dx+c)^qf\right)^r e\right)^2}{(bx+a)^5}\,dx$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^5,x, algorithm="fricas
")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b^5*x^5 + 5*a*b^4*x^4 + 10
*a^2*b^3*x^3 + 10*a^3*b^2*x^2 + 5*a^4*b*x + a^5), x)
```

Sympy [F]

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^5}\,dx = \int \frac{\log\left(e(f(a+bx)^p\,(c+dx)^q)^r\right)^2}{(a+bx)^5}\,dx$$

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(b*x+a)**5,x)
```

```
[Out] Integral(log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)**2/(a + b*x)**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1816 vs. 2(836) = 1672.

Time = 0.35 (sec) , antiderivative size = 1816, normalized size of antiderivative = 2.05

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^5}\,dx = \text{Too large to display}$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^5,x, algorithm="maxima
")
```

```
[Out] -1/24*(12*d^4*f*q*log(b*x + a)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2
 - 4*a^3*b*c*d^3 + a^4*d^4) - 12*d^4*f*q*log(d*x + c)/(b^4*c^4 - 4*a*b^3*c^
```

```
3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) + (12*b^3*d^3*f*q*x^3 -
a*b^2*c^2*d*f*(9*p - 4*q) + a^2*b*c*d^2*f*(9*p - 14*q) - a^3*d^3*f*(3*p - 2
2*q) + 3*b^3*c^3*f*p - 6*(b^3*c*d^2*f*q - 7*a*b^2*d^3*f*q)*x^2 + 4*(b^3*c^2
*d*f*q - 5*a*b^2*c*d^2*f*q + 13*a^2*b*d^3*f*q)*x)/(a^4*b^3*c^3 - 3*a^5*b^2*
c^2*d + 3*a^6*b*c*d^2 - a^7*d^3 + (b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^
2 - a^3*b^4*d^3)*x^4 + 4*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a
^4*b^3*d^3)*x^3 + 6*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*
b^2*d^3)*x^2 + 4*(a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d
^3)*x))*r*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(b*f) + 1/288*(144*(p*q + q^
2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/
(b*c - a*d)))*d^4*f^2/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*
b*c*d^3 + a^4*d^4) + 12*(3*p*q - 22*q^2)*d^4*f^2*log(d*x + c)/(b^4*c^4 - 4*
a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) - (9*b^4*c^4*f^2
*p^2 - 4*(9*p^2 - 7*p*q)*a*b^3*c^3*d*f^2 + 6*(9*p^2 - 23*p*q + 4*q^2)*a^2*b
^2*c^2*d^2*f^2 - 12*(3*p^2 - 31*p*q + 14*q^2)*a^3*b*c*d^3*f^2 + (9*p^2 - 26
2*p*q + 144*q^2)*a^4*d^4*f^2 + 60*((3*p*q - 2*q^2)*b^4*c*d^3*f^2 - (3*p*q -
 2*q^2)*a*b^3*d^4*f^2)*x^3 - 6*((9*p*q - 4*q^2)*b^4*c^2*d^2*f^2 - 4*(27*p*q
 - 17*q^2)*a*b^3*c*d^3*f^2 + (99*p*q - 64*q^2)*a^2*b^2*d^4*f^2)*x^2 - 72*(b
^4*d^4*f^2*p*q*x^4 + 4*a*b^3*d^4*f^2*p*q*x^3 + 6*a^2*b^2*d^4*f^2*p*q*x^2 +
4*a^3*b*d^4*f^2*p*q*x + a^4*d^4*f^2*p*q)*log(b*x + a)^2 + 144*(b^4*d^4*f^2*
p*q*x^4 + 4*a*b^3*d^4*f^2*p*q*x^3 + 6*a^2*b^2*d^4*f^2*p*q*x^2 + 4*a^3*b*d^4
*f^2*p*q*x + a^4*d^4*f^2*p*q)*log(b*x + a)*log(d*x + c) + 72*(b^4*d^4*f^2*q
^2*x^4 + 4*a*b^3*d^4*f^2*q^2*x^3 + 6*a^2*b^2*d^4*f^2*q^2*x^2 + 4*a^3*b*d^4*
f^2*q^2*x + a^4*d^4*f^2*q^2)*log(d*x + c)^2 + 4*(7*b^4*c^3*d*f^2*p*q - 12*(
4*p*q - q^2)*a*b^3*c^2*d^2*f^2 + 6*(35*p*q - 19*q^2)*a^2*b^2*c*d^3*f^2 - (1
69*p*q - 102*q^2)*a^3*b*d^4*f^2)*x + 12*((3*p*q - 22*q^2)*b^4*d^4*f^2*x^4 +
 4*(3*p*q - 22*q^2)*a*b^3*d^4*f^2*x^3 + 6*(3*p*q - 22*q^2)*a^2*b^2*d^4*f^2*
x^2 + 4*(3*p*q - 22*q^2)*a^3*b*d^4*f^2*x + (3*p*q - 22*q^2)*a^4*d^4*f^2)*lo
g(b*x + a))/(a^4*b^4*c^4 - 4*a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*
d^3 + a^8*d^4 + (b^8*c^4 - 4*a*b^7*c^3*d + 6*a^2*b^6*c^2*d^2 - 4*a^3*b^5*c*
d^3 + a^4*b^4*d^4)*x^4 + 4*(a*b^7*c^4 - 4*a^2*b^6*c^3*d + 6*a^3*b^5*c^2*d^2
 - 4*a^4*b^4*c*d^3 + a^5*b^3*d^4)*x^3 + 6*(a^2*b^6*c^4 - 4*a^3*b^5*c^3*d +
6*a^4*b^4*c^2*d^2 - 4*a^5*b^3*c*d^3 + a^6*b^2*d^4)*x^2 + 4*(a^3*b^5*c^4 - 4
*a^4*b^4*c^3*d + 6*a^5*b^3*c^2*d^2 - 4*a^6*b^2*c*d^3 + a^7*b*d^4)*x))*r^2/(
b*f^2) - 1/4*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/((b*x + a)^4*b)
```

Giac [F]

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^5}\,dx=\int \frac{\log\left(\left((bx+a)^p(dx+c)^qf\right)^re\right)^2}{(bx+a)^5}\,dx$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^5,x, algorithm="giac")
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b*x + a)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^5}\,dx = \int \frac{\ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)^2}{(a+b\,x)^5}\,dx$$

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(a + b*x)^5,x)
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(a + b*x)^5, x)
```

3.25 $\int (g+hx)^4 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx$

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Optimal result

Integrand size = 29, antiderivative size = 334

$$\begin{aligned}\int (g+hx)^4 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx &= -\frac{(bg-ah)^4prx}{5b^4} - \frac{(dg-ch)^4qrx}{5d^4} - \frac{(bg-ah)^3pr(g+hx)^2}{10b^3h} - \frac{(dg-ch)^3qr(g+hx)^2}{10d^3h} \\ &\quad - \frac{(bg-ah)^2pr(g+hx)^3}{15b^2h} - \frac{(dg-ch)^2qr(g+hx)^3}{15d^2h} - \frac{(bg-ah)pr(g+hx)^4}{20bh} \\ &\quad - \frac{(dg-ch)qr(g+hx)^4}{20dh} - \frac{pr(g+hx)^5}{25h} - \frac{qr(g+hx)^5}{25h} - \frac{(bg-ah)^5pr\log(a+bx)}{5b^5h} \\ &\quad - \frac{(dg-ch)^5qr\log(c+dx)}{5d^5h} + \frac{(g+hx)^5\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{5h}\end{aligned}$$

```
[Out] -1/5*(-a*h+b*g)^4*p*r*x/b^4-1/5*(-c*h+d*g)^4*q*r*x/d^4-1/10*(-a*h+b*g)^3*p*
r*(h*x+g)^2/b^3/h-1/10*(-c*h+d*g)^3*q*r*(h*x+g)^2/d^3/h-1/15*(-a*h+b*g)^2*p
*r*(h*x+g)^3/b^2/h-1/15*(-c*h+d*g)^2*q*r*(h*x+g)^3/d^2/h-1/20*(-a*h+b*g)*p*
r*(h*x+g)^4/b/h-1/20*(-c*h+d*g)*q*r*(h*x+g)^4/d/h-1/25*p*r*(h*x+g)^5/h-1/25
*q*r*(h*x+g)^5/h-1/5*(-a*h+b*g)^5*p*r*ln(b*x+a)/b^5/h-1/5*(-c*h+d*g)^5*q*r*
ln(d*x+c)/d^5/h+1/5*(h*x+g)^5*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/h
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used

$= \{2581, 45\}$

$$\int (g+hx)^4 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx$$
$$= -\frac{pr(bg-ah)^5 \log(a+bx)}{5b^5h} - \frac{prx(bg-ah)^4}{5b^4} - \frac{pr(g+hx)^2(bg-ah)^3}{10b^3h} - \frac{pr(g+hx)^3(bg-ah)^2}{15b^2h} + \frac{(g+hx)^5 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{5h} - \frac{pr(g+hx)^4(bg-ah)}{20bh} - \frac{qr(dg-ch)^5 \log(c+dx)}{5d^5h} - \frac{qrx(dg-ch)^4}{5d^4} - \frac{qr(g+hx)^2(dg-ch)^3}{10d^3h} - \frac{qr(g+hx)^3(dg-ch)^2}{15d^2h} - \frac{qr(g+hx)^4(dg-ch)}{20dh} - \frac{pr(g+hx)^5}{25h} - \frac{qr(g+hx)^5}{25h}$$

```
[In] Int[(g + h*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] -1/5*((b*g - a*h)^4*p*r*x)/b^4 - ((d*g - c*h)^4*q*r*x)/(5*d^4) - ((b*g - a*
h)^3*p*r*(g + h*x)^2)/(10*b^3*h) - ((d*g - c*h)^3*q*r*(g + h*x)^2)/(10*d^3*
h) - ((b*g - a*h)^2*p*r*(g + h*x)^3)/(15*b^2*h) - ((d*g - c*h)^2*q*r*(g + h
*x)^3)/(15*d^2*h) - ((b*g - a*h)*p*r*(g + h*x)^4)/(20*b*h) - ((d*g - c*h)*q
*r*(g + h*x)^4)/(20*d*h) - (p*r*(g + h*x)^5)/(25*h) - (q*r*(g + h*x)^5)/(25
*h) - ((b*g - a*h)^5*p*r*Log[a + b*x])/(5*b^5*h) - ((d*g - c*h)^5*q*r*Log[c
 + d*x])/(5*d^5*h) + ((g + h*x)^5*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(5*
h)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{(g+hx)^5 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{5h} - \frac{(bpr) \int \frac{(g+hx)^5}{a+bx} \, dx}{5h} - \frac{(dqr) \int \frac{(g+hx)^5}{c+dx} \, dx}{5h}$$

$$= \frac{(g+hx)^5 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{5h} - \frac{(bpr)\int \left(\frac{h(bg-ah)^4}{b^5} + \frac{(bg-ah)^5}{b^5(a+bx)} + \frac{h(bg-ah)^3(g+hx)}{b^4} + \frac{h(bg-ah)^2(g+hx)^2}{b^3} + \frac{h(bg-ah)(g+hx)^3}{b^2} + \frac{h(g+hx)^4}{b}\right) dx}{5h} - \frac{(dqr)\int \left(\frac{h(dg-ch)^4}{d^5} + \frac{(dg-ch)^5}{d^5(c+dx)} + \frac{h(dg-ch)^3(g+hx)}{d^4} + \frac{h(dg-ch)^2(g+hx)^2}{d^3} + \frac{h(dg-ch)(g+hx)^3}{d^2} + \frac{h(g+hx)^4}{d}\right) dx}{5h}$$

$$= -\frac{(bg-ah)^4prx}{5b^4} - \frac{(dg-ch)^4qrx}{5d^4} - \frac{(bg-ah)^3pr(g+hx)^2}{10b^3h} - \frac{(dg-ch)^3qr(g+hx)^2}{10d^3h} - \frac{(bg-ah)^2pr(g+hx)^3}{15b^2h} - \frac{(dg-ch)^2qr(g+hx)^3}{15d^2h} - \frac{(bg-ah)pr(g+hx)^4}{20bh} - \frac{(dg-ch)qr(g+hx)^4}{20dh} - \frac{pr(g+hx)^5}{25h} - \frac{qr(g+hx)^5}{25h} - \frac{(bg-ah)^5pr\log(a+bx)}{5b^5h} - \frac{(dg-ch)^5qr\log(c+dx)}{5d^5h} + \frac{(g+hx)^5\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{5h}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.82

$$\int (g+hx)^4 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) dx$$

$$= -\frac{pr\left(60bh(bg-ah)^4x + 30b^2(bg-ah)^3(g+hx)^2 + 20b^3(bg-ah)^2(g+hx)^3 + 15b^4(bg-ah)(g+hx)^4 + 12b^5(g+hx)^5 + 60(bg-ah)^5\log(a+bx)\right)}{60b^5} - \frac{qr(60a}{}$$

```
[In] Integrate[(g + h*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] (-1/60*(p*r*(60*b*h*(b*g - a*h)^4*x + 30*b^2*(b*g - a*h)^3*(g + h*x)^2 + 20
*b^3*(b*g - a*h)^2*(g + h*x)^3 + 15*b^4*(b*g - a*h)*(g + h*x)^4 + 12*b^5*(g
 + h*x)^5 + 60*(b*g - a*h)^5*Log[a + b*x]))/b^5 - (q*r*(60*d*h*(d*g - c*h)^
4*x + 30*d^2*(d*g - c*h)^3*(g + h*x)^2 + 20*d^3*(d*g - c*h)^2*(g + h*x)^3 +
 15*d^4*(d*g - c*h)*(g + h*x)^4 + 12*d^5*(g + h*x)^5 + 60*(d*g - c*h)^5*Log
[c + d*x]))/(60*d^5) + (g + h*x)^5*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(5
*h)
```

Maple [F]

$$\int (hx+g)^4 \ln \left(e\left(f(bx+a)^p (dx+c)^q\right)^r\right) dx$$

```
[In] int((h*x+g)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

```
[Out] int((h*x+g)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 945 vs. $2(308) = 616$.

Time = 0.33 (sec) , antiderivative size = 945, normalized size of antiderivative = 2.83

$$\int (g+hx)^4 \log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = -\frac{12\,(b^5d^5h^4p+b^5d^5h^4q)rx^5+15\,((5\,b^5d^5gh^3-ab^4d^5h^4)p+(5\,b^5d^5gh^3-b^5cd^4h^4)q)rx^4+20\,((10\,b^5d^5g^2h\ldots}{\ldots}$$

```
[In] integrate((h*x+g)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas")
```

```
[Out] -1/300*(12*(b^5*d^5*h^4*p + b^5*d^5*h^4*q)*r*x^5 + 15*((5*b^5*d^5*g*h^3 - a
*b^4*d^5*h^4)*p + (5*b^5*d^5*g*h^3 - b^5*c*d^4*h^4)*q)*r*x^4 + 20*((10*b^5*
d^5*g^2*h^2 - 5*a*b^4*d^5*g*h^3 + a^2*b^3*d^5*h^4)*p + (10*b^5*d^5*g^2*h^2
- 5*b^5*c*d^4*g*h^3 + b^5*c^2*d^3*h^4)*q)*r*x^3 + 30*((10*b^5*d^5*g^3*h - 1
0*a*b^4*d^5*g^2*h^2 + 5*a^2*b^3*d^5*g*h^3 - a^3*b^2*d^5*h^4)*p + (10*b^5*d^
5*g^3*h - 10*b^5*c*d^4*g^2*h^2 + 5*b^5*c^2*d^3*g*h^3 - b^5*c^3*d^2*h^4)*q)*
r*x^2 + 60*((5*b^5*d^5*g^4 - 10*a*b^4*d^5*g^3*h + 10*a^2*b^3*d^5*g^2*h^2 -
5*a^3*b^2*d^5*g*h^3 + a^4*b*d^5*h^4)*p + (5*b^5*d^5*g^4 - 10*b^5*c*d^4*g^3*
h + 10*b^5*c^2*d^3*g^2*h^2 - 5*b^5*c^3*d^2*g*h^3 + b^5*c^4*d*h^4)*q)*r*x -
60*(b^5*d^5*h^4*p*r*x^5 + 5*b^5*d^5*g*h^3*p*r*x^4 + 10*b^5*d^5*g^2*h^2*p*r*
x^3 + 10*b^5*d^5*g^3*h*p*r*x^2 + 5*b^5*d^5*g^4*p*r*x + (5*a*b^4*d^5*g^4 - 1
0*a^2*b^3*d^5*g^3*h + 10*a^3*b^2*d^5*g^2*h^2 - 5*a^4*b*d^5*g*h^3 + a^5*d^5*
h^4)*p*r)*log(b*x + a) - 60*(b^5*d^5*h^4*q*r*x^5 + 5*b^5*d^5*g*h^3*q*r*x^4
+ 10*b^5*d^5*g^2*h^2*q*r*x^3 + 10*b^5*d^5*g^3*h*q*r*x^2 + 5*b^5*d^5*g^4*q*r
*x + (5*b^5*c*d^4*g^4 - 10*b^5*c^2*d^3*g^3*h + 10*b^5*c^3*d^2*g^2*h^2 - 5*b
^5*c^4*d*g*h^3 + b^5*c^5*h^4)*q*r)*log(d*x + c) - 60*(b^5*d^5*h^4*x^5 + 5*b
^5*d^5*g*h^3*x^4 + 10*b^5*d^5*g^2*h^2*x^3 + 10*b^5*d^5*g^3*h*x^2 + 5*b^5*d^
5*g^4*x)*log(e) - 60*(b^5*d^5*h^4*r*x^5 + 5*b^5*d^5*g*h^3*r*x^4 + 10*b^5*d^
5*g^2*h^2*r*x^3 + 10*b^5*d^5*g^3*h*r*x^2 + 5*b^5*d^5*g^4*r*x)*log(f))/(b^5*
d^5)
```

Sympy [F(-1)]

Timed out.

$$\int (g+hx)^4 \log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \text{Timed out}$$

```
[In] integrate((h*x+g)**4*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 624 vs. 2(308) = 616.

Time = 0.20 (sec) , antiderivative size = 624, normalized size of antiderivative = 1.87

$$\int (g+hx)^4 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right)\, dx$$
$$= \frac{1}{5}\left(h^4x^5+5\,gh^3x^4+10\,g^2h^2x^3+10\,g^3hx^2+5\,g^4x\right)\log\left(\left(((bx+a)^p(dx+c)^q f\right)^r e\right) + \frac{r\left(\frac{60\,(5\,ab^4fg^4p-10\,a^2b^3fg^3hp+10\,a^3b^2fg^2h^2p-5\,a^4bfgh^3p+a^5fh^4p)\log(bx+a)}{b^5}+\frac{60\,(5\,cd^4fg^4q-10\,c^2d^3fg^3hq+10\,c^3d^2fg^2h^2q-5\,c^4dfgh}{d^5}\right.}{}$$

```
[In] integrate((h*x+g)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima")
```

```
[Out] 1/5*(h^4*x^5 + 5*g*h^3*x^4 + 10*g^2*h^2*x^3 + 10*g^3*h*x^2 + 5*g^4*x)*log((
(b*x + a)^p*(d*x + c)^q*f)^r*e) + 1/300*r*(60*(5*a*b^4*f*g^4*p - 10*a^2*b^3
*f*g^3*h*p + 10*a^3*b^2*f*g^2*h^2*p - 5*a^4*b*f*g*h^3*p + a^5*f*h^4*p)*log(
b*x + a)/b^5 + 60*(5*c*d^4*f*g^4*q - 10*c^2*d^3*f*g^3*h*q + 10*c^3*d^2*f*g^
2*h^2*q - 5*c^4*d*f*g*h^3*q + c^5*f*h^4*q)*log(d*x + c)/d^5 - (12*b^4*d^4*f
*h^4*(p + q)*x^5 - 15*(a*b^3*d^4*f*h^4*p - (5*d^4*f*g*h^3*(p + q) - c*d^3*f
*h^4*q)*b^4)*x^4 - 20*(5*a*b^3*d^4*f*g*h^3*p - a^2*b^2*d^4*f*h^4*p - (10*d^
4*f*g^2*h^2*(p + q) - 5*c*d^3*f*g*h^3*q + c^2*d^2*f*h^4*q)*b^4)*x^3 - 30*(1
0*a*b^3*d^4*f*g^2*h^2*p - 5*a^2*b^2*d^4*f*g*h^3*p + a^3*b*d^4*f*h^4*p - (10
*d^4*f*g^3*h*(p + q) - 10*c*d^3*f*g^2*h^2*q + 5*c^2*d^2*f*g*h^3*q - c^3*d*f
*h^4*q)*b^4)*x^2 - 60*(10*a*b^3*d^4*f*g^3*h*p - 10*a^2*b^2*d^4*f*g^2*h^2*p
+ 5*a^3*b*d^4*f*g*h^3*p - a^4*d^4*f*h^4*p - (5*d^4*f*g^4*(p + q) - 10*c*d^3
*f*g^3*h*q + 10*c^2*d^2*f*g^2*h^2*q - 5*c^3*d*f*g*h^3*q + c^4*f*h^4*q)*b^4)
*x)/(b^4*d^4))/f
```

Giac [F]

$$\int (g+hx)^4 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right)\, dx = \int (hx+g)^4 \log\left(\left(((bx+a)^p(dx+c)^q f\right)^r e\right)\, dx$$

```
[In] integrate((h*x+g)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 2.08 (sec) , antiderivative size = 1128, normalized size of antiderivative = 3.38

$$\int (g+hx)^4 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx = \text{Too large to display}$$

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(g + h*x)^4,x)
```

```
[Out] log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(g^4*x + (h^4*x^5)/5 + 2*g^3*h*x^2 + g
*h^3*x^4 + 2*g^2*h^2*x^3) - x^2*(((5*a*d + 5*b*c)*(((5*a*d + 5*b*c)*((h^3*r
*(b*c*h*p + 5*b*d*g*p + a*d*h*q + 5*b*d*g*q))/(5*b*d) - (h^4*r*(p + q)*(5*a
*d + 5*b*c))/(25*b*d)))/(5*b*d) - (g*h^2*r*(b*c*h*p + 2*b*d*g*p + a*d*h*q +
 2*b*d*g*q))/(b*d) + (a*c*h^4*r*(p + q))/(5*b*d)))/(10*b*d) - (a*c*((h^3*r*
(b*c*h*p + 5*b*d*g*p + a*d*h*q + 5*b*d*g*q))/(5*b*d) - (h^4*r*(p + q)*(5*a*
d + 5*b*c))/(25*b*d)))/(2*b*d) + (g^2*h*r*(b*c*h*p + b*d*g*p + a*d*h*q + b*
d*g*q))/(b*d)) - x^4*((h^3*r*(b*c*h*p + 5*b*d*g*p + a*d*h*q + 5*b*d*g*q))/(
20*b*d) - (h^4*r*(p + q)*(5*a*d + 5*b*c))/(100*b*d)) - x*((a*c*(((5*a*d + 5
*b*c)*((h^3*r*(b*c*h*p + 5*b*d*g*p + a*d*h*q + 5*b*d*g*q))/(5*b*d) - (h^4*r
*(p + q)*(5*a*d + 5*b*c))/(25*b*d)))/(5*b*d) - (g*h^2*r*(b*c*h*p + 2*b*d*g*
p + a*d*h*q + 2*b*d*g*q))/(b*d) + (a*c*h^4*r*(p + q))/(5*b*d)))/(b*d) - ((5
*a*d + 5*b*c)*(((5*a*d + 5*b*c)*(((5*a*d + 5*b*c)*((h^3*r*(b*c*h*p + 5*b*d*
g*p + a*d*h*q + 5*b*d*g*q))/(5*b*d) - (h^4*r*(p + q)*(5*a*d + 5*b*c))/(25*b
*d)))/(5*b*d) - (g*h^2*r*(b*c*h*p + 2*b*d*g*p + a*d*h*q + 2*b*d*g*q))/(b*d)
 + (a*c*h^4*r*(p + q))/(5*b*d)))/(5*b*d) - (a*c*((h^3*r*(b*c*h*p + 5*b*d*g*
p + a*d*h*q + 5*b*d*g*q))/(5*b*d) - (h^4*r*(p + q)*(5*a*d + 5*b*c))/(25*b*d
)))/(b*d) + (2*g^2*h*r*(b*c*h*p + b*d*g*p + a*d*h*q + b*d*g*q))/(b*d)))/(5*
b*d) + (g^3*r*(2*b*c*h*p + b*d*g*p + 2*a*d*h*q + b*d*g*q))/(b*d)) + x^3*(((
5*a*d + 5*b*c)*((h^3*r*(b*c*h*p + 5*b*d*g*p + a*d*h*q + 5*b*d*g*q))/(5*b*d)
 - (h^4*r*(p + q)*(5*a*d + 5*b*c))/(25*b*d)))/(15*b*d) - (g*h^2*r*(b*c*h*p
+ 2*b*d*g*p + a*d*h*q + 2*b*d*g*q))/(3*b*d) + (a*c*h^4*r*(p + q))/(15*b*d))
 + (log(a + b*x)*((a^5*h^4*p*r)/5 + a*b^4*g^4*p*r + 2*a^3*b^2*g^2*h^2*p*r -
 a^4*b*g*h^3*p*r - 2*a^2*b^3*g^3*h*p*r))/b^5 + (log(c + d*x)*((c^5*h^4*q*r)
/5 + c*d^4*g^4*q*r + 2*c^3*d^2*g^2*h^2*q*r - c^4*d*g*h^3*q*r - 2*c^2*d^3*g^
3*h*q*r))/d^5 - (h^4*r*x^5*(p + q))/25
```

3.26 $\int (g+hx)^3 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx$

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Optimal result

Integrand size = 29, antiderivative size = 276

$$\begin{aligned}
\int (g+hx)^3 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx
&= -\frac{(bg-ah)^3 prx}{4b^3} - \frac{(dg-ch)^3 qrx}{4d^3} - \frac{(bg-ah)^2 pr(g+hx)^2}{8b^2h} \\
&\quad - \frac{(dg-ch)^2 qr(g+hx)^2}{8d^2h} - \frac{(bg-ah)pr(g+hx)^3}{12bh} - \frac{(dg-ch)qr(g+hx)^3}{12dh} \\
&\quad - \frac{pr(g+hx)^4}{16h} - \frac{qr(g+hx)^4}{16h} - \frac{(bg-ah)^4 pr \log(a+bx)}{4b^4h} \\
&\quad - \frac{(dg-ch)^4 qr \log(c+dx)}{4d^4h} + \frac{(g+hx)^4 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4h}
\end{aligned}$$

```
[Out] -1/4*(-a*h+b*g)^3*p*r*x/b^3-1/4*(-c*h+d*g)^3*q*r*x/d^3-1/8*(-a*h+b*g)^2*p*r
*(h*x+g)^2/b^2/h-1/8*(-c*h+d*g)^2*q*r*(h*x+g)^2/d^2/h-1/12*(-a*h+b*g)*p*r*(
h*x+g)^3/b/h-1/12*(-c*h+d*g)*q*r*(h*x+g)^3/d/h-1/16*p*r*(h*x+g)^4/h-1/16*q*
r*(h*x+g)^4/h-1/4*(-a*h+b*g)^4*p*r*ln(b*x+a)/b^4/h-1/4*(-c*h+d*g)^4*q*r*ln(
d*x+c)/d^4/h+1/4*(h*x+g)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/h
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used

$= \{2581, 45\}$

$$\int (g+hx)^3 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx$$
$$= -\frac{pr(bg-ah)^4 \log(a+bx)}{4b^4h} - \frac{prx(bg-ah)^3}{4b^3} - \frac{pr(g+hx)^2(bg-ah)^2}{8b^2h} + \frac{(g+hx)^4 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4h} - \frac{pr(g+hx)^3(bg-ah)}{12bh} - \frac{qr(dg-ch)^4 \log(c+dx)}{4d^4h} - \frac{qrx(dg-ch)^3}{4d^3} - \frac{qr(g+hx)^2(dg-ch)^2}{8d^2h} - \frac{qr(g+hx)^3(dg-ch)}{12dh} - \frac{pr(g+hx)^4}{16h} - \frac{qr(g+hx)^4}{16h}$$

```
[In] Int[(g + h*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] -1/4*((b*g - a*h)^3*p*r*x)/b^3 - ((d*g - c*h)^3*q*r*x)/(4*d^3) - ((b*g - a*
h)^2*p*r*(g + h*x)^2)/(8*b^2*h) - ((d*g - c*h)^2*q*r*(g + h*x)^2)/(8*d^2*h)
 - ((b*g - a*h)*p*r*(g + h*x)^3)/(12*b*h) - ((d*g - c*h)*q*r*(g + h*x)^3)/(
12*d*h) - (p*r*(g + h*x)^4)/(16*h) - (q*r*(g + h*x)^4)/(16*h) - ((b*g - a*h
)^4*p*r*Log[a + b*x])/(4*b^4*h) - ((d*g - c*h)^4*q*r*Log[c + d*x])/(4*d^4*h
) + ((g + h*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(4*h)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{(g+hx)^4 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4h} - \frac{(bpr) \int \frac{(g+hx)^4}{a+bx} \, dx}{4h} - \frac{(dqr) \int \frac{(g+hx)^4}{c+dx} \, dx}{4h}$$

$$= \frac{(g+hx)^4 \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4h} - \frac{(bpr)\int \left(\frac{h(bg-ah)^3}{b^4} + \frac{(bg-ah)^4}{b^4(a+bx)} + \frac{h(bg-ah)^2(g+hx)}{b^3} + \frac{h(bg-ah)(g+hx)^2}{b^2} + \frac{h(g+hx)^3}{b}\right)dx}{4h} - \frac{(dqr)\int \left(\frac{h(dg-ch)^3}{d^4} + \frac{(dg-ch)^4}{d^4(c+dx)} + \frac{h(dg-ch)^2(g+hx)}{d^3} + \frac{h(dg-ch)(g+hx)^2}{d^2} + \frac{h(g+hx)^3}{d}\right)dx}{4h}$$

$$= -\frac{(bg-ah)^3prx}{4b^3} - \frac{(dg-ch)^3qrx}{4d^3} - \frac{(bg-ah)^2pr(g+hx)^2}{8b^2h} - \frac{(dg-ch)^2qr(g+hx)^2}{8d^2h} - \frac{(bg-ah)pr(g+hx)^3}{12bh} - \frac{(dg-ch)qr(g+hx)^3}{12dh} - \frac{pr(g+hx)^4}{16h} - \frac{qr(g+hx)^4}{16h} - \frac{(bg-ah)^4pr\log(a+bx)}{4b^4h} - \frac{(dg-ch)^4qr\log(c+dx)}{4d^4h} + \frac{(g+hx)^4 \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4h}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.84

$$\int (g+hx)^3 \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) dx$$

$$= \frac{\frac{1}{12}r\left(-\frac{p(12bh(bg-ah)^3x+6b^2(bg-ah)^2(g+hx)^2+4b^3(bg-ah)(g+hx)^3+3b^4(g+hx)^4+12(bg-ah)^4\log(a+bx))}{b^4} - \frac{q(12dh(dg-ch)^3x+6d^2(dg-c}{}\right.}{4h}$$

```
[In] Integrate[(g + h*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] ((r*(-((p*(12*b*h*(b*g - a*h)^3*x + 6*b^2*(b*g - a*h)^2*(g + h*x)^2 + 4*b^3
*(b*g - a*h)*(g + h*x)^3 + 3*b^4*(g + h*x)^4 + 12*(b*g - a*h)^4*Log[a + b*x
]))/b^4) - (q*(12*d*h*(d*g - c*h)^3*x + 6*d^2*(d*g - c*h)^2*(g + h*x)^2 + 4
*d^3*(d*g - c*h)*(g + h*x)^3 + 3*d^4*(g + h*x)^4 + 12*(d*g - c*h)^4*Log[c +
 d*x]))/d^4))/12 + (g + h*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(4*h)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1078 vs. 2(254) = 508.

Time = 291.42 (sec) , antiderivative size = 1079, normalized size of antiderivative = 3.91

method	result	size
parallelrisch	Expression too large to display	1079

```
[In] int((h*x+g)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*(-24*x^2*b^4*c*d^3*g*h^2*q*r-48*a^3*g*h^2*p*r*d^4*b-48*b^4*c^3*d*g*h^
2*q*r+6*a^3*b*c*d^3*h^3*p*r+6*a*b^3*c^3*d*h^3*q*r+6*x^2*a^2*b^2*d^4*h^3*p*r
+6*x^2*b^4*c^2*d^2*h^3*q*r+36*x^2*b^4*d^4*g^2*h*p*r+36*x^2*b^4*d^4*g^2*h*q*
r-96*ln(b*x+a)*a*b^3*d^4*g^3*p*r-12*x*a^3*b*d^4*h^3*p*r-12*x*b^4*c^3*d*h^3*
q*r+72*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*a*b^3*c*d^3*g^2*h-48*ln(b*x+a)*b^4*c
*d^3*g^3*p*r-48*ln(d*x+c)*a*b^3*d^4*g^3*q*r-96*ln(d*x+c)*b^4*c*d^3*g^3*q*r-
72*ln(b*x+a)*a*b^3*c*d^3*g^2*h*p*r-72*ln(d*x+c)*a*b^3*c*d^3*g^2*h*q*r-4*x^3
*a*b^3*d^4*h^3*p*r-4*x^3*b^4*c*d^3*h^3*q*r+16*x^3*b^4*d^4*g*h^2*p*r+16*x^3*
b^4*d^4*g*h^2*q*r+72*a^2*b^2*d^4*p*r*g^2*h+72*b^4*c^2*d^2*q*r*g^2*h-24*a^2*
b^2*c*d^3*g*h^2*p*r-24*a*b^3*c^2*d^2*g*h^2*q*r+36*a*b^3*c*d^3*g^2*h*p*r+36*
a*b^3*c*d^3*g^2*h*q*r+12*a^4*h^3*p*r*d^4+12*b^4*c^4*h^3*q*r-12*x^4*ln(e*(f*
(b*x+a)^p*(d*x+c)^q)^r)*b^4*d^4*h^3-48*x*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*b^
4*d^4*g^3+48*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*a*b^3*d^4*g^3+48*ln(e*(f*(b*x+
a)^p*(d*x+c)^q)^r)*b^4*c*d^3*g^3-48*a*b^3*d^4*p*r*g^3-48*a*b^3*d^4*q*r*g^3-
48*b^4*c*d^3*p*r*g^3-48*b^4*c*d^3*q*r*g^3-72*x^2*ln(e*(f*(b*x+a)^p*(d*x+c)^
q)^r)*b^4*d^4*g^2*h+48*x*b^4*d^4*g^3*p*r+48*x*b^4*d^4*g^3*q*r+12*ln(b*x+a)*
a^4*d^4*h^3*p*r+12*ln(d*x+c)*b^4*c^4*h^3*q*r+3*x^4*b^4*d^4*h^3*p*r+3*x^4*b^
4*d^4*h^3*q*r-48*x^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*b^4*d^4*g*h^2+48*x*a^2
*b^2*d^4*g*h^2*p*r-72*x*a*b^3*d^4*g^2*h*p*r+48*x*b^4*c^2*d^2*g*h^2*q*r-72*x
*b^4*c*d^3*g^2*h*q*r-48*ln(b*x+a)*a^3*b*d^4*g*h^2*p*r+72*ln(b*x+a)*a^2*b^2*
d^4*g^2*h*p*r-48*ln(d*x+c)*b^4*c^3*d*g*h^2*q*r+72*ln(d*x+c)*b^4*c^2*d^2*g^2
*h*q*r-24*x^2*a*b^3*d^4*g*h^2*p*r)/b^4/d^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 679 vs. 2(254) = 508.

Time = 0.30 (sec) , antiderivative size = 679, normalized size of antiderivative = 2.46

$$\int (g+hx)^3 \log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = -\frac{3\,(b^4d^4h^3p+b^4d^4h^3q)rx^4+4\,((4\,b^4d^4gh^2-ab^3d^4h^3)p+(4\,b^4d^4gh^2-b^4cd^3h^3)q)rx^3+6\,((6\,b^4d^4g^2h-4}{}$$

```
[In] integrate((h*x+g)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas")
```

```
[Out] -1/48*(3*(b^4*d^4*h^3*p + b^4*d^4*h^3*q)*r*x^4 + 4*((4*b^4*d^4*g*h^2 - a*b^
3*d^4*h^3)*p + (4*b^4*d^4*g*h^2 - b^4*c*d^3*h^3)*q)*r*x^3 + 6*((6*b^4*d^4*g
^2*h - 4*a*b^3*d^4*g*h^2 + a^2*b^2*d^4*h^3)*p + (6*b^4*d^4*g^2*h - 4*b^4*c*
d^3*g*h^2 + b^4*c^2*d^2*h^3)*q)*r*x^2 + 12*((4*b^4*d^4*g^3 - 6*a*b^3*d^4*g^
2*h + 4*a^2*b^2*d^4*g*h^2 - a^3*b*d^4*h^3)*p + (4*b^4*d^4*g^3 - 6*b^4*c*d^3
*g^2*h + 4*b^4*c^2*d^2*g*h^2 - b^4*c^3*d*h^3)*q)*r*x - 12*(b^4*d^4*h^3*p*r*
x^4 + 4*b^4*d^4*g*h^2*p*r*x^3 + 6*b^4*d^4*g^2*h*p*r*x^2 + 4*b^4*d^4*g^3*p*r
*x + (4*a*b^3*d^4*g^3 - 6*a^2*b^2*d^4*g^2*h + 4*a^3*b*d^4*g*h^2 - a^4*d^4*h
^3)*p*r)*log(b*x + a) - 12*(b^4*d^4*h^3*q*r*x^4 + 4*b^4*d^4*g*h^2*q*r*x^3 +
 6*b^4*d^4*g^2*h*q*r*x^2 + 4*b^4*d^4*g^3*q*r*x + (4*b^4*c*d^3*g^3 - 6*b^4*c
```

```
^2*d^2*g^2*h + 4*b^4*c^3*d*g*h^2 - b^4*c^4*h^3)*q*r)*log(d*x + c) - 12*(b^4
*d^4*h^3*x^4 + 4*b^4*d^4*g*h^2*x^3 + 6*b^4*d^4*g^2*h*x^2 + 4*b^4*d^4*g^3*x)
*log(e) - 12*(b^4*d^4*h^3*r*x^4 + 4*b^4*d^4*g*h^2*r*x^3 + 6*b^4*d^4*g^2*h*r
*x^2 + 4*b^4*d^4*g^3*r*x)*log(f))/(b^4*d^4)
```

Sympy [F(-1)]

Timed out.

$$\int (g+hx)^3 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx = \text{Timed out}$$

```
[In] integrate((h*x+g)**3*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.56

$$\int (g+hx)^3 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx$$
$$= \frac{1}{4}\left(h^3x^4 + 4\,gh^2x^3 + 6\,g^2hx^2 + 4\,g^3x\right)\log\left(((bx+a)^p(dx+c)^q f)^r\, e\right)$$
$$+ \frac{r\left(\frac{12\,(4\,ab^3fg^3p - 6\,a^2b^2fg^2hp + 4\,a^3bfgh^2p - a^4fh^3p)\log(bx+a)}{b^4} + \frac{12\,(4\,cd^3fg^3q - 6\,c^2d^2fg^2hq + 4\,c^3dfgh^2q - c^4fh^3q)\log(dx+c)}{d^4} - \frac{3\,b^3d^3f}{}\right.}{}$$

```
[In] integrate((h*x+g)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima")
```

```
[Out] 1/4*(h^3*x^4 + 4*g*h^2*x^3 + 6*g^2*h*x^2 + 4*g^3*x)*log(((b*x + a)^p*(d*x +
 c)^q*f)^r*e) + 1/48*r*(12*(4*a*b^3*f*g^3*p - 6*a^2*b^2*f*g^2*h*p + 4*a^3*b
*f*g*h^2*p - a^4*f*h^3*p)*log(b*x + a)/b^4 + 12*(4*c*d^3*f*g^3*q - 6*c^2*d^
2*f*g^2*h*q + 4*c^3*d*f*g*h^2*q - c^4*f*h^3*q)*log(d*x + c)/d^4 - (3*b^3*d^
3*f*h^3*(p + q)*x^4 - 4*(a*b^2*d^3*f*h^3*p - (4*d^3*f*g*h^2*(p + q) - c*d^2
*f*h^3*q)*b^3)*x^3 - 6*(4*a*b^2*d^3*f*g*h^2*p - a^2*b*d^3*f*h^3*p - (6*d^3*
f*g^2*h*(p + q) - 4*c*d^2*f*g*h^2*q + c^2*d*f*h^3*q)*b^3)*x^2 - 12*(6*a*b^2
*d^3*f*g^2*h*p - 4*a^2*b*d^3*f*g*h^2*p + a^3*d^3*f*h^3*p - (4*d^3*f*g^3*(p
+ q) - 6*c*d^2*f*g^2*h*q + 4*c^2*d*f*g*h^2*q - c^3*f*h^3*q)*b^3)*x)/(b^3*d^
3))/f
```

Giac [F(-1)]

Timed out.

$$\int (g+hx)^3 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \text{Timed out}$$

```
[In] integrate((h*x+g)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 1.88 (sec) , antiderivative size = 641, normalized size of antiderivative = 2.32

$$\int (g+hx)^3 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$$
$$= \ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)\left(g^3\,x+\frac{3\,g^2\,h\,x^2}{2}+g\,h^2\,x^3+\frac{h^3\,x^4}{4}\right)$$
$$-x\left(\frac{(4\,a\,d+4\,b\,c)\left(\frac{(4\,a\,d+4\,b\,c)\left(\frac{h^2\,r\,(b\,c\,h\,p+4\,b\,d\,g\,p+a\,d\,h\,q+4\,b\,d\,g\,q)}{4\,b\,d}-\frac{h^3\,r\,(p+q)\,(4\,a\,d+4\,b\,c)}{16\,b\,d}\right)}{4\,b\,d}-\frac{g\,h\,r\,(2\,b\,c\,h\,p+3\,b\,d\,g\,p+2\,a\,d\,h\,q+3\,b\cdots}{2\,b\,d}\cdots}{4\,b\,d}\right.$$
$$\left.+\frac{g^2\,r\,(3\,b\,c\,h\,p+2\,b\,d\,g\,p+3\,a\,d\,h\,q+2\,b\,d\,g\,q)}{2\,b\,d}-\frac{a\,c\left(\frac{h^2\,r\,(b\,c\,h\,p+4\,b\,d\,g\,p+a\,d\,h\,q+4\,b\,d\,g\,q)}{4\,b\,d}-\frac{h^3\,r\,(p+q)\,(4\,a\,d+4\,b\,c)}{16\,b\,d}\right)}{b\,d}\right)$$
$$-x^3\left(\frac{h^2\,r\,(b\,c\,h\,p+4\,b\,d\,g\,p+a\,d\,h\,q+4\,b\,d\,g\,q)}{12\,b\,d}-\frac{h^3\,r\,(p+q)\,(4\,a\,d+4\,b\,c)}{48\,b\,d}\right)$$
$$+x^2\left(\frac{(4\,a\,d+4\,b\,c)\left(\frac{h^2\,r\,(b\,c\,h\,p+4\,b\,d\,g\,p+a\,d\,h\,q+4\,b\,d\,g\,q)}{4\,b\,d}-\frac{h^3\,r\,(p+q)\,(4\,a\,d+4\,b\,c)}{16\,b\,d}\right)}{8\,b\,d}\right.$$
$$\left.-\frac{g\,h\,r\,(2\,b\,c\,h\,p+3\,b\,d\,g\,p+2\,a\,d\,h\,q+3\,b\,d\,g\,q)}{4\,b\,d}+\frac{a\,c\,h^3\,r\,(p+q)}{8\,b\,d}\right)$$
$$-\frac{\ln(a+b\,x)\,\left(p\,r\,a^4\,h^3-4\,p\,r\,a^3\,b\,g\,h^2+6\,p\,r\,a^2\,b^2\,g^2\,h-4\,p\,r\,a\,b^3\,g^3\right)}{4\,b^4}$$
$$-\frac{\ln(c+d\,x)\,\left(q\,r\,c^4\,h^3-4\,q\,r\,c^3\,d\,g\,h^2+6\,q\,r\,c^2\,d^2\,g^2\,h-4\,q\,r\,c\,d^3\,g^3\right)}{4\,d^4}-\frac{h^3\,r\,x^4\,(p+q)}{16}$$

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(g + h*x)^3,x)
```

```
[Out] log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(g^3*x + (h^3*x^4)/4 + (3*g^2*h*x^2)/2
 + g*h^2*x^3) - x*(((4*a*d + 4*b*c)*(((4*a*d + 4*b*c)*((h^2*r*(b*c*h*p + 4*
b*d*g*p + a*d*h*q + 4*b*d*g*q))/(4*b*d) - (h^3*r*(p + q)*(4*a*d + 4*b*c))/(
16*b*d)))/(4*b*d) - (g*h*r*(2*b*c*h*p + 3*b*d*g*p + 2*a*d*h*q + 3*b*d*g*q))
/(2*b*d) + (a*c*h^3*r*(p + q))/(4*b*d)))/(4*b*d) + (g^2*r*(3*b*c*h*p + 2*b*
d*g*p + 3*a*d*h*q + 2*b*d*g*q))/(2*b*d) - (a*c*((h^2*r*(b*c*h*p + 4*b*d*g*p
 + a*d*h*q + 4*b*d*g*q))/(4*b*d) - (h^3*r*(p + q)*(4*a*d + 4*b*c))/(16*b*d)
))/(b*d)) - x^3*((h^2*r*(b*c*h*p + 4*b*d*g*p + a*d*h*q + 4*b*d*g*q))/(12*b*
d) - (h^3*r*(p + q)*(4*a*d + 4*b*c))/(48*b*d)) + x^2*(((4*a*d + 4*b*c)*((h^
2*r*(b*c*h*p + 4*b*d*g*p + a*d*h*q + 4*b*d*g*q))/(4*b*d) - (h^3*r*(p + q)*(
4*a*d + 4*b*c))/(16*b*d)))/(8*b*d) - (g*h*r*(2*b*c*h*p + 3*b*d*g*p + 2*a*d*
h*q + 3*b*d*g*q))/(4*b*d) + (a*c*h^3*r*(p + q))/(8*b*d)) - (log(a + b*x)*(a
^4*h^3*p*r - 4*a*b^3*g^3*p*r - 4*a^3*b*g*h^2*p*r + 6*a^2*b^2*g^2*h*p*r))/(4
*b^4) - (log(c + d*x)*(c^4*h^3*q*r - 4*c*d^3*g^3*q*r - 4*c^3*d*g*h^2*q*r +
6*c^2*d^2*g^2*h*q*r))/(4*d^4) - (h^3*r*x^4*(p + q))/16
```

3.27 $\int (g+hx)^2 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx$

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Optimal result

Integrand size = 29, antiderivative size = 218

$$\begin{aligned}\int (g+hx)^2 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx = & -\frac{(bg-ah)^2prx}{3b^2}-\frac{(dg-ch)^2qrx}{3d^2} \\ & -\frac{(bg-ah)pr(g+hx)^2}{6bh} \\ & -\frac{(dg-ch)qr(g+hx)^2}{6dh}-\frac{pr(g+hx)^3}{9h} \\ & -\frac{qr(g+hx)^3}{9h}-\frac{(bg-ah)^3pr\log(a+bx)}{3b^3h} \\ & -\frac{(dg-ch)^3qr\log(c+dx)}{3d^3h} \\ & +\frac{(g+hx)^3\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h}\end{aligned}$$

```
[Out] -1/3*(-a*h+b*g)^2*p*r*x/b^2-1/3*(-c*h+d*g)^2*q*r*x/d^2-1/6*(-a*h+b*g)*p*r*(
h*x+g)^2/b/h-1/6*(-c*h+d*g)*q*r*(h*x+g)^2/d/h-1/9*p*r*(h*x+g)^3/h-1/9*q*r*(
h*x+g)^3/h-1/3*(-a*h+b*g)^3*p*r*ln(b*x+a)/b^3/h-1/3*(-c*h+d*g)^3*q*r*ln(d*x
+c)/d^3/h+1/3*(h*x+g)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/h
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2581, 45}

$$\int (g+hx)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = -\frac{pr(bg-ah)^3\log(a+bx)}{3b^3h}-\frac{prx(bg-ah)^2}{3b^2}+\frac{(g+hx)^3\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h}-\frac{pr(g+hx)^2(bg-ah)}{6bh}-\frac{qr(dg-ch)^3\log(c+dx)}{3d^3h}-\frac{qrx(dg-ch)^2}{3d^2}-\frac{qr(g+hx)^2(dg-ch)}{6dh}-\frac{pr(g+hx)^3}{9h}-\frac{qr(g+hx)^3}{9h}$$

```
[In] Int[(g + h*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] -1/3*((b*g - a*h)^2*p*r*x)/b^2 - ((d*g - c*h)^2*q*r*x)/(3*d^2) - ((b*g - a*
h)*p*r*(g + h*x)^2)/(6*b*h) - ((d*g - c*h)*q*r*(g + h*x)^2)/(6*d*h) - (p*r*
(g + h*x)^3)/(9*h) - (q*r*(g + h*x)^3)/(9*h) - ((b*g - a*h)^3*p*r*Log[a + b
*x])/(3*b^3*h) - ((d*g - c*h)^3*q*r*Log[c + d*x])/(3*d^3*h) + ((g + h*x)^3*
Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(3*h)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{(g+hx)^3\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3h} - \frac{(bpr)\int\frac{(g+hx)^3}{a+bx}\,dx}{3h} - \frac{(dqr)\int\frac{(g+hx)^3}{c+dx}\,dx}{3h}$$

$$= \frac{(g+hx)^3\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3h} - \frac{(bpr)\int\left(\frac{h(bg-ah)^2}{b^3}+\frac{(bg-ah)^3}{b^3(a+bx)}+\frac{h(bg-ah)(g+hx)}{b^2}+\frac{h(g+hx)^2}{b}\right)dx}{3h} - \frac{(dqr)\int\left(\frac{h(dg-ch)^2}{d^3}+\frac{(dg-ch)^3}{d^3(c+dx)}+\frac{h(dg-ch)(g+hx)}{d^2}+\frac{h(g+hx)^2}{d}\right)dx}{3h}$$

$$= -\frac{(bg-ah)^2prx}{3b^2} - \frac{(dg-ch)^2qrx}{3d^2} - \frac{(bg-ah)pr(g+hx)^2}{6bh} - \frac{(dg-ch)qr(g+hx)^2}{6dh} - \frac{pr(g+hx)^3}{9h} - \frac{qr(g+hx)^3}{9h} - \frac{(bg-ah)^3pr\log(a+bx)}{3b^3h} - \frac{(dg-ch)^3qr\log(c+dx)}{3d^3h} + \frac{(g+hx)^3\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3h}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.96

$$\int(g+hx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)dx$$
$$= \frac{-\frac{r\left(6d^3(bg-ah)^3p\log(a+bx)+b\left(6a^2d^3h^3px-3abd^3hp\left(g^2+6ghx+h^2x^2\right)+b^2d\left(6c^2h^3qx-3cdhq\left(g^2+6ghx+h^2x^2\right)+d^2(p+q)\left(5g^3+18g^2hx+9gh^2\right.\right.\right.\right.}{6b^3d^3}}{3h}$$

```
[In] Integrate[(g + h*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] (-1/6*(r*(6*d^3*(b*g - a*h)^3*p*Log[a + b*x] + b*(6*a^2*d^3*h^3*p*x - 3*a*b
*d^3*h*p*(g^2 + 6*g*h*x + h^2*x^2) + b^2*d*(6*c^2*h^3*q*x - 3*c*d*h*q*(g^2
+ 6*g*h*x + h^2*x^2) + d^2*(p + q)*(5*g^3 + 18*g^2*h*x + 9*g*h^2*x^2 + 2*h^
3*x^3)) + 6*b^2*(d*g - c*h)^3*q*Log[c + d*x])))/(b^3*d^3) + (g + h*x)^3*Log
[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(3*h)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 754 vs. $2(200) = 400$.

Time = 68.29 (sec) , antiderivative size = 755, normalized size of antiderivative = 3.46

method	result
parallelrisch	$\frac{-18\ln(bx+a)a^2b\,d^3ghpr-18\ln(dx+c)b^3c^2dghqr+18xa\,b^2d^3ghpr+18x\,b^3c\,d^2ghqr-9ca\,b^2d^2ghpr-9ca\,b^2d^2ghqr+18\ln(bx+a)a}{}$

```
[In] int((h*x+g)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x,method=_RETURNVERBOSE)
```

```
[Out] 1/18*(-18*ln(b*x+a)*a^2*b*d^3*g*h*p*r-18*ln(d*x+c)*b^3*c^2*d*g*h*q*r+18*x*a
*b^2*d^3*g*h*p*r+18*x*b^3*c*d^2*g*h*q*r-9*c*a*b^2*d^2*g*h*p*r-9*c*a*b^2*d^2
*g*h*q*r+18*ln(b*x+a)*a*b^2*c*d^2*g*h*p*r+18*ln(d*x+c)*a*b^2*c*d^2*g*h*q*r+
6*x^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*b^3*d^3*h^2+18*x*ln(e*(f*(b*x+a)^p*(d
*x+c)^q)^r)*b^3*d^3*g^2-18*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*a*b^2*d^3*g^2-18
*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*b^3*c*d^2*g^2-2*x^3*b^3*d^3*h^2*q*r+18*x^2
*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*b^3*d^3*g*h-18*x*b^3*d^3*g^2*p*r-18*x*b^3*
d^3*g^2*q*r+6*ln(b*x+a)*a^3*d^3*h^2*p*r+6*ln(d*x+c)*b^3*c^3*h^2*q*r-2*x^3*b
^3*d^3*h^2*p*r+6*a^3*h^2*p*r*d^3+6*b^3*c^3*h^2*q*r+18*a*b^2*d^3*g^2*p*r+18*
b^3*c*d^2*g^2*q*r+3*x^2*a*b^2*d^3*h^2*p*r+3*x^2*b^3*c*d^2*h^2*q*r-9*x^2*b^3
*d^3*g*h*p*r-9*x^2*b^3*d^3*g*h*q*r-6*x*a^2*b*d^3*h^2*p*r-6*x*b^3*c^2*d*h^2*
q*r-18*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*a*b^2*c*d^2*g*h+36*ln(b*x+a)*a*b^2*d
^3*g^2*p*r+18*ln(b*x+a)*b^3*c*d^2*g^2*p*r+18*ln(d*x+c)*a*b^2*d^3*g^2*q*r+36
*ln(d*x+c)*b^3*c*d^2*g^2*q*r+18*a*b^2*d^3*g^2*q*r+18*b^3*c*d^2*g^2*p*r+3*c*
a^2*b*d^2*h^2*p*r+3*c^2*a*b^2*d*h^2*q*r-18*a^2*g*h*p*r*d^3*b-18*b^3*c^2*d*g
*h*q*r)/b^3/d^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 441 vs. $2(200) = 400$.

Time = 0.32 (sec) , antiderivative size = 441, normalized size of antiderivative = 2.02

$$\int (g+hx)^2 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = -\frac{2\,(b^3d^3h^2p+b^3d^3h^2q)rx^3+3\,((3\,b^3d^3gh-ab^2d^3h^2)p+(3\,b^3d^3gh-b^3cd^2h^2)q)rx^2+6\,((3\,b^3d^3g^2-3\,ab^2d}{}$$

```
[In] integrate((h*x+g)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas")
```

```
[Out] -1/18*(2*(b^3*d^3*h^2*p + b^3*d^3*h^2*q)*r*x^3 + 3*((3*b^3*d^3*g*h - a*b^2*
d^3*h^2)*p + (3*b^3*d^3*g*h - b^3*c*d^2*h^2)*q)*r*x^2 + 6*((3*b^3*d^3*g^2 -
 3*a*b^2*d^3*g*h + a^2*b*d^3*h^2)*p + (3*b^3*d^3*g^2 - 3*b^3*c*d^2*g*h + b^
3*c^2*d*h^2)*q)*r*x - 6*(b^3*d^3*h^2*p*r*x^3 + 3*b^3*d^3*g*h*p*r*x^2 + 3*b^
3*d^3*g^2*p*r*x + (3*a*b^2*d^3*g^2 - 3*a^2*b*d^3*g*h + a^3*d^3*h^2)*p*r)*lo
```

```
g(b*x + a) - 6*(b^3*d^3*h^2*q*r*x^3 + 3*b^3*d^3*g*h*q*r*x^2 + 3*b^3*d^3*g^2
*q*r*x + (3*b^3*c*d^2*g^2 - 3*b^3*c^2*d*g*h + b^3*c^3*h^2)*q*r)*log(d*x + c
) - 6*(b^3*d^3*h^2*x^3 + 3*b^3*d^3*g*h*x^2 + 3*b^3*d^3*g^2*x)*log(e) - 6*(b
^3*d^3*h^2*r*x^3 + 3*b^3*d^3*g*h*r*x^2 + 3*b^3*d^3*g^2*r*x)*log(f))/(b^3*d^
3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 930 vs. $2(192) = 384$.

Time = 166.57 (sec) , antiderivative size = 930, normalized size of antiderivative = 4.27

$$\int (g+hx)^2 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx = \text{Too large to display}$$

```
[In] integrate((h*x+g)**2*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r),x)
```

```
[Out] Piecewise(((g**2*x + g*h*x**2 + h**2*x**3/3)*log(e*(a**p*c**q*f)**r), Eq(b,
 0) & Eq(d, 0)), (c**3*h**2*log(e*(a**p*f*(c + d*x)**q)**r)/(3*d**3) - c**2
*g*h*log(e*(a**p*f*(c + d*x)**q)**r)/d**2 - c**2*h**2*q*r*x/(3*d**2) + c*g*
*2*log(e*(a**p*f*(c + d*x)**q)**r)/d + c*g*h*q*r*x/d + c*h**2*q*r*x**2/(6*d
) - g**2*q*r*x + g**2*x*log(e*(a**p*f*(c + d*x)**q)**r) - g*h*q*r*x**2/2 +
g*h*x**2*log(e*(a**p*f*(c + d*x)**q)**r) - h**2*q*r*x**3/9 + h**2*x**3*log(
e*(a**p*f*(c + d*x)**q)**r)/3, Eq(b, 0)), (a**3*h**2*log(e*(c**q*f*(a + b*x
)**p)**r)/(3*b**3) - a**2*g*h*log(e*(c**q*f*(a + b*x)**p)**r)/b**2 - a**2*h
**2*p*r*x/(3*b**2) + a*g**2*log(e*(c**q*f*(a + b*x)**p)**r)/b + a*g*h*p*r*x
/b + a*h**2*p*r*x**2/(6*b) - g**2*p*r*x + g**2*x*log(e*(c**q*f*(a + b*x)**p
)**r) - g*h*p*r*x**2/2 + g*h*x**2*log(e*(c**q*f*(a + b*x)**p)**r) - h**2*p*
r*x**3/9 + h**2*x**3*log(e*(c**q*f*(a + b*x)**p)**r)/3, Eq(d, 0)), (-a**3*h
**2*q*r*log(c/d + x)/(3*b**3) + a**3*h**2*log(e*(f*(a + b*x)**p*(c + d*x)**
q)**r)/(3*b**3) + a**2*g*h*q*r*log(c/d + x)/b**2 - a**2*g*h*log(e*(f*(a + b
*x)**p*(c + d*x)**q)**r)/b**2 - a**2*h**2*p*r*x/(3*b**2) - a*g**2*q*r*log(c
/d + x)/b + a*g**2*log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)/b + a*g*h*p*r*x/
b + a*h**2*p*r*x**2/(6*b) + c**3*h**2*q*r*log(c/d + x)/(3*d**3) - c**2*g*h*
q*r*log(c/d + x)/d**2 - c**2*h**2*q*r*x/(3*d**2) + c*g**2*q*r*log(c/d + x)/
d + c*g*h*q*r*x/d + c*h**2*q*r*x**2/(6*d) - g**2*p*r*x - g**2*q*r*x + g**2*
x*log(e*(f*(a + b*x)**p*(c + d*x)**q)**r) - g*h*p*r*x**2/2 - g*h*q*r*x**2/2
 + g*h*x**2*log(e*(f*(a + b*x)**p*(c + d*x)**q)**r) - h**2*p*r*x**3/9 - h**
2*q*r*x**3/9 + h**2*x**3*log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)/3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.23

$$\int (g+hx)^2 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$$
$$= \frac{1}{3}\left(h^2x^3+3\,ghx^2+3\,g^2x\right)\log\left(\left((bx+a)^p(dx+c)^qf\right)^r e\right) + \frac{r\left(\frac{6\,(3\,ab^2fg^2p-3\,a^2bfghp+a^3fh^2p)\log(bx+a)}{b^3}+\frac{6\,(3\,cd^2fg^2q-3\,c^2dfghq+c^3fh^2q)\log(dx+c)}{d^3}-\frac{2\,b^2d^2fh^2(p+q)x^3-3\,(abd^2fh^2p-(3\,d^2\cdots}{\cdots}\right.}{18\,f}$$

```
[In] integrate((h*x+g)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima")
```

```
[Out] 1/3*(h^2*x^3 + 3*g*h*x^2 + 3*g^2*x)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e) +
1/18*r*(6*(3*a*b^2*f*g^2*p - 3*a^2*b*f*g*h*p + a^3*f*h^2*p)*log(b*x + a)/b^
3 + 6*(3*c*d^2*f*g^2*q - 3*c^2*d*f*g*h*q + c^3*f*h^2*q)*log(d*x + c)/d^3 -
(2*b^2*d^2*f*h^2*(p + q)*x^3 - 3*(a*b*d^2*f*h^2*p - (3*d^2*f*g*h*(p + q) -
c*d*f*h^2*q)*b^2)*x^2 - 6*(3*a*b*d^2*f*g*h*p - a^2*d^2*f*h^2*p - (3*d^2*f*g
^2*(p + q) - 3*c*d*f*g*h*q + c^2*f*h^2*q)*b^2)*x)/(b^2*d^2))/f
```

Giac [A] (verification not implemented)

none

Time = 58.02 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.67

$$\int (g+hx)^2 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$$
$$= -\frac{1}{9}\left(h^2pr+h^2qr-3\,h^2r\log(f)-3\,h^2\log(e)\right)x^3 + \frac{1}{3}\left(h^2prx^3+3\,ghprx^2+3\,g^2prx\right)\log(bx+a) + \frac{1}{3}\left(h^2qrx^3+3\,ghqrx^2+3\,g^2qrx\right)\log(dx+c) - \frac{(3\,bdghpr-adh^2pr+3\,bdghqr-bch^2qr-6\,bdghr\log(f)-6\,bdgh\log(e))x^2}{6\,bd} + \frac{(3\,ab^2g^2pr-3\,a^2bghpr+a^3h^2pr)\log(bx+a)}{3\,b^3} + \frac{(3\,cd^2g^2qr-3\,c^2dghqr+c^3h^2qr)\log(-dx-c)}{3\,d^3} - \frac{(3\,b^2d^2g^2pr-3\,abd^2ghpr+a^2d^2h^2pr+3\,b^2d^2g^2qr-3\,b^2cdghqr+b^2c^2h^2qr-3\,b^2d^2g^2r\log(f)-3\,b^2d^2g\cdots}{3\,b^2d^2}$$

```
[In] integrate((h*x+g)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")
```

```
[Out] -1/9*(h^2*p*r + h^2*q*r - 3*h^2*r*log(f) - 3*h^2*log(e))*x^3 + 1/3*(h^2*p*r
*x^3 + 3*g*h*p*r*x^2 + 3*g^2*p*r*x)*log(b*x + a) + 1/3*(h^2*q*r*x^3 + 3*g*h
*q*r*x^2 + 3*g^2*q*r*x)*log(d*x + c) - 1/6*(3*b*d*g*h*p*r - a*d*h^2*p*r + 3
*b*d*g*h*q*r - b*c*h^2*q*r - 6*b*d*g*h*r*log(f) - 6*b*d*g*h*log(e))*x^2/(b*
d) + 1/3*(3*a*b^2*g^2*p*r - 3*a^2*b*g*h*p*r + a^3*h^2*p*r)*log(b*x + a)/b^3
 + 1/3*(3*c*d^2*g^2*q*r - 3*c^2*d*g*h*q*r + c^3*h^2*q*r)*log(-d*x - c)/d^3
- 1/3*(3*b^2*d^2*g^2*p*r - 3*a*b*d^2*g*h*p*r + a^2*d^2*h^2*p*r + 3*b^2*d^2*
g^2*q*r - 3*b^2*c*d*g*h*q*r + b^2*c^2*h^2*q*r - 3*b^2*d^2*g^2*r*log(f) - 3*
b^2*d^2*g^2*log(e))*x/(b^2*d^2)
```

Mupad [B] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.50

$$\int (g+hx)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$$
$$= x\left(\frac{\left(\frac{hr(bchp+3bdgp+adhq+3bdgq)}{3bd}-\frac{h^2r(p+q)(3ad+3bc)}{9bd}\right)(3ad+3bc)}{3bd} - \frac{gr(bchp+bdgp+adhq+bdgq)}{bd}+\frac{ach^2r(p+q)}{3bd}\right)$$
$$- x^2\left(\frac{hr(bchp+3bdgp+adhq+3bdgq)}{6bd}-\frac{h^2r(p+q)(3ad+3bc)}{18bd}\right)$$
$$+ \ln\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\left(g^2x+ghx^2+\frac{h^2x^3}{3}\right)$$
$$+ \frac{\ln(a+bx)\left(pra^3h^2-3pra^2bgh+3prab^2g^2\right)}{3b^3}$$
$$+ \frac{\ln(c+dx)\left(qrc^3h^2-3qrc^2dgh+3qrcd^2g^2\right)}{3d^3}-\frac{h^2rx^3(p+q)}{9}$$

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(g + h*x)^2,x)
```

```
[Out] x*((((h*r*(b*c*h*p + 3*b*d*g*p + a*d*h*q + 3*b*d*g*q))/(3*b*d) - (h^2*r*(p
+ q)*(3*a*d + 3*b*c))/(9*b*d))*(3*a*d + 3*b*c))/(3*b*d) - (g*r*(b*c*h*p + b
*d*g*p + a*d*h*q + b*d*g*q))/(b*d) + (a*c*h^2*r*(p + q))/(3*b*d)) - x^2*((h
*r*(b*c*h*p + 3*b*d*g*p + a*d*h*q + 3*b*d*g*q))/(6*b*d) - (h^2*r*(p + q)*(3
*a*d + 3*b*c))/(18*b*d)) + log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(g^2*x + (h
^2*x^3)/3 + g*h*x^2) + (log(a + b*x)*(a^3*h^2*p*r + 3*a*b^2*g^2*p*r - 3*a^2
*b*g*h*p*r))/(3*b^3) + (log(c + d*x)*(c^3*h^2*q*r + 3*c*d^2*g^2*q*r - 3*c^2
*d*g*h*q*r))/(3*d^3) - (h^2*r*x^3*(p + q))/9
```

3.28 $\int (g+hx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx$

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Optimal result

Integrand size = 27, antiderivative size = 160

$$\int (g+hx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx=-\frac{(bg-ah)prx}{2b}-\frac{(dg-ch)qrx}{2d}-\frac{pr(g+hx)^2}{4h}-\frac{qr(g+hx)^2}{4h}-\frac{(bg-ah)^2pr\log(a+bx)}{2b^2h}-\frac{(dg-ch)^2qr\log(c+dx)}{2d^2h}+\frac{(g+hx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h}$$

```
[Out] -1/2*(-a*h+b*g)*p*r*x/b-1/2*(-c*h+d*g)*q*r*x/d-1/4*p*r*(h*x+g)^2/h-1/4*q*r*
(h*x+g)^2/h-1/2*(-a*h+b*g)^2*p*r*ln(b*x+a)/b^2/h-1/2*(-c*h+d*g)^2*q*r*ln(d*
x+c)/d^2/h+1/2*(h*x+g)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/h
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2581, 45}

$$\int (g+hx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx=-\frac{pr(bg-ah)^2\log(a+bx)}{2b^2h}+\frac{(g+hx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h}-\frac{prx(bg-ah)}{2b}-\frac{qr(dg-ch)^2\log(c+dx)}{2d^2h}-\frac{qrx(dg-ch)}{2d}-\frac{pr(g+hx)^2}{4h}-\frac{qr(g+hx)^2}{4h}$$

```
[In] Int[(g + h*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] -1/2*((b*g - a*h)*p*r*x)/b - ((d*g - c*h)*q*r*x)/(2*d) - (p*r*(g + h*x)^2)/
(4*h) - (q*r*(g + h*x)^2)/(4*h) - ((b*g - a*h)^2*p*r*Log[a + b*x])/(2*b^2*h
) - ((d*g - c*h)^2*q*r*Log[c + d*x])/(2*d^2*h) + ((g + h*x)^2*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/(2*h)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(g+hx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h} - \frac{(bpr)\int\frac{(g+hx)^2}{a+bx}\,dx}{2h} - \frac{(dqr)\int\frac{(g+hx)^2}{c+dx}\,dx}{2h} \\
&= \frac{(g+hx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h} - \frac{(bpr)\int\left(\frac{h(bg-ah)}{b^2}+\frac{(bg-ah)^2}{b^2(a+bx)}+\frac{h(g+hx)}{b}\right)dx}{2h} \\
&\quad - \frac{(dqr)\int\left(\frac{h(dg-ch)}{d^2}+\frac{(dg-ch)^2}{d^2(c+dx)}+\frac{h(g+hx)}{d}\right)dx}{2h} \\
&= -\frac{(bg-ah)prx}{2b} - \frac{(dg-ch)qrx}{2d} - \frac{pr(g+hx)^2}{4h} - \frac{qr(g+hx)^2}{4h} \\
&\quad - \frac{(bg-ah)^2pr\log(a+bx)}{2b^2h} - \frac{(dg-ch)^2qr\log(c+dx)}{2d^2h} \\
&\quad + \frac{(g+hx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.75

$$\int (g+hx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx =$$
$$-\frac{2ad^2(-2bg+ah)pr\log(a+bx)+b(2bc(-2dg+ch)qr\log(c+dx)+dx(r(-2adhp-2bchq+bd(p+q)(}{4b^2d^2}$$

```
[In] Integrate[(g + h*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] -1/4*(2*a*d^2*(-2*b*g + a*h)*p*r*Log[a + b*x] + b*(2*b*c*(-2*d*g + c*h)*q*r
*Log[c + d*x] + d*x*(r*(-2*a*d*h*p - 2*b*c*h*q + b*d*(p + q)*(4*g + h*x)) -
 2*b*d*(2*g + h*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])))/(b^2*d^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 437 vs. 2(146) = 292.

Time = 11.89 (sec) , antiderivative size = 438, normalized size of antiderivative = 2.74

method	result
parallelrisch	$-\frac{-2\ln(bx+a)abcdhpr-2\ln(dx+c)abcdhqr+abcdhpr+abcdhqr-2xabd^2hpr-2xb^2cdhqr-8\ln(bx+a)abd^2gpr-4\ln(bx+a)b^2cd}{}$

```
[In] int((h*x+g)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(-2*ln(b*x+a)*a*b*c*d*h*p*r-2*ln(d*x+c)*a*b*c*d*h*q*r+a*b*c*d*h*p*r+a*
b*c*d*h*q*r-2*x*a*b*d^2*h*p*r-2*x*b^2*c*d*h*q*r-8*ln(b*x+a)*a*b*d^2*g*p*r-4
*ln(b*x+a)*b^2*c*d*g*p*r-4*ln(d*x+c)*a*b*d^2*g*q*r-8*ln(d*x+c)*b^2*c*d*g*q*
r-2*x^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*b^2*d^2*h-4*x*ln(e*(f*(b*x+a)^p*(d*
x+c)^q)^r)*b^2*d^2*g+4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*a*b*d^2*g+4*ln(e*(f*
(b*x+a)^p*(d*x+c)^q)^r)*b^2*c*d*g+x^2*b^2*d^2*h*p*r+x^2*b^2*d^2*h*q*r+4*x*b
^2*d^2*g*p*r+4*x*b^2*d^2*g*q*r+2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*a*b*c*d*h+
2*ln(b*x+a)*a^2*d^2*h*p*r+2*ln(d*x+c)*b^2*c^2*h*q*r+2*a^2*h*p*r*d^2+2*b^2*c
^2*h*q*r-4*a*b*d^2*g*q*r-4*b^2*c*d*g*p*r-4*a*b*d^2*g*p*r-4*b^2*c*d*g*q*r)/b
^2/d^2
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.51

$$\int (g+hx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx =$$
$$-\frac{(b^2d^2hp+b^2d^2hq)rx^2+2\left((2\,b^2d^2g-abd^2h)p+(2\,b^2d^2g-b^2cdh)q\right)rx-2\left(b^2d^2hprx^2+2\,b^2d^2gprx+\right.}{}$$

```
[In] integrate((h*x+g)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas")
```

```
[Out] -1/4*((b^2*d^2*h*p + b^2*d^2*h*q)*r*x^2 + 2*((2*b^2*d^2*g - a*b*d^2*h)*p +
(2*b^2*d^2*g - b^2*c*d*h)*q)*r*x - 2*(b^2*d^2*h*p*r*x^2 + 2*b^2*d^2*g*p*r*x
 + (2*a*b*d^2*g - a^2*d^2*h)*p*r)*log(b*x + a) - 2*(b^2*d^2*h*q*r*x^2 + 2*b
^2*d^2*g*q*r*x + (2*b^2*c*d*g - b^2*c^2*h)*q*r)*log(d*x + c) - 2*(b^2*d^2*h
*x^2 + 2*b^2*d^2*g*x)*log(e) - 2*(b^2*d^2*h*r*x^2 + 2*b^2*d^2*g*r*x)*log(f)
)/(b^2*d^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 502 vs. 2(141) = 282.

Time = 25.80 (sec) , antiderivative size = 502, normalized size of antiderivative = 3.14

$$\int (g+hx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$$
$$=\begin{cases}\left(gx+\frac{hx^2}{2}\right)\log\left(e(a^pc^qf)^r\right)\\ -\frac{c^2h\log\left(e(a^pf(c+dx)^q)^r\right)}{2d^2}+\frac{cg\log\left(e(a^pf(c+dx)^q)^r\right)}{d}+\frac{chqrx}{2d}-gqrx+gx\log\left(e(a^pf(c+dx)^q)^r\right)-\frac{hqrx^2}{4}+\frac{hx^2\log(e}{} \\ -\frac{a^2h\log\left(e(c^qf(a+bx)^p)^r\right)}{2b^2}+\frac{ag\log\left(e(c^qf(a+bx)^p)^r\right)}{b}+\frac{ahprx}{2b}-gprx+gx\log\left(e(c^qf(a+bx)^p)^r\right)-\frac{hprx^2}{4}+\frac{hx^2\log(e}{} \\ \frac{a^2hqr\log\left(\frac{c}{d}+x\right)}{2b^2}-\frac{a^2h\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b^2}-\frac{agqr\log\left(\frac{c}{d}+x\right)}{b}+\frac{ag\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b}+\frac{ahprx}{2b}-\frac{c^2hqr\log\left(\frac{c}{d}+x\right)}{2d^2}\end{cases}$$

```
[In] integrate((h*x+g)*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r),x)
```

```
[Out] Piecewise(((g*x + h*x**2/2)*log(e*(a**p*c**q*f)**r), Eq(b, 0) & Eq(d, 0)),
(-c**2*h*log(e*(a**p*f*(c + d*x)**q)**r)/(2*d**2) + c*g*log(e*(a**p*f*(c +
d*x)**q)**r)/d + c*h*q*r*x/(2*d) - g*q*r*x + g*x*log(e*(a**p*f*(c + d*x)**q
)**r) - h*q*r*x**2/4 + h*x**2*log(e*(a**p*f*(c + d*x)**q)**r)/2, Eq(b, 0)),
 (-a**2*h*log(e*(c**q*f*(a + b*x)**p)**r)/(2*b**2) + a*g*log(e*(c**q*f*(a +
 b*x)**p)**r)/b + a*h*p*r*x/(2*b) - g*p*r*x + g*x*log(e*(c**q*f*(a + b*x)**
p)**r) - h*p*r*x**2/4 + h*x**2*log(e*(c**q*f*(a + b*x)**p)**r)/2, Eq(d, 0))
, (a**2*h*q*r*log(c/d + x)/(2*b**2) - a**2*h*log(e*(f*(a + b*x)**p*(c + d*x
)**q)**r)/(2*b**2) - a*g*q*r*log(c/d + x)/b + a*g*log(e*(f*(a + b*x)**p*(c
```

```
+ d*x)**q)**r)/b + a*h*p*r*x/(2*b) - c**2*h*q*r*log(c/d + x)/(2*d**2) + c*g
*q*r*log(c/d + x)/d + c*h*q*r*x/(2*d) - g*p*r*x - g*q*r*x + g*x*log(e*(f*(a
 + b*x)**p*(c + d*x)**q)**r) - h*p*r*x**2/4 - h*q*r*x**2/4 + h*x**2*log(e*(
f*(a + b*x)**p*(c + d*x)**q)**r)/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.89

$$\int (g+hx)\log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx = \frac{1}{2} \left(hx^2 + 2\,gx\right) \log \left(\left((bx+a)^p(dx+c)^q f\right)^r e\right) + \frac{r\left(\frac{2\,(2\,abfgp-a^2fhp)\log(bx+a)}{b^2} + \frac{2\,(2\,cdfgq-c^2fhq)\log(dx+c)}{d^2} - \frac{bdfh(p+q)x^2-2\,(adfhp-(2\,dfg(p+q)-cfhq)b)x}{bd}\right)}{4\,f}$$

```
[In] integrate((h*x+g)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima")
```

```
[Out] 1/2*(h*x^2 + 2*g*x)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e) + 1/4*r*(2*(2*a*b*
f*g*p - a^2*f*h*p)*log(b*x + a)/b^2 + 2*(2*c*d*f*g*q - c^2*f*h*q)*log(d*x +
 c)/d^2 - (b*d*f*h*(p + q)*x^2 - 2*(a*d*f*h*p - (2*d*f*g*(p + q) - c*f*h*q)
*b)*x)/(b*d))/f
```

Giac [A] (verification not implemented)

none

Time = 3.02 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.12

$$\begin{aligned}\int (g+hx)\log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx &= -\frac{1}{4}\,(hpr + hqr - 2\,hr\log(f) - 2\,h\log(e))x^2 \\ &\quad + \frac{1}{2}\left(hprx^2 + 2\,gprx\right)\log(bx+a) + \frac{1}{2}\left(hqrx^2 + 2\,gqrx\right)\log(dx+c) \\ &\quad - \frac{(2\,bdgpr - adhpr + 2\,bdgqr - bchqr - 2\,bdgr\log(f) - 2\,bdg\log(e))x}{2\,bd} \\ &\quad + \frac{(2\,abgpr - a^2hpr)\log(-bx-a)}{2\,b^2} + \frac{(2\,cdgqr - c^2hqr)\log(dx+c)}{2\,d^2}\end{aligned}$$

```
[In] integrate((h*x+g)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")
```

```
[Out] -1/4*(h*p*r + h*q*r - 2*h*r*log(f) - 2*h*log(e))*x^2 + 1/2*(h*p*r*x^2 + 2*g
*p*r*x)*log(b*x + a) + 1/2*(h*q*r*x^2 + 2*g*q*r*x)*log(d*x + c) - 1/2*(2*b*
d*g*p*r - a*d*h*p*r + 2*b*d*g*q*r - b*c*h*q*r - 2*b*d*g*r*log(f) - 2*b*d*g*
log(e))*x/(b*d) + 1/2*(2*a*b*g*p*r - a^2*h*p*r)*log(-b*x - a)/b^2 + 1/2*(2*
c*d*g*q*r - c^2*h*q*r)*log(d*x + c)/d^2
```

Mupad [B] (verification not implemented)

Time = 1.52 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.96

$$\int (g+hx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)\,\left(\frac{h\,x^2}{2}+g\,x\right) - x\left(\frac{r\,(b\,c\,h\,p+2\,b\,d\,g\,p+a\,d\,h\,q+2\,b\,d\,g\,q)}{2\,b\,d} - \frac{h\,r\,(p+q)\,\left(2\,a\,d+2\,b\,c\right)}{4\,b\,d}\right) - \frac{\ln\left(a+b\,x\right)\,\left(a^2\,h\,p\,r-2\,a\,b\,g\,p\,r\right)}{2\,b^2} - \frac{\ln\left(c+d\,x\right)\,\left(c^2\,h\,q\,r-2\,c\,d\,g\,q\,r\right)}{2\,d^2} - \frac{h\,r\,x^2\,(p+q)}{4}$$

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(g + h*x),x)
```

```
[Out] log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(g*x + (h*x^2)/2) - x*((r*(b*c*h*p + 2
*b*d*g*p + a*d*h*q + 2*b*d*g*q))/(2*b*d) - (h*r*(p + q)*(2*a*d + 2*b*c))/(4
*b*d)) - (log(a + b*x)*(a^2*h*p*r - 2*a*b*g*p*r))/(2*b^2) - (log(c + d*x)*(
c^2*h*q*r - 2*c*d*g*q*r))/(2*d^2) - (h*r*x^2*(p + q))/4
```

3.29 $\int \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx$

Optimal result 284
Rubi [A] (verified) 284
Mathematica [A] (verified) 285
Maple [A] (verified) 285
Fricas [A] (verification not implemented) 286
Sympy [B] (verification not implemented) 286
Maxima [A] (verification not implemented) 287
Giac [A] (verification not implemented) 287
Mupad [B] (verification not implemented) 287

Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx = -((p+q)rx) + \frac{(bc-ad)qr\log(c+dx)}{bd} + \frac{(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b}$$

```
[Out] -(p+q)*r*x+(-a*d+b*c)*q*r*ln(d*x+c)/b/d+(b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q
)^r)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2579, 31, 8}

$$\int \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx = \frac{(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b} + \frac{qr(bc-ad)\log(c+dx)}{bd} - (rx(p+q))$$

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] -((p + q)*r*x) + ((b*c - a*d)*q*r*Log[c + d*x])/(b*d) + ((a + b*x)*Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2579

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[(a + b*x)*(Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s/b), x] + (Dist[q*r*s*((b*c - a*d)/b), Int[Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r]^(s - 1)/(c + d*x), x], x] - Dist[r*s*(p + q), Int[Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]^(s - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p, q,
r, s}, x] && NeQ[b*c - a*d, 0] && NeQ[p + q, 0] && IGtQ[s, 0] && LtQ[s, 4]
```

Rubi steps

$$\text{integral} = \frac{(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b} + \frac{((bc-ad)qr)\int\frac{1}{c+dx}\,dx}{b} - ((p+q)r)\int 1\,dx$$

$$= -((p+q)rx) + \frac{(bc-ad)qr\log(c+dx)}{bd} + \frac{(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93

$$\int \log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \frac{apr\log(a+bx)}{b} + \frac{cqr\log(c+dx)}{d} + x\left(-((p+q)r) + \log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)$$

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
[Out] (a*p*r*Log[a + b*x])/b + (c*q*r*Log[c + d*x])/d + x*(-((p + q)*r) + Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r])
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

method	result
default	$\ln\left(e(f(bx+a)^p(dx+c)^q)^r\right)x - r\left(xp+xq-\frac{cq\ln(dx+c)}{d}-\frac{ap\ln(bx+a)}{b}\right)$
parallelrisch	$\frac{\ln(bx+a)adpq\,r^2-\ln(bx+a)bcpq\,r^2-xbdpq\,r^2-xbd\,q^2r^2+x\ln(e(f(bx+a)^p(dx+c)^q)^r)bdqr+adpq\,r^2+ad\,q^2r^2+bcpq\,r^2+bc\,r^2q^2}{bdqr}$

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x,method=_RETURNVERBOSE)
```

```
[Out] ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*x-r*(x*p+x*q-c*q/d*ln(d*x+c)-a*p/b*ln(b*x+a
))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.18

$$\int \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) dx = \frac{bdrx\log(f)+bdx\log(e)-(bdp+bdq)rx+(bdprx+adpr)\log(bx+a)+(bdqrx+bcqr)\log(dx+c)}{bd}$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas")
```

```
[Out] (b*d*r*x*log(f) + b*d*x*log(e) - (b*d*p + b*d*q)*r*x + (b*d*p*r*x + a*d*p*r
)*log(b*x + a) + (b*d*q*r*x + b*c*q*r)*log(d*x + c))/(b*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(54) = 108.

Time = 5.06 (sec) , antiderivative size = 184, normalized size of antiderivative = 3.02

$$\int \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) dx = \begin{cases} x\log\left(e(a^pc^qf)^r\right) & \text{for } \\ \frac{c\log\left(e(a^pf(c+dx)^q)^r\right)}{d} - qrx + x\log\left(e(a^pf(c+dx)^q)^r\right) & \text{for } \\ \frac{a\log\left(e(c^qf(a+bx)^p)^r\right)}{b} - prx + x\log\left(e(c^qf(a+bx)^p)^r\right) & \text{for } \\ -\frac{aqr\log(c+dx)}{b} + \frac{a\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b} + \frac{cqr\log(c+dx)}{d} - prx - qrx + x\log\left(e(f(a+bx)^p(c+dx)^q)^r\right) & \text{othe} \end{cases}$$

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r),x)
```

```
[Out] Piecewise((x*log(e*(a**p*c**q*f)**r), Eq(b, 0) & Eq(d, 0)), (c*log(e*(a**p*
f*(c + d*x)**q)**r)/d - q*r*x + x*log(e*(a**p*f*(c + d*x)**q)**r), Eq(b, 0)
), (a*log(e*(c**q*f*(a + b*x)**p)**r)/b - p*r*x + x*log(e*(c**q*f*(a + b*x)
**p)**r), Eq(d, 0)), (-a*q*r*log(c + d*x)/b + a*log(e*(f*(a + b*x)**p*(c +
d*x)**q)**r)/b + c*q*r*log(c + d*x)/d - p*r*x - q*r*x + x*log(e*(f*(a + b*x
)**p*(c + d*x)**q)**r), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.23

$$\int \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx = x \log \left(((bx+a)^p(dx+c)^q f)^r e\right) - \frac{\left(bfp\left(\frac{x}{b} - \frac{a \log(bx+a)}{b^2}\right) + dfq\left(\frac{x}{d} - \frac{c \log(dx+c)}{d^2}\right)\right)r}{f}$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima")
```

```
[Out] x*log(((b*x + a)^p*(d*x + c)^q*f)^r*e) - (b*f*p*(x/b - a*log(b*x + a)/b^2)
+ d*f*q*(x/d - c*log(d*x + c)/d^2))*r/f
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.13

$$\int \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx = prx \log (bx+a) + qrx \log (dx+c) + \frac{apr \log (bx+a)}{b} + \frac{cqr \log (-dx-c)}{d} - (pr + qr - r \log (f) - \log (e))x$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")
```

```
[Out] p*r*x*log(b*x + a) + q*r*x*log(d*x + c) + a*p*r*log(b*x + a)/b + c*q*r*log(
-d*x - c)/d - (p*r + q*r - r*log(f) - log(e))*x
```

Mupad [B] (verification not implemented)

Time = 1.23 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98

$$\int \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx = x \ln \left(e \left(f \left(a+b\,x\right)^p \left(c+d\,x\right)^q\right)^r\right) - p\,r\,x - q\,r\,x + \frac{a\,p\,r\,\ln (a+b\,x)}{b} + \frac{c\,q\,r\,\ln (c+d\,x)}{d}$$

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r),x)
```

```
[Out] x*log(e*(f*(a + b*x)^p*(c + d*x)^q)^r) - p*r*x - q*r*x + (a*p*r*log(a + b*x
))/b + (c*q*r*log(c + d*x))/d
```

3.30 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{g+hx}\,dx$

Optimal result 288
Rubi [A] (verified) 288
Mathematica [A] (verified) 290
Maple [A] (verified) 291
Fricas [F] 291
Sympy [F(-1)] 291
Maxima [A] (verification not implemented) 291
Giac [F] 292
Mupad [F(-1)] 292

Optimal result

Integrand size = 29, antiderivative size = 148

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{g+hx}\,dx = -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(g+hx)}{h} - \frac{qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log(g+hx)}{h} + \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\log(g+hx)}{h} - \frac{pr\,\text{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)}{h} - \frac{qr\,\text{PolyLog}\left(2,\frac{d(g+hx)}{dg-ch}\right)}{h}$$

```
[Out] -p*r*ln(-h*(b*x+a)/(-a*h+b*g))*ln(h*x+g)/h-q*r*ln(-h*(d*x+c)/(-c*h+d*g))*ln
(h*x+g)/h+ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*ln(h*x+g)/h-p*r*polylog(2,b*(h*x+
g)/(-a*h+b*g))/h-q*r*polylog(2,d*(h*x+g)/(-c*h+d*g))/h
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used

= {2580, 2441, 2440, 2438}

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{g+hx}\,dx = \frac{\log(g+hx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h} - \frac{pr\,\text{PolyLog}\left(2, \frac{b(g+hx)}{bg-ah}\right)}{h} - \frac{pr\log(g+hx)\log\left(-\frac{h(a+bx)}{bg-ah}\right)}{h} - \frac{qr\,\text{PolyLog}\left(2, \frac{d(g+hx)}{dg-ch}\right)}{h} - \frac{qr\log(g+hx)\log\left(-\frac{h(c+dx)}{dg-ch}\right)}{h}$$

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x),x]
```

```
[Out] -((p*r*Log[-((h*(a + b*x))/(b*g - a*h))]*Log[g + h*x])/h) - (q*r*Log[-((h*(
c + d*x))/(d*g - c*h))]*Log[g + h*x])/h + (Log[e*(f*(a + b*x)^p*(c + d*x)^q
)^r]*Log[g + h*x])/h - (p*r*PolyLog[2, (b*(g + h*x))/(b*g - a*h)])/h - (q*r
*PolyLog[2, (d*(g + h*x))/(d*g - c*h)])/h
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2580

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[Log[g + h*x]*(Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]/h), x] + (-Dist[b*p*(r/h), Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[d*q*(r/h), Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
```

```
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\log(g+hx)}{h} - \frac{(bpr)\int \frac{\log(g+hx)}{a+bx}\,dx}{h} - \frac{(dqr)\int \frac{\log(g+hx)}{c+dx}\,dx}{h} \\
&= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(g+hx)}{h} - \frac{qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log(g+hx)}{h} \\
&\quad + \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\log(g+hx)}{h} \\
&\quad + (pr)\int \frac{\log\left(\frac{h(a+bx)}{-bg+ah}\right)}{g+hx}\,dx + (qr)\int \frac{\log\left(\frac{h(c+dx)}{-dg+ch}\right)}{g+hx}\,dx \\
&= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(g+hx)}{h} - \frac{qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log(g+hx)}{h} \\
&\quad + \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\log(g+hx)}{h} + \frac{(pr)\text{Subst}\left(\int \frac{\log\left(1+\frac{bx}{-bg+ah}\right)}{x}\,dx, x, g+hx\right)}{h} \\
&\quad + \frac{(qr)\text{Subst}\left(\int \frac{\log\left(1+\frac{dx}{-dg+ch}\right)}{x}\,dx, x, g+hx\right)}{h} \\
&= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(g+hx)}{h} - \frac{qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log(g+hx)}{h} \\
&\quad + \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\log(g+hx)}{h} - \frac{pr\text{Li}_2\left(\frac{b(g+hx)}{bg-ah}\right)}{h} - \frac{qr\text{Li}_2\left(\frac{d(g+hx)}{dg-ch}\right)}{h}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.10

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{g+hx}\,dx = \frac{-pr\log(a+bx)\log(g+hx) - qr\log(c+dx)\log(g+hx) + \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\log(g+hx) + pr}{}$$

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x),x]
```

```
[Out] (-(p*r*Log[a + b*x]*Log[g + h*x]) - q*r*Log[c + d*x]*Log[g + h*x] + Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r]*Log[g + h*x] + p*r*Log[a + b*x]*Log[(b*(g + h
*x))/(b*g - a*h)] + q*r*Log[c + d*x]*Log[(d*(g + h*x))/(d*g - c*h)] + p*r*P
olyLog[2, (h*(a + b*x))/(-(b*g) + a*h)] + q*r*PolyLog[2, (h*(c + d*x))/(-(d
*g) + c*h)])/h
```

Maple [A] (verified)

Time = 18.98 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.26

method	result
parts	$\frac{\ln(e(f(bx+a)^p(dx+c)^q)^r)\ln(hx+g)}{h} - \frac{r\left(bph\left(\frac{\operatorname{dilog}\left(\frac{(hx+g)b+ah-bg}{ah-bg}\right)}{b}+\frac{\ln(hx+g)\ln\left(\frac{(hx+g)b+ah-bg}{ah-bg}\right)}{b}\right)+dqh\left(\frac{\operatorname{dilog}\left(\frac{d(hx+g)+ch}{ch-dg}\right)}{d}\right.\right.}{h^2}$

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g),x,method=_RETURNVERBOSE)
```

```
[Out] ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*ln(h*x+g)/h-1/h^2*r*(b*p*h*(dilog(((h*x+g)*
b+a*h-b*g)/(a*h-b*g))/b+ln(h*x+g)*ln(((h*x+g)*b+a*h-b*g)/(a*h-b*g))/b)+d*q*
h*(dilog((d*(h*x+g)+c*h-d*g)/(c*h-d*g))/d+ln(h*x+g)*ln((d*(h*x+g)+c*h-d*g)/
(c*h-d*g))/d))
```

Fricas [F]

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{g+hx}\,dx = \int \frac{\log\left(\left(\left(bx+a\right)^p\left(dx+c\right)^q f\right)^r e\right)}{hx+g}\,dx$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g),x, algorithm="fricas")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(h*x + g), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{g+hx}\,dx = \text{Timed out}$$

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(h*x+g),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.26

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{g+hx}\,dx = \frac{\left(\frac{\left(\log(bx+a)\log\left(\frac{bhx+ah}{bg-ah}+1\right)+\mathrm{Li}_2\left(-\frac{bhx+ah}{bg-ah}\right)\right)fp}{h}+\frac{\left(\log(dx+c)\log\left(\frac{dhx+ch}{dg-ch}+1\right)+\mathrm{Li}_2\left(-\frac{dhx+ch}{dg-ch}\right)\right)fq}{h}\right)r}{f} - \frac{(fp\log(bx+a)+fq\log(dx+c))r\log(hx+g)}{fh} + \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)\log(hx+g)}{h}$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g),x, algorithm="maxima")
```

```
[Out] ((log(b*x + a)*log((b*h*x + a*h)/(b*g - a*h) + 1) + dilog(-(b*h*x + a*h)/(b
*g - a*h)))*f*p/h + (log(d*x + c)*log((d*h*x + c*h)/(d*g - c*h) + 1) + dilo
g(-(d*h*x + c*h)/(d*g - c*h)))*f*q/h)*r/f - (f*p*log(b*x + a) + f*q*log(d*x
 + c))*r*log(h*x + g)/(f*h) + log(((b*x + a)^p*(d*x + c)^q*f)^r*e)*log(h*x
+ g)/h
```

Giac [F]

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{g+hx}\,dx = \int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{hx+g}\,dx$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g),x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(h*x + g), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{g+hx}\,dx = \int \frac{\ln\left(e\,{\left(f\,{\left(a+b\,x\right)}^p\,{\left(c+d\,x\right)}^q\right)}^r\right)}{g+h\,x}\,dx$$

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(g + h*x),x)
```

```
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(g + h*x), x)
```

3.31 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(g+hx)^2}\,dx$

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Optimal result

Integrand size = 29, antiderivative size = 128

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^2}\,dx = \frac{bpr\log(a+bx)}{h(bg-ah)}+\frac{dqr\log(c+dx)}{h(dg-ch)} - \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h(g+hx)} - \frac{bpr\log(g+hx)}{h(bg-ah)}-\frac{dqr\log(g+hx)}{h(dg-ch)}$$

```
[Out] b*p*r*ln(b*x+a)/h/(-a*h+b*g)+d*q*r*ln(d*x+c)/h/(-c*h+d*g)-ln(e*(f*(b*x+a)^p
*(d*x+c)^q)^r)/h/(h*x+g)-b*p*r*ln(h*x+g)/h/(-a*h+b*g)-d*q*r*ln(h*x+g)/h/(-c
*h+d*g)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2581, 36, 31}

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^2}\,dx = -\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h(g+hx)}+\frac{bpr\log(a+bx)}{h(bg-ah)} - \frac{bpr\log(g+hx)}{h(bg-ah)}+\frac{dqr\log(c+dx)}{h(dg-ch)}-\frac{dqr\log(g+hx)}{h(dg-ch)}$$

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x)^2,x]
```

```
[Out] (b*p*r*Log[a + b*x])/(h*(b*g - a*h)) + (d*q*r*Log[c + d*x])/(h*(d*g - c*h))
 - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(g + h*x)) - (b*p*r*Log[g + h*x]
)/(h*(b*g - a*h)) - (d*q*r*Log[g + h*x])/(h*(d*g - c*h))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h(g+hx)} + \frac{(bpr)\int\frac{1}{(a+bx)(g+hx)}\,dx}{h} + \frac{(dqr)\int\frac{1}{(c+dx)(g+hx)}\,dx}{h} \\
&= -\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h(g+hx)} - \frac{(bpr)\int\frac{1}{g+hx}\,dx}{bg-ah} \\
&\quad + \frac{(b^2pr)\int\frac{1}{a+bx}\,dx}{h(bg-ah)} - \frac{(dqr)\int\frac{1}{g+hx}\,dx}{dg-ch} + \frac{(d^2qr)\int\frac{1}{c+dx}\,dx}{h(dg-ch)} \\
&= \frac{bpr\log(a+bx)}{h(bg-ah)} + \frac{dqr\log(c+dx)}{h(dg-ch)} - \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h(g+hx)} \\
&\quad - \frac{bpr\log(g+hx)}{h(bg-ah)} - \frac{dqr\log(g+hx)}{h(dg-ch)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.73

$$\int\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^2}\,dx = \frac{-\frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{g+hx} + \frac{bpr(\log(a+bx)-\log(g+hx))}{bg-ah} + \frac{dqr(\log(c+dx)-\log(g+hx))}{dg-ch}}{h}$$

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x)^2,x]
```

```
[Out] (-(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x)) + (b*p*r*(Log[a + b*x] -
 Log[g + h*x]))/(b*g - a*h) + (d*q*r*(Log[c + d*x] - Log[g + h*x]))/(d*g -
c*h))/h
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. 2(128) = 256.

Time = 84.84 (sec) , antiderivative size = 479, normalized size of antiderivative = 3.74

method	result
parallelrisch	$-\frac{\ln(hx+g)xabcd\,g^2hpr+\ln(hx+g)xabcd\,g^2hqr-\ln(dx+c)ab\,c^2g^2hqr-\ln(hx+g)a^2cd\,g^2hqr-\ln(hx+g)ab\,c^2g^2hpr+\ln(hx+g)}{}$

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(ln(h*x+g)*x*a*b*c*d*g^2*h*p*r+ln(h*x+g)*x*a*b*c*d*g^2*h*q*r-ln(d*x+c)*a*b
*c^2*g^2*h*q*r-ln(h*x+g)*a^2*c*d*g^2*h*q*r-ln(h*x+g)*a*b*c^2*g^2*h*p*r+ln(h
*x+g)*a*b*c*d*g^3*p*r+ln(h*x+g)*a*b*c*d*g^3*q*r-x*ln(e*(f*(b*x+a)^p*(d*x+c)
^q)^r)*a*b*c*d*g^2*h-ln(b*x+a)*a^2*c*d*g^2*h*p*r-ln(b*x+a)*x*a^2*c*d*g*h^2*
p*r-ln(d*x+c)*x*a*b*c^2*g*h^2*q*r-ln(h*x+g)*x*a^2*c*d*g*h^2*q*r-ln(h*x+g)*x
*a*b*c^2*g*h^2*p*r-x*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*a^2*c^2*h^3+ln(b*x+a)*
x*a^2*c^2*h^3*p*r+ln(d*x+c)*x*a^2*c^2*h^3*q*r+ln(b*x+a)*a^2*c^2*g*h^2*p*r+l
n(d*x+c)*a^2*c^2*g*h^2*q*r+x*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*a^2*c*d*g*h^2+
x*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*a*b*c^2*g*h^2)/(a*c*h^2-a*d*g*h-b*c*g*h+b
*d*g^2)/h/(h*x+g)/a/c/g
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(128) = 256.

Time = 72.61 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.19

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^2}\,dx = -\frac{(bdg^2+ach^2-(bc+ad)gh)r\log(f)-((bdgh-bch^2)prx+(adgh-ach^2)pr)\log(bx+a)-((bdgh-}{}$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^2,x, algorithm="fricas")
```

```
[Out] -((b*d*g^2 + a*c*h^2 - (b*c + a*d)*g*h)*r*log(f) - ((b*d*g*h - b*c*h^2)*p*r
*x + (a*d*g*h - a*c*h^2)*p*r)*log(b*x + a) - ((b*d*g*h - a*d*h^2)*q*r*x + (
b*c*g*h - a*c*h^2)*q*r)*log(d*x + c) + (((b*d*g*h - b*c*h^2)*p + (b*d*g*h -
 a*d*h^2)*q)*r*x + ((b*d*g^2 - b*c*g*h)*p + (b*d*g^2 - a*d*g*h)*q)*r)*log(h
*x + g) + (b*d*g^2 + a*c*h^2 - (b*c + a*d)*g*h)*log(e))/(b*d*g^3*h + a*c*g*
h^3 - (b*c + a*d)*g^2*h^2 + (b*d*g^2*h^2 + a*c*h^4 - (b*c + a*d)*g*h^3)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^2}\,dx = \text{Timed out}$$

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(h*x+g)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.96

$$\begin{aligned}\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^2}\,dx &= \frac{\left(bfp\left(\frac{\log(bx+a)}{bg-ah}-\frac{\log(hx+g)}{bg-ah}\right)+dfq\left(\frac{\log(dx+c)}{dg-ch}-\frac{\log(hx+g)}{dg-ch}\right)\right)r}{fh}\\&\quad-\frac{\log\left(((bx+a)^p(dx+c)^qf)^re\right)}{(hx+g)h}\end{aligned}$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^2,x, algorithm="maxima")
[Out] (b*f*p*(log(b*x + a)/(b*g - a*h) - log(h*x + g)/(b*g - a*h)) + d*f*q*(log(d
*x + c)/(d*g - c*h) - log(h*x + g)/(d*g - c*h)))*r/(f*h) - log(((b*x + a)^p
*(d*x + c)^q*f)^r*e)/((h*x + g)*h)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.49

$$\begin{aligned}\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^2}\,dx &= \frac{b^2pr\log\left(|-bx-a|\right)}{b^2gh-abh^2}+\frac{d^2qr\log\left(|dx+c|\right)}{d^2gh-cdh^2}\\&\quad-\frac{pr\log\left(bx+a\right)}{h^2x+gh}-\frac{qr\log\left(dx+c\right)}{h^2x+gh}\\&\quad-\frac{(bdgpr-bchpr+bdgqr-adhqr)\log\left(hx+g\right)}{bdg^2h-bcgh^2-adgh^2+ach^3}\\&\quad-\frac{r\log\left(f\right)+\log\left(e\right)}{h^2x+gh}\end{aligned}$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^2,x, algorithm="giac")
```

```
[Out] b^2*p*r*log(abs(-b*x - a))/(b^2*g*h - a*b*h^2) + d^2*q*r*log(abs(d*x + c))/
(d^2*g*h - c*d*h^2) - p*r*log(b*x + a)/(h^2*x + g*h) - q*r*log(d*x + c)/(h^
2*x + g*h) - (b*d*g*p*r - b*c*h*p*r + b*d*g*q*r - a*d*h*q*r)*log(h*x + g)/(
b*d*g^2*h - b*c*g*h^2 - a*d*g*h^2 + a*c*h^3) - (r*log(f) + log(e))/(h^2*x +
 g*h)
```

Mupad [B] (verification not implemented)

Time = 2.29 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.19

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^2}\,dx = \frac{\ln\left(g+h\,x\right)\,\left(b\,c\,h\,p\,r-g\,\left(b\,d\,p\,r+b\,d\,q\,r\right)+a\,d\,h\,q\,r\right)}{a\,c\,h^3-a\,d\,g\,h^2-b\,c\,g\,h^2+b\,d\,g^2\,h} - \frac{\ln\left(e\,\left(f\,\left(a+b\,x\right)^p\,\left(c+d\,x\right)^q\right)^r\right)\,\left(x+\frac{g}{h}\right)}{\left(g+h\,x\right)^2} - \frac{b\,p\,r\,\ln\left(a+b\,x\right)}{a\,h^2-b\,g\,h} - \frac{d\,q\,r\,\ln\left(c+d\,x\right)}{c\,h^2-d\,g\,h}$$

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(g + h*x)^2,x)
```

```
[Out] (log(g + h*x)*(b*c*h*p*r - g*(b*d*p*r + b*d*q*r) + a*d*h*q*r))/(a*c*h^3 - a
*d*g*h^2 - b*c*g*h^2 + b*d*g^2*h) - (log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(
x + g/h))/(g + h*x)^2 - (b*p*r*log(a + b*x))/(a*h^2 - b*g*h) - (d*q*r*log(c
 + d*x))/(c*h^2 - d*g*h)
```

3.32 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(g+hx)^3} dx$

Optimal result 298
Rubi [A] (verified) 298
Mathematica [A] (verified) 300
Maple [F] 300
Fricas [F(-1)] 300
Sympy [F(-1)] 301
Maxima [A] (verification not implemented) 301
Giac [B] (verification not implemented) 301
Mupad [B] (verification not implemented) 302

Optimal result

Integrand size = 29, antiderivative size = 202

$$\begin{aligned}\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^3} dx &= \frac{bpr}{2h(bg-ah)(g+hx)} + \frac{dqr}{2h(dg-ch)(g+hx)} \\ &\quad + \frac{b^2pr\log(a+bx)}{2h(bg-ah)^2} + \frac{d^2qr\log(c+dx)}{2h(dg-ch)^2} \\ &\quad - \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h(g+hx)^2} \\ &\quad - \frac{b^2pr\log(g+hx)}{2h(bg-ah)^2} - \frac{d^2qr\log(g+hx)}{2h(dg-ch)^2}\end{aligned}$$

```
[Out] 1/2*b*p*r/h/(-a*h+b*g)/(h*x+g)+1/2*d*q*r/h/(-c*h+d*g)/(h*x+g)+1/2*b^2*p*r*l
n(b*x+a)/h/(-a*h+b*g)^2+1/2*d^2*q*r*ln(d*x+c)/h/(-c*h+d*g)^2-1/2*ln(e*(f*(b
*x+a)^p*(d*x+c)^q)^r)/h/(h*x+g)^2-1/2*b^2*p*r*ln(h*x+g)/h/(-a*h+b*g)^2-1/2*
d^2*q*r*ln(h*x+g)/h/(-c*h+d*g)^2
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used

$= \{2581, 46\}$

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^3}\,dx = \frac{b^2pr\log(a+bx)}{2h(bg-ah)^2} - \frac{b^2pr\log(g+hx)}{2h(bg-ah)^2} - \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h(g+hx)^2} + \frac{bpr}{2h(g+hx)(bg-ah)} + \frac{d^2qr\log(c+dx)}{2h(dg-ch)^2} - \frac{d^2qr\log(g+hx)}{2h(dg-ch)^2} + \frac{dqr}{2h(g+hx)(dg-ch)}$$

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x)^3,x]
```

```
[Out] (b*p*r)/(2*h*(b*g - a*h)*(g + h*x)) + (d*q*r)/(2*h*(d*g - c*h)*(g + h*x)) +
 (b^2*p*r*Log[a + b*x])/(2*h*(b*g - a*h)^2) + (d^2*q*r*Log[c + d*x])/(2*h*(
d*g - c*h)^2) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(2*h*(g + h*x)^2) - (b
^2*p*r*Log[g + h*x])/(2*h*(b*g - a*h)^2) - (d^2*q*r*Log[g + h*x])/(2*h*(d*g
 - c*h)^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h(g+hx)^2} + \frac{(bpr)\int \frac{1}{(a+bx)(g+hx)^2}\,dx}{2h} + \frac{(dqr)\int \frac{1}{(c+dx)(g+hx)^2}\,dx}{2h}$$

$$= -\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h(g+hx)^2} + \frac{(bpr)\int \left(\frac{b^2}{(bg-ah)^2(a+bx)} - \frac{h}{(bg-ah)(g+hx)^2} - \frac{bh}{(bg-ah)^2(g+hx)}\right)dx}{2h} + \frac{(dqr)\int \left(\frac{d^2}{(dg-ch)^2(c+dx)} - \frac{h}{(dg-ch)(g+hx)^2} - \frac{dh}{(dg-ch)^2(g+hx)}\right)dx}{2h}$$

$$= \frac{bpr}{2h(bg-ah)(g+hx)} + \frac{dqr}{2h(dg-ch)(g+hx)} + \frac{b^2pr\log(a+bx)}{2h(bg-ah)^2} + \frac{d^2qr\log(c+dx)}{2h(dg-ch)^2} - \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h(g+hx)^2} - \frac{b^2pr\log(g+hx)}{2h(bg-ah)^2} - \frac{d^2qr\log(g+hx)}{2h(dg-ch)^2}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.02

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^3}\,dx$$
$$= \frac{-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right) + \frac{r(g+hx)((bc-ad)(bg-ah)(dg-ch)(bdg(p+q)-h(bcp+adq))-(g+hx)(-b^2(bc-ad)(dg-ch)^2p(\log(a}{(bc-ad)(bg-ah)^2(dg-ch)^2}}{2h(g+hx)^2}$$

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x)^3,x]
```

```
[Out] (-Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (r*(g + h*x)*((b*c - a*d)*(b*g - a
*h)*(d*g - c*h)*(b*d*g*(p + q) - h*(b*c*p + a*d*q)) - (g + h*x)*(-(b^2*(b*c
 - a*d)*(d*g - c*h)^2*p*(Log[a + b*x] - Log[g + h*x])) + d^2*(-(b*c) + a*d)
*(b*g - a*h)^2*q*(Log[c + d*x] - Log[g + h*x]))))/((b*c - a*d)*(b*g - a*h)^
2*(d*g - c*h)^2))/(2*h*(g + h*x)^2)
```

Maple [F]

$$\int \frac{\ln\left(e\left(f\left(bx+a\right)^{p}\left(dx+c\right)^{q}\right)^{r}\right)}{\left(hx+g\right)^{3}}dx$$

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^3,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^3,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^3}\,dx = \text{Timed out}$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^3}\,dx = \text{Timed out}$$

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(h*x+g)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.15

$$\int \frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^3}\,dx = \frac{\left(bfp\left(\frac{b\log(bx+a)}{b^2g^2-2\,abgh+a^2h^2} - \frac{b\log(hx+g)}{b^2g^2-2\,abgh+a^2h^2} + \frac{1}{bg^2-agh+(bgh-ah^2)x}\right) + dfq\left(\frac{d\log(dx+c)}{d^2g^2-2\,cdgh+c^2h^2} - \frac{d\log(hx+g)}{d^2g^2-2\,cdgh+c^2h^2} + \frac{1}{dg\cdots}\right.\right.}{2\,fh} - \frac{\log\left(\left((bx+a)^p(dx+c)^qf\right)^r e\right)}{2\,(hx+g)^2h}$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^3,x, algorithm="maxima")
[Out] 1/2*(b*f*p*(b*log(b*x + a)/(b^2*g^2 - 2*a*b*g*h + a^2*h^2) - b*log(h*x + g)
/(b^2*g^2 - 2*a*b*g*h + a^2*h^2) + 1/(b*g^2 - a*g*h + (b*g*h - a*h^2)*x)) +
 d*f*q*(d*log(d*x + c)/(d^2*g^2 - 2*c*d*g*h + c^2*h^2) - d*log(h*x + g)/(d^
2*g^2 - 2*c*d*g*h + c^2*h^2) + 1/(d*g^2 - c*g*h + (d*g*h - c*h^2)*x)))*r/(f
*h) - 1/2*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/((h*x + g)^2*h)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 603 vs. 2(188) = 376.

Time = 0.45 (sec) , antiderivative size = 603, normalized size of antiderivative = 2.99

$$\int \frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^3}\,dx = \frac{b^3pr\log\left(|bx+a|\right)}{2\left(b^3g^2h-2\,ab^2gh^2+a^2bh^3\right)} + \frac{d^3qr\log\left(|dx+c|\right)}{2\left(d^3g^2h-2\,cd^2gh^2+c^2dh^3\right)} - \frac{pr\log\left(bx+a\right)}{2\left(h^3x^2+2\,gh^2x+g^2h\right)} - \frac{qr\log\left(dx+c\right)}{2\left(h^3x^2+2\,gh^2x+g^2h\right)} - \frac{\left(b^2d^2g^2pr-2\,b^2cdghpr+b^2c^2h^2pr+b^2d^2g^2qr-2\,abd^2ghqr+a^2d^2h^2qr\right)\log\left(hx+g\right)}{2\left(b^2d^2g^4h-2\,b^2cdg^3h^2-2\,abd^2g^3h^2+b^2c^2g^2h^3+4\,abcdg^2h^3+a^2d^2g^2h^3-2\,abc^2gh^4-2\,a^2cdgh^4+a^2\cdots\right)} + \frac{bdghprx-bch^2prx+bdghqrx-adh^2qrx+bdg^2pr-bcghpr+bdg^2qr-adghqr-bdg^2r\log\left(f\right)+bcg\cdots}{2\left(bdg^2h^3x^2-bcgh^4x^2-adgh^4x^2+ach^5x^2+2\,bdg^3h^2x-2\,bcg^2h^3\cdots\right)}$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^3,x, algorithm="giac")
```

```
[Out] 1/2*b^3*p*r*log(abs(b*x + a))/(b^3*g^2*h - 2*a*b^2*g*h^2 + a^2*b*h^3) + 1/2
*d^3*q*r*log(abs(d*x + c))/(d^3*g^2*h - 2*c*d^2*g*h^2 + c^2*d*h^3) - 1/2*p*
r*log(b*x + a)/(h^3*x^2 + 2*g*h^2*x + g^2*h) - 1/2*q*r*log(d*x + c)/(h^3*x^
2 + 2*g*h^2*x + g^2*h) - 1/2*(b^2*d^2*g^2*p*r - 2*b^2*c*d*g*h*p*r + b^2*c^2
*h^2*p*r + b^2*d^2*g^2*q*r - 2*a*b*d^2*g*h*q*r + a^2*d^2*h^2*q*r)*log(h*x +
 g)/(b^2*d^2*g^4*h - 2*b^2*c*d*g^3*h^2 - 2*a*b*d^2*g^3*h^2 + b^2*c^2*g^2*h^
3 + 4*a*b*c*d*g^2*h^3 + a^2*d^2*g^2*h^3 - 2*a*b*c^2*g*h^4 - 2*a^2*c*d*g*h^4
 + a^2*c^2*h^5) + 1/2*(b*d*g*h*p*r*x - b*c*h^2*p*r*x + b*d*g*h*q*r*x - a*d*
h^2*q*r*x + b*d*g^2*p*r - b*c*g*h*p*r + b*d*g^2*q*r - a*d*g*h*q*r - b*d*g^2
*r*log(f) + b*c*g*h*r*log(f) + a*d*g*h*r*log(f) - a*c*h^2*r*log(f) - b*d*g^
2*log(e) + b*c*g*h*log(e) + a*d*g*h*log(e) - a*c*h^2*log(e))/(b*d*g^2*h^3*x
^2 - b*c*g*h^4*x^2 - a*d*g*h^4*x^2 + a*c*h^5*x^2 + 2*b*d*g^3*h^2*x - 2*b*c*
g^2*h^3*x - 2*a*d*g^2*h^3*x + 2*a*c*g*h^4*x + b*d*g^4*h - b*c*g^3*h^2 - a*d
*g^3*h^2 + a*c*g^2*h^3)
```

Mupad [B] (verification not implemented)

Time = 3.87 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.90

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^3}\,dx = \frac{b^2\,p\,r\,\ln\left(a+b\,x\right)}{2\,a^2\,h^3-4\,a\,b\,g\,h^2+2\,b^2\,g^2\,h} - \frac{\ln\left(g+h\,x\right)\,\left(h^2\,\left(q\,r\,a^2\,d^2+p\,r\,b^2\,c^2\right)-h\,\left(2\,c\,g\,p\,r\,b^2\,d+2\,a\,g\,q\,r\,b\,d^2\right)+b^2\,d^2\,g^2\,p\,r+b^2\,d^2\,g\cdots\right.}{2\,a^2\,c^2\,h^5-4\,a^2\,c\,d\,g\,h^4+2\,a^2\,d^2\,g^2\,h^3-4\,a\,b\,c^2\,g\,h^4+8\,a\,b\,c\,d\,g^2\,h^3-4\,a\,b\,d^2\,g^3\,h^2+2\,b^2\,c^2\,g^2\,h^3-4\,b^2\,c\cdots} - \frac{\ln\left(e\,\left(f\,\left(a+b\,x\right)^p\,\left(c+d\,x\right)^q\right)^r\right)\,\left(\frac{x}{2}+\frac{g}{2\,h}\right)}{\left(g+h\,x\right)^3} - \frac{b\,c\,h\,p\,r-b\,d\,g\,p\,r+a\,d\,h\,q\,r-b\,d\,g\,q\,r}{\left(2\,x\,h^2+2\,g\,h\right)\,\left(a\,c\,h^2+b\,d\,g^2-a\,d\,g\,h-b\,c\,g\,h\right)} + \frac{d^2\,q\,r\,\ln\left(c+d\,x\right)}{2\,c^2\,h^3-4\,c\,d\,g\,h^2+2\,d^2\,g^2\,h}$$

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(g + h*x)^3,x)
```

```
[Out] (b^2*p*r*log(a + b*x))/(2*a^2*h^3 + 2*b^2*g^2*h - 4*a*b*g*h^2) - (log(g + h
*x)*(h^2*(b^2*c^2*p*r + a^2*d^2*q*r) - h*(2*a*b*d^2*g*q*r + 2*b^2*c*d*g*p*r
) + b^2*d^2*g^2*p*r + b^2*d^2*g^2*q*r))/(2*a^2*c^2*h^5 + 2*b^2*d^2*g^4*h +
2*a^2*d^2*g^2*h^3 + 2*b^2*c^2*g^2*h^3 - 4*a*b*c^2*g*h^4 - 4*a^2*c*d*g*h^4 -
 4*a*b*d^2*g^3*h^2 - 4*b^2*c*d*g^3*h^2 + 8*a*b*c*d*g^2*h^3) - (log(e*(f*(a
+ b*x)^p*(c + d*x)^q)^r)*(x/2 + g/(2*h)))/(g + h*x)^3 - (b*c*h*p*r - b*d*g*
p*r + a*d*h*q*r - b*d*g*q*r)/((2*g*h + 2*h^2*x)*(a*c*h^2 + b*d*g^2 - a*d*g*
h - b*c*g*h)) + (d^2*q*r*log(c + d*x))/(2*c^2*h^3 + 2*d^2*g^2*h - 4*c*d*g*h
^2)
```

3.33 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(g+hx)^4} dx$

Optimal result . . . 303
Rubi [A] (verified) . . . 304
Mathematica [A] (verified) . . . 305
Maple [F] . . . 306
Fricas [F(-1)] . . . 306
Sympy [F(-1)] . . . 306
Maxima [A] (verification not implemented) . . . 306
Giac [B] (verification not implemented) . . . 307
Mupad [B] (verification not implemented) . . . 308

Optimal result

Integrand size = 29, antiderivative size = 260

$$\begin{aligned}\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^4} dx &= \frac{bpr}{6h(bg-ah)(g+hx)^2} + \frac{dqr}{6h(dg-ch)(g+hx)^2} \\ &+ \frac{b^2pr}{3h(bg-ah)^2(g+hx)} + \frac{d^2qr}{3h(dg-ch)^2(g+hx)} \\ &+ \frac{b^3pr\log(a+bx)}{3h(bg-ah)^3} + \frac{d^3qr\log(c+dx)}{3h(dg-ch)^3} \\ &- \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h(g+hx)^3} \\ &- \frac{b^3pr\log(g+hx)}{3h(bg-ah)^3} - \frac{d^3qr\log(g+hx)}{3h(dg-ch)^3}\end{aligned}$$

```
[Out] 1/6*b*p*r/h/(-a*h+b*g)/(h*x+g)^2+1/6*d*q*r/h/(-c*h+d*g)/(h*x+g)^2+1/3*b^2*p
*r/h/(-a*h+b*g)^2/(h*x+g)+1/3*d^2*q*r/h/(-c*h+d*g)^2/(h*x+g)+1/3*b^3*p*r*ln
(b*x+a)/h/(-a*h+b*g)^3+1/3*d^3*q*r*ln(d*x+c)/h/(-c*h+d*g)^3-1/3*ln(e*(f*(b*
x+a)^p*(d*x+c)^q)^r)/h/(h*x+g)^3-1/3*b^3*p*r*ln(h*x+g)/h/(-a*h+b*g)^3-1/3*d
^3*q*r*ln(h*x+g)/h/(-c*h+d*g)^3
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2581, 46}

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^4}\,dx = \frac{b^3pr\log(a+bx)}{3h(bg-ah)^3} - \frac{b^3pr\log(g+hx)}{3h(bg-ah)^3} + \frac{b^2pr}{3h(g+hx)(bg-ah)^2} - \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h(g+hx)^3} + \frac{bpr}{6h(g+hx)^2(bg-ah)} + \frac{d^3qr\log(c+dx)}{3h(dg-ch)^3} - \frac{d^3qr\log(g+hx)}{3h(dg-ch)^3} + \frac{d^2qr}{3h(g+hx)(dg-ch)^2} + \frac{dqr}{6h(g+hx)^2(dg-ch)}$$

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x)^4,x]
```

```
[Out] (b*p*r)/(6*h*(b*g - a*h)*(g + h*x)^2) + (d*q*r)/(6*h*(d*g - c*h)*(g + h*x)^
2) + (b^2*p*r)/(3*h*(b*g - a*h)^2*(g + h*x)) + (d^2*q*r)/(3*h*(d*g - c*h)^2
*(g + h*x)) + (b^3*p*r*Log[a + b*x])/(3*h*(b*g - a*h)^3) + (d^3*q*r*Log[c +
 d*x])/(3*h*(d*g - c*h)^3) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(3*h*(g +
 h*x)^3) - (b^3*p*r*Log[g + h*x])/(3*h*(b*g - a*h)^3) - (d^3*q*r*Log[g + h*
x])/(3*h*(d*g - c*h)^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3h(g+hx)^3}+\frac{(bpr)\int \frac{1}{(a+bx)(g+hx)^3}\,dx}{3h}+\frac{(dqr)\int \frac{1}{(c+dx)(g+hx)^3}\,dx}{3h}\\
&= -\frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3h(g+hx)^3}\\
&\quad+\frac{(bpr)\int \left(\frac{b^3}{(bg-ah)^3(a+bx)}-\frac{h}{(bg-ah)(g+hx)^3}-\frac{bh}{(bg-ah)^2(g+hx)^2}-\frac{b^2h}{(bg-ah)^3(g+hx)}\right)dx}{3h}\\
&\quad+\frac{(dqr)\int \left(\frac{d^3}{(dg-ch)^3(c+dx)}-\frac{h}{(dg-ch)(g+hx)^3}-\frac{dh}{(dg-ch)^2(g+hx)^2}-\frac{d^2h}{(dg-ch)^3(g+hx)}\right)dx}{3h}\\
&= \frac{bpr}{6h(bg-ah)(g+hx)^2}+\frac{dqr}{6h(dg-ch)(g+hx)^2}+\frac{b^2pr}{3h(bg-ah)^2(g+hx)}\\
&\quad+\frac{d^2qr}{3h(dg-ch)^2(g+hx)}+\frac{b^3pr\log(a+bx)}{3h(bg-ah)^3}+\frac{d^3qr\log(c+dx)}{3h(dg-ch)^3}\\
&\quad-\frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3h(g+hx)^3}-\frac{b^3pr\log(g+hx)}{3h(bg-ah)^3}-\frac{d^3qr\log(g+hx)}{3h(dg-ch)^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.98

$$
\int \frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^4}\,dx
$$

$$
= \frac{-2\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)+\frac{r(g+hx)\left((bg-ah)^2(dg-ch)^2(bdg(p+q)-h(bcp+adq))-(g+hx)\left((bg-ah)(dg-ch)\left(4abd^2ghq-2a\right.\right.\right.}{}}{6h(g+h}
$$

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x)^4,x]
```

```
[Out] (-2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (r*(g + h*x)*((b*g - a*h)^2*(d*g
 - c*h)^2*(b*d*g*(p + q) - h*(b*c*p + a*d*q)) - (g + h*x)*((b*g - a*h)*(d*g
 - c*h)*(4*a*b*d^2*g*h*q - 2*a^2*d^2*h^2*q - 2*b^2*(-2*c*d*g*h*p + c^2*h^2*
p + d^2*g^2*(p + q))) - 2*(g + h*x)*(b^3*(d*g - c*h)^3*p*(Log[a + b*x] - Lo
g[g + h*x]) + d^3*(b*g - a*h)^3*q*(Log[c + d*x] - Log[g + h*x])))))/((b*g -
 a*h)^3*(d*g - c*h)^3))/(6*h*(g + h*x)^3)
```

Maple [F]

$$\int \frac{\ln \left(e{\left(f{\left(bx+a\right)}^{p} \left(dx+c\right)^{q}\right)}^{r}\right)}{\left(hx+g\right)^{4}}dx$$

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^4,x)
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^4,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^4} \, dx = \text{Timed out}$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^4,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^4} \, dx = \text{Timed out}$$

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(h*x+g)**4,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.75

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^4} \, dx$$
$$= \frac{\left(\left(\frac{2\,b^2\log(bx+a)}{b^3g^3-3\,ab^2g^2h+3\,a^2bgh^2-a^3h^3} - \frac{2\,b^2\log(hx+g)}{b^3g^3-3\,ab^2g^2h+3\,a^2bgh^2-a^3h^3} + \frac{2\,bhx+3\,bg-ah}{b^2g^4-2\,abg^3h+a^2g^2h^2+(b^2g^2h^2-2\,abgh^3+a^2h^4)x^2+2\,(b^2g^3h-2\,ab\cdots}\right.\right.}{\cdots}$$
$$- \frac{\log \left(\left(\left(bx+a\right)^{p}\left(dx+c\right)^{q}f\right)^{r}e\right)}{3\left(hx+g\right)^{3}h}$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^4,x, algorithm="maxima")
[Out] 1/6*((2*b^2*log(b*x + a)/(b^3*g^3 - 3*a*b^2*g^2*h + 3*a^2*b*g*h^2 - a^3*h^3
) - 2*b^2*log(h*x + g)/(b^3*g^3 - 3*a*b^2*g^2*h + 3*a^2*b*g*h^2 - a^3*h^3)
```

```
+ (2*b*h*x + 3*b*g - a*h)/(b^2*g^4 - 2*a*b*g^3*h + a^2*g^2*h^2 + (b^2*g^2*h
^2 - 2*a*b*g*h^3 + a^2*h^4)*x^2 + 2*(b^2*g^3*h - 2*a*b*g^2*h^2 + a^2*g*h^3)
*x))*b*f*p + (2*d^2*log(d*x + c)/(d^3*g^3 - 3*c*d^2*g^2*h + 3*c^2*d*g*h^2 -
 c^3*h^3) - 2*d^2*log(h*x + g)/(d^3*g^3 - 3*c*d^2*g^2*h + 3*c^2*d*g*h^2 - c
^3*h^3) + (2*d*h*x + 3*d*g - c*h)/(d^2*g^4 - 2*c*d*g^3*h + c^2*g^2*h^2 + (d
^2*g^2*h^2 - 2*c*d*g*h^3 + c^2*h^4)*x^2 + 2*(d^2*g^3*h - 2*c*d*g^2*h^2 + c^
2*g*h^3)*x))*d*f*q)*r/(f*h) - 1/3*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/((h*
x + g)^3*h)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1783 vs. $2(242) = 484$.

Time = 0.52 (sec) , antiderivative size = 1783, normalized size of antiderivative = 6.86

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^4}\,dx = \text{Too large to display}$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^4,x, algorithm="giac")
```

```
[Out] 1/3*b^4*p*r*log(abs(b*x + a))/(b^4*g^3*h - 3*a*b^3*g^2*h^2 + 3*a^2*b^2*g*h^
3 - a^3*b*h^4) + 1/3*d^4*q*r*log(abs(d*x + c))/(d^4*g^3*h - 3*c*d^3*g^2*h^2
 + 3*c^2*d^2*g*h^3 - c^3*d*h^4) - 1/3*p*r*log(b*x + a)/(h^4*x^3 + 3*g*h^3*x
^2 + 3*g^2*h^2*x + g^3*h) - 1/3*q*r*log(d*x + c)/(h^4*x^3 + 3*g*h^3*x^2 + 3
*g^2*h^2*x + g^3*h) - 1/3*(b^3*d^3*g^3*p*r - 3*b^3*c*d^2*g^2*h*p*r + 3*b^3*
c^2*d*g*h^2*p*r - b^3*c^3*h^3*p*r + b^3*d^3*g^3*q*r - 3*a*b^2*d^3*g^2*h*q*r
 + 3*a^2*b*d^3*g*h^2*q*r - a^3*d^3*h^3*q*r)*log(h*x + g)/(b^3*d^3*g^6*h - 3
*b^3*c*d^2*g^5*h^2 - 3*a*b^2*d^3*g^5*h^2 + 3*b^3*c^2*d*g^4*h^3 + 9*a*b^2*c*
d^2*g^4*h^3 + 3*a^2*b*d^3*g^4*h^3 - b^3*c^3*g^3*h^4 - 9*a*b^2*c^2*d*g^3*h^4
 - 9*a^2*b*c*d^2*g^3*h^4 - a^3*d^3*g^3*h^4 + 3*a*b^2*c^3*g^2*h^5 + 9*a^2*b*
c^2*d*g^2*h^5 + 3*a^3*c*d^2*g^2*h^5 - 3*a^2*b*c^3*g*h^6 - 3*a^3*c^2*d*g*h^6
 + a^3*c^3*h^7) + 1/6*(2*b^2*d^2*g^2*h^2*p*r*x^2 - 4*b^2*c*d*g*h^3*p*r*x^2
+ 2*b^2*c^2*h^4*p*r*x^2 + 2*b^2*d^2*g^2*h^2*q*r*x^2 - 4*a*b*d^2*g*h^3*q*r*x
^2 + 2*a^2*d^2*h^4*q*r*x^2 + 5*b^2*d^2*g^3*h*p*r*x - 10*b^2*c*d*g^2*h^2*p*r
*x - a*b*d^2*g^2*h^2*p*r*x + 5*b^2*c^2*g*h^3*p*r*x + 2*a*b*c*d*g*h^3*p*r*x
- a*b*c^2*h^4*p*r*x + 5*b^2*d^2*g^3*h*q*r*x - b^2*c*d*g^2*h^2*q*r*x - 10*a*
b*d^2*g^2*h^2*q*r*x + 2*a*b*c*d*g*h^3*q*r*x + 5*a^2*d^2*g*h^3*q*r*x - a^2*c
*d*h^4*q*r*x + 3*b^2*d^2*g^4*p*r - 6*b^2*c*d*g^3*h*p*r - a*b*d^2*g^3*h*p*r
+ 3*b^2*c^2*g^2*h^2*p*r + 2*a*b*c*d*g^2*h^2*p*r - a*b*c^2*g*h^3*p*r + 3*b^2
*d^2*g^4*q*r - b^2*c*d*g^3*h*q*r - 6*a*b*d^2*g^3*h*q*r + 2*a*b*c*d*g^2*h^2*
q*r + 3*a^2*d^2*g^2*h^2*q*r - a^2*c*d*g*h^3*q*r - 2*b^2*d^2*g^4*r*log(f) +
4*b^2*c*d*g^3*h*r*log(f) + 4*a*b*d^2*g^3*h*r*log(f) - 2*b^2*c^2*g^2*h^2*r*l
og(f) - 8*a*b*c*d*g^2*h^2*r*log(f) - 2*a^2*d^2*g^2*h^2*r*log(f) + 4*a*b*c^2
*g*h^3*r*log(f) + 4*a^2*c*d*g*h^3*r*log(f) - 2*a^2*c^2*h^4*r*log(f) - 2*b^2
*d^2*g^4*log(e) + 4*b^2*c*d*g^3*h*log(e) + 4*a*b*d^2*g^3*h*log(e) - 2*b^2*c
^2*g^2*h^2*log(e) - 8*a*b*c*d*g^2*h^2*log(e) - 2*a^2*d^2*g^2*h^2*log(e) + 4
```

```
*a*b*c^2*g*h^3*log(e) + 4*a^2*c*d*g*h^3*log(e) - 2*a^2*c^2*h^4*log(e))/(b^2
*d^2*g^4*h^4*x^3 - 2*b^2*c*d*g^3*h^5*x^3 - 2*a*b*d^2*g^3*h^5*x^3 + b^2*c^2*
g^2*h^6*x^3 + 4*a*b*c*d*g^2*h^6*x^3 + a^2*d^2*g^2*h^6*x^3 - 2*a*b*c^2*g*h^7
*x^3 - 2*a^2*c*d*g*h^7*x^3 + a^2*c^2*h^8*x^3 + 3*b^2*d^2*g^5*h^3*x^2 - 6*b^
2*c*d*g^4*h^4*x^2 - 6*a*b*d^2*g^4*h^4*x^2 + 3*b^2*c^2*g^3*h^5*x^2 + 12*a*b*
c*d*g^3*h^5*x^2 + 3*a^2*d^2*g^3*h^5*x^2 - 6*a*b*c^2*g^2*h^6*x^2 - 6*a^2*c*d
*g^2*h^6*x^2 + 3*a^2*c^2*g*h^7*x^2 + 3*b^2*d^2*g^6*h^2*x - 6*b^2*c*d*g^5*h^
3*x - 6*a*b*d^2*g^5*h^3*x + 3*b^2*c^2*g^4*h^4*x + 12*a*b*c*d*g^4*h^4*x + 3*
a^2*d^2*g^4*h^4*x - 6*a*b*c^2*g^3*h^5*x - 6*a^2*c*d*g^3*h^5*x + 3*a^2*c^2*g
^2*h^6*x + b^2*d^2*g^7*h - 2*b^2*c*d*g^6*h^2 - 2*a*b*d^2*g^6*h^2 + b^2*c^2*
g^5*h^3 + 4*a*b*c*d*g^5*h^3 + a^2*d^2*g^5*h^3 - 2*a*b*c^2*g^4*h^4 - 2*a^2*c
*d*g^4*h^4 + a^2*c^2*g^3*h^5)
```

Mupad [B] (verification not implemented)

Time = 6.87 (sec) , antiderivative size = 977, normalized size of antiderivative = 3.76

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^4}\,dx$$
$$= \frac{\frac{3\,b^2\,d^2\,g^3\,p\,r+3\,b^2\,d^2\,g^3\,q\,r-a\,b\,c^2\,h^3\,p\,r-a^2\,c\,d\,h^3\,q\,r+3\,b^2\,c^2\,g\,h^2\,p\,r+3\,a^2\,d^2\,g\,h^2\,q\,r-a\,b\,d^2\,g^2\,h\,p\,r-6\,a\,b\,d^2\,g^2\,h\,q\,r-6\,b^2\,c\,d\,g^2\,h\,p\,r-b^2\,c\,d\,g^2\,h\,q\,r}{2\,(a^2\,c^2\,h^4-2\,a^2\,c\,d\,g\,h^3+a^2\,d^2\,g^2\,h^2-2\,a\,b\,c^2\,g\,h^3+4\,a\,b\,c\,d\,g^2\,h^2-2\,a\,b\,d^2\,g^3\,h+b^2\,c^2\,g^2\,h^2-2\,b^2\,c\,d\,g^3\,h+b^2\,d^2\,g^4)}}{3\,g^2\,h+6\,g\,h}$$
$$+\frac{\ln\left(g+h\,x\right)\,\left(g^2\,\left(3\,c\,h\,p\,r\,b^3\,d^2+3\,a\,h\,q\,r\,b^2\,d^3\right)}{3\,a^3\,c^3\,h^7-9\,a^3\,c^2\,d\,g\,h^6+9\,a^3\,c\,d^2\,g^2\,h^5-3\,a^3\,d^3\,g^3\,h^4-9\,a^2\,b\,c^3\,g\,h^6+27\,a^2\,b\,c^2\,d\,g^2\,h^5-27\,a^2\,b\,c\,d^2\,g^3}$$
$$-\frac{\ln\left(e\,\left(f\,{\left(a+b\,x\right)}^p\,{\left(c+d\,x\right)}^q\right)^r\right)\,\left(\frac{x}{3}+\frac{g}{3\,h}\right)}{{\left(g+h\,x\right)}^4}-\frac{b^3\,p\,r\,\ln\left(a+b\,x\right)}{3\,a^3\,h^4-9\,a^2\,b\,g\,h^3+9\,a\,b^2\,g^2\,h^2-3\,b^3\,g^3\,h}$$
$$-\frac{d^3\,q\,r\,\ln\left(c+d\,x\right)}{3\,c^3\,h^4-9\,c^2\,d\,g\,h^3+9\,c\,d^2\,g^2\,h^2-3\,d^3\,g^3\,h}$$

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(g + h*x)^4,x)
```

```
[Out] ((3*b^2*d^2*g^3*p*r + 3*b^2*d^2*g^3*q*r - a*b*c^2*h^3*p*r - a^2*c*d*h^3*q*r
 + 3*b^2*c^2*g*h^2*p*r + 3*a^2*d^2*g*h^2*q*r - a*b*d^2*g^2*h*p*r - 6*a*b*d^
2*g^2*h*q*r - 6*b^2*c*d*g^2*h*p*r - b^2*c*d*g^2*h*q*r + 2*a*b*c*d*g*h^2*p*r
 + 2*a*b*c*d*g*h^2*q*r)/(2*(a^2*c^2*h^4 + b^2*d^2*g^4 + a^2*d^2*g^2*h^2 + b
^2*c^2*g^2*h^2 - 2*a*b*c^2*g*h^3 - 2*a*b*d^2*g^3*h - 2*a^2*c*d*g*h^3 - 2*b^
2*c*d*g^3*h + 4*a*b*c*d*g^2*h^2)) + (x*(b^2*c^2*h^3*p*r + a^2*d^2*h^3*q*r +
 b^2*d^2*g^2*h*p*r + b^2*d^2*g^2*h*q*r - 2*a*b*d^2*g*h^2*q*r - 2*b^2*c*d*g*
h^2*p*r))/(a^2*c^2*h^4 + b^2*d^2*g^4 + a^2*d^2*g^2*h^2 + b^2*c^2*g^2*h^2 -
2*a*b*c^2*g*h^3 - 2*a*b*d^2*g^3*h - 2*a^2*c*d*g*h^3 - 2*b^2*c*d*g^3*h + 4*a
*b*c*d*g^2*h^2))/(3*g^2*h + 3*h^3*x^2 + 6*g*h^2*x) + (log(g + h*x)*(g^2*(3*
a*b^2*d^3*h*q*r + 3*b^3*c*d^2*h*p*r) - g^3*(b^3*d^3*p*r + b^3*d^3*q*r) - g*
(3*a^2*b*d^3*h^2*q*r + 3*b^3*c^2*d*h^2*p*r) + b^3*c^3*h^3*p*r + a^3*d^3*h^3
*q*r))/(3*a^3*c^3*h^7 + 3*b^3*d^3*g^6*h - 3*a^3*d^3*g^3*h^4 - 3*b^3*c^3*g^3
*h^4 - 9*a^2*b*c^3*g*h^6 - 9*a^3*c^2*d*g*h^6 + 9*a*b^2*c^3*g^2*h^5 - 9*a*b^
```

```
2*d^3*g^5*h^2 + 9*a^2*b*d^3*g^4*h^3 + 9*a^3*c*d^2*g^2*h^5 - 9*b^3*c*d^2*g^5
*h^2 + 9*b^3*c^2*d*g^4*h^3 + 27*a*b^2*c*d^2*g^4*h^3 - 27*a*b^2*c^2*d*g^3*h^
4 - 27*a^2*b*c*d^2*g^3*h^4 + 27*a^2*b*c^2*d*g^2*h^5) - (log(e*(f*(a + b*x)^
p*(c + d*x)^q)^r)*(x/3 + g/(3*h)))/(g + h*x)^4 - (b^3*p*r*log(a + b*x))/(3*
a^3*h^4 - 3*b^3*g^3*h + 9*a*b^2*g^2*h^2 - 9*a^2*b*g*h^3) - (d^3*q*r*log(c +
 d*x))/(3*c^3*h^4 - 3*d^3*g^3*h + 9*c*d^2*g^2*h^2 - 9*c^2*d*g*h^3)
```

3.34 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(g+hx)^5} dx$

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Optimal result

Integrand size = 29, antiderivative size = 318

$$\begin{aligned}\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^5}\,dx &= \frac{bpr}{12h(bg-ah)(g+hx)^3}+\frac{dqr}{12h(dg-ch)(g+hx)^3}\\ &+\frac{b^2pr}{8h(bg-ah)^2(g+hx)^2}+\frac{d^2qr}{8h(dg-ch)^2(g+hx)^2}\\ &+\frac{b^3pr}{4h(bg-ah)^3(g+hx)}+\frac{d^3qr}{4h(dg-ch)^3(g+hx)}\\ &+\frac{b^4pr\log(a+bx)}{4h(bg-ah)^4}+\frac{d^4qr\log(c+dx)}{4h(dg-ch)^4}\\ &-\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4h(g+hx)^4}\\ &-\frac{b^4pr\log(g+hx)}{4h(bg-ah)^4}-\frac{d^4qr\log(g+hx)}{4h(dg-ch)^4}\end{aligned}$$

```
[Out] 1/12*b*p*r/h/(-a*h+b*g)/(h*x+g)^3+1/12*d*q*r/h/(-c*h+d*g)/(h*x+g)^3+1/8*b^2
*p*r/h/(-a*h+b*g)^2/(h*x+g)^2+1/8*d^2*q*r/h/(-c*h+d*g)^2/(h*x+g)^2+1/4*b^3*
p*r/h/(-a*h+b*g)^3/(h*x+g)+1/4*d^3*q*r/h/(-c*h+d*g)^3/(h*x+g)+1/4*b^4*p*r*l
n(b*x+a)/h/(-a*h+b*g)^4+1/4*d^4*q*r*ln(d*x+c)/h/(-c*h+d*g)^4-1/4*ln(e*(f*(b
*x+a)^p*(d*x+c)^q)^r)/h/(h*x+g)^4-1/4*b^4*p*r*ln(h*x+g)/h/(-a*h+b*g)^4-1/4*
d^4*q*r*ln(h*x+g)/h/(-c*h+d*g)^4
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2581, 46}

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^5}\,dx = \frac{b^4pr\log(a+bx)}{4h(bg-ah)^4} - \frac{b^4pr\log(g+hx)}{4h(bg-ah)^4} + \frac{b^3pr}{4h(g+hx)(bg-ah)^3} + \frac{b^2pr}{8h(g+hx)^2(bg-ah)^2} - \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4h(g+hx)^4} + \frac{bpr}{12h(g+hx)^3(bg-ah)} + \frac{d^4qr\log(c+dx)}{4h(dg-ch)^4} - \frac{d^4qr\log(g+hx)}{4h(dg-ch)^4} + \frac{d^3qr}{4h(g+hx)(dg-ch)^3} + \frac{d^2qr}{8h(g+hx)^2(dg-ch)^2} + \frac{dqr}{12h(g+hx)^3(dg-ch)}$$

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x)^5,x]
```

```
[Out] (b*p*r)/(12*h*(b*g - a*h)*(g + h*x)^3) + (d*q*r)/(12*h*(d*g - c*h)*(g + h*x
)^3) + (b^2*p*r)/(8*h*(b*g - a*h)^2*(g + h*x)^2) + (d^2*q*r)/(8*h*(d*g - c*
h)^2*(g + h*x)^2) + (b^3*p*r)/(4*h*(b*g - a*h)^3*(g + h*x)) + (d^3*q*r)/(4*
h*(d*g - c*h)^3*(g + h*x)) + (b^4*p*r*Log[a + b*x])/(4*h*(b*g - a*h)^4) + (
d^4*q*r*Log[c + d*x])/(4*h*(d*g - c*h)^4) - Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]/(4*h*(g + h*x)^4) - (b^4*p*r*Log[g + h*x])/(4*h*(b*g - a*h)^4) - (d^4
*q*r*Log[g + h*x])/(4*h*(d*g - c*h)^4)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4h(g+hx)^4} + \frac{(bpr)\int \frac{1}{(a+bx)(g+hx)^4}\,dx}{4h} + \frac{(dqr)\int \frac{1}{(c+dx)(g+hx)^4}\,dx}{4h}$$

$$= -\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4h(g+hx)^4} + \frac{(bpr)\int \left(\frac{b^4}{(bg-ah)^4(a+bx)} - \frac{h}{(bg-ah)(g+hx)^4} - \frac{bh}{(bg-ah)^2(g+hx)^3} - \frac{b^2h}{(bg-ah)^3(g+hx)^2} - \frac{b^3h}{(bg-ah)^4(g+hx)}\right) dx}{4h} + \frac{(dqr)\int \left(\frac{d^4}{(dg-ch)^4(c+dx)} - \frac{h}{(dg-ch)(g+hx)^4} - \frac{dh}{(dg-ch)^2(g+hx)^3} - \frac{d^2h}{(dg-ch)^3(g+hx)^2} - \frac{d^3h}{(dg-ch)^4(g+hx)}\right) dx}{4h}$$

$$= \frac{bpr}{12h(bg-ah)(g+hx)^3} + \frac{dqr}{12h(dg-ch)(g+hx)^3} + \frac{b^2pr}{8h(bg-ah)^2(g+hx)^2} + \frac{d^2qr}{8h(dg-ch)^2(g+hx)^2} + \frac{b^3pr}{4h(bg-ah)^3(g+hx)} + \frac{d^3qr}{4h(dg-ch)^3(g+hx)} + \frac{b^4pr\log(a+bx)}{4h(bg-ah)^4} + \frac{d^4qr\log(c+dx)}{4h(dg-ch)^4} - \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4h(g+hx)^4} - \frac{b^4pr\log(g+hx)}{4h(bg-ah)^4} - \frac{d^4qr\log(g+hx)}{4h(dg-ch)^4}$$

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.51

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^5}\,dx = \frac{-6\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) + \frac{r(g+hx)\left(2(bg-ah)^3(dg-ch)^3(bdg(p+q)-h(bcp+adq))-(g+hx)\left((bg-ah)^2(dg-ch)^2(6abd^2ghq-\cdots\right.\right.}{\cdots}}{\cdots}$$

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x)^5,x]
```

```
[Out] (-6*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (r*(g + h*x)*(2*(b*g - a*h)^3*(d
*g - c*h)^3*(b*d*g*(p + q) - h*(b*c*p + a*d*q)) - (g + h*x)*((b*g - a*h)^2*
(d*g - c*h)^2*(6*a*b*d^2*g*h*q - 3*a^2*d^2*h^2*q - 3*b^2*(-2*c*d*g*h*p + c^
2*h^2*p + d^2*g^2*(p + q))) + 6*(g + h*x)*(-((b*g - a*h)*(-(d*g) + c*h)*(3*
a*b^2*d^3*g^2*h*q - 3*a^2*b*d^3*g*h^2*q + a^3*d^3*h^3*q - b^3*(-3*c*d^2*g^2
*h*p + 3*c^2*d*g*h^2*p - c^3*h^3*p + d^3*g^3*(p + q)))) - (g + h*x)*(b^4*(d
*g - c*h)^4*p*Log[a + b*x] + d^4*(b*g - a*h)^4*q*Log[c + d*x] - (-4*a*b^3*d
^4*g^3*h*q + 6*a^2*b^2*d^4*g^2*h^2*q - 4*a^3*b*d^4*g*h^3*q + a^4*d^4*h^4*q
+ b^4*(-4*c*d^3*g^3*h*p + 6*c^2*d^2*g^2*h^2*p - 4*c^3*d*g*h^3*p + c^4*h^4*p
 + d^4*g^4*(p + q)))*Log[g + h*x])))))/((b*g - a*h)^4*(d*g - c*h)^4))/(24*h
*(g + h*x)^4)
```

Maple [F]

$$\int \frac{\ln \left(e {\left(f \left(b x +a \right)^{p} \left(d x +c \right)^{q}\right)}^{r}\right)}{\left(h x +g \right)^{5}}d x$$

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^5,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^5,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^5} \, dx = \text{Timed out}$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^5,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^5} \, dx = \text{Timed out}$$

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(h*x+g)**5,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 776 vs. $2(296) = 592$.

Time = 0.23 (sec) , antiderivative size = 776, normalized size of antiderivative = 2.44

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^5} \, dx = \frac{\left(\left(\frac{6\, b^{3} \log\left(bx+a\right)}{b^{4} g^{4}-4\, a b^{3} g^{3} h+6\, a^{2} b^{2} g^{2} h^{2}-4\, a^{3} b g h^{3}+a^{4} h^{4}} - \frac{6\, b^{3} \log\left(hx+g\right)}{b^{4} g^{4}-4\, a b^{3} g^{3} h+6\, a^{2} b^{2} g^{2} h^{2}-4\, a^{3} b g h^{3}+a^{4} h^{4}} + \frac{}{b^{3} g^{6}-3\, a b^{2} g^{5} h+3\, a^{2} b g^{4} h^{2}-a^{3} g^{3} h^{3}+(b^{3}}\right.\right.}{} - \frac{\log\left(\left(\left(bx+a\right)^{p} \left(dx+c\right)^{q} f\right)^{r} e\right)}{4\, \left(hx+g\right)^{4} h}$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^5,x, algorithm="maxima")
```

```
[Out] 1/24*((6*b^3*log(b*x + a)/(b^4*g^4 - 4*a*b^3*g^3*h + 6*a^2*b^2*g^2*h^2 - 4*
a^3*b*g*h^3 + a^4*h^4) - 6*b^3*log(h*x + g)/(b^4*g^4 - 4*a*b^3*g^3*h + 6*a^
```

```
2*b^2*g^2*h^2 - 4*a^3*b*g*h^3 + a^4*h^4) + (6*b^2*h^2*x^2 + 11*b^2*g^2 - 7*
a*b*g*h + 2*a^2*h^2 + 3*(5*b^2*g*h - a*b*h^2)*x)/(b^3*g^6 - 3*a*b^2*g^5*h +
 3*a^2*b*g^4*h^2 - a^3*g^3*h^3 + (b^3*g^3*h^3 - 3*a*b^2*g^2*h^4 + 3*a^2*b*g
*h^5 - a^3*h^6)*x^3 + 3*(b^3*g^4*h^2 - 3*a*b^2*g^3*h^3 + 3*a^2*b*g^2*h^4 -
a^3*g*h^5)*x^2 + 3*(b^3*g^5*h - 3*a*b^2*g^4*h^2 + 3*a^2*b*g^3*h^3 - a^3*g^2
*h^4)*x))*b*f*p + (6*d^3*log(d*x + c)/(d^4*g^4 - 4*c*d^3*g^3*h + 6*c^2*d^2*
g^2*h^2 - 4*c^3*d*g*h^3 + c^4*h^4) - 6*d^3*log(h*x + g)/(d^4*g^4 - 4*c*d^3*
g^3*h + 6*c^2*d^2*g^2*h^2 - 4*c^3*d*g*h^3 + c^4*h^4) + (6*d^2*h^2*x^2 + 11*
d^2*g^2 - 7*c*d*g*h + 2*c^2*h^2 + 3*(5*d^2*g*h - c*d*h^2)*x)/(d^3*g^6 - 3*c
*d^2*g^5*h + 3*c^2*d*g^4*h^2 - c^3*g^3*h^3 + (d^3*g^3*h^3 - 3*c*d^2*g^2*h^4
 + 3*c^2*d*g*h^5 - c^3*h^6)*x^3 + 3*(d^3*g^4*h^2 - 3*c*d^2*g^3*h^3 + 3*c^2*
d*g^2*h^4 - c^3*g*h^5)*x^2 + 3*(d^3*g^5*h - 3*c*d^2*g^4*h^2 + 3*c^2*d*g^3*h
^3 - c^3*g^2*h^4)*x))*d*f*q)*r/(f*h) - 1/4*log(((b*x + a)^p*(d*x + c)^q*f)^
r*e)/((h*x + g)^4*h)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3943 vs. 2(296) = 592.

Time = 0.60 (sec) , antiderivative size = 3943, normalized size of antiderivative = 12.40

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^5}\,dx = \text{Too large to display}$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^5,x, algorithm="giac")
```

```
[Out] 1/4*b^5*p*r*log(abs(b*x + a))/(b^5*g^4*h - 4*a*b^4*g^3*h^2 + 6*a^2*b^3*g^2*
h^3 - 4*a^3*b^2*g*h^4 + a^4*b*h^5) + 1/4*d^5*q*r*log(abs(d*x + c))/(d^5*g^4
*h - 4*c*d^4*g^3*h^2 + 6*c^2*d^3*g^2*h^3 - 4*c^3*d^2*g*h^4 + c^4*d*h^5) - 1
/4*p*r*log(b*x + a)/(h^5*x^4 + 4*g*h^4*x^3 + 6*g^2*h^3*x^2 + 4*g^3*h^2*x +
g^4*h) - 1/4*q*r*log(d*x + c)/(h^5*x^4 + 4*g*h^4*x^3 + 6*g^2*h^3*x^2 + 4*g^
3*h^2*x + g^4*h) - 1/4*(b^4*d^4*g^4*p*r - 4*b^4*c*d^3*g^3*h*p*r + 6*b^4*c^2
*d^2*g^2*h^2*p*r - 4*b^4*c^3*d*g*h^3*p*r + b^4*c^4*h^4*p*r + b^4*d^4*g^4*q*
r - 4*a*b^3*d^4*g^3*h*q*r + 6*a^2*b^2*d^4*g^2*h^2*q*r - 4*a^3*b*d^4*g*h^3*q
*r + a^4*d^4*h^4*q*r)*log(h*x + g)/(b^4*d^4*g^8*h - 4*b^4*c*d^3*g^7*h^2 - 4
*a*b^3*d^4*g^7*h^2 + 6*b^4*c^2*d^2*g^6*h^3 + 16*a*b^3*c*d^3*g^6*h^3 + 6*a^2
*b^2*d^4*g^6*h^3 - 4*b^4*c^3*d*g^5*h^4 - 24*a*b^3*c^2*d^2*g^5*h^4 - 24*a^2*
b^2*c*d^3*g^5*h^4 - 4*a^3*b*d^4*g^5*h^4 + b^4*c^4*g^4*h^5 + 16*a*b^3*c^3*d*
g^4*h^5 + 36*a^2*b^2*c^2*d^2*g^4*h^5 + 16*a^3*b*c*d^3*g^4*h^5 + a^4*d^4*g^4
*h^5 - 4*a*b^3*c^4*g^3*h^6 - 24*a^2*b^2*c^3*d*g^3*h^6 - 24*a^3*b*c^2*d^2*g^
3*h^6 - 4*a^4*c*d^3*g^3*h^6 + 6*a^2*b^2*c^4*g^2*h^7 + 16*a^3*b*c^3*d*g^2*h^
7 + 6*a^4*c^2*d^2*g^2*h^7 - 4*a^3*b*c^4*g*h^8 - 4*a^4*c^3*d*g*h^8 + a^4*c^4
*h^9) + 1/24*(6*b^3*d^3*g^3*h^3*p*r*x^3 - 18*b^3*c*d^2*g^2*h^4*p*r*x^3 + 18
*b^3*c^2*d*g*h^5*p*r*x^3 - 6*b^3*c^3*h^6*p*r*x^3 + 6*b^3*d^3*g^3*h^3*q*r*x^
3 - 18*a*b^2*d^3*g^2*h^4*q*r*x^3 + 18*a^2*b*d^3*g*h^5*q*r*x^3 - 6*a^3*d^3*h
^6*q*r*x^3 + 21*b^3*d^3*g^4*h^2*p*r*x^2 - 63*b^3*c*d^2*g^3*h^3*p*r*x^2 - 3*
```

a*b^2*d^3*g^3*h^3*p*r*x^2 + 63*b^3*c^2*d*g^2*h^4*p*r*x^2 + 9*a*b^2*c*d^2*g^
2*h^4*p*r*x^2 - 21*b^3*c^3*g*h^5*p*r*x^2 - 9*a*b^2*c^2*d*g*h^5*p*r*x^2 + 3*
a*b^2*c^3*h^6*p*r*x^2 + 21*b^3*d^3*g^4*h^2*q*r*x^2 - 3*b^3*c*d^2*g^3*h^3*q*
r*x^2 - 63*a*b^2*d^3*g^3*h^3*q*r*x^2 + 9*a*b^2*c*d^2*g^2*h^4*q*r*x^2 + 63*a
^2*b*d^3*g^2*h^4*q*r*x^2 - 9*a^2*b*c*d^2*g*h^5*q*r*x^2 - 21*a^3*d^3*g*h^5*q
*r*x^2 + 3*a^3*c*d^2*h^6*q*r*x^2 + 26*b^3*d^3*g^5*h*p*r*x - 78*b^3*c*d^2*g^
4*h^2*p*r*x - 10*a*b^2*d^3*g^4*h^2*p*r*x + 78*b^3*c^2*d*g^3*h^3*p*r*x + 30*
a*b^2*c*d^2*g^3*h^3*p*r*x + 2*a^2*b*d^3*g^3*h^3*p*r*x - 26*b^3*c^3*g^2*h^4*
p*r*x - 30*a*b^2*c^2*d*g^2*h^4*p*r*x - 6*a^2*b*c*d^2*g^2*h^4*p*r*x + 10*a*b
^2*c^3*g*h^5*p*r*x + 6*a^2*b*c^2*d*g*h^5*p*r*x - 2*a^2*b*c^3*h^6*p*r*x + 26
*b^3*d^3*g^5*h*q*r*x - 10*b^3*c*d^2*g^4*h^2*q*r*x - 78*a*b^2*d^3*g^4*h^2*q*
r*x + 2*b^3*c^2*d*g^3*h^3*q*r*x + 30*a*b^2*c*d^2*g^3*h^3*q*r*x + 78*a^2*b*d
^3*g^3*h^3*q*r*x - 6*a*b^2*c^2*d*g^2*h^4*q*r*x - 30*a^2*b*c*d^2*g^2*h^4*q*r
*x - 26*a^3*d^3*g^2*h^4*q*r*x + 6*a^2*b*c^2*d*g*h^5*q*r*x + 10*a^3*c*d^2*g*
h^5*q*r*x - 2*a^3*c^2*d*h^6*q*r*x + 11*b^3*d^3*g^6*p*r - 33*b^3*c*d^2*g^5*h
*p*r - 7*a*b^2*d^3*g^5*h*p*r + 33*b^3*c^2*d*g^4*h^2*p*r + 21*a*b^2*c*d^2*g^
4*h^2*p*r + 2*a^2*b*d^3*g^4*h^2*p*r - 11*b^3*c^3*g^3*h^3*p*r - 21*a*b^2*c^2
*d*g^3*h^3*p*r - 6*a^2*b*c*d^2*g^3*h^3*p*r + 7*a*b^2*c^3*g^2*h^4*p*r + 6*a^
2*b*c^2*d*g^2*h^4*p*r - 2*a^2*b*c^3*g*h^5*p*r + 11*b^3*d^3*g^6*q*r - 7*b^3*
c*d^2*g^5*h*q*r - 33*a*b^2*d^3*g^5*h*q*r + 2*b^3*c^2*d*g^4*h^2*q*r + 21*a*b
^2*c*d^2*g^4*h^2*q*r + 33*a^2*b*d^3*g^4*h^2*q*r - 6*a*b^2*c^2*d*g^3*h^3*q*r
 - 21*a^2*b*c*d^2*g^3*h^3*q*r - 11*a^3*d^3*g^3*h^3*q*r + 6*a^2*b*c^2*d*g^2*
h^4*q*r + 7*a^3*c*d^2*g^2*h^4*q*r - 2*a^3*c^2*d*g*h^5*q*r - 6*b^3*d^3*g^6*r
*log(f) + 18*b^3*c*d^2*g^5*h*r*log(f) + 18*a*b^2*d^3*g^5*h*r*log(f) - 18*b^
3*c^2*d*g^4*h^2*r*log(f) - 54*a*b^2*c*d^2*g^4*h^2*r*log(f) - 18*a^2*b*d^3*g
^4*h^2*r*log(f) + 6*b^3*c^3*g^3*h^3*r*log(f) + 54*a*b^2*c^2*d*g^3*h^3*r*log
(f) + 54*a^2*b*c*d^2*g^3*h^3*r*log(f) + 6*a^3*d^3*g^3*h^3*r*log(f) - 18*a*b
^2*c^3*g^2*h^4*r*log(f) - 54*a^2*b*c^2*d*g^2*h^4*r*log(f) - 18*a^3*c*d^2*g^
2*h^4*r*log(f) + 18*a^2*b*c^3*g*h^5*r*log(f) + 18*a^3*c^2*d*g*h^5*r*log(f)
 - 6*a^3*c^3*h^6*r*log(f) - 6*b^3*d^3*g^6*log(e) + 18*b^3*c*d^2*g^5*h*log(e)
 + 18*a*b^2*d^3*g^5*h*log(e) - 18*b^3*c^2*d*g^4*h^2*log(e) - 54*a*b^2*c*d^2
*g^4*h^2*log(e) - 18*a^2*b*d^3*g^4*h^2*log(e) + 6*b^3*c^3*g^3*h^3*log(e) +
54*a*b^2*c^2*d*g^3*h^3*log(e) + 54*a^2*b*c*d^2*g^3*h^3*log(e) + 6*a^3*d^3*g
^3*h^3*log(e) - 18*a*b^2*c^3*g^2*h^4*log(e) - 54*a^2*b*c^2*d*g^2*h^4*log(e)
 - 18*a^3*c*d^2*g^2*h^4*log(e) + 18*a^2*b*c^3*g*h^5*log(e) + 18*a^3*c^2*d*g
*h^5*log(e) - 6*a^3*c^3*h^6*log(e))/(b^3*d^3*g^6*h^5*x^4 - 3*b^3*c*d^2*g^5*
h^6*x^4 - 3*a*b^2*d^3*g^5*h^6*x^4 + 3*b^3*c^2*d*g^4*h^7*x^4 + 9*a*b^2*c*d^2
*g^4*h^7*x^4 + 3*a^2*b*d^3*g^4*h^7*x^4 - b^3*c^3*g^3*h^8*x^4 - 9*a*b^2*c^2*
d*g^3*h^8*x^4 - 9*a^2*b*c*d^2*g^3*h^8*x^4 - a^3*d^3*g^3*h^8*x^4 + 3*a*b^2*c
^3*g^2*h^9*x^4 + 9*a^2*b*c^2*d*g^2*h^9*x^4 + 3*a^3*c*d^2*g^2*h^9*x^4 - 3*a^
2*b*c^3*g*h^10*x^4 - 3*a^3*c^2*d*g*h^10*x^4 + a^3*c^3*h^11*x^4 + 4*b^3*d^3*
g^7*h^4*x^3 - 12*b^3*c*d^2*g^6*h^5*x^3 - 12*a*b^2*d^3*g^6*h^5*x^3 + 12*b^3*
c^2*d*g^5*h^6*x^3 + 36*a*b^2*c*d^2*g^5*h^6*x^3 + 12*a^2*b*d^3*g^5*h^6*x^3 -
 4*b^3*c^3*g^4*h^7*x^3 - 36*a*b^2*c^2*d*g^4*h^7*x^3 - 36*a^2*b*c*d^2*g^4*h^
7*x^3 - 4*a^3*d^3*g^4*h^7*x^3 + 12*a*b^2*c^3*g^3*h^8*x^3 + 36*a^2*b*c^2*d*g

```
^3*h^8*x^3 + 12*a^3*c*d^2*g^3*h^8*x^3 - 12*a^2*b*c^3*g^2*h^9*x^3 - 12*a^3*c
^2*d*g^2*h^9*x^3 + 4*a^3*c^3*g*h^10*x^3 + 6*b^3*d^3*g^8*h^3*x^2 - 18*b^3*c*
d^2*g^7*h^4*x^2 - 18*a*b^2*d^3*g^7*h^4*x^2 + 18*b^3*c^2*d*g^6*h^5*x^2 + 54*
a*b^2*c*d^2*g^6*h^5*x^2 + 18*a^2*b*d^3*g^6*h^5*x^2 - 6*b^3*c^3*g^5*h^6*x^2
- 54*a*b^2*c^2*d*g^5*h^6*x^2 - 54*a^2*b*c*d^2*g^5*h^6*x^2 - 6*a^3*d^3*g^5*h
^6*x^2 + 18*a*b^2*c^3*g^4*h^7*x^2 + 54*a^2*b*c^2*d*g^4*h^7*x^2 + 18*a^3*c*d
^2*g^4*h^7*x^2 - 18*a^2*b*c^3*g^3*h^8*x^2 - 18*a^3*c^2*d*g^3*h^8*x^2 + 6*a^
3*c^3*g^2*h^9*x^2 + 4*b^3*d^3*g^9*h^2*x - 12*b^3*c*d^2*g^8*h^3*x - 12*a*b^2
*d^3*g^8*h^3*x + 12*b^3*c^2*d*g^7*h^4*x + 36*a*b^2*c*d^2*g^7*h^4*x + 12*a^2
*b*d^3*g^7*h^4*x - 4*b^3*c^3*g^6*h^5*x - 36*a*b^2*c^2*d*g^6*h^5*x - 36*a^2*
b*c*d^2*g^6*h^5*x - 4*a^3*d^3*g^6*h^5*x + 12*a*b^2*c^3*g^5*h^6*x + 36*a^2*b
*c^2*d*g^5*h^6*x + 12*a^3*c*d^2*g^5*h^6*x - 12*a^2*b*c^3*g^4*h^7*x - 12*a^3
*c^2*d*g^4*h^7*x + 4*a^3*c^3*g^3*h^8*x + b^3*d^3*g^10*h - 3*b^3*c*d^2*g^9*h
^2 - 3*a*b^2*d^3*g^9*h^2 + 3*b^3*c^2*d*g^8*h^3 + 9*a*b^2*c*d^2*g^8*h^3 + 3*
a^2*b*d^3*g^8*h^3 - b^3*c^3*g^7*h^4 - 9*a*b^2*c^2*d*g^7*h^4 - 9*a^2*b*c*d^2
*g^7*h^4 - a^3*d^3*g^7*h^4 + 3*a*b^2*c^3*g^6*h^5 + 9*a^2*b*c^2*d*g^6*h^5 +
3*a^3*c*d^2*g^6*h^5 - 3*a^2*b*c^3*g^5*h^6 - 3*a^3*c^2*d*g^5*h^6 + a^3*c^3*g
^4*h^7)
```

Mupad [B] (verification not implemented)

Time = 11.66 (sec) , antiderivative size = 2215, normalized size of antiderivative = 6.97

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^5}\,dx = \text{Too large to display}$$

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(g + h*x)^5,x)
```

```
[Out] ((11*b^3*d^3*g^5*p*r + 11*b^3*d^3*g^5*q*r - 11*b^3*c^3*g^2*h^3*p*r - 11*a^3
*d^3*g^2*h^3*q*r - 2*a^2*b*c^3*h^5*p*r - 2*a^3*c^2*d*h^5*q*r + 7*a*b^2*c^3*
g*h^4*p*r - 7*a*b^2*d^3*g^4*h*p*r - 33*a*b^2*d^3*g^4*h*q*r + 7*a^3*c*d^2*g*
h^4*q*r - 33*b^3*c*d^2*g^4*h*p*r - 7*b^3*c*d^2*g^4*h*q*r + 2*a^2*b*d^3*g^3*
h^2*p*r + 33*a^2*b*d^3*g^3*h^2*q*r + 33*b^3*c^2*d*g^3*h^2*p*r + 2*b^3*c^2*d
*g^3*h^2*q*r + 21*a*b^2*c*d^2*g^3*h^2*p*r - 21*a*b^2*c^2*d*g^2*h^3*p*r - 6*
a^2*b*c*d^2*g^2*h^3*p*r + 21*a*b^2*c*d^2*g^3*h^2*q*r - 6*a*b^2*c^2*d*g^2*h^
3*q*r - 21*a^2*b*c*d^2*g^2*h^3*q*r + 6*a^2*b*c^2*d*g*h^4*p*r + 6*a^2*b*c^2*
d*g*h^4*q*r)/(6*(a^3*c^3*h^6 + b^3*d^3*g^6 - a^3*d^3*g^3*h^3 - b^3*c^3*g^3*
h^3 - 3*a^2*b*c^3*g*h^5 - 3*a*b^2*d^3*g^5*h - 3*a^3*c^2*d*g*h^5 - 3*b^3*c*d
^2*g^5*h + 3*a*b^2*c^3*g^2*h^4 + 3*a^2*b*d^3*g^4*h^2 + 3*a^3*c*d^2*g^2*h^4
+ 3*b^3*c^2*d*g^4*h^2 + 9*a*b^2*c*d^2*g^4*h^2 - 9*a*b^2*c^2*d*g^3*h^3 - 9*a
^2*b*c*d^2*g^3*h^3 + 9*a^2*b*c^2*d*g^2*h^4)) - (x^2*(b^3*c^3*h^5*p*r + a^3*
d^3*h^5*q*r - b^3*d^3*g^3*h^2*p*r - b^3*d^3*g^3*h^2*q*r - 3*a^2*b*d^3*g*h^4
*q*r - 3*b^3*c^2*d*g*h^4*p*r + 3*a*b^2*d^3*g^2*h^3*q*r + 3*b^3*c*d^2*g^2*h^
3*p*r))/(a^3*c^3*h^6 + b^3*d^3*g^6 - a^3*d^3*g^3*h^3 - b^3*c^3*g^3*h^3 - 3*
a^2*b*c^3*g*h^5 - 3*a*b^2*d^3*g^5*h - 3*a^3*c^2*d*g*h^5 - 3*b^3*c*d^2*g^5*h
```

+ 3*a*b^2*c^3*g^2*h^4 + 3*a^2*b*d^3*g^4*h^2 + 3*a^3*c*d^2*g^2*h^4 + 3*b^3*
c^2*d*g^4*h^2 + 9*a*b^2*c*d^2*g^4*h^2 - 9*a*b^2*c^2*d*g^3*h^3 - 9*a^2*b*c*d
^2*g^3*h^3 + 9*a^2*b*c^2*d*g^2*h^4) + (x*(a*b^2*c^3*h^5*p*r + a^3*c*d^2*h^5
*q*r - 5*b^3*c^3*g*h^4*p*r - 5*a^3*d^3*g*h^4*q*r + 5*b^3*d^3*g^4*h*p*r + 5*
b^3*d^3*g^4*h*q*r - a*b^2*d^3*g^3*h^2*p*r - 15*a*b^2*d^3*g^3*h^2*q*r + 15*a
^2*b*d^3*g^2*h^3*q*r - 15*b^3*c*d^2*g^3*h^2*p*r + 15*b^3*c^2*d*g^2*h^3*p*r
- b^3*c*d^2*g^3*h^2*q*r + 3*a*b^2*c*d^2*g^2*h^3*p*r + 3*a*b^2*c*d^2*g^2*h^3
*q*r - 3*a*b^2*c^2*d*g*h^4*p*r - 3*a^2*b*c*d^2*g*h^4*q*r))/(2*(a^3*c^3*h^6
+ b^3*d^3*g^6 - a^3*d^3*g^3*h^3 - b^3*c^3*g^3*h^3 - 3*a^2*b*c^3*g*h^5 - 3*a
*b^2*d^3*g^5*h - 3*a^3*c^2*d*g*h^5 - 3*b^3*c*d^2*g^5*h + 3*a*b^2*c^3*g^2*h^
4 + 3*a^2*b*d^3*g^4*h^2 + 3*a^3*c*d^2*g^2*h^4 + 3*b^3*c^2*d*g^4*h^2 + 9*a*b
^2*c*d^2*g^4*h^2 - 9*a*b^2*c^2*d*g^3*h^3 - 9*a^2*b*c*d^2*g^3*h^3 + 9*a^2*b*
c^2*d*g^2*h^4)))/(4*g^3*h + 4*h^4*x^3 + 12*g^2*h^2*x + 12*g*h^3*x^2) - (log
(g + h*x)*(h^4*(b^4*c^4*p*r + a^4*d^4*q*r) - h*(4*a*b^3*d^4*g^3*q*r + 4*b^4
*c*d^3*g^3*p*r) + h^2*(6*a^2*b^2*d^4*g^2*q*r + 6*b^4*c^2*d^2*g^2*p*r) - h^3
*(4*a^3*b*d^4*g*q*r + 4*b^4*c^3*d*g*p*r) + b^4*d^4*g^4*p*r + b^4*d^4*g^4*q*
r))/(4*a^4*c^4*h^9 + 4*b^4*d^4*g^8*h + 4*a^4*d^4*g^4*h^5 + 4*b^4*c^4*g^4*h^
5 + 24*a^2*b^2*c^4*g^2*h^7 + 24*a^2*b^2*d^4*g^6*h^3 + 24*a^4*c^2*d^2*g^2*h^
7 + 24*b^4*c^2*d^2*g^6*h^3 - 16*a^3*b*c^4*g*h^8 - 16*a^4*c^3*d*g*h^8 - 16*a
*b^3*c^4*g^3*h^6 - 16*a*b^3*d^4*g^7*h^2 - 16*a^3*b*d^4*g^5*h^4 - 16*a^4*c*d
^3*g^3*h^6 - 16*b^4*c*d^3*g^7*h^2 - 16*b^4*c^3*d*g^5*h^4 + 64*a*b^3*c*d^3*g
^6*h^3 + 64*a*b^3*c^3*d*g^4*h^5 + 64*a^3*b*c*d^3*g^4*h^5 + 64*a^3*b*c^3*d*g
^2*h^7 - 96*a*b^3*c^2*d^2*g^5*h^4 - 96*a^2*b^2*c*d^3*g^5*h^4 - 96*a^2*b^2*c
^3*d*g^3*h^6 - 96*a^3*b*c^2*d^2*g^3*h^6 + 144*a^2*b^2*c^2*d^2*g^4*h^5) - (l
og(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(x/4 + g/(4*h)))/(g + h*x)^5 + (b^4*p*r
*log(a + b*x))/(4*a^4*h^5 + 4*b^4*g^4*h - 16*a*b^3*g^3*h^2 + 24*a^2*b^2*g^2
*h^3 - 16*a^3*b*g*h^4) + (d^4*q*r*log(c + d*x))/(4*c^4*h^5 + 4*d^4*g^4*h -
16*c*d^3*g^3*h^2 + 24*c^2*d^2*g^2*h^3 - 16*c^3*d*g*h^4)

3.35 $\int (g+hx)^3 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$

Optimal result 319
Rubi [A] (verified) 321
Mathematica [A] (verified) 327
Maple [F] 328
Fricas [F] 328
Sympy [F] 329
Maxima [A] (verification not implemented) 329
Giac [F] 330
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Optimal result

Integrand size = 31, antiderivative size = 2240

$$\int (g+hx)^3\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx=\frac{2(bg-ah)^3p^2r^2x}{b^3}+\frac{5(bg-ah)^3pqr^2x}{8b^3}$$

$$+\frac{5(bg-ah)^2(dg-ch)pqr^2x}{12b^2d}+\frac{5(bg-ah)(dg-ch)^2pqr^2x}{12bd^2}+\frac{5(dg-ch)^3pqr^2x}{8d^3}$$

$$+\frac{2(dg-ch)^3q^2r^2x}{d^3}+\frac{3h(bg-ah)^2p^2r^2(a+bx)^2}{4b^4}+\frac{2h^2(bg-ah)p^2r^2(a+bx)^3}{9b^4}$$

$$+\frac{h^3p^2r^2(a+bx)^4}{32b^4}+\frac{3h(dg-ch)^2q^2r^2(c+dx)^2}{4d^4}+\frac{2h^2(dg-ch)q^2r^2(c+dx)^3}{9d^4}$$

$$+\frac{h^3q^2r^2(c+dx)^4}{32d^4}+\frac{3(bg-ah)^2pqr^2(g+hx)^2}{16b^2h}+\frac{(bg-ah)(dg-ch)pqr^2(g+hx)^2}{6bdh}$$

$$+\frac{3(dg-ch)^2pqr^2(g+hx)^2}{16d^2h}+\frac{7(bg-ah)pqr^2(g+hx)^3}{72bh}+\frac{7(dg-ch)pqr^2(g+hx)^3}{72dh}$$

$$+\frac{pqr^2(g+hx)^4}{16h}+\frac{(bg-ah)^4pqr^2\log(a+bx)}{8b^4h}+\frac{(bg-ah)^3(dg-ch)pqr^2\log(a+bx)}{6b^3dh}$$

$$+\frac{(bg-ah)^2(dg-ch)^2pqr^2\log(a+bx)}{4b^2d^2h}-\frac{2(bg-ah)^3p^2r^2(a+bx)\log(a+bx)}{b^4}$$

$$-\frac{(dg-ch)^3pqr^2(a+bx)\log(a+bx)}{2bd^3}-\frac{3h(bg-ah)^2p^2r^2(a+bx)^2\log(a+bx)}{2b^4}$$

$$-\frac{2h^2(bg-ah)p^2r^2(a+bx)^3\log(a+bx)}{3b^4}-\frac{h^3p^2r^2(a+bx)^4\log(a+bx)}{8b^4}$$

$$-\frac{(dg-ch)^2pqr^2(g+hx)^2\log(a+bx)}{4d^2h}-\frac{(dg-ch)pqr^2(g+hx)^3\log(a+bx)}{6dh}$$

$$-\frac{pqr^2(g+hx)^4\log(a+bx)}{8h}-\frac{(bg-ah)^4p^2r^2\log^2(a+bx)}{4b^4h}$$

$$+\frac{(bg-ah)^2(dg-ch)^2pqr^2\log(c+dx)}{4b^2d^2h}+\frac{(bg-ah)(dg-ch)^3pqr^2\log(c+dx)}{6bd^3h}$$

$$+\frac{(dg-ch)^4pqr^2\log(c+dx)}{8d^4h}-\frac{(bg-ah)^3pqr^2(c+dx)\log(c+dx)}{2b^3d}$$

$$-\frac{2(dg-ch)^3q^2r^2(c+dx)\log(c+dx)}{d^4}-\frac{3h(dg-ch)^2q^2r^2(c+dx)^2\log(c+dx)}{2d^4}$$

$$-\frac{2h^2(dg-ch)q^2r^2(c+dx)^3\log(c+dx)}{3d^4}-\frac{h^3q^2r^2(c+dx)^4\log(c+dx)}{8d^4}$$

$$-\frac{(bg-ah)^2pqr^2(g+hx)^2\log(c+dx)}{4b^2h}-\frac{(bg-ah)pqr^2(g+hx)^3\log(c+dx)}{6bh}$$

$$-\frac{pqr^2(g+hx)^4\log(c+dx)}{8h}-\frac{(bg-ah)^4pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{2b^4h}$$

$$-\frac{(dg-ch)^4q^2r^2\log^2(c+dx)}{4d^4h}-\frac{(dg-ch)^4pqr^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{2d^4h}$$

$$+\frac{(bg-ah)^3prx(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right))}{2b^3}$$

$$+\frac{(dg-ch)^3qrx(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right))}{2d^3}$$

$$+\frac{(bg-ah)^2pr(g+hx)^2\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{4b^2h}$$

$$+\frac{(dg-ch)^2qr(g+hx)^2\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{4d^2h}$$

```
[Out] -1/2*(-c*h+d*g)^3*p*q*r^2*(b*x+a)*ln(b*x+a)/b/d^3-1/4*(-c*h+d*g)^2*p*q*r^2*
(h*x+g)^2*ln(b*x+a)/d^2/h+1/6*(-a*h+b*g)^3*(-c*h+d*g)*p*q*r^2*ln(b*x+a)/b^3
/d/h+1/4*(-a*h+b*g)^2*(-c*h+d*g)^2*p*q*r^2*ln(b*x+a)/b^2/d^2/h+1/4*(-a*h+b*
g)^2*(-c*h+d*g)^2*p*q*r^2*ln(d*x+c)/b^2/d^2/h+1/6*(-a*h+b*g)*(-c*h+d*g)^3*p
*q*r^2*ln(d*x+c)/b/d^3/h+1/6*(-a*h+b*g)*(-c*h+d*g)*p*q*r^2*(h*x+g)^2/b/d/h+
1/4*(h*x+g)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/h+3/16*(-c*h+d*g)^2*p*q*r^2
*(h*x+g)^2/d^2/h+7/72*(-a*h+b*g)*p*q*r^2*(h*x+g)^3/b/h+7/72*(-c*h+d*g)*p*q*
r^2*(h*x+g)^3/d/h-1/2*(-c*h+d*g)^4*p*q*r^2*polylog(2,-d*(b*x+a)/(-a*d+b*c))
/d^4/h-1/2*(-a*h+b*g)^4*p*q*r^2*polylog(2,b*(d*x+c)/(-a*d+b*c))/b^4/h-2*(-a
*h+b*g)^3*p^2*r^2*(b*x+a)*ln(b*x+a)/b^4-1/8*h^3*p^2*r^2*(b*x+a)^4*ln(b*x+a)
/b^4-1/8*p*q*r^2*(h*x+g)^4*ln(b*x+a)/h-1/4*(-a*h+b*g)^4*p^2*r^2*ln(b*x+a)^2
/b^4/h-2*(-c*h+d*g)^3*q^2*r^2*(d*x+c)*ln(d*x+c)/d^4-1/8*h^3*q^2*r^2*(d*x+c)
^4*ln(d*x+c)/d^4-1/6*(-c*h+d*g)*p*q*r^2*(h*x+g)^3*ln(b*x+a)/d/h-1/2*(-a*h+b
*g)^3*p*q*r^2*(d*x+c)*ln(d*x+c)/b^3/d-1/4*(-a*h+b*g)^2*p*q*r^2*(h*x+g)^2*ln
(d*x+c)/b^2/h-1/6*(-a*h+b*g)*p*q*r^2*(h*x+g)^3*ln(d*x+c)/b/h-1/2*(-a*h+b*g)
^4*p*q*r^2*ln(-d*(b*x+a)/(-a*d+b*c))*ln(d*x+c)/b^4/h-1/2*(-c*h+d*g)^4*p*q*r
^2*ln(b*x+a)*ln(b*(d*x+c)/(-a*d+b*c))/d^4/h+5/12*(-a*h+b*g)^2*(-c*h+d*g)*p*
q*r^2*x/b^2/d+5/12*(-a*h+b*g)*(-c*h+d*g)^2*p*q*r^2*x/b/d^2-1/8*p*q*r^2*(h*x
+g)^4*ln(d*x+c)/h-1/4*(-c*h+d*g)^4*q^2*r^2*ln(d*x+c)^2/d^4/h+1/2*(-a*h+b*g)
^3*p*r*x*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/b^3+
1/2*(-c*h+d*g)^3*q*r*x*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+
c)^q)^r))/d^3+3/4*h*(-c*h+d*g)^2*q^2*r^2*(d*x+c)^2/d^4+2/9*h^2*(-c*h+d*g)*q
^2*r^2*(d*x+c)^3/d^4+5/8*(-a*h+b*g)^3*p*q*r^2*x/b^3+5/8*(-c*h+d*g)^3*p*q*r^
2*x/d^3+3/4*h*(-a*h+b*g)^2*p^2*r^2*(b*x+a)^2/b^4+2/9*h^2*(-a*h+b*g)*p^2*r^2
*(b*x+a)^3/b^4+1/8*p*r*(h*x+g)^4*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+
a)^p*(d*x+c)^q)^r))/h+1/8*q*r*(h*x+g)^4*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(
f*(b*x+a)^p*(d*x+c)^q)^r))/h+2*(-a*h+b*g)^3*p^2*r^2*x/b^3+2*(-c*h+d*g)^3*q^
2*r^2*x/d^3+1/32*h^3*p^2*r^2*(b*x+a)^4/b^4+1/32*h^3*q^2*r^2*(d*x+c)^4/d^4+1
/16*p*q*r^2*(h*x+g)^4/h+1/8*(-a*h+b*g)^4*p*q*r^2*ln(b*x+a)/b^4/h-3/2*h*(-a*
h+b*g)^2*p^2*r^2*(b*x+a)^2*ln(b*x+a)/b^4-2/3*h^2*(-a*h+b*g)*p^2*r^2*(b*x+a)
^3*ln(b*x+a)/b^4+1/8*(-c*h+d*g)^4*p*q*r^2*ln(d*x+c)/d^4/h-3/2*h*(-c*h+d*g)^
2*q^2*r^2*(d*x+c)^2*ln(d*x+c)/d^4-2/3*h^2*(-c*h+d*g)*q^2*r^2*(d*x+c)^3*ln(d
*x+c)/d^4+1/4*(-a*h+b*g)^2*p*r*(h*x+g)^2*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*
(f*(b*x+a)^p*(d*x+c)^q)^r))/b^2/h+1/4*(-c*h+d*g)^2*q*r*(h*x+g)^2*(p*r*ln(b*
x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/d^2/h+1/6*(-a*h+b*g)*p*
r*(h*x+g)^3*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/b
/h+1/6*(-c*h+d*g)*q*r*(h*x+g)^3*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a
)^p*(d*x+c)^q)^r))/d/h+1/2*(-a*h+b*g)^4*p*r*ln(b*x+a)*(p*r*ln(b*x+a)+q*r*ln
(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/b^4/h+1/2*(-c*h+d*g)^4*q*r*ln(d*x+
c)*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/d^4/h+3/16
*(-a*h+b*g)^2*p*q*r^2*(h*x+g)^2/b^2/h
```

Rubi [A] (verified)

Time = 1.63 (sec) , antiderivative size = 2240, normalized size of antiderivative = 1.00, number of steps used = 49, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.452$, Rules

used = {2584, 2593, 2458, 45, 2372, 12, 2338, 2465, 2436, 2332, 2441, 2440, 2438, 2442}

$$\int (g+hx)^3 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right) dx = -\frac{p^2r^2\log^2(a+bx)(bg-ah)^4}{4b^4h}$$

$$+\frac{pqr^2\log(a+bx)(bg-ah)^4}{8b^4h} - \frac{pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)(bg-ah)^4}{2b^4h}$$

$$+\frac{pr\log(a+bx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)(bg-ah)^4}{2b^4h}$$

$$-\frac{pqr^2\,\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)(bg-ah)^4}{2b^4h} + \frac{2p^2r^2x(bg-ah)^3}{b^3}$$

$$+\frac{5pqr^2x(bg-ah)^3}{8b^3} + \frac{(dg-ch)pqr^2\log(a+bx)(bg-ah)^3}{6b^3dh}$$

$$-\frac{2p^2r^2(a+bx)\log(a+bx)(bg-ah)^3}{b^4} - \frac{pqr^2(c+dx)\log(c+dx)(bg-ah)^3}{2b^3d}$$

$$+\frac{prx\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)(bg-ah)^3}{2b^3}$$

$$+\frac{3hp^2r^2(a+bx)^2(bg-ah)^2}{4b^4} + \frac{3pqr^2(g+hx)^2(bg-ah)^2}{16b^2h} + \frac{5(dg-ch)pqr^2x(bg-ah)^2}{12b^2d}$$

$$+\frac{(dg-ch)^2pqr^2\log(a+bx)(bg-ah)^2}{4b^2d^2h} - \frac{3hp^2r^2(a+bx)^2\log(a+bx)(bg-ah)^2}{2b^4}$$

$$+\frac{(dg-ch)^2pqr^2\log(c+dx)(bg-ah)^2}{4b^2d^2h} - \frac{pqr^2(g+hx)^2\log(c+dx)(bg-ah)^2}{4b^2h}$$

$$+\frac{pr(g+hx)^2\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)(bg-ah)^2}{4b^2h}$$

$$+\frac{2h^2p^2r^2(a+bx)^3(bg-ah)}{9b^4} + \frac{7pqr^2(g+hx)^3(bg-ah)}{72bh}$$

$$+\frac{(dg-ch)pqr^2(g+hx)^2(bg-ah)}{6bdh} + \frac{5(dg-ch)^2pqr^2x(bg-ah)}{12bd^2}$$

$$-\frac{2h^2p^2r^2(a+bx)^3\log(a+bx)(bg-ah)}{3b^4}$$

$$-\frac{pqr^2(g+hx)^3\log(c+dx)(bg-ah)}{6bh} + \frac{(dg-ch)^3pqr^2\log(c+dx)(bg-ah)}{6bd^3h}$$

$$+\frac{pr(g+hx)^3\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)(bg-ah)}{6bh}$$

$$+\frac{h^3p^2r^2(a+bx)^4}{32b^4} + \frac{h^3q^2r^2(c+dx)^4}{32d^4} + \frac{pqr^2(g+hx)^4}{16h} + \frac{2h^2(dg-ch)q^2r^2(c+dx)^3}{9d^4}$$

$$+\frac{7(dg-ch)pqr^2(g+hx)^3}{72dh} + \frac{3h(dg-ch)^2q^2r^2(c+dx)^2}{4d^4}$$

$$+\frac{3(dg-ch)^2pqr^2(g+hx)^2}{16d^2h} - \frac{(dg-ch)^4q^2r^2\log^2(c+dx)}{4d^4h}$$

$$+\frac{(g+hx)^4\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4h} + \frac{2(dg-ch)^3q^2r^2x}{d^3}$$

$$+\frac{5(dg-ch)^3pqr^2x}{8d^3} - \frac{h^3p^2r^2(a+bx)^4\log(a+bx)}{8b^4} - \frac{pqr^2(g+hx)^4\log(a+bx)}{8h}$$

$$-\frac{(dg-ch)pqr^2(g+hx)^3\log(a+bx)}{6dh} - \frac{(dg-ch)^2pqr^2(g+hx)^2\log(a+bx)}{4d^2h}$$

$$-\frac{(dg-ch)^3pqr^2(a+bx)\log(a+bx)}{2bd^3} - \frac{h^3q^2r^2(c+dx)^4\log(c+dx)}{8d^4}$$

$$pqr^2(g+hx)^4\log(c+dx) \quad 2h^2(dg-ch)q^2r^2(c+dx)^3\log(c+dx)$$

```
[In] Int[(g + h*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]

[Out] (2*(b*g - a*h)^3*p^2*r^2*x)/b^3 + (5*(b*g - a*h)^3*p*q*r^2*x)/(8*b^3) + (5*
(b*g - a*h)^2*(d*g - c*h)*p*q*r^2*x)/(12*b^2*d) + (5*(b*g - a*h)*(d*g - c*h
)^2*p*q*r^2*x)/(12*b*d^2) + (5*(d*g - c*h)^3*p*q*r^2*x)/(8*d^3) + (2*(d*g -
 c*h)^3*q^2*r^2*x)/d^3 + (3*h*(b*g - a*h)^2*p^2*r^2*(a + b*x)^2)/(4*b^4) +
(2*h^2*(b*g - a*h)*p^2*r^2*(a + b*x)^3)/(9*b^4) + (h^3*p^2*r^2*(a + b*x)^4)
/(32*b^4) + (3*h*(d*g - c*h)^2*q^2*r^2*(c + d*x)^2)/(4*d^4) + (2*h^2*(d*g -
 c*h)*q^2*r^2*(c + d*x)^3)/(9*d^4) + (h^3*q^2*r^2*(c + d*x)^4)/(32*d^4) + (
3*(b*g - a*h)^2*p*q*r^2*(g + h*x)^2)/(16*b^2*h) + ((b*g - a*h)*(d*g - c*h)*
p*q*r^2*(g + h*x)^2)/(6*b*d*h) + (3*(d*g - c*h)^2*p*q*r^2*(g + h*x)^2)/(16*
d^2*h) + (7*(b*g - a*h)*p*q*r^2*(g + h*x)^3)/(72*b*h) + (7*(d*g - c*h)*p*q*
r^2*(g + h*x)^3)/(72*d*h) + (p*q*r^2*(g + h*x)^4)/(16*h) + ((b*g - a*h)^4*p
*q*r^2*Log[a + b*x])/(8*b^4*h) + ((b*g - a*h)^3*(d*g - c*h)*p*q*r^2*Log[a +
 b*x])/(6*b^3*d*h) + ((b*g - a*h)^2*(d*g - c*h)^2*p*q*r^2*Log[a + b*x])/(4*
b^2*d^2*h) - (2*(b*g - a*h)^3*p^2*r^2*(a + b*x)*Log[a + b*x])/b^4 - ((d*g -
 c*h)^3*p*q*r^2*(a + b*x)*Log[a + b*x])/(2*b*d^3) - (3*h*(b*g - a*h)^2*p^2*
r^2*(a + b*x)^2*Log[a + b*x])/(2*b^4) - (2*h^2*(b*g - a*h)*p^2*r^2*(a + b*x
)^3*Log[a + b*x])/(3*b^4) - (h^3*p^2*r^2*(a + b*x)^4*Log[a + b*x])/(8*b^4)
- ((d*g - c*h)^2*p*q*r^2*(g + h*x)^2*Log[a + b*x])/(4*d^2*h) - ((d*g - c*h)
*p*q*r^2*(g + h*x)^3*Log[a + b*x])/(6*d*h) - (p*q*r^2*(g + h*x)^4*Log[a + b
*x])/(8*h) - ((b*g - a*h)^4*p^2*r^2*Log[a + b*x]^2)/(4*b^4*h) + ((b*g - a*h
)^2*(d*g - c*h)^2*p*q*r^2*Log[c + d*x])/(4*b^2*d^2*h) + ((b*g - a*h)*(d*g -
 c*h)^3*p*q*r^2*Log[c + d*x])/(6*b*d^3*h) + ((d*g - c*h)^4*p*q*r^2*Log[c +
d*x])/(8*d^4*h) - ((b*g - a*h)^3*p*q*r^2*(c + d*x)*Log[c + d*x])/(2*b^3*d)
- (2*(d*g - c*h)^3*q^2*r^2*(c + d*x)*Log[c + d*x])/d^4 - (3*h*(d*g - c*h)^2
*q^2*r^2*(c + d*x)^2*Log[c + d*x])/(2*d^4) - (2*h^2*(d*g - c*h)*q^2*r^2*(c
+ d*x)^3*Log[c + d*x])/(3*d^4) - (h^3*q^2*r^2*(c + d*x)^4*Log[c + d*x])/(8*
d^4) - ((b*g - a*h)^2*p*q*r^2*(g + h*x)^2*Log[c + d*x])/(4*b^2*h) - ((b*g -
 a*h)*p*q*r^2*(g + h*x)^3*Log[c + d*x])/(6*b*h) - (p*q*r^2*(g + h*x)^4*Log[
c + d*x])/(8*h) - ((b*g - a*h)^4*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*
Log[c + d*x])/(2*b^4*h) - ((d*g - c*h)^4*q^2*r^2*Log[c + d*x]^2)/(4*d^4*h)
- ((d*g - c*h)^4*p*q*r^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(2*d^
4*h) + ((b*g - a*h)^3*p*r*x*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]))/(2*b^3) + ((d*g - c*h)^3*q*r*x*(p*r*Log[a +
b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(2*d^3) +
((b*g - a*h)^2*p*r*(g + h*x)^2*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e
*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(4*b^2*h) + ((d*g - c*h)^2*q*r*(g + h*x)^
2*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^
r]))/(4*d^2*h) + ((b*g - a*h)*p*r*(g + h*x)^3*(p*r*Log[a + b*x] + q*r*Log[c
 + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(6*b*h) + ((d*g - c*h)*q*r
*(g + h*x)^3*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c
 + d*x)^q)^r]))/(6*d*h) + (p*r*(g + h*x)^4*(p*r*Log[a + b*x] + q*r*Log[c +
d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(8*h) + (q*r*(g + h*x)^4*(p*r
*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(
```

```
8*h) + ((b*g - a*h)^4*p*r*Log[a + b*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x]
 - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(2*b^4*h) + ((d*g - c*h)^4*q*r*Lo
g[c + d*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r]))/(2*d^4*h) + ((g + h*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^2)/(4*h) - ((d*g - c*h)^4*p*q*r^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))]
)/(2*d^4*h) - ((b*g - a*h)^4*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])
/(2*b^4*h)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2584

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1
)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(h*(m + 1))), x] + (-Dist[b*p*r*(
s/(h*(m + 1))), Int[(g + h*x)^(m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1)/(a + b*x)), x], x] - Dist[d*q*r*(s/(h*(m + 1))), Int[(g + h*x)^(m
+ 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(c + d*x)), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
```

```
tQ[s, 0] && NeQ[m, -1]
```

Rule 2593

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(g+hx)^4\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4h} - \frac{(bpr)\int \frac{(g+hx)^4\log(e(f(a+bx)^p(c+dx)^q)^r)}{a+bx}\,dx}{2h} - \frac{(dqr)\int \frac{(g+hx)^4\log(e(f(a+bx)^p(c+dx)^q)^r)}{c+dx}\,dx}{2h}\\
&= \frac{(g+hx)^4\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4h} - \frac{(bp^2r^2)\int \frac{(g+hx)^4\log(a+bx)}{a+bx}\,dx}{2h} - \frac{(bpqr^2)\int \frac{(g+hx)^4\log(c+dx)}{a+bx}\,dx}{2h}\\
&\quad - \frac{(dpqr^2)\int \frac{(g+hx)^4\log(a+bx)}{c+dx}\,dx}{2h} - \frac{(dq^2r^2)\int \frac{(g+hx)^4\log(c+dx)}{c+dx}\,dx}{2h}\\
&\quad + \frac{(bpr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)))\int \frac{(g+hx)^4}{a+bx}\,dx}{2h}\\
&\quad + \frac{(dqr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)))\int \frac{(g+hx)^4}{c+dx}\,dx}{2h}
\end{aligned}$$

$$= \frac{(g+hx)^4 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4h} - \frac{(p^2r^2)\,\text{Subst}\left(\int \frac{\left(\frac{bg-ah}{b}+\frac{hx}{b}\right)^4 \log(x)}{x}\,dx, x, a+bx\right)}{2h} - \frac{(bpqr^2)\int \left(\frac{h(bg-ah)^3\log(c+dx)}{b^4} + \frac{(bg-ah)^4\log(c+dx)}{b^4(a+bx)} + \frac{h(bg-ah)^2(g+hx)\log(c+dx)}{b^3} + \frac{h(bg-ah)(g+hx)^2\log(c+dx)}{b^2}\right.}{2h} - \frac{(dpqr^2)\int \left(\frac{h(dg-ch)^3\log(a+bx)}{d^4} + \frac{(dg-ch)^4\log(a+bx)}{d^4(c+dx)} + \frac{h(dg-ch)^2(g+hx)\log(a+bx)}{d^3} + \frac{h(dg-ch)(g+hx)^2\log(a+bx)}{d^2}\right.}{2h} - \frac{(q^2r^2)\,\text{Subst}\left(\int \frac{\left(\frac{dg-ch}{d}+\frac{hx}{d}\right)^4 \log(x)}{x}\,dx, x, c+dx\right)}{2h} + \frac{(bpr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)))\int \left(\frac{h(bg-ah)^3}{b^4} + \frac{(bg-ah)^4}{b^4(a+bx)} + \right.}{2h} + \frac{(dqr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)))\int \left(\frac{h(dg-ch)^3}{d^4} + \frac{(dg-ch)^4}{d^4(c+dx)} + \right.}{2h}$$

$$= \text{Too large to display}$$

Mathematica [A] (verified)

Time = 1.63 (sec) , antiderivative size = 1386, normalized size of antiderivative = 0.62

$$\int (g+hx)^3 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$$

$$= \frac{72ad^4(-4b^3g^3+6ab^2g^2h-4a^2bgh^2+a^3h^3)\,p^2r^2\log^2(a+bx) + 12pr\log(a+bx)\left(12b^4c(-4d^3g^3+6cd^2g^2\right.}{}$$

```
[In] Integrate[(g + h*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] (72*a*d^4*(-4*b^3*g^3 + 6*a*b^2*g^2*h - 4*a^2*b*g*h^2 + a^3*h^3)*p^2*r^2*Lo
g[a + b*x]^2 + 12*p*r*Log[a + b*x]*(12*b^4*c*(-4*d^3*g^3 + 6*c*d^2*g^2*h -
4*c^2*d*g*h^2 + c^3*h^3)*q*r*Log[c + d*x] - 12*(4*a*b^3*d^4*g^3 - 6*a^2*b^2
*d^4*g^2*h + 4*a^3*b*d^4*g*h^2 - a^4*d^4*h^3 + b^4*c*(-4*d^3*g^3 + 6*c*d^2*
g^2*h - 4*c^2*d*g*h^2 + c^3*h^3))*q*r*Log[(b*(c + d*x))/(b*c - a*d)] + a*d*
((12*b^3*(-4*d^3*g^3 + 6*c*d^2*g^2*h - 4*c^2*d*g*h^2 + c^3*h^3)*q + a^3*d^3
*h^3*(25*p + 3*q) - 4*a^2*b*d^2*h^2*(22*d*g*p + 4*d*g*q - c*h*q) + 6*a*b^2*
d*h*(-4*c*d*g*h*q + c^2*h^2*q + 6*d^2*g^2*(3*p + q)))*r + 12*d^3*(4*b^3*g^3
 - 6*a*b^2*g^2*h + 4*a^2*b*g*h^2 - a^3*h^3)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r])) + b*(72*b^3*c*(-4*d^3*g^3 + 6*c*d^2*g^2*h - 4*c^2*d*g*h^2 + c^3*h^3
)*q^2*r^2*Log[c + d*x]^2 + 12*q*r*Log[c + d*x]*((12*a^3*c*d^3*h^3*p + 6*a^2
*b*c*d^2*h^2*(-8*d*g + c*h)*p + 4*a*b^2*d*(12*d^3*g^3 + 18*c*d^2*g^2*h - 6*
```

```
c^2*d*g*h^2 + c^3*h^3)*p + b^3*c*(-48*d^3*g^3*(p + q) + 36*c*d^2*g^2*h*(p +
 3*q) - 8*c^2*d*g*h^2*(2*p + 11*q) + c^3*h^3*(3*p + 25*q)))*r - 12*b^3*c*(-
4*d^3*g^3 + 6*c*d^2*g^2*h - 4*c^2*d*g*h^2 + c^3*h^3)*Log[e*(f*(a + b*x)^p*(
c + d*x)^q)^r]) + d*(r^2*(-60*a^3*d^3*h^3*p*(5*p + 3*q)*x + 6*a^2*b*d^2*h^2
*p*x*(-20*c*h*q + 16*d*g*(11*p + 8*q) + d*h*(13*p + 9*q)*x) + b^3*x*(-60*c^
3*h^3*q*(3*p + 5*q) + 6*c^2*d*h^2*q*(16*g*(8*p + 11*q) + h*(9*p + 13*q)*x)
- 4*c*d^2*h*q*(p + q)*(324*g^2 + 60*g*h*x + 7*h^2*x^2) + d^3*(p + q)^2*(576
*g^3 + 216*g^2*h*x + 64*g*h^2*x^2 + 9*h^3*x^3)) - 4*a*b^2*p*(36*c^3*h^3*q +
 6*c^2*d*h^2*q*(-24*g + 5*h*x) - 12*c*d^2*h*q*(-18*g^2 + 12*g*h*x + h^2*x^2
) + d^3*(-144*g^3*q + 324*g^2*h*(p + q)*x + 60*g*h^2*(p + q)*x^2 + 7*h^3*(p
 + q)*x^3))) + 12*r*(12*a^3*d^3*h^3*p*x - 6*a^2*b*d^3*h^2*p*x*(8*g + h*x) +
 4*a*b^2*d^3*p*(-12*g^3 + 18*g^2*h*x + 6*g*h^2*x^2 + h^3*x^3) - b^3*x*(-12*
c^3*h^3*q + 6*c^2*d*h^2*q*(8*g + h*x) - 4*c*d^2*h*q*(18*g^2 + 6*g*h*x + h^2
*x^2) + d^3*(p + q)*(48*g^3 + 36*g^2*h*x + 16*g*h^2*x^2 + 3*h^3*x^3)))*Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 72*b^3*d^3*x*(4*g^3 + 6*g^2*h*x + 4*g*h^
2*x^2 + h^3*x^3)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)) - 144*(4*a*b^3*d^
4*g^3 - 6*a^2*b^2*d^4*g^2*h + 4*a^3*b*d^4*g*h^2 - a^4*d^4*h^3 + b^4*c*(-4*d
^3*g^3 + 6*c*d^2*g^2*h - 4*c^2*d*g*h^2 + c^3*h^3))*p*q*r^2*PolyLog[2, (d*(a
 + b*x))/(-(b*c) + a*d)])/(288*b^4*d^4)
```

Maple [F]

$$\int (hx+g)^3 \ln \left(e(f(bx+a)^p (dx+c)^q)^r\right)^2 dx$$

```
[In] int((h*x+g)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

```
[Out] int((h*x+g)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

Fricas [F]

$$\int (g+hx)^3 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right) dx = \int (hx+g)^3 \log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2 dx$$

```
[In] integrate((h*x+g)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="fricas
")
```

```
[Out] integral((h^3*x^3 + 3*g*h^2*x^2 + 3*g^2*h*x + g^3)*log(((b*x + a)^p*(d*x +
c)^q*f)^r*e)^2, x)
```

Sympy [F]

$$\int (g+hx)^3 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right) dx = \int (g+hx)^3 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)^2 dx$$

```
[In] integrate((h*x+g)**3*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2,x)
[Out] Integral((g + h*x)**3*log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 1799, normalized size of antiderivative = 0.80

$$\int (g+hx)^3 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right) dx = \text{Too large to display}$$

```
[In] integrate((h*x+g)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="maxima
")
[Out] 1/4*(h^3*x^4 + 4*g*h^2*x^3 + 6*g^2*h*x^2 + 4*g^3*x)*log(((b*x + a)^p*(d*x +
 c)^q*f)^r*e)^2 + 1/24*r*(12*(4*a*b^3*f*g^3*p - 6*a^2*b^2*f*g^2*h*p + 4*a^3
*b*f*g*h^2*p - a^4*f*h^3*p)*log(b*x + a)/b^4 + 12*(4*c*d^3*f*g^3*q - 6*c^2*
d^2*f*g^2*h*q + 4*c^3*d*f*g*h^2*q - c^4*f*h^3*q)*log(d*x + c)/d^4 - (3*b^3*
d^3*f*h^3*(p + q)*x^4 - 4*(a*b^2*d^3*f*h^3*p - (4*d^3*f*g*h^2*(p + q) - c*d
^2*f*h^3*q)*b^3)*x^3 - 6*(4*a*b^2*d^3*f*g*h^2*p - a^2*b*d^3*f*h^3*p - (6*d^
3*f*g^2*h*(p + q) - 4*c*d^2*f*g*h^2*q + c^2*d*f*h^3*q)*b^3)*x^2 - 12*(6*a*b
^2*d^3*f*g^2*h*p - 4*a^2*b*d^3*f*g*h^2*p + a^3*d^3*f*h^3*p - (4*d^3*f*g^3*(
p + q) - 6*c*d^2*f*g^2*h*q + 4*c^2*d*f*g*h^2*q - c^3*f*h^3*q)*b^3)*x)/(b^3*
d^3))*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/f + 1/288*r^2*(12*(12*a^3*c*d^3*
f^2*h^3*p*q - 6*(8*c*d^3*f^2*g*h^2*p*q - c^2*d^2*f^2*h^3*p*q)*a^2*b + 4*(18
*c*d^3*f^2*g^2*h*p*q - 6*c^2*d^2*f^2*g*h^2*p*q + c^3*d*f^2*h^3*p*q)*a*b^2 -
 (48*(p*q + q^2)*c*d^3*f^2*g^3 - 36*(p*q + 3*q^2)*c^2*d^2*f^2*g^2*h + 8*(2*
p*q + 11*q^2)*c^3*d*f^2*g*h^2 - (3*p*q + 25*q^2)*c^4*f^2*h^3)*b^3)*log(d*x
+ c)/(b^3*d^4) - 144*(4*a*b^3*d^4*f^2*g^3*p*q - 6*a^2*b^2*d^4*f^2*g^2*h*p*q
 + 4*a^3*b*d^4*f^2*g*h^2*p*q - a^4*d^4*f^2*h^3*p*q - (4*c*d^3*f^2*g^3*p*q -
 6*c^2*d^2*f^2*g^2*h*p*q + 4*c^3*d*f^2*g*h^2*p*q - c^4*f^2*h^3*p*q)*b^4)*(l
og(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c
- a*d)))/(b^4*d^4) + (9*(p^2 + 2*p*q + q^2)*b^4*d^4*f^2*h^3*x^4 - 144*(4*c*
d^3*f^2*g^3*p*q - 6*c^2*d^2*f^2*g^2*h*p*q + 4*c^3*d*f^2*g*h^2*p*q - c^4*f^2
*h^3*p*q)*b^4*log(b*x + a)*log(d*x + c) - 72*(4*c*d^3*f^2*g^3*q^2 - 6*c^2*d
^2*f^2*g^2*h*q^2 + 4*c^3*d*f^2*g*h^2*q^2 - c^4*f^2*h^3*q^2)*b^4*log(d*x + c
)^2 - 4*(7*(p^2 + p*q)*a*b^3*d^4*f^2*h^3 - (16*(p^2 + 2*p*q + q^2)*d^4*f^2*
g*h^2 - 7*(p*q + q^2)*c*d^3*f^2*h^3)*b^4)*x^3 + 6*((13*p^2 + 9*p*q)*a^2*b^2
```

```
*d^4*f^2*h^3 + 8*(c*d^3*f^2*h^3*p*q - 5*(p^2 + p*q)*d^4*f^2*g*h^2)*a*b^3 +
(36*(p^2 + 2*p*q + q^2)*d^4*f^2*g^2*h - 40*(p*q + q^2)*c*d^3*f^2*g*h^2 + (9
*p*q + 13*q^2)*c^2*d^2*f^2*h^3)*b^4)*x^2 - 72*(4*a*b^3*d^4*f^2*g^3*p^2 - 6*
a^2*b^2*d^4*f^2*g^2*h*p^2 + 4*a^3*b*d^4*f^2*g*h^2*p^2 - a^4*d^4*f^2*h^3*p^2
)*log(b*x + a)^2 - 12*(5*(5*p^2 + 3*p*q)*a^3*b*d^4*f^2*h^3 + 2*(5*c*d^3*f^2
*h^3*p*q - 4*(11*p^2 + 8*p*q)*d^4*f^2*g*h^2)*a^2*b^2 - 2*(24*c*d^3*f^2*g*h^
2*p*q - 5*c^2*d^2*f^2*h^3*p*q - 54*(p^2 + p*q)*d^4*f^2*g^2*h)*a*b^3 - (48*(
p^2 + 2*p*q + q^2)*d^4*f^2*g^3 - 108*(p*q + q^2)*c*d^3*f^2*g^2*h + 8*(8*p*q
 + 11*q^2)*c^2*d^2*f^2*g*h^2 - 5*(3*p*q + 5*q^2)*c^3*d*f^2*h^3)*b^4)*x + 12
*((25*p^2 + 3*p*q)*a^4*d^4*f^2*h^3 + 4*(c*d^3*f^2*h^3*p*q - 2*(11*p^2 + 2*p
*q)*d^4*f^2*g*h^2)*a^3*b - 6*(4*c*d^3*f^2*g*h^2*p*q - c^2*d^2*f^2*h^3*p*q -
 6*(3*p^2 + p*q)*d^4*f^2*g^2*h)*a^2*b^2 + 12*(6*c*d^3*f^2*g^2*h*p*q - 4*c^2
*d^2*f^2*g*h^2*p*q + c^3*d*f^2*h^3*p*q - 4*(p^2 + p*q)*d^4*f^2*g^3)*a*b^3)*
log(b*x + a))/(b^4*d^4))/f^2
```

Giac [F]

$$\int (g+hx)^3 \log^2 \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx = \int (hx+g)^3 \log \left(\left(((bx+a)^p(dx+c)^q f\right)^r e\right)^2 \, dx$$

```
[In] integrate((h*x+g)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (g+hx)^3 \log^2 \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx = \int \ln \left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)^2 (g+h\,x)^3 \, dx$$

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(g + h*x)^3,x)
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(g + h*x)^3, x)
```

3.36 $\int (g+hx)^2 \log^2 \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx$

Optimal result 332
Rubi [A] (verified) 333
Mathematica [A] (verified) 340
Maple [F] 340
Fricas [F] 341
Sympy [F] 341
Maxima [A] (verification not implemented) 341
Giac [F(-1)] 342
Mupad [F(-1)] 342

Optimal result

Integrand size = 31, antiderivative size = 1645

$$\int (g+hx)^2 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right) dx = \frac{2(bg-ah)^2p^2r^2x}{b^2} + \frac{8(bg-ah)^2pqr^2x}{9b^2}$$
$$+ \frac{2(bg-ah)(dg-ch)pqr^2x}{3bd} + \frac{8(dg-ch)^2pqr^2x}{9d^2} + \frac{2(dg-ch)^2q^2r^2x}{d^2}$$
$$+ \frac{h(bg-ah)p^2r^2(a+bx)^2}{2b^3} + \frac{2h^2p^2r^2(a+bx)^3}{27b^3} + \frac{h(dg-ch)q^2r^2(c+dx)^2}{2d^3}$$
$$+ \frac{2h^2q^2r^2(c+dx)^3}{27d^3} + \frac{5(bg-ah)pqr^2(g+hx)^2}{18bh} + \frac{5(dg-ch)pqr^2(g+hx)^2}{18dh}$$
$$+ \frac{4pqr^2(g+hx)^3}{27h} + \frac{2(bg-ah)^3pqr^2\log(a+bx)}{9b^3h} + \frac{(bg-ah)^2(dg-ch)pqr^2\log(a+bx)}{3b^2dh}$$
$$- \frac{2(bg-ah)^2p^2r^2(a+bx)\log(a+bx)}{b^3} - \frac{2(dg-ch)^2pqr^2(a+bx)\log(a+bx)}{3bd^2}$$
$$- \frac{h(bg-ah)p^2r^2(a+bx)^2\log(a+bx)}{b^3} - \frac{2h^2p^2r^2(a+bx)^3\log(a+bx)}{9b^3}$$
$$- \frac{(dg-ch)pqr^2(g+hx)^2\log(a+bx)}{3dh} - \frac{2pqr^2(g+hx)^3\log(a+bx)}{9h}$$
$$- \frac{(bg-ah)^3p^2r^2\log^2(a+bx)}{3b^3h} + \frac{(bg-ah)(dg-ch)^2pqr^2\log(c+dx)}{3bd^2h}$$
$$+ \frac{2(dg-ch)^3pqr^2\log(c+dx)}{9d^3h} - \frac{2(bg-ah)^2pqr^2(c+dx)\log(c+dx)}{3b^2d}$$
$$- \frac{2(dg-ch)^2q^2r^2(c+dx)\log(c+dx)}{d^3} - \frac{h(dg-ch)q^2r^2(c+dx)^2\log(c+dx)}{d^3}$$
$$- \frac{2h^2q^2r^2(c+dx)^3\log(c+dx)}{9d^3} - \frac{(bg-ah)pqr^2(g+hx)^2\log(c+dx)}{3bh}$$
$$- \frac{2pqr^2(g+hx)^3\log(c+dx)}{9h} - \frac{2(bg-ah)^3pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{3b^3h}$$
$$- \frac{(dg-ch)^3q^2r^2\log^2(c+dx)}{3d^3h} - \frac{2(dg-ch)^3pqr^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{3d^3h}$$
$$+ \frac{2(bg-ah)^2prx(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right))}{3b^2}$$
$$+ \frac{2(dg-ch)^2qrx(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right))}{3d^2}$$
$$+ \frac{(bg-ah)pr(g+hx)^2\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{3bh}$$
$$+ \frac{(dg-ch)qr(g+hx)^2\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{3dh}$$
$$+ \frac{2pr(g+hx)^3\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{9h}$$
$$+ \frac{2qr(g+hx)^3\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{9h}$$
$$+ \frac{2(bg-ah)^3pr\log(a+bx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{3b^3h}$$
$$+ \frac{2(dg-ch)^3qr\log(c+dx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{3d^3h}$$
$$+ \frac{(g+hx)^3\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h}$$

```
[Out] 1/3*(-a*h+b*g)^2*(-c*h+d*g)*p*q*r^2*ln(b*x+a)/b^2/d/h+1/3*(-a*h+b*g)*(-c*h+
d*g)^2*p*q*r^2*ln(d*x+c)/b/d^2/h+1/3*(h*x+g)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)
^r)^2/h+2/9*(-a*h+b*g)^3*p*q*r^2*ln(b*x+a)/b^3/h+2/9*(-c*h+d*g)^3*p*q*r^2*l
n(d*x+c)/d^3/h+1/3*(-a*h+b*g)*p*r*(h*x+g)^2*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln
(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/b/h+1/3*(-c*h+d*g)*q*r*(h*x+g)^2*(p*r*ln(b*x
+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/d/h+2/3*(-a*h+b*g)^3*p*r
*ln(b*x+a)*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/b^
3/h+2/3*(-c*h+d*g)^3*q*r*ln(d*x+c)*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*
x+a)^p*(d*x+c)^q)^r))/d^3/h+5/18*(-a*h+b*g)*p*q*r^2*(h*x+g)^2/b/h+5/18*(-c*
h+d*g)*p*q*r^2*(h*x+g)^2/d/h-2/3*(-c*h+d*g)^3*p*q*r^2*polylog(2,-d*(b*x+a)/
(-a*d+b*c))/d^3/h-2/3*(-a*h+b*g)^3*p*q*r^2*polylog(2,b*(d*x+c)/(-a*d+b*c))/
b^3/h-2/3*(-c*h+d*g)^2*p*q*r^2*(b*x+a)*ln(b*x+a)/b/d^2-1/3*(-c*h+d*g)*p*q*r
^2*(h*x+g)^2*ln(b*x+a)/d/h-2/3*(-a*h+b*g)^2*p*q*r^2*(d*x+c)*ln(d*x+c)/b^2/d
-1/3*(-a*h+b*g)*p*q*r^2*(h*x+g)^2*ln(d*x+c)/b/h-2/3*(-a*h+b*g)^3*p*q*r^2*ln
(-d*(b*x+a)/(-a*d+b*c))*ln(d*x+c)/b^3/h-2/3*(-c*h+d*g)^3*p*q*r^2*ln(b*x+a)*
ln(b*(d*x+c)/(-a*d+b*c))/d^3/h+2/3*(-a*h+b*g)*(-c*h+d*g)*p*q*r^2*x/b/d-2*(-
a*h+b*g)^2*p^2*r^2*(b*x+a)*ln(b*x+a)/b^3-2/9*h^2*p^2*r^2*(b*x+a)^3*ln(b*x+a
)/b^3-2/9*p*q*r^2*(h*x+g)^3*ln(b*x+a)/h-1/3*(-a*h+b*g)^3*p^2*r^2*ln(b*x+a)^
2/b^3/h-2*(-c*h+d*g)^2*q^2*r^2*(d*x+c)*ln(d*x+c)/d^3-2/9*h^2*q^2*r^2*(d*x+c
)^3*ln(d*x+c)/d^3-2/9*p*q*r^2*(h*x+g)^3*ln(d*x+c)/h-1/3*(-c*h+d*g)^3*q^2*r^
2*ln(d*x+c)^2/d^3/h+2/3*(-a*h+b*g)^2*p*r*x*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(
e*(f*(b*x+a)^p*(d*x+c)^q)^r))/b^2+2/3*(-c*h+d*g)^2*q*r*x*(p*r*ln(b*x+a)+q*r
*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/d^2+8/9*(-a*h+b*g)^2*p*q*r^2*x/
b^2+8/9*(-c*h+d*g)^2*p*q*r^2*x/d^2+1/2*h*(-a*h+b*g)*p^2*r^2*(b*x+a)^2/b^3+1
/2*h*(-c*h+d*g)*q^2*r^2*(d*x+c)^2/d^3-h*(-c*h+d*g)*q^2*r^2*(d*x+c)^2*ln(d*x
+c)/d^3-h*(-a*h+b*g)*p^2*r^2*(b*x+a)^2*ln(b*x+a)/b^3+2/9*p*r*(h*x+g)^3*(p*r
*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/h+2/9*q*r*(h*x+g)
^3*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/h+2*(-a*h+
b*g)^2*p^2*r^2*x/b^2+2*(-c*h+d*g)^2*q^2*r^2*x/d^2+2/27*h^2*p^2*r^2*(b*x+a)^
3/b^3+2/27*h^2*q^2*r^2*(d*x+c)^3/d^3+4/27*p*q*r^2*(h*x+g)^3/h
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 1645, normalized size of antiderivative = 1.00, number of steps used = 47, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.484$, Rules

used = {2584, 2593, 2458, 45, 2372, 12, 14, 2338, 2465, 2436, 2332, 2441, 2440, 2438, 2442}

$$
\begin{aligned}
\int (g+hx)^2 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx &= -\frac{p^2r^2\log^2(a+bx)(bg-ah)^3}{3b^3h} \\
&+ \frac{2pqr^2\log(a+bx)(bg-ah)^3}{9b^3h} - \frac{2pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)(bg-ah)^3}{3b^3h} \\
&+ \frac{2pr\log(a+bx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)(bg-ah)^3}{3b^3h} \\
&- \frac{2pqr^2\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)(bg-ah)^3}{3b^3h} + \frac{2p^2r^2x(bg-ah)^2}{b^2} \\
&+ \frac{8pqr^2x(bg-ah)^2}{9b^2} + \frac{(dg-ch)pqr^2\log(a+bx)(bg-ah)^2}{3b^2dh} \\
&- \frac{2p^2r^2(a+bx)\log(a+bx)(bg-ah)^2}{b^3} - \frac{2pqr^2(c+dx)\log(c+dx)(bg-ah)^2}{3b^2d} \\
&+ \frac{2prx(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right))(bg-ah)^2}{3b^2} \\
&+ \frac{hp^2r^2(a+bx)^2(bg-ah)}{2b^3} + \frac{5pqr^2(g+hx)^2(bg-ah)}{18bh} \\
&+ \frac{2(dg-ch)pqr^2x(bg-ah)}{3bd} - \frac{hp^2r^2(a+bx)^2\log(a+bx)(bg-ah)}{b^3} \\
&+ \frac{(dg-ch)^2pqr^2\log(c+dx)(bg-ah)}{3bd^2h} - \frac{pqr^2(g+hx)^2\log(c+dx)(bg-ah)}{3bh} \\
&+ \frac{pr(g+hx)^2\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)(bg-ah)}{3bh} \\
&+ \frac{2h^2p^2r^2(a+bx)^3}{27b^3} + \frac{2h^2q^2r^2(c+dx)^3}{27d^3} + \frac{4pqr^2(g+hx)^3}{27h} \\
&+ \frac{h(dg-ch)q^2r^2(c+dx)^2}{2d^3} + \frac{5(dg-ch)pqr^2(g+hx)^2}{18dh} - \frac{(dg-ch)^3q^2r^2\log^2(c+dx)}{3d^3h} \\
&+ \frac{(g+hx)^3\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h} + \frac{2(dg-ch)^2q^2r^2x}{d^2} \\
&+ \frac{8(dg-ch)^2pqr^2x}{9d^2} - \frac{2h^2p^2r^2(a+bx)^3\log(a+bx)}{9b^3} - \frac{2pqr^2(g+hx)^3\log(a+bx)}{9h} \\
&- \frac{(dg-ch)pqr^2(g+hx)^2\log(a+bx)}{3dh} - \frac{2(dg-ch)^2pqr^2(a+bx)\log(a+bx)}{3bd^2} \\
&- \frac{2h^2q^2r^2(c+dx)^3\log(c+dx)}{9d^3} - \frac{2pqr^2(g+hx)^3\log(c+dx)}{9h} \\
&+ \frac{2(dg-ch)^3pqr^2\log(c+dx)}{9d^3h} - \frac{h(dg-ch)q^2r^2(c+dx)^2\log(c+dx)}{d^3} \\
&- \frac{2(dg-ch)^2q^2r^2(c+dx)\log(c+dx)}{d^3} - \frac{2(dg-ch)^3pqr^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{3d^3h} \\
&+ \frac{2pr(g+hx)^3\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{9h} \\
&+ \frac{2qr(g+hx)^3\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{9h} \\
&+ \frac{(dg-ch)qr(g+hx)^2\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{3dh} \\
&+ \frac{2(dg-ch)^2qrx(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right))}{3d^2}
\end{aligned}
$$

```
[In] Int[(g + h*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] (2*(b*g - a*h)^2*p^2*r^2*x)/b^2 + (8*(b*g - a*h)^2*p*q*r^2*x)/(9*b^2) + (2*
(b*g - a*h)*(d*g - c*h)*p*q*r^2*x)/(3*b*d) + (8*(d*g - c*h)^2*p*q*r^2*x)/(9
*d^2) + (2*(d*g - c*h)^2*q^2*r^2*x)/d^2 + (h*(b*g - a*h)*p^2*r^2*(a + b*x)^
2)/(2*b^3) + (2*h^2*p^2*r^2*(a + b*x)^3)/(27*b^3) + (h*(d*g - c*h)*q^2*r^2*
(c + d*x)^2)/(2*d^3) + (2*h^2*q^2*r^2*(c + d*x)^3)/(27*d^3) + (5*(b*g - a*h
)*p*q*r^2*(g + h*x)^2)/(18*b*h) + (5*(d*g - c*h)*p*q*r^2*(g + h*x)^2)/(18*d
*h) + (4*p*q*r^2*(g + h*x)^3)/(27*h) + (2*(b*g - a*h)^3*p*q*r^2*Log[a + b*x
])/(9*b^3*h) + ((b*g - a*h)^2*(d*g - c*h)*p*q*r^2*Log[a + b*x])/(3*b^2*d*h)
 - (2*(b*g - a*h)^2*p^2*r^2*(a + b*x)*Log[a + b*x])/b^3 - (2*(d*g - c*h)^2*
p*q*r^2*(a + b*x)*Log[a + b*x])/(3*b*d^2) - (h*(b*g - a*h)*p^2*r^2*(a + b*x
)^2*Log[a + b*x])/b^3 - (2*h^2*p^2*r^2*(a + b*x)^3*Log[a + b*x])/(9*b^3) -
((d*g - c*h)*p*q*r^2*(g + h*x)^2*Log[a + b*x])/(3*d*h) - (2*p*q*r^2*(g + h*
x)^3*Log[a + b*x])/(9*h) - ((b*g - a*h)^3*p^2*r^2*Log[a + b*x]^2)/(3*b^3*h)
 + ((b*g - a*h)*(d*g - c*h)^2*p*q*r^2*Log[c + d*x])/(3*b*d^2*h) + (2*(d*g -
 c*h)^3*p*q*r^2*Log[c + d*x])/(9*d^3*h) - (2*(b*g - a*h)^2*p*q*r^2*(c + d*x
)*Log[c + d*x])/(3*b^2*d) - (2*(d*g - c*h)^2*q^2*r^2*(c + d*x)*Log[c + d*x]
)/d^3 - (h*(d*g - c*h)*q^2*r^2*(c + d*x)^2*Log[c + d*x])/d^3 - (2*h^2*q^2*r
^2*(c + d*x)^3*Log[c + d*x])/(9*d^3) - ((b*g - a*h)*p*q*r^2*(g + h*x)^2*Log
[c + d*x])/(3*b*h) - (2*p*q*r^2*(g + h*x)^3*Log[c + d*x])/(9*h) - (2*(b*g -
 a*h)^3*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(3*b^3*h) -
 ((d*g - c*h)^3*q^2*r^2*Log[c + d*x]^2)/(3*d^3*h) - (2*(d*g - c*h)^3*p*q*r^
2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(3*d^3*h) + (2*(b*g - a*h)^2
*p*r*x*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x
)^q)^r]))/(3*b^2) + (2*(d*g - c*h)^2*q*r*x*(p*r*Log[a + b*x] + q*r*Log[c +
d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(3*d^2) + ((b*g - a*h)*p*r*(g
 + h*x)^2*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]))/(3*b*h) + ((d*g - c*h)*q*r*(g + h*x)^2*(p*r*Log[a + b*x] + q*r
*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(3*d*h) + (2*p*r*(g
+ h*x)^3*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d
*x)^q)^r]))/(9*h) + (2*q*r*(g + h*x)^3*(p*r*Log[a + b*x] + q*r*Log[c + d*x]
 - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(9*h) + (2*(b*g - a*h)^3*p*r*Log[
a + b*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d
*x)^q)^r]))/(3*b^3*h) + (2*(d*g - c*h)^3*q*r*Log[c + d*x]*(p*r*Log[a + b*x]
 + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(3*d^3*h) + ((
g + h*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)/(3*h) - (2*(d*g - c*h)^3
*p*q*r^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(3*d^3*h) - (2*(b*g - a*
h)^3*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(3*b^3*h)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2584

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1
)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(h*(m + 1))), x] + (-Dist[b*p*r*(
s/(h*(m + 1))), Int[(g + h*x)^(m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1)/(a + b*x)), x], x] - Dist[d*q*r*(s/(h*(m + 1))), Int[(g + h*x)^(m
+ 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(c + d*x)), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2593

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
```

```
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$\text{integral} = \frac{(g+hx)^3\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h} - \frac{(2bpr)\int \frac{(g+hx)^3\log(e(f(a+bx)^p(c+dx)^q)^r)}{a+bx}\,dx}{3h} - \frac{(2dqr)\int \frac{(g+hx)^3\log(e(f(a+bx)^p(c+dx)^q)^r)}{c+dx}\,dx}{3h}$$

$$= \frac{(g+hx)^3\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h} - \frac{(2bp^2r^2)\int \frac{(g+hx)^3\log(a+bx)}{a+bx}\,dx}{3h} - \frac{(2bpqr^2)\int \frac{(g+hx)^3\log(c+dx)}{a+bx}\,dx}{3h} - \frac{(2dpqr^2)\int \frac{(g+hx)^3\log(a+bx)}{c+dx}\,dx}{3h} - \frac{(2dq^2r^2)\int \frac{(g+hx)^3\log(c+dx)}{c+dx}\,dx}{3h} + \frac{(2bpr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)))\int \frac{(g+hx)^3}{a+bx}\,dx}{3h} + \frac{(2dqr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)))\int \frac{(g+hx)^3}{c+dx}\,dx}{3h}$$

$$= \frac{(g+hx)^3\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h} - \frac{(2p^2r^2)\,\text{Subst}\left(\int \frac{\left(\frac{bg-ah}{b}+\frac{hx}{b}\right)^3\log(x)}{x}\,dx, x, a+bx\right)}{3h} - \frac{(2bpqr^2)\int \left(\frac{h(bg-ah)^2\log(c+dx)}{b^3}+\frac{(bg-ah)^3\log(c+dx)}{b^3(a+bx)}+\frac{h(bg-ah)(g+hx)\log(c+dx)}{b^2}+\frac{h(g+hx)^2\log(c+dx)}{b}\right)dx}{3h} - \frac{(2dpqr^2)\int \left(\frac{h(dg-ch)^2\log(a+bx)}{d^3}+\frac{(dg-ch)^3\log(a+bx)}{d^3(c+dx)}+\frac{h(dg-ch)(g+hx)\log(a+bx)}{d^2}+\frac{h(g+hx)^2\log(a+bx)}{d}\right)dx}{3h} - \frac{(2q^2r^2)\,\text{Subst}\left(\int \frac{\left(\frac{dg-ch}{d}+\frac{hx}{d}\right)^3\log(x)}{x}\,dx, x, c+dx\right)}{3h} + \frac{(2bpr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)))\int \left(\frac{h(bg-ah)^2}{b^3}+\frac{(bg-ah)^3}{b^3(a+bx)}+\right.}{3h} + \frac{(2dqr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)))\int \left(\frac{h(dg-ch)^2}{d^3}+\frac{(dg-ch)^3}{d^3(c+dx)}+\right.}{3h}$$

$$= -\frac{2(bg-ah)^2p^2r^2(a+bx)\log(a+bx)}{b^3} - \frac{h(bg-ah)p^2r^2(a+bx)^2\log(a+bx)}{b^3} - \frac{2h^2p^2r^2(a+bx)^3\log(a+bx)}{9b^3} - \frac{2(bg-ah)^3p^2r^2\log^2(a+bx)}{3b^3h} - \frac{2(dg-ch)^2q^2r^2(c+dx)\log(c+dx)}{d^3} - \frac{h(dg-ch)q^2r^2(c+dx)^2\log(c+dx)}{d^3} - \frac{2h^2q^2r^2(c+dx)^3\log(c+dx)}{9d^3} - \frac{2(dg-ch)^3q^2r^2\log^2(c+dx)}{3d^3h} + \frac{2(bg-ah)^2prx(pr\log(a+bx)+qr\log(c+dx)-\log(e(f(a+bx)^p(c+dx)^q)^r))}{3b^2} + \frac{2(dg-ch)^2qrx(pr\log(a+bx)+qr\log(c+dx)-\log(e(f(a+bx)^p(c+dx)^q)^r))}{3d^2} + \frac{(bg-ah)pr(g+hx)^2(pr\log(a+bx)+qr\log(c+dx)-\log(e(f(a+bx)^p(c+dx)^q)^r))}{3bh} + \frac{(dg-ch)qr(g+hx)^2(pr\log(a+bx)+qr\log(c+dx)-\log(e(f(a+bx)^p(c+dx)^q)^r))}{3dh} + \frac{2pr(g+hx)^3(pr\log(a+bx)+qr\log(c+dx)-\log(e(f(a+bx)^p(c+dx)^q)^r))}{9h} + \frac{2qr(g+hx)^3(pr\log(a+bx)+qr\log(c+dx)-\log(e(f(a+bx)^p(c+dx)^q)^r))}{9h} + \frac{2(bg-ah)^3pr\log(a+bx)(pr\log(a+bx)+qr\log(c+dx)-\log(e(f(a+bx)^p(c+dx)^q)^r))}{3b^3h} + \frac{2(dg-ch)^3qr\log(c+dx)(pr\log(a+bx)+qr\log(c+dx)-\log(e(f(a+bx)^p(c+dx)^q)^r))}{3d^3h} + \frac{(g+hx)^3\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{3h} + \frac{(2p^2r^2)\,\text{Subst}\left(\int \frac{hx(18b^2g^2+9bgh(-4a+x)+h^2(18a^2-9ax+2x^2))+6(bg-ah)^3\log(x)}{6b^3x}\,dx, x, a+bx\right)}{3h} - \frac{1}{3}(2pqr^2)\int (g+hx)^2\log(a+bx)\,dx - \frac{1}{3}(2pqr^2)\int (g+hx)^2\log(c+dx)\,dx - \frac{(2(bg-ah)pqr^2)\int (g+hx)\log(c+dx)\,dx}{3b} - \frac{(2(bg-ah)^2pqr^2)\int \log(c+dx)\,dx}{3b^2} - \frac{(2(bg-ah)^3pqr^2)\int \frac{\log(c+dx)}{a+bx}\,dx}{3b^2h} - \frac{(2(dg-ch)pqr^2)\int (g+hx)\log(a+bx)\,dx}{3d} - \frac{(2(dg-ch)^2pqr^2)\int \log(a+bx)\,dx}{3d^2} - \frac{(2(dg-ch)^3pqr^2)\int \frac{\log(a+bx)}{c+dx}\,dx}{3d^2h} + \frac{(2q^2r^2)\,\text{Subst}\left(\int \frac{hx(18d^2g^2+9dgh(-4c+x)+h^2(18c^2-9cx+2x^2))+6(dg-ch)^3\log(x)}{6d^3x}\,dx, x, c+dx\right)}{3h}$$

$= \text{Too large to display}$

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 899, normalized size of antiderivative = 0.55

$$\int (g+hx)^2 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$$

$$= \frac{-18ad^3(3b^2g^2-3abgh+a^2h^2)\,p^2r^2\log^2(a+bx)-6pr\log(a+bx)\left(6b^3c(3d^2g^2-3cdgh+c^2h^2)\,qr\log(c+\right.}{}$$

```
[In] Integrate[(g + h*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] (-18*a*d^3*(3*b^2*g^2 - 3*a*b*g*h + a^2*h^2)*p^2*r^2*Log[a + b*x]^2 - 6*p*r
*Log[a + b*x]*(6*b^3*c*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2)*q*r*Log[c + d*x] -
 6*(b*c - a*d)*(a^2*d^2*h^2 + a*b*d*h*(-3*d*g + c*h) + b^2*(3*d^2*g^2 - 3*c
*d*g*h + c^2*h^2))*q*r*Log[(b*(c + d*x))/(b*c - a*d)] + a*d*((6*b^2*(3*d^2*
g^2 - 3*c*d*g*h + c^2*h^2)*q + a^2*d^2*h^2*(11*p + 2*q) - 3*a*b*d*h*(-(c*h*
q) + 3*d*g*(3*p + q)))*r - 6*d^2*(3*b^2*g^2 - 3*a*b*g*h + a^2*h^2)*Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r])) + b*(-18*b^2*c*(3*d^2*g^2 - 3*c*d*g*h + c^2*
h^2)*q^2*r^2*Log[c + d*x]^2 - 6*q*r*Log[c + d*x]*((6*a^2*c*d^2*h^2*p - 3*a*
b*d*(6*d^2*g^2 + 6*c*d*g*h - c^2*h^2)*p + b^2*c*(18*d^2*g^2*(p + q) - 9*c*d
*g*h*(p + 3*q) + c^2*h^2*(2*p + 11*q)))*r - 6*b^2*c*(3*d^2*g^2 - 3*c*d*g*h
+ c^2*h^2)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]) + d*(r^2*(6*a^2*d^2*h^2*p*
(11*p + 8*q)*x + b^2*x*(6*c^2*h^2*q*(8*p + 11*q) - 3*c*d*h*q*(p + q)*(54*g
+ 5*h*x) + d^2*(p + q)^2*(108*g^2 + 27*g*h*x + 4*h^2*x^2)) - 3*a*b*p*(-12*c
^2*h^2*q - 12*c*d*h*q*(-3*g + h*x) + d^2*(-36*g^2*q + 54*g*h*(p + q)*x + 5*
h^2*(p + q)*x^2))) - 6*r*(6*a^2*d^2*h^2*p*x + 3*a*b*d^2*p*(6*g^2 - 6*g*h*x
- h^2*x^2) + b^2*x*(6*c^2*h^2*q - 3*c*d*h*q*(6*g + h*x) + d^2*(p + q)*(18*g
^2 + 9*g*h*x + 2*h^2*x^2)))*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 18*b^2*d
^2*x*(3*g^2 + 3*g*h*x + h^2*x^2)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)) +
 36*(b*c - a*d)*(a^2*d^2*h^2 + a*b*d*h*(-3*d*g + c*h) + b^2*(3*d^2*g^2 - 3*
c*d*g*h + c^2*h^2))*p*q*r^2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/(54*b
^3*d^3)
```

Maple [F]

$$\int (hx+g)^2 \ln\left(e(f(bx+a)^p\,(dx+c)^q)^r\right)^2 dx$$

```
[In] int((h*x+g)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

```
[Out] int((h*x+g)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

Fricas [F]

$$\int (g+hx)^2 \log^2 \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx = \int (hx+g)^2 \log \left(((bx+a)^p(dx+c)^q f)^r e\right)^2 \, dx$$

```
[In] integrate((h*x+g)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="fricas
")
[Out] integral((h^2*x^2 + 2*g*h*x + g^2)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2,
x)
```

Sympy [F]

$$\int (g+hx)^2 \log^2 \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx = \int (g+hx)^2 \log \left(e(f(a+bx)^p (c+dx)^q)^r\right)^2 dx$$

```
[In] integrate((h*x+g)**2*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2,x)
[Out] Integral((g + h*x)**2*log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 1123, normalized size of antiderivative = 0.68

$$\int (g+hx)^2 \log^2 \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx = \text{Too large to display}$$

```
[In] integrate((h*x+g)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="maxima
")
[Out] 1/3*(h^2*x^3 + 3*g*h*x^2 + 3*g^2*x)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2
+ 1/9*r*(6*(3*a*b^2*f*g^2*p - 3*a^2*b*f*g*h*p + a^3*f*h^2*p)*log(b*x + a)/b
^3 + 6*(3*c*d^2*f*g^2*q - 3*c^2*d*f*g*h*q + c^3*f*h^2*q)*log(d*x + c)/d^3 -
 (2*b^2*d^2*f*h^2*(p + q)*x^3 - 3*(a*b*d^2*f*h^2*p - (3*d^2*f*g*h*(p + q) -
 c*d*f*h^2*q)*b^2)*x^2 - 6*(3*a*b*d^2*f*g*h*p - a^2*d^2*f*h^2*p - (3*d^2*f*
g^2*(p + q) - 3*c*d*f*g*h*q + c^2*f*h^2*q)*b^2)*x)/(b^2*d^2))*log(((b*x + a
)^p*(d*x + c)^q*f)^r*e)/f - 1/54*r^2*(6*(6*a^2*c*d^2*f^2*h^2*p*q - 3*(6*c*d
^2*f^2*g*h*p*q - c^2*d*f^2*h^2*p*q)*a*b + (18*(p*q + q^2)*c*d^2*f^2*g^2 - 9
*(p*q + 3*q^2)*c^2*d*f^2*g*h + (2*p*q + 11*q^2)*c^3*f^2*h^2)*b^2)*log(d*x +
 c)/(b^2*d^3) + 36*(3*a*b^2*d^3*f^2*g^2*p*q - 3*a^2*b*d^3*f^2*g*h*p*q + a^3
*d^3*f^2*h^2*p*q - (3*c*d^2*f^2*g^2*p*q - 3*c^2*d*f^2*g*h*p*q + c^3*f^2*h^2
*p*q)*b^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x
```

```
 + a*d)/(b*c - a*d)))/(b^3*d^3) - (4*(p^2 + 2*p*q + q^2)*b^3*d^3*f^2*h^2*x^
3 - 36*(3*c*d^2*f^2*g^2*p*q - 3*c^2*d*f^2*g*h*p*q + c^3*f^2*h^2*p*q)*b^3*lo
g(b*x + a)*log(d*x + c) - 18*(3*c*d^2*f^2*g^2*q^2 - 3*c^2*d*f^2*g*h*q^2 + c
^3*f^2*h^2*q^2)*b^3*log(d*x + c)^2 - 3*(5*(p^2 + p*q)*a*b^2*d^3*f^2*h^2 - (
9*(p^2 + 2*p*q + q^2)*d^3*f^2*g*h - 5*(p*q + q^2)*c*d^2*f^2*h^2)*b^3)*x^2 -
 18*(3*a*b^2*d^3*f^2*g^2*p^2 - 3*a^2*b*d^3*f^2*g*h*p^2 + a^3*d^3*f^2*h^2*p^
2)*log(b*x + a)^2 + 6*((11*p^2 + 8*p*q)*a^2*b*d^3*f^2*h^2 + 3*(2*c*d^2*f^2*
h^2*p*q - 9*(p^2 + p*q)*d^3*f^2*g*h)*a*b^2 + (18*(p^2 + 2*p*q + q^2)*d^3*f^
2*g^2 - 27*(p*q + q^2)*c*d^2*f^2*g*h + (8*p*q + 11*q^2)*c^2*d*f^2*h^2)*b^3)
*x - 6*((11*p^2 + 2*p*q)*a^3*d^3*f^2*h^2 + 3*(c*d^2*f^2*h^2*p*q - 3*(3*p^2
+ p*q)*d^3*f^2*g*h)*a^2*b - 6*(3*c*d^2*f^2*g*h*p*q - c^2*d*f^2*h^2*p*q - 3*
(p^2 + p*q)*d^3*f^2*g^2)*a*b^2)*log(b*x + a))/(b^3*d^3))/f^2
```

Giac [F(-1)]

Timed out.

$$\int (g+hx)^2 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \text{Timed out}$$

```
[In] integrate((h*x+g)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int (g+hx)^2 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \int \ln\left(e\left(f\left(a+b\,x\right)^p\left(c+d\,x\right)^q\right)^r\right)^2 \left(g+h\,x\right)^2\,dx$$

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(g + h*x)^2,x)
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(g + h*x)^2, x)
```

3.37 $\int (g+hx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$

Optimal result 344
Rubi [A] (verified) 345
Mathematica [A] (verified) 356
Maple [F] 356
Fricas [F] 356
Sympy [F] 357
Maxima [A] (verification not implemented) 357
Giac [F] 358
Mupad [F(-1)] 358

Optimal result

Integrand size = 29, antiderivative size = 1063

$$\int (g+hx)\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx$$
$$= \frac{3(bg-ah)pqr^2x}{2b} + \frac{3(dg-ch)pqr^2x}{2d} + \frac{pqr^2(g+hx)^2}{2h} + \frac{p^2r^2(4bg-3ah+bhx)^2}{4b^2h}$$
$$+ \frac{q^2r^2(4dg-3ch+dhx)^2}{4d^2h} + \frac{(bg-ah)^2pqr^2\log(a+bx)}{2b^2h}$$
$$- \frac{2(bg-ah)p^2r^2(a+bx)\log(a+bx)}{b^2} - \frac{(dg-ch)pqr^2(a+bx)\log(a+bx)}{bd}$$
$$- \frac{hp^2r^2(a+bx)^2\log(a+bx)}{2b^2} - \frac{pqr^2(g+hx)^2\log(a+bx)}{2h} - \frac{(bg-ah)^2p^2r^2\log^2(a+bx)}{2b^2h}$$
$$+ \frac{(dg-ch)^2pqr^2\log(c+dx)}{2d^2h} - \frac{(bg-ah)pqr^2(c+dx)\log(c+dx)}{bd}$$
$$- \frac{2(dg-ch)q^2r^2(c+dx)\log(c+dx)}{d^2} - \frac{hq^2r^2(c+dx)^2\log(c+dx)}{2d^2}$$
$$- \frac{pqr^2(g+hx)^2\log(c+dx)}{2h} - \frac{(bg-ah)^2pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{b^2h}$$
$$- \frac{(dg-ch)^2q^2r^2\log^2(c+dx)}{2d^2h} - \frac{(dg-ch)^2pqr^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{d^2h}$$
$$+ \frac{(bg-ah)prx\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\right)}{b}$$
$$+ \frac{(dg-ch)qrx\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\right)}{d}$$
$$+ \frac{pr(g+hx)^2\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\right)}{2h}$$
$$+ \frac{qr(g+hx)^2\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\right)}{2h}$$
$$+ \frac{(bg-ah)^2pr\log(a+bx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\right)}{b^2h}$$
$$+ \frac{(dg-ch)^2qr\log(c+dx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\right)}{d^2h}$$
$$+ \frac{(g+hx)^2\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h}$$
$$- \frac{(dg-ch)^2pqr^2\,\mathrm{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{d^2h} - \frac{(bg-ah)^2pqr^2\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{b^2h}$$

```
[Out] -(-c*h+d*g)*p*q*r^2*(b*x+a)*ln(b*x+a)/b/d-(-a*h+b*g)*p*q*r^2*(d*x+c)*ln(d*x
+c)/b/d-(-a*h+b*g)^2*p*q*r^2*ln(-d*(b*x+a)/(-a*d+b*c))*ln(d*x+c)/b^2/h-(-c*
h+d*g)^2*p*q*r^2*ln(b*x+a)*ln(b*(d*x+c)/(-a*d+b*c))/d^2/h+1/2*(h*x+g)^2*ln(
e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/h-(-a*h+b*g)^2*p*q*r^2*polylog(2,b*(d*x+c)/(
```

```
-a*d+b*c))/b^2/h+(-a*h+b*g)^2*p*r*ln(b*x+a)*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln
(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/b^2/h+(-c*h+d*g)^2*q*r*ln(d*x+c)*(p*r*ln(b*x
+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/d^2/h-(-c*h+d*g)^2*p*q*r
^2*polylog(2,-d*(b*x+a)/(-a*d+b*c))/d^2/h+(-a*h+b*g)*p*r*x*(p*r*ln(b*x+a)+q
*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/b+(-c*h+d*g)*q*r*x*(p*r*ln(b*
x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/d+1/2*(-a*h+b*g)^2*p*q*
r^2*ln(b*x+a)/b^2/h+1/2*(-c*h+d*g)^2*p*q*r^2*ln(d*x+c)/d^2/h+3/2*(-a*h+b*g)
*p*q*r^2*x/b+3/2*(-c*h+d*g)*p*q*r^2*x/d+1/2*p*r*(h*x+g)^2*(p*r*ln(b*x+a)+q*
r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/h+1/2*q*r*(h*x+g)^2*(p*r*ln(b*
x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/h+1/2*p*q*r^2*(h*x+g)^2
/h+1/4*p^2*r^2*(b*h*x-3*a*h+4*b*g)^2/b^2/h+1/4*q^2*r^2*(d*h*x-3*c*h+4*d*g)^
2/d^2/h-2*(-a*h+b*g)*p^2*r^2*(b*x+a)*ln(b*x+a)/b^2-1/2*h*p^2*r^2*(b*x+a)^2*
ln(b*x+a)/b^2-1/2*p*q*r^2*(h*x+g)^2*ln(b*x+a)/h-1/2*(-a*h+b*g)^2*p^2*r^2*ln
(b*x+a)^2/b^2/h-2*(-c*h+d*g)*q^2*r^2*(d*x+c)*ln(d*x+c)/d^2-1/2*h*q^2*r^2*(d
*x+c)^2*ln(d*x+c)/d^2-1/2*p*q*r^2*(h*x+g)^2*ln(d*x+c)/h-1/2*(-c*h+d*g)^2*q^
2*r^2*ln(d*x+c)^2/d^2/h
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 1063, normalized size of antiderivative = 1.00, number of steps used = 39, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.517$, Rules

used = {2584, 2593, 2458, 45, 2372, 12, 14, 2338, 2465, 2436, 2332, 2441, 2440, 2438, 2442}

$$\int (g+hx)\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx = -\frac{p^2r^2\log^2(a+bx)(bg-ah)^2}{2b^2h}$$
$$+\frac{pqr^2\log(a+bx)(bg-ah)^2}{2b^2h}-\frac{pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)(bg-ah)^2}{b^2h}$$
$$+\frac{pr\log(a+bx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\right)(bg-ah)^2}{b^2h}$$
$$-\frac{pqr^2\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)(bg-ah)^2}{b^2h}+\frac{3pqr^2x(bg-ah)}{2b}$$
$$-\frac{2p^2r^2(a+bx)\log(a+bx)(bg-ah)}{b^2}-\frac{pqr^2(c+dx)\log(c+dx)(bg-ah)}{bd}$$
$$+\frac{prx\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\right)(bg-ah)}{b}$$
$$+\frac{pqr^2(g+hx)^2}{2h}+\frac{p^2r^2(4bg-3ah+bhx)^2}{4b^2h}+\frac{q^2r^2(4dg-3ch+dhx)^2}{4d^2h}$$
$$-\frac{(dg-ch)^2q^2r^2\log^2(c+dx)}{2d^2h}+\frac{(g+hx)^2\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h}$$
$$+\frac{3(dg-ch)pqr^2x}{2d}-\frac{hp^2r^2(a+bx)^2\log(a+bx)}{2b^2}-\frac{pqr^2(g+hx)^2\log(a+bx)}{2h}$$
$$-\frac{(dg-ch)pqr^2(a+bx)\log(a+bx)}{bd}+\frac{(dg-ch)^2pqr^2\log(c+dx)}{2d^2h}$$
$$-\frac{hq^2r^2(c+dx)^2\log(c+dx)}{2d^2}-\frac{pqr^2(g+hx)^2\log(c+dx)}{2h}$$
$$-\frac{2(dg-ch)q^2r^2(c+dx)\log(c+dx)}{d^2}-\frac{(dg-ch)^2pqr^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{d^2h}$$
$$+\frac{pr(g+hx)^2\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\right)}{2h}$$
$$+\frac{qr(g+hx)^2\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\right)}{2h}$$
$$+\frac{(dg-ch)qrx\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\right)}{d}$$
$$+\frac{(dg-ch)^2qr\log(c+dx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\right)}{d^2h}$$
$$-\frac{(dg-ch)^2pqr^2\,\mathrm{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{d^2h}$$

```
[In] Int[(g + h*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] (3*(b*g - a*h)*p*q*r^2*x)/(2*b) + (3*(d*g - c*h)*p*q*r^2*x)/(2*d) + (p*q*r^
2*(g + h*x)^2)/(2*h) + (p^2*r^2*(4*b*g - 3*a*h + b*h*x)^2)/(4*b^2*h) + (q^2
*r^2*(4*d*g - 3*c*h + d*h*x)^2)/(4*d^2*h) + ((b*g - a*h)^2*p*q*r^2*Log[a +
b*x])/(2*b^2*h) - (2*(b*g - a*h)*p^2*r^2*(a + b*x)*Log[a + b*x])/b^2 - ((d*
```

```
g - c*h)*p*q*r^2*(a + b*x)*Log[a + b*x])/(b*d) - (h*p^2*r^2*(a + b*x)^2*Log
[a + b*x])/(2*b^2) - (p*q*r^2*(g + h*x)^2*Log[a + b*x])/(2*h) - ((b*g - a*h
)^2*p^2*r^2*Log[a + b*x]^2)/(2*b^2*h) + ((d*g - c*h)^2*p*q*r^2*Log[c + d*x]
)/(2*d^2*h) - ((b*g - a*h)*p*q*r^2*(c + d*x)*Log[c + d*x])/(b*d) - (2*(d*g
- c*h)*q^2*r^2*(c + d*x)*Log[c + d*x])/d^2 - (h*q^2*r^2*(c + d*x)^2*Log[c +
 d*x])/(2*d^2) - (p*q*r^2*(g + h*x)^2*Log[c + d*x])/(2*h) - ((b*g - a*h)^2*
p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(b^2*h) - ((d*g - c
*h)^2*q^2*r^2*Log[c + d*x]^2)/(2*d^2*h) - ((d*g - c*h)^2*p*q*r^2*Log[a + b*
x]*Log[(b*(c + d*x))/(b*c - a*d)])/(d^2*h) + ((b*g - a*h)*p*r*x*(p*r*Log[a
+ b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/b + ((d*
g - c*h)*q*r*x*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*
(c + d*x)^q)^r]))/d + (p*r*(g + h*x)^2*(p*r*Log[a + b*x] + q*r*Log[c + d*x]
 - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(2*h) + (q*r*(g + h*x)^2*(p*r*Log
[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(2*h)
 + ((b*g - a*h)^2*p*r*Log[a + b*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - L
og[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(b^2*h) + ((d*g - c*h)^2*q*r*Log[c +
d*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]))/(d^2*h) + ((g + h*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)/(2*h
) - ((d*g - c*h)^2*p*q*r^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(d^2*h
) - ((b*g - a*h)^2*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(b^2*h)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2584

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1
)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(h*(m + 1))), x] + (-Dist[b*p*r*(
s/(h*(m + 1))), Int[(g + h*x)^(m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1)/(a + b*x)), x], x] - Dist[d*q*r*(s/(h*(m + 1))), Int[(g + h*x)^(m
+ 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(c + d*x)), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2593

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$\text{integral} = \frac{(g+hx)^2\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h} - \frac{(bpr)\int\frac{(g+hx)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{a+bx}\,dx}{h} - \frac{(dqr)\int\frac{(g+hx)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{c+dx}\,dx}{h}$$

$$= \frac{(g+hx)^2 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h} - \frac{(bp^2r^2)\int \frac{(g+hx)^2\log(a+bx)}{a+bx}\,dx}{h} - \frac{(bpqr^2)\int \frac{(g+hx)^2\log(c+dx)}{a+bx}\,dx}{h} - \frac{(dpqr^2)\int \frac{(g+hx)^2\log(a+bx)}{c+dx}\,dx}{h} - \frac{(dq^2r^2)\int \frac{(g+hx)^2\log(c+dx)}{c+dx}\,dx}{h} + \frac{(bpr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)))\int \frac{(g+hx)^2}{a+bx}\,dx}{h} + \frac{(dqr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)))\int \frac{(g+hx)^2}{c+dx}\,dx}{h}$$

$$= \frac{(g+hx)^2 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h} - \frac{(p^2r^2)\,\text{Subst}\left(\int \frac{\left(\frac{bg-ah}{b}+\frac{hx}{b}\right)^2\log(x)}{x}\,dx, x, a+bx\right)}{h} - \frac{(bpqr^2)\int \left(\frac{h(bg-ah)\log(c+dx)}{b^2}+\frac{(bg-ah)^2\log(c+dx)}{b^2(a+bx)}+\frac{h(g+hx)\log(c+dx)}{b}\right)dx}{h} - \frac{(dpqr^2)\int \left(\frac{h(dg-ch)\log(a+bx)}{d^2}+\frac{(dg-ch)^2\log(a+bx)}{d^2(c+dx)}+\frac{h(g+hx)\log(a+bx)}{d}\right)dx}{h} - \frac{(q^2r^2)\,\text{Subst}\left(\int \frac{\left(\frac{dg-ch}{d}+\frac{hx}{d}\right)^2\log(x)}{x}\,dx, x, c+dx\right)}{h} + \frac{(bpr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)))\int \left(\frac{h(bg-ah)}{b^2}+\frac{(bg-ah)^2}{b^2(a+bx)}+ h(\right.}{h} + \frac{(dqr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)))\int \left(\frac{h(dg-ch)}{d^2}+\frac{(dg-ch)^2}{d^2(c+dx)}+ h\right.}{h}$$

$$= -\frac{2(bg-ah)p^2r^2(a+bx)\log(a+bx)}{b^2} - \frac{hp^2r^2(a+bx)^2\log(a+bx)}{2b^2}$$
$$- \frac{(bg-ah)^2p^2r^2\log^2(a+bx)}{b^2h} - \frac{2(dg-ch)q^2r^2(c+dx)\log(c+dx)}{d^2}$$
$$- \frac{hq^2r^2(c+dx)^2\log(c+dx)}{2d^2} - \frac{(dg-ch)^2q^2r^2\log^2(c+dx)}{d^2h}$$
$$+ \frac{(bg-ah)prx(pr\log(a+bx)+qr\log(c+dx)-\log(e(f(a+bx)^p(c+dx)^q)^r))}{b}$$
$$+ \frac{(dg-ch)qrx(pr\log(a+bx)+qr\log(c+dx)-\log(e(f(a+bx)^p(c+dx)^q)^r))}{d}$$
$$+ \frac{pr(g+hx)^2(pr\log(a+bx)+qr\log(c+dx)-\log(e(f(a+bx)^p(c+dx)^q)^r))}{2h}$$
$$+ \frac{qr(g+hx)^2(pr\log(a+bx)+qr\log(c+dx)-\log(e(f(a+bx)^p(c+dx)^q)^r))}{2h}$$
$$+ \frac{(bg-ah)^2pr\log(a+bx)(pr\log(a+bx)+qr\log(c+dx)-\log(e(f(a+bx)^p(c+dx)^q)^r))}{b^2h}$$
$$+ \frac{(dg-ch)^2qr\log(c+dx)(pr\log(a+bx)+qr\log(c+dx)-\log(e(f(a+bx)^p(c+dx)^q)^r))}{d^2h}$$
$$+ \frac{(g+hx)^2\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{2h}$$
$$+ \frac{(p^2r^2)\,\mathrm{Subst}\left(\int \frac{hx(4bg+h(-4a+x))+2(bg-ah)^2\log(x)}{2b^2x}\,dx, x, a+bx\right)}{h}$$
$$- (pqr^2)\int (g+hx)\log(a+bx)\,dx - (pqr^2)\int (g+hx)\log(c+dx)\,dx$$
$$- \frac{((bg-ah)pqr^2)\int \log(c+dx)\,dx}{b} - \frac{((bg-ah)^2pqr^2)\int \frac{\log(c+dx)}{a+bx}\,dx}{bh}$$
$$- \frac{((dg-ch)pqr^2)\int \log(a+bx)\,dx}{d} - \frac{((dg-ch)^2pqr^2)\int \frac{\log(a+bx)}{c+dx}\,dx}{dh}$$
$$+ \frac{(q^2r^2)\,\mathrm{Subst}\left(\int \frac{hx(4dg+h(-4c+x))+2(dg-ch)^2\log(x)}{2d^2x}\,dx, x, c+dx\right)}{h}$$

$$
\begin{aligned}
&= -\frac{2(bg-ah)p^2r^2(a+bx)\log(a+bx)}{b^2} - \frac{hp^2r^2(a+bx)^2\log(a+bx)}{2b^2} \\
&- \frac{pqr^2(g+hx)^2\log(a+bx)}{2h} - \frac{(bg-ah)^2p^2r^2\log^2(a+bx)}{b^2h} \\
&- \frac{2(dg-ch)q^2r^2(c+dx)\log(c+dx)}{d^2} - \frac{hq^2r^2(c+dx)^2\log(c+dx)}{2d^2} \\
&- \frac{pqr^2(g+hx)^2\log(c+dx)}{2h} - \frac{(bg-ah)^2pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{b^2h} \\
&- \frac{(dg-ch)^2q^2r^2\log^2(c+dx)}{d^2h} - \frac{(dg-ch)^2pqr^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{d^2h} \\
&+ \frac{(bg-ah)prx(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right))}{b} \\
&+ \frac{(dg-ch)qrx(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right))}{d} \\
&+ \frac{pr(g+hx)^2\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{2h} \\
&+ \frac{qr(g+hx)^2\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{2h} \\
&+ \frac{(bg-ah)^2pr\log(a+bx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{b^2h} \\
&+ \frac{(dg-ch)^2qr\log(c+dx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{d^2h} \\
&+ \frac{(g+hx)^2\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h} \\
&+ \frac{(p^2r^2)\operatorname{Subst}\left(\int \frac{hx(4bg+h(-4a+x))+2(bg-ah)^2\log(x)}{x}\,dx, x, a+bx\right)}{2b^2h} \\
&+ \frac{(bpqr^2)\int \frac{(g+hx)^2}{a+bx}\,dx}{2h} + \frac{(dpqr^2)\int \frac{(g+hx)^2}{c+dx}\,dx}{2h} \\
&- \frac{((bg-ah)pqr^2)\operatorname{Subst}(\int \log(x)\,dx, x, c+dx)}{bd} \\
&+ \frac{(d(bg-ah)^2pqr^2)\int \frac{\log\left(\frac{d(a+bx)}{-bc+ad}\right)}{c+dx}\,dx}{b^2h} \\
&- \frac{((dg-ch)pqr^2)\operatorname{Subst}(\int \log(x)\,dx, x, a+bx)}{bd} \\
&+ \frac{(b(dg-ch)^2pqr^2)\int \frac{\log\left(\frac{b(c+dx)}{bc-ad}\right)}{a+bx}\,dx}{d^2h} \\
&+ \frac{(q^2r^2)\operatorname{Subst}\left(\int \frac{hx(4dg+h(-4c+x))+2(dg-ch)^2\log(x)}{x}\,dx, x, c+dx\right)}{2d^2h}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(bg-ah)pqr^2x}{b} + \frac{(dg-ch)pqr^2x}{d} - \frac{2(bg-ah)p^2r^2(a+bx)\log(a+bx)}{b^2} \\
&- \frac{(dg-ch)pqr^2(a+bx)\log(a+bx)}{bd} - \frac{hp^2r^2(a+bx)^2\log(a+bx)}{2b^2} \\
&- \frac{pqr^2(g+hx)^2\log(a+bx)}{2h} - \frac{(bg-ah)^2p^2r^2\log^2(a+bx)}{b^2h} \\
&- \frac{(bg-ah)pqr^2(c+dx)\log(c+dx)}{bd} - \frac{2(dg-ch)q^2r^2(c+dx)\log(c+dx)}{d^2} \\
&- \frac{hq^2r^2(c+dx)^2\log(c+dx)}{2d^2} - \frac{pqr^2(g+hx)^2\log(c+dx)}{2h} \\
&- \frac{(bg-ah)^2pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{b^2h} \\
&- \frac{(dg-ch)^2q^2r^2\log^2(c+dx)}{d^2h} - \frac{(dg-ch)^2pqr^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{d^2h} \\
&+ \frac{(bg-ah)prx(pr\log(a+bx)+qr\log(c+dx)-\log(e(f(a+bx)^p(c+dx)^q)^r))}{b} \\
&+ \frac{(dg-ch)qrx(pr\log(a+bx)+qr\log(c+dx)-\log(e(f(a+bx)^p(c+dx)^q)^r))}{d} \\
&+ \frac{pr(g+hx)^2\left(pr\log(a+bx)+qr\log(c+dx)-\log(e(f(a+bx)^p(c+dx)^q)^r)\right)}{2h} \\
&+ \frac{qr(g+hx)^2\left(pr\log(a+bx)+qr\log(c+dx)-\log(e(f(a+bx)^p(c+dx)^q)^r)\right)}{2h} \\
&+ \frac{(bg-ah)^2pr\log(a+bx)\left(pr\log(a+bx)+qr\log(c+dx)-\log(e(f(a+bx)^p(c+dx)^q)^r)\right)}{b^2h} \\
&+ \frac{(dg-ch)^2qr\log(c+dx)\left(pr\log(a+bx)+qr\log(c+dx)-\log(e(f(a+bx)^p(c+dx)^q)^r)\right)}{d^2h} \\
&+ \frac{(g+hx)^2\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h} \\
&+ \frac{(p^2r^2)\,\text{Subst}\left(\int\left(-h(-4bg+4ah-hx)+\frac{2(bg-ah)^2\log(x)}{x}\right)dx,x,a+bx\right)}{2b^2h} \\
&+ \frac{(bpqr^2)\int\left(\frac{h(bg-ah)}{b^2}+\frac{(bg-ah)^2}{b^2(a+bx)}+\frac{h(g+hx)}{b}\right)dx}{2h} \\
&+ \frac{(dpqr^2)\int\left(\frac{h(dg-ch)}{d^2}+\frac{(dg-ch)^2}{d^2(c+dx)}+\frac{h(g+hx)}{d}\right)dx}{2h} \\
&+ \frac{((bg-ah)^2pqr^2)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{bx}{-bc+ad}\right)}{x}dx,x,c+dx\right)}{b^2h} \\
&+ \frac{((dg-ch)^2pqr^2)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{dx}{bc-ad}\right)}{x}dx,x,a+bx\right)}{d^2h} \\
&+ \frac{(q^2r^2)\,\text{Subst}\left(\int\left(-h(-4dg+4ch-hx)+\frac{2(dg-ch)^2\log(x)}{x}\right)dx,x,c+dx\right)}{2d^2h}
\end{aligned}
$$

$$= \frac{3(bg-ah)pqr^2x}{2b} + \frac{3(dg-ch)pqr^2x}{2d} + \frac{pqr^2(g+hx)^2}{2h} + \frac{p^2r^2(4bg-3ah+bhx)^2}{4b^2h} + \frac{q^2r^2(4dg-3ch+dhx)^2}{4d^2h} + \frac{(bg-ah)^2pqr^2\log(a+bx)}{2b^2h} - \frac{2(bg-ah)p^2r^2(a+bx)\log(a+bx)}{b^2} - \frac{(dg-ch)pqr^2(a+bx)\log(a+bx)}{bd} - \frac{hp^2r^2(a+bx)^2\log(a+bx)}{2b^2} - \frac{pqr^2(g+hx)^2\log(a+bx)}{2h} - \frac{(bg-ah)^2p^2r^2\log^2(a+bx)}{b^2h} + \frac{(dg-ch)^2pqr^2\log(c+dx)}{2d^2h} - \frac{(bg-ah)pqr^2(c+dx)\log(c+dx)}{bd} - \frac{2(dg-ch)q^2r^2(c+dx)\log(c+dx)}{d^2} - \frac{hq^2r^2(c+dx)^2\log(c+dx)}{2d^2} - \frac{pqr^2(g+hx)^2\log(c+dx)}{2h} - \frac{(bg-ah)^2pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{b^2h} - \frac{(dg-ch)^2q^2r^2\log^2(c+dx)}{d^2h} - \frac{(dg-ch)^2pqr^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{d^2h} + \frac{(bg-ah)prx(pr\log(a+bx)+qr\log(c+dx)-\log(e(f(a+bx)^p(c+dx)^q)^r))}{b} + \frac{(dg-ch)qrx(pr\log(a+bx)+qr\log(c+dx)-\log(e(f(a+bx)^p(c+dx)^q)^r))}{d} + \frac{pr(g+hx)^2\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{2h} + \frac{qr(g+hx)^2\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{2h} + \frac{(bg-ah)^2pr\log(a+bx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{b^2h} + \frac{(dg-ch)^2qr\log(c+dx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{d^2h} + \frac{(g+hx)^2\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h} - \frac{(dg-ch)^2pqr^2\text{Li}_2\left(-\frac{d(a+bx)}{bc-ad}\right)}{d^2h} - \frac{(bg-ah)^2pqr^2\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)}{b^2h} + \frac{((bg-ah)^2p^2r^2)\,\text{Subst}\left(\int\frac{\log(x)}{x}\,dx,x,a+bx\right)}{b^2h} + \frac{((dg-ch)^2q^2r^2)\,\text{Subst}\left(\int\frac{\log(x)}{x}\,dx,x,c+dx\right)}{d^2h}$$

$$
\begin{aligned}
&= \frac{3(bg-ah)pqr^2x}{2b} + \frac{3(dg-ch)pqr^2x}{2d} + \frac{pqr^2(g+hx)^2}{2h} \\
&+ \frac{p^2r^2(4bg-3ah+bhx)^2}{4b^2h} + \frac{q^2r^2(4dg-3ch+dhx)^2}{4d^2h} \\
&+ \frac{(bg-ah)^2pqr^2\log(a+bx)}{2b^2h} - \frac{2(bg-ah)p^2r^2(a+bx)\log(a+bx)}{b^2} \\
&- \frac{(dg-ch)pqr^2(a+bx)\log(a+bx)}{bd} - \frac{hp^2r^2(a+bx)^2\log(a+bx)}{2b^2} \\
&- \frac{pqr^2(g+hx)^2\log(a+bx)}{2h} - \frac{(bg-ah)^2p^2r^2\log^2(a+bx)}{2b^2h} \\
&+ \frac{(dg-ch)^2pqr^2\log(c+dx)}{2d^2h} - \frac{(bg-ah)pqr^2(c+dx)\log(c+dx)}{bd} \\
&- \frac{2(dg-ch)q^2r^2(c+dx)\log(c+dx)}{d^2} - \frac{hq^2r^2(c+dx)^2\log(c+dx)}{2d^2} \\
&- \frac{pqr^2(g+hx)^2\log(c+dx)}{2h} - \frac{(bg-ah)^2pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{b^2h} \\
&- \frac{(dg-ch)^2q^2r^2\log^2(c+dx)}{2d^2h} - \frac{(dg-ch)^2pqr^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{d^2h} \\
&+ \frac{(bg-ah)prx(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right))}{b} \\
&+ \frac{(dg-ch)qrx(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right))}{d} \\
&+ \frac{pr(g+hx)^2\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{2h} \\
&+ \frac{qr(g+hx)^2\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{2h} \\
&+ \frac{(bg-ah)^2pr\log(a+bx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{b^2h} \\
&+ \frac{(dg-ch)^2qr\log(c+dx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{d^2h} \\
&+ \frac{(g+hx)^2\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h} \\
&- \frac{(dg-ch)^2pqr^2\operatorname{Li}_2\left(-\frac{d(a+bx)}{bc-ad}\right)}{d^2h} - \frac{(bg-ah)^2pqr^2\operatorname{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)}{b^2h}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 480, normalized size of antiderivative = 0.45

$$\int (g+hx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$$

$$= \frac{2ad^2(-2bg+ah)p^2r^2\log^2(a+bx)+2pr\log(a+bx)\left(2b^2c(-2dg+ch)qr\log(c+dx)-2(bc-ad)(-2bdg\right.}{}$$

```
[In] Integrate[(g + h*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] (2*a*d^2*(-2*b*g + a*h)*p^2*r^2*Log[a + b*x]^2 + 2*p*r*Log[a + b*x]*(2*b^2*
c*(-2*d*g + c*h)*q*r*Log[c + d*x] - 2*(b*c - a*d)*(-2*b*d*g + b*c*h + a*d*h
)*q*r*Log[(b*(c + d*x))/(b*c - a*d)] + a*d*(2*b*(-2*d*g + c*h)*q*r + a*d*h*
(3*p + q)*r + (4*b*d*g - 2*a*d*h)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])) +
b*(2*b*c*(-2*d*g + c*h)*q^2*r^2*Log[c + d*x]^2 + 2*q*r*Log[c + d*x]*(2*a*d*
(2*d*g + c*h)*p*r + b*c*(-4*d*g*(p + q) + c*h*(p + 3*q))*r - 2*b*c*(-2*d*g
+ c*h)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]) + d*(r^2*(-2*a*p*(-4*d*g*q + 2
*c*h*q + 3*d*h*(p + q)*x) + b*(p + q)*x*(-6*c*h*q + d*(p + q)*(8*g + h*x)))
 - 2*r*(2*a*d*p*(2*g - h*x) + b*x*(-2*c*h*q + d*(p + q)*(4*g + h*x)))*Log[e
*(f*(a + b*x)^p*(c + d*x)^q)^r] + 2*b*d*x*(2*g + h*x)*Log[e*(f*(a + b*x)^p*
(c + d*x)^q)^r]^2)) - 4*(b*c - a*d)*(-2*b*d*g + b*c*h + a*d*h)*p*q*r^2*Poly
Log[2, (d*(a + b*x))/(-(b*c) + a*d)])/(4*b^2*d^2)
```

Maple [F]

$$\int (hx+g)\ln\left(e\left(f\left(bx+a\right)^{p}\left(dx+c\right)^{q}\right)^{r}\right)^{2}dx$$

```
[In] int((h*x+g)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

```
[Out] int((h*x+g)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

Fricas [F]

$$\int (g+hx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \int (hx+g)\log\left(\left(\left((bx+a)^p(dx+c)^qf\right)^re\right)^2\,dx\right.$$

```
[In] integrate((h*x+g)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="fricas")
```

```
[Out] integral((h*x + g)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

Sympy [F]

$$\int (g+hx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx=\int (g+hx)\log\left(e(f(a+bx)^p\,(c+dx)^q)^r\right)^2dx$$

```
[In] integrate((h*x+g)*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2,x)
```

```
[Out] Integral((g + h*x)*log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 623, normalized size of antiderivative = 0.59

$$\int (g+hx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx=\frac{1}{2}\left(hx^2+2\,gx\right)\log\left(\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2$$

$$+\frac{r\left(\frac{2\,(2\,abfgp-a^2fhp)\log(bx+a)}{b^2}+\frac{2\,(2\,cdfgq-c^2fhq)\log(dx+c)}{d^2}-\frac{bdfh(p+q)x^2-2\,(adfhp-(2\,dfg(p+q)-cfhq)b)x}{bd}\right)\log\left(\left((bx+a\cdots\right.\right.}{2\,f}$$

$$+\frac{r^2\left(\frac{2\,(2\,acdf^2hpq-(4\,(pq+q^2)cdf^2g-(pq+3\,q^2)c^2f^2h)b)\log(dx+c)}{bd^2}-\frac{4\,(2\,abd^2f^2gpq-a^2d^2f^2hpq-(2\,cdf^2gpq-c^2f^2hpq)b^2)\left(\log(bx+a)\cdots\right.}{b^2d^2}\cdots\right.}{\cdots}$$

```
[In] integrate((h*x+g)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(h*x^2 + 2*g*x)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2 + 1/2*r*(2*(2*a*
b*f*g*p - a^2*f*h*p)*log(b*x + a)/b^2 + 2*(2*c*d*f*g*q - c^2*f*h*q)*log(d*x
 + c)/d^2 - (b*d*f*h*(p + q)*x^2 - 2*(a*d*f*h*p - (2*d*f*g*(p + q) - c*f*h*
q)*b)*x)/(b*d))*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/f + 1/4*r^2*(2*(2*a*c*
d*f^2*h*p*q - (4*(p*q + q^2)*c*d*f^2*g - (p*q + 3*q^2)*c^2*f^2*h)*b)*log(d*
x + c)/(b*d^2) - 4*(2*a*b*d^2*f^2*g*p*q - a^2*d^2*f^2*h*p*q - (2*c*d*f^2*g*
p*q - c^2*f^2*h*p*q)*b^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1)
+ dilog(-(b*d*x + a*d)/(b*c - a*d)))/(b^2*d^2) + ((p^2 + 2*p*q + q^2)*b^2*d
^2*f^2*h*x^2 - 4*(2*c*d*f^2*g*p*q - c^2*f^2*h*p*q)*b^2*log(b*x + a)*log(d*x
 + c) - 2*(2*c*d*f^2*g*q^2 - c^2*f^2*h*q^2)*b^2*log(d*x + c)^2 - 2*(2*a*b*d
^2*f^2*g*p^2 - a^2*d^2*f^2*h*p^2)*log(b*x + a)^2 - 2*(3*(p^2 + p*q)*a*b*d^2
*f^2*h - (4*(p^2 + 2*p*q + q^2)*d^2*f^2*g - 3*(p*q + q^2)*c*d*f^2*h)*b^2)*x
 + 2*((3*p^2 + p*q)*a^2*d^2*f^2*h + 2*(c*d*f^2*h*p*q - 2*(p^2 + p*q)*d^2*f^
2*g)*a*b)*log(b*x + a))/(b^2*d^2))/f^2
```

Giac [F]

$$\int (g+hx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \int (hx+g)\log\left(\left(((bx+a)^p(dx+c)^q f\right)^r e\right)^2\,dx$$

```
[In] integrate((h*x+g)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="giac")
[Out] integrate((h*x + g)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (g+hx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \int \ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)^2\,(g+h\,x)\,dx$$

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(g + h*x),x)
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(g + h*x), x)
```

3.38 $\int \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$

Optimal result 359
Rubi [A] (verified) 360
Mathematica [A] (verified) 363
Maple [F] 363
Fricas [F] 364
Sympy [F] 364
Maxima [A] (verification not implemented) 364
Giac [F] 365
Mupad [F(-1)] 365

Optimal result

Integrand size = 23, antiderivative size = 269

$$
\begin{aligned}
&\int \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx \\
&= 2(p+q)^2r^2x-\frac{2(bc-ad)q(p+q)r^2\log(c+dx)}{bd} \\
&\quad-\frac{2(bc-ad)pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{bd}-\frac{(bc-ad)q^2r^2\log^2(c+dx)}{bd} \\
&\quad-\frac{2(p+q)r(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b} \\
&\quad+\frac{2(bc-ad)qr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{bd} \\
&\quad+\frac{(a+bx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b}-\frac{2(bc-ad)pqr^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{bd}
\end{aligned}
$$

```
[Out] 2*(p+q)^2*r^2*x-2*(-a*d+b*c)*q*(p+q)*r^2*ln(d*x+c)/b/d-2*(-a*d+b*c)*p*q*r^2
*ln(-d*(b*x+a)/(-a*d+b*c))*ln(d*x+c)/b/d-(-a*d+b*c)*q^2*r^2*ln(d*x+c)^2/b/d
-2*(p+q)*r*(b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b+2*(-a*d+b*c)*q*r*ln(d*
x+c)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d+(b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^
q)^r)^2/b-2*(-a*d+b*c)*p*q*r^2*polylog(2,b*(d*x+c)/(-a*d+b*c))/b/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {2579, 2580, 2441, 2440, 2438, 2437, 2338, 31, 8}

$$\int \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right) dx = \frac{(a+bx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b} - \frac{2r(p+q)(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b} + \frac{2qr(bc-ad)\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{bd} - \frac{2pqr^2(bc-ad)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{bd} - \frac{2qr^2(p+q)(bc-ad)\log(c+dx)}{bd} - \frac{2pqr^2(bc-ad)\log(c+dx)\log\left(-\frac{d(a+bx)}{bc-ad}\right)}{bd} - \frac{q^2r^2(bc-ad)\log^2(c+dx)}{bd} + 2r^2x(p+q)^2$$

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] 2*(p + q)^2*r^2*x - (2*(b*c - a*d)*q*(p + q)*r^2*Log[c + d*x])/(b*d) - (2*(
b*c - a*d)*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(b*d) -
((b*c - a*d)*q^2*r^2*Log[c + d*x]^2)/(b*d) - (2*(p + q)*r*(a + b*x)*Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r])/b + (2*(b*c - a*d)*q*r*Log[c + d*x]*Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r])/(b*d) + ((a + b*x)*Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^2)/b - (2*(b*c - a*d)*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*
d)])/(b*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
```

qQ[e*f - d*g, 0]

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2579

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[(a + b*x)*(Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s/b), x] + (Dist[q*r*s*((b*c - a*d)/b), Int[Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r]^(s - 1)/(c + d*x), x], x] - Dist[r*s*(p + q), Int[Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]^(s - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p, q,
r, s}, x] && NeQ[b*c - a*d, 0] && NeQ[p + q, 0] && IGtQ[s, 0] && LtQ[s, 4]
```

Rule 2580

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[Log[g + h*x]*(Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]/h), x] + (-Dist[b*p*(r/h), Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[d*q*(r/h), Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\text{integral} = \frac{(a+bx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b} + \frac{(2(bc-ad)qr)\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{c+dx}\,dx}{b}$$
$$-(2(p+q)r)\int \log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$$

$$= -\frac{2(p+q)r(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b}$$
$$+ \frac{2(bc-ad)qr\log(c+dx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{bd}$$
$$+ \frac{(a+bx)\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b}$$
$$- \frac{(2(bc-ad)pqr^2)\int\frac{\log(c+dx)}{a+bx}\,dx}{d} - \frac{(2(bc-ad)q^2r^2)\int\frac{\log(c+dx)}{c+dx}\,dx}{b}$$
$$- \frac{(2(bc-ad)q(p+q)r^2)\int\frac{1}{c+dx}\,dx}{b} + \left(2(p+q)^2r^2\right)\int 1\,dx$$

$$= 2(p+q)^2r^2x - \frac{2(bc-ad)q(p+q)r^2\log(c+dx)}{bd}$$
$$- \frac{2(bc-ad)pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{bd}$$
$$- \frac{2(p+q)r(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b}$$
$$+ \frac{2(bc-ad)qr\log(c+dx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{bd}$$
$$+ \frac{(a+bx)\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b} + \frac{(2(bc-ad)pqr^2)\int\frac{\log\left(\frac{d(a+bx)}{-bc+ad}\right)}{c+dx}\,dx}{b}$$
$$- \frac{(2(bc-ad)q^2r^2)\,\mathrm{Subst}\left(\int\frac{\log(x)}{x}\,dx, x, c+dx\right)}{bd}$$

$$= 2(p+q)^2r^2x - \frac{2(bc-ad)q(p+q)r^2\log(c+dx)}{bd}$$
$$- \frac{2(bc-ad)pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{bd} - \frac{(bc-ad)q^2r^2\log^2(c+dx)}{bd}$$
$$- \frac{2(p+q)r(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b}$$
$$+ \frac{2(bc-ad)qr\log(c+dx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{bd}$$
$$+ \frac{(a+bx)\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b}$$
$$+ \frac{(2(bc-ad)pqr^2)\,\mathrm{Subst}\left(\int\frac{\log\left(1+\frac{bx}{-bc+ad}\right)}{x}\,dx, x, c+dx\right)}{bd}$$

$$= 2(p+q)^2r^2x - \frac{2(bc-ad)q(p+q)r^2\log(c+dx)}{bd} - \frac{2(bc-ad)pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{bd} - \frac{(bc-ad)q^2r^2\log^2(c+dx)}{bd} - \frac{2(p+q)r(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b} + \frac{2(bc-ad)qr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{bd} + \frac{(a+bx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b} - \frac{2(bc-ad)pqr^2\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)}{bd}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.45

$$\int \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right) dx = \frac{2adpqr^2 + 2bdp^2r^2x + 4bdpqr^2x + 2bdq^2r^2x - adp^2r^2\log^2(a+bx) - 2bcpqr^2\log(c+dx) + 2adpqr^2\log(c}{}$$

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] (2*a*d*p*q*r^2 + 2*b*d*p^2*r^2*x + 4*b*d*p*q*r^2*x + 2*b*d*q^2*r^2*x - a*d*
p^2*r^2*Log[a + b*x]^2 - 2*b*c*p*q*r^2*Log[c + d*x] + 2*a*d*p*q*r^2*Log[c +
 d*x] - 2*b*c*q^2*r^2*Log[c + d*x] - b*c*q^2*r^2*Log[c + d*x]^2 - 2*p*r*Log
[a + b*x]*(b*c*q*r*Log[c + d*x] + (-(b*c) + a*d)*q*r*Log[(b*(c + d*x))/(b*c
 - a*d)] + a*d*(q*r - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])) - 2*a*d*p*r*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 2*b*d*p*r*x*Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r] - 2*b*d*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 2*b*c*q*r*L
og[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + b*d*x*Log[e*(f*(a + b*x)
^p*(c + d*x)^q)^r]^2 + 2*(b*c - a*d)*p*q*r^2*PolyLog[2, (d*(a + b*x))/(-(b*
c) + a*d)])/(b*d)
```

Maple [F]

$$\int \ln\left(e\left(f(bx+a)^p\,(dx+c)^q\right)^r\right)^2 dx$$

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

Fricas [F]

$$\int \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \int \log\left(\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2\right)\,dx$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="fricas")
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

Sympy [F]

$$\int \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \int \log\left(e\left(f(a+bx)^p\,(c+dx)^q\right)^r\right)^2\,dx$$

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2,x)
[Out] Integral(log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.11

$$\int \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = x\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2 - \frac{2\left(f(p+q)x - \frac{afp\log(bx+a)}{b} - \frac{cfq\log(dx+c)}{d}\right)r\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{f} - \frac{\left(\frac{2\,(pq+q^2)cf^2\log(dx+c)}{d} - \frac{2\,(bcf^2pq-adf^2pq)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)}{bd} + \frac{adf^2p^2\log(bx+a)^2+2\,bcf^2pq\log(bx+a)\,\mathrm{l}}{}\right.}{f^2}$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="maxima")
[Out] x*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2 - 2*(f*(p + q)*x - a*f*p*log(b*x +
 a)/b - c*f*q*log(d*x + c)/d)*r*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/f - (2
*(p*q + q^2)*c*f^2*log(d*x + c)/d - 2*(b*c*f^2*p*q - a*d*f^2*p*q)*(log(b*x
+ a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d))
)/(b*d) + (a*d*f^2*p^2*log(b*x + a)^2 + 2*b*c*f^2*p*q*log(b*x + a)*log(d*x
+ c) + b*c*f^2*q^2*log(d*x + c)^2 - 2*(p^2 + 2*p*q + q^2)*b*d*f^2*x + 2*(p^
2 + p*q)*a*d*f^2*log(b*x + a))/(b*d))*r^2/f^2
```

Giac [F]

$$\int \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \int \log\left(\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2\,dx\right.$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="giac")
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \int \ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)^2\,dx$$

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2,x)
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2, x)
```

3.39 $\int \frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{g+hx} \, dx$

Optimal result 367
Rubi [A] (verified) 368
Mathematica [A] (verified) 375
Maple [F] 376
Fricas [F] 377
Sympy [F(-1)] 377
Maxima [F] 377
Giac [F] 377
Mupad [F(-1)] 378

Optimal result

Integrand size = 31, antiderivative size = 1471

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{g+hx}\,dx$$
$$= \frac{pqr^2\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log^2\left(\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}\right)}{h}+\frac{p^2r^2\log^2(a+bx)\log(g+hx)}{h}$$
$$+\frac{2pqr^2\log(a+bx)\log(c+dx)\log(g+hx)}{h}+\frac{q^2r^2\log^2(c+dx)\log(g+hx)}{h}$$
$$-\frac{2pr\log(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\log(g+hx)}{h}$$
$$-\frac{2qr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\log(g+hx)}{h}$$
$$+\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\log(g+hx)}{h}-\frac{p^2r^2\log^2(a+bx)\log\left(\frac{b(g+hx)}{bg-ah}\right)}{h}$$
$$-\frac{2pqr^2\log(a+bx)\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log\left(\frac{b(g+hx)}{bg-ah}\right)}{h}+\frac{pqr^2\log^2\left(-\frac{h(c+dx)}{dg-ch}\right)\log\left(\frac{b(g+hx)}{bg-ah}\right)}{h}$$
$$-\frac{2pqr^2\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log\left(\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}\right)\log\left(\frac{b(g+hx)}{bg-ah}\right)}{h}$$
$$+\frac{pqr^2\log^2\left(\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}\right)\log\left(\frac{b(g+hx)}{bg-ah}\right)}{h}$$
$$+\frac{2pr\log(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\log\left(\frac{b(g+hx)}{bg-ah}\right)}{h}$$
$$-\frac{2pqr^2\log(a+bx)\log(c+dx)\log\left(\frac{d(g+hx)}{dg-ch}\right)}{h}-\frac{q^2r^2\log^2(c+dx)\log\left(\frac{d(g+hx)}{dg-ch}\right)}{h}$$
$$+\frac{2pqr^2\log(a+bx)\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log\left(\frac{d(g+hx)}{dg-ch}\right)}{h}-\frac{pqr^2\log^2\left(-\frac{h(c+dx)}{dg-ch}\right)\log\left(\frac{d(g+hx)}{dg-ch}\right)}{h}$$
$$+\frac{2pqr^2\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log\left(\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}\right)\log\left(\frac{d(g+hx)}{dg-ch}\right)}{h}$$
$$+\frac{2qr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\log\left(\frac{d(g+hx)}{dg-ch}\right)}{h}$$
$$-\frac{pqr^2\log^2\left(\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}\right)\log\left(-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}\right)}{h}$$
$$-\frac{2pr\left(qr\log\left(\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}\right)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)\operatorname{PolyLog}\left(2,-\frac{h(a+bx)}{bg-ah}\right)}{h}$$
$$+\frac{2qr\left(pr\log\left(\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}\right)+\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)\operatorname{PolyLog}\left(2,-\frac{h(c+dx)}{dg-ch}\right)}{h}$$
$$+\frac{2pqr^2\log\left(\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{h}$$
$$-\frac{2pqr^2\log\left(\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}\right)\operatorname{PolyLog}\left(2,\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}\right)}{h}$$

```
[Out] -2*p^2*r^2*polylog(3,-h*(b*x+a)/(-a*h+b*g))/h-2*q^2*r^2*polylog(3,-h*(d*x+c
)/(-c*h+d*g))/h+p*q*r^2*ln((a*d-b*c)/d/(b*x+a))*ln((-a*h+b*g)*(d*x+c)/(-c*h
+d*g)/(b*x+a))^2/h+p*q*r^2*ln(-h*(d*x+c)/(-c*h+d*g))^2*ln(b*(h*x+g)/(-a*h+b
*g))/h+p*q*r^2*ln((-a*h+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))^2*ln(b*(h*x+g)/(-a
*h+b*g))/h-p*q*r^2*ln(-h*(d*x+c)/(-c*h+d*g))^2*ln(d*(h*x+g)/(-c*h+d*g))/h-p
*q*r^2*ln((-a*h+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))^2*ln(-(-a*d+b*c)*(h*x+g)/(
-c*h+d*g)/(b*x+a))/h+p^2*r^2*ln(b*x+a)^2*ln(h*x+g)/h+2*p*q*r^2*ln(b*x+a)*ln
(d*x+c)*ln(h*x+g)/h-2*p*q*r^2*ln(b*x+a)*ln(-h*(d*x+c)/(-c*h+d*g))*ln(b*(h*x
+g)/(-a*h+b*g))/h-2*p*q*r^2*ln(-h*(d*x+c)/(-c*h+d*g))*ln((-a*h+b*g)*(d*x+c)
/(-c*h+d*g)/(b*x+a))*ln(b*(h*x+g)/(-a*h+b*g))/h-2*p*q*r^2*ln(b*x+a)*ln(d*x+
c)*ln(d*(h*x+g)/(-c*h+d*g))/h+2*p*q*r^2*ln(b*x+a)*ln(-h*(d*x+c)/(-c*h+d*g))
*ln(d*(h*x+g)/(-c*h+d*g))/h+2*p*q*r^2*ln(-h*(d*x+c)/(-c*h+d*g))*ln((-a*h+b*
g)*(d*x+c)/(-c*h+d*g)/(b*x+a))*ln(d*(h*x+g)/(-c*h+d*g))/h+q^2*r^2*ln(d*x+c)
^2*ln(h*x+g)/h-p^2*r^2*ln(b*x+a)^2*ln(b*(h*x+g)/(-a*h+b*g))/h-q^2*r^2*ln(d*
x+c)^2*ln(d*(h*x+g)/(-c*h+d*g))/h+ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2*ln(h*x+
g)/h-2*p*r*ln(b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*ln(h*x+g)/h-2*q*r*ln(d
*x+c)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*ln(h*x+g)/h+2*p*r*ln(b*x+a)*ln(e*(f*(
b*x+a)^p*(d*x+c)^q)^r)*ln(b*(h*x+g)/(-a*h+b*g))/h+2*q*r*ln(d*x+c)*ln(e*(f*(
b*x+a)^p*(d*x+c)^q)^r)*ln(d*(h*x+g)/(-c*h+d*g))/h+2*p*q*r^2*ln((-a*h+b*g)*(
d*x+c)/(-c*h+d*g)/(b*x+a))*polylog(2,b*(d*x+c)/d/(b*x+a))/h-2*p*q*r^2*ln((-
a*h+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))*polylog(2,(-a*h+b*g)*(d*x+c)/(-c*h+d*g
)/(b*x+a))/h-2*p*r*(q*r*ln((-a*h+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))-ln(e*(f*(
b*x+a)^p*(d*x+c)^q)^r))*polylog(2,-h*(b*x+a)/(-a*h+b*g))/h+2*q*r*(p*r*ln((-
a*h+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))+ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))*polyl
og(2,-h*(d*x+c)/(-c*h+d*g))/h-2*p*q*r^2*polylog(3,-h*(b*x+a)/(-a*h+b*g))/h-
2*p*q*r^2*polylog(3,-h*(d*x+c)/(-c*h+d*g))/h-2*p*q*r^2*polylog(3,b*(d*x+c)/
d/(b*x+a))/h+2*p*q*r^2*polylog(3,(-a*h+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))/h
```

Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 2096, normalized size of antiderivative = 1.42, number of steps used = 29, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.452$, Rules

used = {2583, 2586, 2441, 2440, 2438, 2481, 2422, 2354, 2421, 6724, 2490, 2487, 2485, 2352}

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{g+hx}\,dx$$

$$= -\frac{pq\left(\log\left(\frac{b(c+dx)}{bc-ad}\right)+\log\left(\frac{bg-ah}{b(g+hx)}\right)-\log\left(\frac{(bg-ah)(c+dx)}{(bc-ad)(g+hx)}\right)\right)\log^2\left(-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}\right)r^2}{h}$$

$$+\frac{pq\left(\log\left(\frac{b(c+dx)}{bc-ad}\right)-\log\left(-\frac{h(c+dx)}{dg-ch}\right)\right)\left(\log(a+bx)+\log\left(-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}\right)\right)^2 r^2}{h}$$

$$-\frac{pq\left(\log\left(-\frac{d(a+bx)}{bc-ad}\right)+\log\left(\frac{dg-ch}{d(g+hx)}\right)-\log\left(-\frac{(dg-ch)(a+bx)}{(bc-ad)(g+hx)}\right)\right)\log^2\left(\frac{(bc-ad)(g+hx)}{(bg-ah)(c+dx)}\right)r^2}{h}$$

$$+\frac{pq\left(\log\left(-\frac{d(a+bx)}{bc-ad}\right)-\log\left(-\frac{h(a+bx)}{bg-ah}\right)\right)\left(\log(c+dx)+\log\left(\frac{(bc-ad)(g+hx)}{(bg-ah)(c+dx)}\right)\right)^2 r^2}{h}$$

$$-\frac{2pq\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)\log(g+hx)r^2}{h}$$

$$-\frac{2pq\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(g+hx)r^2}{h}$$

$$-\frac{2pq\left(\log(g+hx)-\log\left(-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}\right)\right)\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)r^2}{h}$$

$$-\frac{2pq\left(\log(g+hx)-\log\left(\frac{(bc-ad)(g+hx)}{(bg-ah)(c+dx)}\right)\right)\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)r^2}{h}$$

$$+\frac{2pq\log\left(-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}\right)\text{PolyLog}\left(2,\frac{h(a+bx)}{b(g+hx)}\right)r^2}{h}$$

$$-\frac{2pq\log\left(-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}\right)\text{PolyLog}\left(2,-\frac{(dg-ch)(a+bx)}{(bc-ad)(g+hx)}\right)r^2}{h}$$

$$+\frac{2pq\log\left(\frac{(bc-ad)(g+hx)}{(bg-ah)(c+dx)}\right)\text{PolyLog}\left(2,\frac{h(c+dx)}{d(g+hx)}\right)r^2}{h}$$

$$-\frac{2pq\log\left(\frac{(bc-ad)(g+hx)}{(bg-ah)(c+dx)}\right)\text{PolyLog}\left(2,\frac{(bg-ah)(c+dx)}{(bc-ad)(g+hx)}\right)r^2}{h}$$

$$-\frac{2pq\left(\log(c+dx)+\log\left(\frac{(bc-ad)(g+hx)}{(bg-ah)(c+dx)}\right)\right)\text{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)r^2}{h}$$

$$-\frac{2pq\left(\log(a+bx)+\log\left(-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}\right)\right)\text{PolyLog}\left(2,\frac{d(g+hx)}{dg-ch}\right)r^2}{h}$$

$$+\frac{2pq\,\text{PolyLog}\left(3,-\frac{d(a+bx)}{bc-ad}\right)r^2}{h}-\frac{2p^2\,\text{PolyLog}\left(3,-\frac{h(a+bx)}{bg-ah}\right)r^2}{h}$$

$$+\frac{2pq\,\text{PolyLog}\left(3,\frac{b(c+dx)}{bc-ad}\right)r^2}{h}-\frac{2q^2\,\text{PolyLog}\left(3,-\frac{h(c+dx)}{dg-ch}\right)r^2}{h}$$

$$+\frac{2pq\,\text{PolyLog}\left(3,\frac{h(a+bx)}{b(g+hx)}\right)r^2}{h}-\frac{2pq\,\text{PolyLog}\left(3,-\frac{(dg-ch)(a+bx)}{(bc-ad)(g+hx)}\right)r^2}{h}$$

$$+\frac{2pq\,\text{PolyLog}\left(3,\frac{h(c+dx)}{d(g+hx)}\right)r^2}{}-\frac{2pq\,\text{PolyLog}\left(3,\frac{(bg-ah)(c+dx)}{(bc-ad)(g+hx)}\right)r^2}{}$$

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(g + h*x),x]

[Out] -((Log[(a + b*x)^(p*r)]^2*Log[g + h*x])/h) - (2*p*q*r^2*Log[-((d*(a + b*x))
/(b*c - a*d))]*Log[c + d*x]*Log[g + h*x])/h - (2*p*q*r^2*Log[a + b*x]*Log[(
b*(c + d*x))/(b*c - a*d)]*Log[g + h*x])/h + (2*q*r*(p*r*Log[a + b*x] - Log[
(a + b*x)^(p*r)])*Log[-((h*(c + d*x))/(d*g - c*h))]*Log[g + h*x])/h + (2*p*
r*Log[-((h*(a + b*x))/(b*g - a*h))]*(q*r*Log[c + d*x] - Log[(c + d*x)^(q*r)
])*Log[g + h*x])/h - (Log[(c + d*x)^(q*r)]^2*Log[g + h*x])/h + (2*p*r*Log[-
((h*(a + b*x))/(b*g - a*h))]*(Log[(a + b*x)^(p*r)] + Log[(c + d*x)^(q*r)] -
 Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])*Log[g + h*x])/h + (2*q*r*Log[-((h*(c
 + d*x))/(d*g - c*h))]*(Log[(a + b*x)^(p*r)] + Log[(c + d*x)^(q*r)] - Log[e
*(f*(a + b*x)^p*(c + d*x)^q)^r])*Log[g + h*x])/h + (Log[e*(f*(a + b*x)^p*(c
 + d*x)^q)^r]^2*Log[g + h*x])/h + (Log[(a + b*x)^(p*r)]^2*Log[(b*(g + h*x))
/(b*g - a*h)])/h + (Log[(c + d*x)^(q*r)]^2*Log[(d*(g + h*x))/(d*g - c*h)])/
h - (p*q*r^2*(Log[(b*(c + d*x))/(b*c - a*d)] + Log[(b*g - a*h)/(b*(g + h*x)
)] - Log[((b*g - a*h)*(c + d*x))/((b*c - a*d)*(g + h*x))])*Log[-(((b*c - a*
d)*(g + h*x))/((d*g - c*h)*(a + b*x)))]^2)/h + (p*q*r^2*(Log[(b*(c + d*x))/
(b*c - a*d)] - Log[-((h*(c + d*x))/(d*g - c*h))])*(Log[a + b*x] + Log[-(((b
*c - a*d)*(g + h*x))/((d*g - c*h)*(a + b*x)))])^2)/h - (p*q*r^2*(Log[-((d*(
a + b*x))/(b*c - a*d))] + Log[(d*g - c*h)/(d*(g + h*x))] - Log[-(((d*g - c*
h)*(a + b*x))/((b*c - a*d)*(g + h*x)))])*Log[((b*c - a*d)*(g + h*x))/((b*g
- a*h)*(c + d*x))]^2)/h + (p*q*r^2*(Log[-((d*(a + b*x))/(b*c - a*d))] - Log
[-((h*(a + b*x))/(b*g - a*h))])*(Log[c + d*x] + Log[((b*c - a*d)*(g + h*x))
/((b*g - a*h)*(c + d*x))])^2)/h - (2*p*q*r^2*(Log[g + h*x] - Log[-(((b*c -
a*d)*(g + h*x))/((d*g - c*h)*(a + b*x)))])*PolyLog[2, -((d*(a + b*x))/(b*c
- a*d))])/h + (2*p*r*Log[(a + b*x)^(p*r)]*PolyLog[2, -((h*(a + b*x))/(b*g -
 a*h))])/h - (2*p*q*r^2*(Log[g + h*x] - Log[((b*c - a*d)*(g + h*x))/((b*g -
 a*h)*(c + d*x))])*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/h + (2*q*r*Log[(c
 + d*x)^(q*r)]*PolyLog[2, -((h*(c + d*x))/(d*g - c*h))])/h + (2*p*q*r^2*Log
[-(((b*c - a*d)*(g + h*x))/((d*g - c*h)*(a + b*x)))]*PolyLog[2, (h*(a + b*x
))/(b*(g + h*x))])/h - (2*p*q*r^2*Log[-(((b*c - a*d)*(g + h*x))/((d*g - c*h
)*(a + b*x)))]*PolyLog[2, -(((d*g - c*h)*(a + b*x))/((b*c - a*d)*(g + h*x))
)])/h + (2*p*q*r^2*Log[((b*c - a*d)*(g + h*x))/((b*g - a*h)*(c + d*x))]*Pol
yLog[2, (h*(c + d*x))/(d*(g + h*x))])/h - (2*p*q*r^2*Log[((b*c - a*d)*(g +
h*x))/((b*g - a*h)*(c + d*x))]*PolyLog[2, ((b*g - a*h)*(c + d*x))/((b*c - a
*d)*(g + h*x))])/h + (2*p*r*(q*r*Log[c + d*x] - Log[(c + d*x)^(q*r)])*PolyL
og[2, (b*(g + h*x))/(b*g - a*h)])/h + (2*p*r*(Log[(a + b*x)^(p*r)] + Log[(c
 + d*x)^(q*r)] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])*PolyLog[2, (b*(g + h
*x))/(b*g - a*h)])/h - (2*p*q*r^2*(Log[c + d*x] + Log[((b*c - a*d)*(g + h*x
))/((b*g - a*h)*(c + d*x))])*PolyLog[2, (b*(g + h*x))/(b*g - a*h)])/h + (2*
q*r*(p*r*Log[a + b*x] - Log[(a + b*x)^(p*r)])*PolyLog[2, (d*(g + h*x))/(d*g
 - c*h)])/h + (2*q*r*(Log[(a + b*x)^(p*r)] + Log[(c + d*x)^(q*r)] - Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r])*PolyLog[2, (d*(g + h*x))/(d*g - c*h)])/h - (
2*p*q*r^2*(Log[a + b*x] + Log[-(((b*c - a*d)*(g + h*x))/((d*g - c*h)*(a + b
*x)))])*PolyLog[2, (d*(g + h*x))/(d*g - c*h)])/h + (2*p*q*r^2*PolyLog[3, -(
```

```
(d*(a + b*x))/(b*c - a*d))])/h - (2*p^2*r^2*PolyLog[3, -((h*(a + b*x))/(b*g
 - a*h))])/h + (2*p*q*r^2*PolyLog[3, (b*(c + d*x))/(b*c - a*d)])/h - (2*q^2
*r^2*PolyLog[3, -((h*(c + d*x))/(d*g - c*h))])/h + (2*p*q*r^2*PolyLog[3, (h
*(a + b*x))/(b*(g + h*x))])/h - (2*p*q*r^2*PolyLog[3, -(((d*g - c*h)*(a + b
*x))/((b*c - a*d)*(g + h*x)))])/h + (2*p*q*r^2*PolyLog[3, (h*(c + d*x))/(d*
(g + h*x))])/h - (2*p*q*r^2*PolyLog[3, ((b*g - a*h)*(c + d*x))/((b*c - a*d)
*(g + h*x))])/h + (2*p*q*r^2*PolyLog[3, (b*(g + h*x))/(b*g - a*h)])/h + (2*
p*q*r^2*PolyLog[3, (d*(g + h*x))/(d*g - c*h)])/h
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2422

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[f*m*(r/(b*n*(p + 1))), Int[x^(m -
 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
```

```
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2485

```
Int[(Log[(a_) + (b_.)*(x_)]*Log[(c_) + (d_.)*(x_)])/(x_), x_Symbol] :> Simp
[Log[(-b)*(x/a)]*Log[a + b*x]*Log[c + d*x], x] + (Simp[(1/2)*(Log[(-b)*(x/a
)] - Log[(-(b*c - a*d))*(x/(a*(c + d*x)))] + Log[(b*c - a*d)/(b*(c + d*x))]
)*Log[a*((c + d*x)/(c*(a + b*x)))]^2, x] - Simp[(1/2)*(Log[(-b)*(x/a)] - Lo
g[(-d)*(x/c)])*(Log[a + b*x] + Log[a*((c + d*x)/(c*(a + b*x)))])^2, x] + Si
mp[(Log[c + d*x] - Log[a*((c + d*x)/(c*(a + b*x)))])*PolyLog[2, 1 + b*(x/a)
], x] + Simp[(Log[a + b*x] + Log[a*((c + d*x)/(c*(a + b*x)))])*PolyLog[2, 1
 + d*(x/c)], x] + Simp[Log[a*((c + d*x)/(c*(a + b*x)))]*PolyLog[2, c*((a +
b*x)/(a*(c + d*x)))], x] - Simp[Log[a*((c + d*x)/(c*(a + b*x)))]*PolyLog[2,
 d*((a + b*x)/(b*(c + d*x)))], x] - Simp[PolyLog[3, 1 + b*(x/a)], x] - Simp
[PolyLog[3, 1 + d*(x/c)], x] + Simp[PolyLog[3, c*((a + b*x)/(a*(c + d*x)))]
, x] - Simp[PolyLog[3, d*((a + b*x)/(b*(c + d*x)))], x]) /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2487

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*Log[(h_.)*((i_.) + (j_.)*(x_))^(m
_.)])/(x_), x_Symbol] :> Dist[m, Int[Log[i + j*x]*(Log[c*(d + e*x)^n]/x), x
], x] - Dist[m*Log[i + j*x] - Log[h*(i + j*x)^m], Int[Log[c*(d + e*x)^n]/x,
 x], x] /; FreeQ[{c, d, e, h, i, j, m, n}, x] && NeQ[e*i - d*j, 0] && NeQ[i
 + j*x, h*(i + j*x)^m]
```

Rule 2490

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.)
*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_) + (l_.)*(x_))^(r_.), x_Symbol] :>
 Dist[1/l, Subst[Int[x^r*(a + b*Log[c*(-(e*k - d*l)/l + e*(x/l))^n])*(f + g
```

```
*Log[h*(-(j*k - i*l)/l + j*(x/l))^m]), x], x, k + l*x], x] /; FreeQ[{a, b,
c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]
```

Rule 2583

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^2/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[Log[g + h*x]*(Log[e*(f*(a
 + b*x)^p*(c + d*x)^q)^r]^2/h), x] + (-Dist[2*b*p*(r/h), Int[Log[g + h*x]*(
Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)), x], x] - Dist[2*d*q*(r/h),
 Int[Log[g + h*x]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(c + d*x)), x], x])
 /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rule 2586

```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.)))/((j_.) + (k
_.)*(x_)), x_Symbol] :> Dist[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - Log[(a
+ b*x)^(p*r)] - Log[(c + d*x)^(q*r)], Int[(s + t*Log[i*(g + h*x)^n])/(j + k
*x), x], x] + (Int[(Log[(a + b*x)^(p*r)]*(s + t*Log[i*(g + h*x)^n]))/(j + k
*x), x] + Int[(Log[(c + d*x)^(q*r)]*(s + t*Log[i*(g + h*x)^n]))/(j + k*x),
x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, n, p, q, r}, x] && NeQ
[b*c - a*d, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} = & \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\log(g+hx)}{h} \\
& - \frac{(2bpr)\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\log(g+hx)}{a+bx}\,dx}{h} \\
& - \frac{(2dqr)\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\log(g+hx)}{c+dx}\,dx}{h}
\end{aligned}
$$

$$= \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\log(g+hx)}{h} - \frac{(2bpr)\int \frac{\log((a+bx)^{pr})\log(g+hx)}{a+bx}\,dx}{h} - \frac{(2bpr)\int \frac{\log((c+dx)^{qr})\log(g+hx)}{a+bx}\,dx}{h} - \frac{(2dqr)\int \frac{\log((a+bx)^{pr})\log(g+hx)}{c+dx}\,dx}{h} - \frac{(2dqr)\int \frac{\log((c+dx)^{qr})\log(g+hx)}{c+dx}\,dx}{h} - \frac{(2bpr(-\log((a+bx)^{pr}) - \log((c+dx)^{qr}) + \log(e(f(a+bx)^p(c+dx)^q)^r)))\int \frac{\log(g+hx)}{a+bx}\,dx}{h} - \frac{(2dqr(-\log((a+bx)^{pr}) - \log((c+dx)^{qr}) + \log(e(f(a+bx)^p(c+dx)^q)^r)))\int \frac{\log(g+hx)}{c+dx}\,dx}{h}$$

$$= \frac{2pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\left(\log((a+bx)^{pr}) + \log((c+dx)^{qr}) - \log(e(f(a+bx)^p(c+dx)^q)^r)\right)\log(g+hx)}{h} + \frac{2qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)\left(\log((a+bx)^{pr}) + \log((c+dx)^{qr}) - \log(e(f(a+bx)^p(c+dx)^q)^r)\right)\log(g+h}{h} + \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\log(g+hx)}{h} - \frac{(2pr)\text{Subst}\left(\int \frac{\log(x^{pr})\log\left(\frac{bg-ah}{b}+\frac{hx}{b}\right)}{x}\,dx, x, a+bx\right)}{h} - \frac{(2pr)\text{Subst}\left(\int \frac{\log\left(\left(\frac{bc-ad}{b}+\frac{dx}{b}\right)^{qr}\right)\log\left(\frac{bg-ah}{b}+\frac{hx}{b}\right)}{x}\,dx, x, a+bx\right)}{h} - \frac{(2qr)\text{Subst}\left(\int \frac{\log(x^{qr})\log\left(\frac{dg-ch}{d}+\frac{hx}{d}\right)}{x}\,dx, x, c+dx\right)}{h} - \frac{(2qr)\text{Subst}\left(\int \frac{\log\left(\left(\frac{-bc+ad}{d}+\frac{bx}{d}\right)^{pr}\right)\log\left(\frac{dg-ch}{d}+\frac{hx}{d}\right)}{x}\,dx, x, c+dx\right)}{h} + (2pr(-\log((a+bx)^{pr}) - \log((c+dx)^{qr}) + \log(e(f(a+bx)^p(c+dx)^q)^r)))\int \frac{\log\left(\frac{h(a+bx)}{-bg+ah}\right)}{g+hx}\,dx + (2qr(-\log((a+bx)^{pr}) - \log((c+dx)^{qr}) + \log(e(f(a+bx)^p(c+dx)^q)^r)))\int \frac{\log\left(\frac{h(c+dx)}{-dg+ch}\right)}{g+hx}\,dx$$

$$= -\frac{\log^2((a+bx)^{pr})\log(g+hx)}{h} - \frac{\log^2((c+dx)^{qr})\log(g+hx)}{h}$$
$$+ \frac{2pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\left(\log((a+bx)^{pr})+\log((c+dx)^{qr})-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)\log(g+}{h}$$
$$+ \frac{2qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)\left(\log((a+bx)^{pr})+\log((c+dx)^{qr})-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)\log(g+}{h}$$
$$+ \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\log(g+hx)}{h}$$
$$+ \frac{\text{Subst}\left(\int \frac{\log^2(x^{pr})}{\frac{bg-ah}{b}+\frac{hx}{b}}dx, x, a+bx\right)}{b} + \frac{\text{Subst}\left(\int \frac{\log^2(x^{qr})}{\frac{dg-ch}{d}+\frac{hx}{d}}dx, x, c+dx\right)}{d}$$
$$- \frac{(2pqr^2)\,\text{Subst}\left(\int \frac{\log\left(\frac{bc-ad}{b}+\frac{dx}{b}\right)\log\left(\frac{bg-ah}{b}+\frac{hx}{b}\right)}{x}dx, x, a+bx\right)}{h}$$
$$- \frac{(2pqr^2)\,\text{Subst}\left(\int \frac{\log\left(\frac{-bc+ad}{d}+\frac{bx}{d}\right)\log\left(\frac{dg-ch}{d}+\frac{hx}{d}\right)}{x}dx, x, c+dx\right)}{h}$$
$$+ \frac{(2qr(pr\log(a+bx)-\log((a+bx)^{pr})))\,\text{Subst}\left(\int \frac{\log\left(\frac{dg-ch}{d}+\frac{hx}{d}\right)}{x}dx, x, c+dx\right)}{h}$$
$$+ \frac{(2pr(qr\log(c+dx)-\log((c+dx)^{qr})))\,\text{Subst}\left(\int \frac{\log\left(\frac{bg-ah}{b}+\frac{hx}{b}\right)}{x}dx, x, a+bx\right)}{h}$$
$$+ \frac{(2pr(-\log((a+bx)^{pr})-\log((c+dx)^{qr})+\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)))\,\text{Subst}\left(\int \frac{\log\left(1+\frac{bx}{-bg+}\right.}{x}\right.}{h}$$
$$+ \frac{(2qr(-\log((a+bx)^{pr})-\log((c+dx)^{qr})+\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)))\,\text{Subst}\left(\int \frac{\log\left(1+\frac{dx}{-dg+}\right.}{x}\right.}{h}$$

= Too large to display

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 1370, normalized size of antiderivative = 0.93

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{g+hx}dx$$
$$= \frac{pqr^2\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log^2\left(\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}\right)+p^2r^2\log^2(a+bx)\log(g+hx)+2pqr^2\log(a+bx)\log(c+dx)\log}{}$$

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(g + h*x),x]
```

```
[Out] (p*q*r^2*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*g - a*h)*(c + d*x))/((d*
g - c*h)*(a + b*x))]^2 + p^2*r^2*Log[a + b*x]^2*Log[g + h*x] + 2*p*q*r^2*Lo
g[a + b*x]*Log[c + d*x]*Log[g + h*x] + q^2*r^2*Log[c + d*x]^2*Log[g + h*x]
- 2*p*r*Log[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[g + h*x] - 2*
q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[g + h*x] + Log[e*
(f*(a + b*x)^p*(c + d*x)^q)^r]^2*Log[g + h*x] - p^2*r^2*Log[a + b*x]^2*Log[
(b*(g + h*x))/(b*g - a*h)] - 2*p*q*r^2*Log[a + b*x]*Log[(h*(c + d*x))/(-(d*
g) + c*h)]*Log[(b*(g + h*x))/(b*g - a*h)] + p*q*r^2*Log[(h*(c + d*x))/(-(d*
g) + c*h)]^2*Log[(b*(g + h*x))/(b*g - a*h)] - 2*p*q*r^2*Log[(h*(c + d*x))/(
-(d*g) + c*h)]*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Log[(b*
(g + h*x))/(b*g - a*h)] + p*q*r^2*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*
(a + b*x))]^2*Log[(b*(g + h*x))/(b*g - a*h)] + 2*p*r*Log[a + b*x]*Log[e*(f*
(a + b*x)^p*(c + d*x)^q)^r]*Log[(b*(g + h*x))/(b*g - a*h)] - 2*p*q*r^2*Log[
a + b*x]*Log[c + d*x]*Log[(d*(g + h*x))/(d*g - c*h)] - q^2*r^2*Log[c + d*x]
^2*Log[(d*(g + h*x))/(d*g - c*h)] + 2*p*q*r^2*Log[a + b*x]*Log[(h*(c + d*x)
)/(-(d*g) + c*h)]*Log[(d*(g + h*x))/(d*g - c*h)] - p*q*r^2*Log[(h*(c + d*x)
)/(-(d*g) + c*h)]^2*Log[(d*(g + h*x))/(d*g - c*h)] + 2*p*q*r^2*Log[(h*(c +
d*x))/(-(d*g) + c*h)]*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*
Log[(d*(g + h*x))/(d*g - c*h)] + 2*q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c
 + d*x)^q)^r]*Log[(d*(g + h*x))/(d*g - c*h)] - p*q*r^2*Log[((b*g - a*h)*(c
+ d*x))/((d*g - c*h)*(a + b*x))]^2*Log[((-(b*c) + a*d)*(g + h*x))/((d*g - c
*h)*(a + b*x))] + 2*p*r*(-(q*r*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a
+ b*x))]) + Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])*PolyLog[2, (h*(a + b*x))/
(-(b*g) + a*h)] + 2*q*r*(p*r*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a +
b*x))] + Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])*PolyLog[2, (h*(c + d*x))/(-(
d*g) + c*h)] + 2*p*q*r^2*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x)
)]*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] - 2*p*q*r^2*Log[((b*g - a*h)*(c
+ d*x))/((d*g - c*h)*(a + b*x))]*PolyLog[2, ((b*g - a*h)*(c + d*x))/((d*g -
 c*h)*(a + b*x))] - 2*p^2*r^2*PolyLog[3, (h*(a + b*x))/(-(b*g) + a*h)] - 2*
p*q*r^2*PolyLog[3, (h*(a + b*x))/(-(b*g) + a*h)] - 2*p*q*r^2*PolyLog[3, (h*
(c + d*x))/(-(d*g) + c*h)] - 2*q^2*r^2*PolyLog[3, (h*(c + d*x))/(-(d*g) + c
*h)] - 2*p*q*r^2*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))] + 2*p*q*r^2*PolyLo
g[3, ((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))])/h
```

Maple [F]

$$\int \frac{\ln \left(e\left(f(bx+a)^p\,(dx+c)^q\right)^r\right)^2}{hx+g}dx$$

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g),x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g),x)
```

Fricas [F]

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{g+hx}\,dx=\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{hx+g}\,dx$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g),x, algorithm="fricas")
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(h*x + g), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{g+hx}\,dx=\text{Timed out}$$

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(h*x+g),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{g+hx}\,dx=\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{hx+g}\,dx$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g),x, algorithm="maxima")
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(h*x + g), x)
```

Giac [F]

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{g+hx}\,dx=\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{hx+g}\,dx$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g),x, algorithm="giac")
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(h*x + g), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{g+hx}\,dx = \int \frac{\ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)^2}{g+h\,x}\,dx$$

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(g + h*x),x)
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(g + h*x), x)
```

3.40 $\int \frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{(g+hx)^2}\,dx$

Optimal result 380
Rubi [A] (verified) 381
Mathematica [B] (verified) 388
Maple [F] 390
Fricas [F] 390
Sympy [F(-1)] 391
Maxima [A] (verification not implemented) 391
Giac [F] 392
Mupad [F(-1)] 392

Optimal result

Integrand size = 31, antiderivative size = 832

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^2}\,dx$$
$$= \frac{2bpqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{h(bg-ah)} + \frac{2dpqr^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{h(dg-ch)}$$
$$- \frac{2bpr\log(a+bx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{h(bg-ah)}$$
$$- \frac{2dqr\log(c+dx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{h(dg-ch)}$$
$$- \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h(g+hx)}$$
$$+ \frac{2bpr\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)\log(g+hx)}{h(bg-ah)}$$
$$+ \frac{2dqr\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)\log(g+hx)}{h(dg-ch)}$$
$$- \frac{2dpqr^2\log(a+bx)\log\left(\frac{b(g+hx)}{bg-ah}\right)}{h(dg-ch)} - \frac{2bpqr^2\log(c+dx)\log\left(\frac{d(g+hx)}{dg-ch}\right)}{h(bg-ah)}$$
$$- \frac{2bp^2r^2\log(a+bx)\log\left(1+\frac{bg-ah}{h(a+bx)}\right)}{h(bg-ah)} - \frac{2dq^2r^2\log(c+dx)\log\left(1+\frac{dg-ch}{h(c+dx)}\right)}{h(dg-ch)}$$
$$+ \frac{2bp^2r^2\,\text{PolyLog}\left(2,-\frac{bg-ah}{h(a+bx)}\right)}{h(bg-ah)} + \frac{2dpqr^2\,\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{h(dg-ch)}$$
$$- \frac{2dpqr^2\,\text{PolyLog}\left(2,-\frac{h(a+bx)}{bg-ah}\right)}{h(dg-ch)} + \frac{2dq^2r^2\,\text{PolyLog}\left(2,-\frac{dg-ch}{h(c+dx)}\right)}{h(dg-ch)}$$
$$+ \frac{2bpqr^2\,\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{h(bg-ah)} - \frac{2bpqr^2\,\text{PolyLog}\left(2,-\frac{h(c+dx)}{dg-ch}\right)}{h(bg-ah)}$$

```
[Out] 2*b*p*q*r^2*ln(-d*(b*x+a)/(-a*d+b*c))*ln(d*x+c)/h/(-a*h+b*g)+2*d*p*q*r^2*ln
(b*x+a)*ln(b*(d*x+c)/(-a*d+b*c))/h/(-c*h+d*g)-2*b*p*r*ln(b*x+a)*(p*r*ln(b*x
+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/h/(-a*h+b*g)-2*d*q*r*ln(
d*x+c)*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/h/(-c*
h+d*g)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/h/(h*x+g)+2*b*p*r*(p*r*ln(b*x+a)+q
*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))*ln(h*x+g)/h/(-a*h+b*g)+2*d*q*
r*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))*ln(h*x+g)/h
/(-c*h+d*g)-2*d*p*q*r^2*ln(b*x+a)*ln(b*(h*x+g)/(-a*h+b*g))/h/(-c*h+d*g)-2*b
*p*q*r^2*ln(d*x+c)*ln(d*(h*x+g)/(-c*h+d*g))/h/(-a*h+b*g)-2*b*p^2*r^2*ln(b*x
+a)*ln(1+(-a*h+b*g)/h/(b*x+a))/h/(-a*h+b*g)-2*d*q^2*r^2*ln(d*x+c)*ln(1+(-c*
```

```
h+d*g)/h/(d*x+c))/h/(-c*h+d*g)+2*b*p^2*r^2*polylog(2,(a*h-b*g)/h/(b*x+a))/h
/(-a*h+b*g)+2*d*p*q*r^2*polylog(2,-d*(b*x+a)/(-a*d+b*c))/h/(-c*h+d*g)-2*d*p
*q*r^2*polylog(2,-h*(b*x+a)/(-a*h+b*g))/h/(-c*h+d*g)+2*d*q^2*r^2*polylog(2,
(c*h-d*g)/h/(d*x+c))/h/(-c*h+d*g)+2*b*p*q*r^2*polylog(2,b*(d*x+c)/(-a*d+b*c
))/h/(-a*h+b*g)-2*b*p*q*r^2*polylog(2,-h*(d*x+c)/(-c*h+d*g))/h/(-a*h+b*g)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 832, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {2584, 2593, 2458, 2379, 2438, 2465, 2441, 2440, 36, 31}

$$
\begin{aligned}
&\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^2}\,dx \\
&= \frac{2bpq\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)r^2}{h(bg-ah)} + \frac{2dpq\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)r^2}{h(dg-ch)} \\
&\quad - \frac{2dpq\log(a+bx)\log\left(\frac{b(g+hx)}{bg-ah}\right)r^2}{h(dg-ch)} - \frac{2bpq\log(c+dx)\log\left(\frac{d(g+hx)}{dg-ch}\right)r^2}{h(bg-ah)} \\
&\quad - \frac{2bp^2\log(a+bx)\log\left(\frac{bg-ah}{h(a+bx)}+1\right)r^2}{h(bg-ah)} - \frac{2dq^2\log(c+dx)\log\left(\frac{dg-ch}{h(c+dx)}+1\right)r^2}{h(dg-ch)} \\
&\quad + \frac{2bp^2\,\text{PolyLog}\left(2,-\frac{bg-ah}{h(a+bx)}\right)r^2}{h(bg-ah)} + \frac{2dpq\,\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)r^2}{h(dg-ch)} \\
&\quad - \frac{2dpq\,\text{PolyLog}\left(2,-\frac{h(a+bx)}{bg-ah}\right)r^2}{h(dg-ch)} + \frac{2dq^2\,\text{PolyLog}\left(2,-\frac{dg-ch}{h(c+dx)}\right)r^2}{h(dg-ch)} \\
&\quad + \frac{2bpq\,\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)r^2}{h(bg-ah)} - \frac{2bpq\,\text{PolyLog}\left(2,-\frac{h(c+dx)}{dg-ch}\right)r^2}{h(bg-ah)} \\
&\quad - \frac{2bp\log(a+bx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)r}{h(bg-ah)} \\
&\quad - \frac{2dq\log(c+dx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)r}{h(dg-ch)} \\
&\quad + \frac{2bp\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)\log(g+hx)r}{h(bg-ah)} \\
&\quad + \frac{2dq\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)\log(g+hx)r}{h(dg-ch)} \\
&\quad - \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h(g+hx)}
\end{aligned}
$$

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(g + h*x)^2,x]
```

```
[Out] (2*b*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(h*(b*g - a*h)
) + (2*d*p*q*r^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(h*(d*g - c*h
)) - (2*b*p*r*Log[a + b*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*
(a + b*x)^p*(c + d*x)^q)^r]))/(h*(b*g - a*h)) - (2*d*q*r*Log[c + d*x]*(p*r*
Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(h
*(d*g - c*h)) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(h*(g + h*x)) + (2*b
*p*r*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r])*Log[g + h*x])/(h*(b*g - a*h)) + (2*d*q*r*(p*r*Log[a + b*x] + q*r*Log
[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])*Log[g + h*x])/(h*(d*g - c
*h)) - (2*d*p*q*r^2*Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)])/(h*(d*g -
c*h)) - (2*b*p*q*r^2*Log[c + d*x]*Log[(d*(g + h*x))/(d*g - c*h)])/(h*(b*g -
 a*h)) - (2*b*p^2*r^2*Log[a + b*x]*Log[1 + (b*g - a*h)/(h*(a + b*x))])/(h*(
b*g - a*h)) - (2*d*q^2*r^2*Log[c + d*x]*Log[1 + (d*g - c*h)/(h*(c + d*x))])
/(h*(d*g - c*h)) + (2*b*p^2*r^2*PolyLog[2, -((b*g - a*h)/(h*(a + b*x)))])/(
h*(b*g - a*h)) + (2*d*p*q*r^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(h*
(d*g - c*h)) - (2*d*p*q*r^2*PolyLog[2, -((h*(a + b*x))/(b*g - a*h))])/(h*(d
*g - c*h)) + (2*d*q^2*r^2*PolyLog[2, -((d*g - c*h)/(h*(c + d*x)))])/(h*(d*g
 - c*h)) + (2*b*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(h*(b*g - a*
h)) - (2*b*p*q*r^2*PolyLog[2, -((h*(c + d*x))/(d*g - c*h))])/(h*(b*g - a*h)
)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
```

```
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2584

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1
)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(h*(m + 1))), x] + (-Dist[b*p*r*(
s/(h*(m + 1))), Int[(g + h*x)^(m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1)/(a + b*x)), x], x] - Dist[d*q*r*(s/(h*(m + 1))), Int[(g + h*x)^(m
+ 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(c + d*x)), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2593

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h(g+hx)}+\frac{(2bpr)\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)(g+hx)}\,dx}{h}\\
&\quad+\frac{(2dqr)\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(c+dx)(g+hx)}\,dx}{h}\\
&= -\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h(g+hx)}+\frac{\left(2bp^2r^2\right)\int \frac{\log(a+bx)}{(a+bx)(g+hx)}\,dx}{h}\\
&\quad+\frac{\left(2bpqr^2\right)\int \frac{\log(c+dx)}{(a+bx)(g+hx)}\,dx}{h}+\frac{\left(2dpqr^2\right)\int \frac{\log(a+bx)}{(c+dx)(g+hx)}\,dx}{h}+\frac{\left(2dq^2r^2\right)\int \frac{\log(c+dx)}{(c+dx)(g+hx)}\,dx}{h}\\
&\quad-\frac{\left(2bpr\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)\right)\int \frac{1}{(a+bx)(g+hx)}\,dx}{h}\\
&\quad-\frac{\left(2dqr\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)\right)\int \frac{1}{(c+dx)(g+hx)}\,dx}{h}\\
&= -\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h(g+hx)}+\frac{\left(2p^2r^2\right)\text{Subst}\left(\int \frac{\log(x)}{x\left(\frac{bg-ah}{b}+\frac{hx}{b}\right)}\,dx,x,a+bx\right)}{h}\\
&\quad+\frac{\left(2bpqr^2\right)\int \left(\frac{b\log(c+dx)}{(bg-ah)(a+bx)}-\frac{h\log(c+dx)}{(bg-ah)(g+hx)}\right)dx}{h}\\
&\quad+\frac{\left(2dpqr^2\right)\int \left(\frac{d\log(a+bx)}{(dg-ch)(c+dx)}-\frac{h\log(a+bx)}{(dg-ch)(g+hx)}\right)dx}{h}\\
&\quad+\frac{\left(2q^2r^2\right)\text{Subst}\left(\int \frac{\log(x)}{x\left(\frac{dg-ch}{d}+\frac{hx}{d}\right)}\,dx,x,c+dx\right)}{h}\\
&\quad+\frac{\left(2bpr\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)\right)\int \frac{1}{g+hx}\,dx}{bg-ah}\\
&\quad-\frac{\left(2b^2pr\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)\right)\int \frac{1}{a+bx}\,dx}{h(bg-ah)}\\
&\quad+\frac{\left(2dqr\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)\right)\int \frac{1}{g+hx}\,dx}{dg-ch}\\
&\quad-\frac{\left(2d^2qr\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)\right)\int \frac{1}{c+dx}\,dx}{h(dg-ch)}
\end{aligned}
$$

$$= -\frac{2bpr\log(a+bx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{h(bg-ah)} - \frac{2dqr\log(c+dx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{h(dg-ch)} - \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h(g+hx)} + \frac{2bpr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right))\log(g+hx)}{h(bg-ah)} + \frac{2dqr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right))\log(g+hx)}{h(dg-ch)} - \frac{2bp^2r^2\log(a+bx)\log\left(1+\frac{bg-ah}{h(a+bx)}\right)}{h(bg-ah)} - \frac{2dq^2r^2\log(c+dx)\log\left(1+\frac{dg-ch}{h(c+dx)}\right)}{h(dg-ch)} + \frac{(2bp^2r^2)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{bg-ah}{hx}\right)}{x}\,dx,x,a+bx\right)}{h(bg-ah)} - \frac{(2bpqr^2)\int\frac{\log(c+dx)}{g+hx}\,dx}{bg-ah} + \frac{(2b^2pqr^2)\int\frac{\log(c+dx)}{a+bx}\,dx}{h(bg-ah)} - \frac{(2dpqr^2)\int\frac{\log(a+bx)}{g+hx}\,dx}{dg-ch} + \frac{(2d^2pqr^2)\int\frac{\log(a+bx)}{c+dx}\,dx}{h(dg-ch)} + \frac{(2dq^2r^2)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{dg-ch}{hx}\right)}{x}\,dx,x,c+dx\right)}{h(dg-ch)}$$

$$
\begin{aligned}
&= \frac{2bpqr^2 \log \left(-\frac{d(a+bx)}{bc-ad}\right) \log(c+dx)}{h(bg-ah)} + \frac{2dpqr^2 \log(a+bx) \log \left(\frac{b(c+dx)}{bc-ad}\right)}{h(dg-ch)} \\
&\quad - \frac{2bpr \log(a+bx) \left(pr \log(a+bx) + qr \log(c+dx) - \log \left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{h(bg-ah)} \\
&\quad - \frac{2dqr \log(c+dx) \left(pr \log(a+bx) + qr \log(c+dx) - \log \left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{h(dg-ch)} \\
&\quad - \frac{\log^2 \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h(g+hx)} \\
&\quad + \frac{2bpr(pr \log(a+bx) + qr \log(c+dx) - \log \left(e(f(a+bx)^p(c+dx)^q)^r\right)) \log(g+hx)}{h(bg-ah)} \\
&\quad + \frac{2dqr(pr \log(a+bx) + qr \log(c+dx) - \log \left(e(f(a+bx)^p(c+dx)^q)^r\right)) \log(g+hx)}{h(dg-ch)} \\
&\quad - \frac{2dpqr^2 \log(a+bx) \log \left(\frac{b(g+hx)}{bg-ah}\right)}{h(dg-ch)} - \frac{2bpqr^2 \log(c+dx) \log \left(\frac{d(g+hx)}{dg-ch}\right)}{h(bg-ah)} \\
&\quad - \frac{2bp^2r^2 \log(a+bx) \log \left(1 + \frac{bg-ah}{h(a+bx)}\right)}{h(bg-ah)} - \frac{2dq^2r^2 \log(c+dx) \log \left(1 + \frac{dg-ch}{h(c+dx)}\right)}{h(dg-ch)} \\
&\quad + \frac{2bp^2r^2 \operatorname{Li}_2\left(-\frac{bg-ah}{h(a+bx)}\right)}{h(bg-ah)} + \frac{2dq^2r^2 \operatorname{Li}_2\left(-\frac{dg-ch}{h(c+dx)}\right)}{h(dg-ch)} \\
&\quad - \frac{(2bdpqr^2) \int \frac{\log\left(\frac{d(a+bx)}{-bc+ad}\right)}{c+dx}\, dx}{h(bg-ah)} + \frac{(2bdpqr^2) \int \frac{\log\left(\frac{d(g+hx)}{dg-ch}\right)}{c+dx}\, dx}{h(bg-ah)} \\
&\quad - \frac{(2bdpqr^2) \int \frac{\log\left(\frac{b(c+dx)}{bc-ad}\right)}{a+bx}\, dx}{h(dg-ch)} + \frac{(2bdpqr^2) \int \frac{\log\left(\frac{b(g+hx)}{bg-ah}\right)}{a+bx}\, dx}{h(dg-ch)}
\end{aligned}
$$

$$= \frac{2bpqr^2 \log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{h(bg-ah)} + \frac{2dpqr^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{h(dg-ch)}$$
$$- \frac{2bpr\log(a+bx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{h(bg-ah)}$$
$$- \frac{2dqr\log(c+dx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{h(dg-ch)}$$
$$- \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h(g+hx)}$$
$$+ \frac{2bpr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right))\log(g+hx)}{h(bg-ah)}$$
$$+ \frac{2dqr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right))\log(g+hx)}{h(dg-ch)}$$
$$- \frac{2dpqr^2\log(a+bx)\log\left(\frac{b(g+hx)}{bg-ah}\right)}{h(dg-ch)} - \frac{2bpqr^2\log(c+dx)\log\left(\frac{d(g+hx)}{dg-ch}\right)}{h(bg-ah)}$$
$$- \frac{2bp^2r^2\log(a+bx)\log\left(1+\frac{bg-ah}{h(a+bx)}\right)}{h(bg-ah)} - \frac{2dq^2r^2\log(c+dx)\log\left(1+\frac{dg-ch}{h(c+dx)}\right)}{h(dg-ch)}$$
$$+ \frac{2bp^2r^2\text{Li}_2\left(-\frac{bg-ah}{h(a+bx)}\right)}{h(bg-ah)} + \frac{2dq^2r^2\text{Li}_2\left(-\frac{dg-ch}{h(c+dx)}\right)}{h(dg-ch)}$$
$$- \frac{(2bpqr^2)\,\text{Subst}\left(\int \frac{\log\left(1+\frac{bx}{-bc+ad}\right)}{x}\,dx, x, c+dx\right)}{h(bg-ah)}$$
$$+ \frac{(2bpqr^2)\,\text{Subst}\left(\int \frac{\log\left(1+\frac{hx}{dg-ch}\right)}{x}\,dx, x, c+dx\right)}{h(bg-ah)}$$
$$- \frac{(2dpqr^2)\,\text{Subst}\left(\int \frac{\log\left(1+\frac{dx}{bc-ad}\right)}{x}\,dx, x, a+bx\right)}{h(dg-ch)}$$
$$+ \frac{(2dpqr^2)\,\text{Subst}\left(\int \frac{\log\left(1+\frac{hx}{bg-ah}\right)}{x}\,dx, x, a+bx\right)}{h(dg-ch)}$$

$$= \frac{2bpqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{h(bg-ah)} + \frac{2dpqr^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{h(dg-ch)}$$
$$- \frac{2bpr\log(a+bx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\right)}{h(bg-ah)}$$
$$- \frac{2dqr\log(c+dx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\right)}{h(dg-ch)}$$
$$- \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{h(g+hx)}$$
$$+ \frac{2bpr\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\right)\log(g+hx)}{h(bg-ah)}$$
$$+ \frac{2dqr\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\right)\log(g+hx)}{h(dg-ch)}$$
$$- \frac{2dpqr^2\log(a+bx)\log\left(\frac{b(g+hx)}{bg-ah}\right)}{h(dg-ch)} - \frac{2bpqr^2\log(c+dx)\log\left(\frac{d(g+hx)}{dg-ch}\right)}{h(bg-ah)}$$
$$- \frac{2bp^2r^2\log(a+bx)\log\left(1+\frac{bg-ah}{h(a+bx)}\right)}{h(bg-ah)} - \frac{2dq^2r^2\log(c+dx)\log\left(1+\frac{dg-ch}{h(c+dx)}\right)}{h(dg-ch)}$$
$$+ \frac{2bp^2r^2\operatorname{Li}_2\left(-\frac{bg-ah}{h(a+bx)}\right)}{h(bg-ah)} + \frac{2dpqr^2\operatorname{Li}_2\left(-\frac{d(a+bx)}{bc-ad}\right)}{h(dg-ch)} - \frac{2dpqr^2\operatorname{Li}_2\left(-\frac{h(a+bx)}{bg-ah}\right)}{h(dg-ch)}$$
$$+ \frac{2dq^2r^2\operatorname{Li}_2\left(-\frac{dg-ch}{h(c+dx)}\right)}{h(dg-ch)} + \frac{2bpqr^2\operatorname{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)}{h(bg-ah)} - \frac{2bpqr^2\operatorname{Li}_2\left(-\frac{h(c+dx)}{dg-ch}\right)}{h(bg-ah)}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2930 vs. 2(832) = 1664.

Time = 0.53 (sec) , antiderivative size = 2930, normalized size of antiderivative = 3.52

$$\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^2}\,dx = \text{Result too large to show}$$

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(g + h*x)^2,x]
```

```
[Out] (-(b*d*g^2*p^2*r^2*Log[a + b*x]^2) + b*c*g*h*p^2*r^2*Log[a + b*x]^2 - b*d*g
*h*p^2*r^2*x*Log[a + b*x]^2 + b*c*h^2*p^2*r^2*x*Log[a + b*x]^2 - 2*b*d*g^2*
p*q*r^2*Log[a + b*x]*Log[c + d*x] + 2*a*d*g*h*p*q*r^2*Log[a + b*x]*Log[c +
d*x] - 2*b*d*g*h*p*q*r^2*x*Log[a + b*x]*Log[c + d*x] + 2*a*d*h^2*p*q*r^2*x*
Log[a + b*x]*Log[c + d*x] - b*d*g^2*q^2*r^2*Log[c + d*x]^2 + a*d*g*h*q^2*r^
2*Log[c + d*x]^2 - b*d*g*h*q^2*r^2*x*Log[c + d*x]^2 + a*d*h^2*q^2*r^2*x*Log
[c + d*x]^2 + 2*b*c*g*h*p*q*r^2*Log[a + b*x]*Log[(h*(c + d*x))/(-(d*g) + c*
h)] - 2*a*d*g*h*p*q*r^2*Log[a + b*x]*Log[(h*(c + d*x))/(-(d*g) + c*h)] + 2*
b*c*h^2*p*q*r^2*x*Log[a + b*x]*Log[(h*(c + d*x))/(-(d*g) + c*h)] - 2*a*d*h^
```

2*p*q*r^2*x*Log[a + b*x]*Log[(h*(c + d*x))/(-(d*g) + c*h)] - b*c*g*h*p*q*r^
2*Log[(h*(c + d*x))/(-(d*g) + c*h)]^2 + a*d*g*h*p*q*r^2*Log[(h*(c + d*x))/(
-(d*g) + c*h)]^2 - b*c*h^2*p*q*r^2*x*Log[(h*(c + d*x))/(-(d*g) + c*h)]^2 +
a*d*h^2*p*q*r^2*x*Log[(h*(c + d*x))/(-(d*g) + c*h)]^2 + 2*b*c*g*h*p*q*r^2*L
og[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(
a + b*x))] - 2*a*d*g*h*p*q*r^2*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*g
- a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))] + 2*b*c*h^2*p*q*r^2*x*Log[(-(b*c
) + a*d)/(d*(a + b*x))]*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))
] - 2*a*d*h^2*p*q*r^2*x*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*g - a*h)*
(c + d*x))/((d*g - c*h)*(a + b*x))] + 2*b*c*g*h*p*q*r^2*Log[(h*(c + d*x))/(
-(d*g) + c*h)]*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))] - 2*a*d
*g*h*p*q*r^2*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[((b*g - a*h)*(c + d*x))/
((d*g - c*h)*(a + b*x))] + 2*b*c*h^2*p*q*r^2*x*Log[(h*(c + d*x))/(-(d*g) +
c*h)]*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))] - 2*a*d*h^2*p*q*
r^2*x*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[((b*g - a*h)*(c + d*x))/((d*g -
c*h)*(a + b*x))] - b*c*g*h*p*q*r^2*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h
)*(a + b*x))]^2 + a*d*g*h*p*q*r^2*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*
(a + b*x))]^2 - b*c*h^2*p*q*r^2*x*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*
(a + b*x))]^2 + a*d*h^2*p*q*r^2*x*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*
(a + b*x))]^2 + 2*b*d*g^2*p*r*Log[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q
)^r] - 2*b*c*g*h*p*r*Log[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 2*
b*d*g*h*p*r*x*Log[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 2*b*c*h^2
*p*r*x*Log[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 2*b*d*g^2*q*r*Lo
g[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 2*a*d*g*h*q*r*Log[c + d*x
]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 2*b*d*g*h*q*r*x*Log[c + d*x]*Log[e
(f(a + b*x)^p*(c + d*x)^q)^r] - 2*a*d*h^2*q*r*x*Log[c + d*x]*Log[e*(f*(a
+ b*x)^p*(c + d*x)^q)^r] - b*d*g^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 +
b*c*g*h*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 + a*d*g*h*Log[e*(f*(a + b*x
)^p*(c + d*x)^q)^r]^2 - a*c*h^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 - 2*
b*d*g^2*p*q*r^2*Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)] + 2*a*d*g*h*p*q
*r^2*Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)] - 2*b*d*g*h*p*q*r^2*x*Log[
a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)] + 2*a*d*h^2*p*q*r^2*x*Log[a + b*x]*
Log[(b*(g + h*x))/(b*g - a*h)] + 2*b*d*g^2*p*q*r^2*Log[(h*(c + d*x))/(-(d*g
) + c*h)]*Log[(b*(g + h*x))/(b*g - a*h)] - 2*b*c*g*h*p*q*r^2*Log[(h*(c + d*
x))/(-(d*g) + c*h)]*Log[(b*(g + h*x))/(b*g - a*h)] + 2*b*d*g*h*p*q*r^2*x*Lo
g[(h*(c + d*x))/(-(d*g) + c*h)]*Log[(b*(g + h*x))/(b*g - a*h)] - 2*b*c*h^2*
p*q*r^2*x*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[(b*(g + h*x))/(b*g - a*h)]
- 2*b*d*g^2*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[(b*(g + h*x))/(b*g
- a*h)] + 2*b*c*g*h*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[(b*(g + h
*x))/(b*g - a*h)] - 2*b*d*g*h*p*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*Lo
g[(b*(g + h*x))/(b*g - a*h)] + 2*b*c*h^2*p*r*x*Log[e*(f*(a + b*x)^p*(c + d*
x)^q)^r]*Log[(b*(g + h*x))/(b*g - a*h)] + 2*b*d*g^2*p*q*r^2*Log[a + b*x]*Lo
g[(d*(g + h*x))/(d*g - c*h)] - 2*a*d*g*h*p*q*r^2*Log[a + b*x]*Log[(d*(g + h
*x))/(d*g - c*h)] + 2*b*d*g*h*p*q*r^2*x*Log[a + b*x]*Log[(d*(g + h*x))/(d*g
- c*h)] - 2*a*d*h^2*p*q*r^2*x*Log[a + b*x]*Log[(d*(g + h*x))/(d*g - c*h)]

```
- 2*b*d*g^2*p*q*r^2*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[(d*(g + h*x))/(d*
g - c*h)] + 2*b*c*g*h*p*q*r^2*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[(d*(g +
 h*x))/(d*g - c*h)] - 2*b*d*g*h*p*q*r^2*x*Log[(h*(c + d*x))/(-(d*g) + c*h)]
*Log[(d*(g + h*x))/(d*g - c*h)] + 2*b*c*h^2*p*q*r^2*x*Log[(h*(c + d*x))/(-(
d*g) + c*h)]*Log[(d*(g + h*x))/(d*g - c*h)] - 2*b*d*g^2*q*r*Log[e*(f*(a + b
*x)^p*(c + d*x)^q)^r]*Log[(d*(g + h*x))/(d*g - c*h)] + 2*a*d*g*h*q*r*Log[e*
(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[(d*(g + h*x))/(d*g - c*h)] - 2*b*d*g*h*q
*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[(d*(g + h*x))/(d*g - c*h)] +
2*a*d*h^2*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[(d*(g + h*x))/(d*g
 - c*h)] + 2*p*(b*c*h*p + a*d*h*q - b*d*g*(p + q))*r^2*(g + h*x)*PolyLog[2,
 (h*(a + b*x))/(-(b*g) + a*h)] + 2*q*(b*c*h*p + a*d*h*q - b*d*g*(p + q))*r^
2*(g + h*x)*PolyLog[2, (h*(c + d*x))/(-(d*g) + c*h)] + 2*b*c*g*h*p*q*r^2*Po
lyLog[2, (b*(c + d*x))/(d*(a + b*x))] - 2*a*d*g*h*p*q*r^2*PolyLog[2, (b*(c
+ d*x))/(d*(a + b*x))] + 2*b*c*h^2*p*q*r^2*x*PolyLog[2, (b*(c + d*x))/(d*(a
 + b*x))] - 2*a*d*h^2*p*q*r^2*x*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(h
*(-(b*g) + a*h)*(-(d*g) + c*h)*(g + h*x))
```

Maple [F]

$$\int \frac{\ln\left(e(f(bx+a)^p\,(dx+c)^q)^r\right)^2}{(hx+g)^2}dx$$

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^2,x)

[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^2,x)
```

Fricas [F]

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^2}\,dx = \int \frac{\log\left(((bx+a)^p(dx+c)^q f)^r\,e\right)^2}{(hx+g)^2}\,dx$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^2,x, algorithm="fricas
")

[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(h^2*x^2 + 2*g*h*x + g^2),
x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^2}\,dx=\text{Timed out}$$

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(h*x+g)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 745, normalized size of antiderivative = 0.90

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^2}\,dx$$
$$=\frac{2\left(\frac{bfp\log(bx+a)}{bg-ah}+\frac{dfq\log(dx+c)}{dg-ch}-\frac{(adfhq-(dfg(p+q)-cfhp)b)\log(hx+g)}{(dgh-ch^2)a-(dg^2-cgh)b}\right)r\log\left(\left((bx+a)^p(dx+c)^qf\right)^re\right)}{fh}$$
$$-\frac{\left(\frac{2\,(bcf^2hpq-adf^2hpq)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)}{(dgh-ch^2)a-(dg^2-cgh)b}+\frac{2\,(adf^2hpq+(cf^2hp^2-(p^2+pq)df^2g)b)\left(\log(bx+a)\log\left(\frac{bhx+ah}{bg-ah}+\right.\right.}{(dgh-ch^2)a-(dg^2-cgh)b}\right.}{}$$
$$-\frac{\log\left(\left((bx+a)^p(dx+c)^qf\right)^re\right)^2}{(hx+g)h}$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^2,x, algorithm="maxima
")
```

```
[Out] 2*(b*f*p*log(b*x + a)/(b*g - a*h) + d*f*q*log(d*x + c)/(d*g - c*h) - (a*d*f
*h*q - (d*f*g*(p + q) - c*f*h*p)*b)*log(h*x + g)/((d*g*h - c*h^2)*a - (d*g^
2 - c*g*h)*b))*r*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(f*h) - (2*(b*c*f^2*h
*p*q - a*d*f^2*h*p*q)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + di
log(-(b*d*x + a*d)/(b*c - a*d)))/((d*g*h - c*h^2)*a - (d*g^2 - c*g*h)*b) +
2*(a*d*f^2*h*p*q + (c*f^2*h*p^2 - (p^2 + p*q)*d*f^2*g)*b)*(log(b*x + a)*log
((b*h*x + a*h)/(b*g - a*h) + 1) + dilog(-(b*h*x + a*h)/(b*g - a*h)))/((d*g*
h - c*h^2)*a - (d*g^2 - c*g*h)*b) + 2*(a*d*f^2*h*q^2 + (c*f^2*h*p*q - (p*q
+ q^2)*d*f^2*g)*b)*(log(d*x + c)*log((d*h*x + c*h)/(d*g - c*h) + 1) + dilog
(-(d*h*x + c*h)/(d*g - c*h)))/((d*g*h - c*h^2)*a - (d*g^2 - c*g*h)*b) - ((d
*f^2*g*p^2 - c*f^2*h*p^2)*b*log(b*x + a)^2 + 2*(b*d*f^2*g*p*q - a*d*f^2*h*p
*q)*log(b*x + a)*log(d*x + c) + (b*d*f^2*g*q^2 - a*d*f^2*h*q^2)*log(d*x + c
)^2 + 2*((a*d*f^2*h*p*q + (c*f^2*h*p^2 - (p^2 + p*q)*d*f^2*g)*b)*log(b*x +
a) + (a*d*f^2*h*q^2 + (c*f^2*h*p*q - (p*q + q^2)*d*f^2*g)*b)*log(d*x + c))*
log(h*x + g))/((d*g*h - c*h^2)*a - (d*g^2 - c*g*h)*b))*r^2/(f^2*h) - log(((
b*x + a)^p*(d*x + c)^q*f)^r*e)^2/((h*x + g)*h)
```

Giac [F]

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^2}\,dx = \int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{\left(hx+g\right)^2}\,dx$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^2,x, algorithm="giac")
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(h*x + g)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^2}\,dx = \int \frac{\ln\left(e\left(f\left(a+b\,x\right)^p\left(c+d\,x\right)^q\right)^r\right)^2}{\left(g+h\,x\right)^2}\,dx$$

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(g + h*x)^2,x)
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(g + h*x)^2, x)
```

3.41 $\int \frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{(g+hx)^3} \, dx$

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Optimal result

Integrand size = 31, antiderivative size = 1304

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^3}\,dx$$
$$= -\frac{bdpqr^2\log(a+bx)}{h(bg-ah)(dg-ch)} + \frac{dpqr^2\log(a+bx)}{h(dg-ch)(g+hx)} - \frac{bp^2r^2(a+bx)\log(a+bx)}{(bg-ah)^2(g+hx)}$$
$$- \frac{bdpqr^2\log(c+dx)}{h(bg-ah)(dg-ch)} + \frac{bpqr^2\log(c+dx)}{h(bg-ah)(g+hx)} - \frac{dq^2r^2(c+dx)\log(c+dx)}{(dg-ch)^2(g+hx)}$$
$$+ \frac{b^2pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{h(bg-ah)^2} + \frac{d^2pqr^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{h(dg-ch)^2}$$
$$- \frac{bpr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right))}{h(bg-ah)(g+hx)}$$
$$- \frac{dqr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right))}{h(dg-ch)(g+hx)}$$
$$- \frac{b^2pr\log(a+bx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{h(bg-ah)^2}$$
$$- \frac{d^2qr\log(c+dx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{h(dg-ch)^2}$$
$$- \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h(g+hx)^2} + \frac{b^2p^2r^2\log(g+hx)}{h(bg-ah)^2}$$
$$+ \frac{2bdpqr^2\log(g+hx)}{h(bg-ah)(dg-ch)} + \frac{d^2q^2r^2\log(g+hx)}{h(dg-ch)^2}$$
$$+ \frac{b^2pr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right))\log(g+hx)}{h(bg-ah)^2}$$
$$+ \frac{d^2qr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right))\log(g+hx)}{h(dg-ch)^2}$$
$$- \frac{d^2pqr^2\log(a+bx)\log\left(\frac{b(g+hx)}{bg-ah}\right)}{h(dg-ch)^2} - \frac{b^2pqr^2\log(c+dx)\log\left(\frac{d(g+hx)}{dg-ch}\right)}{h(bg-ah)^2}$$
$$- \frac{b^2p^2r^2\log(a+bx)\log\left(1+\frac{bg-ah}{h(a+bx)}\right)}{h(bg-ah)^2} - \frac{d^2q^2r^2\log(c+dx)\log\left(1+\frac{dg-ch}{h(c+dx)}\right)}{h(dg-ch)^2}$$
$$+ \frac{b^2p^2r^2\,\text{PolyLog}\left(2,-\frac{bg-ah}{h(a+bx)}\right)}{h(bg-ah)^2} + \frac{d^2pqr^2\,\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{h(dg-ch)^2}$$
$$- \frac{d^2pqr^2\,\text{PolyLog}\left(2,-\frac{h(a+bx)}{bg-ah}\right)}{h(dg-ch)^2} + \frac{d^2q^2r^2\,\text{PolyLog}\left(2,-\frac{dg-ch}{h(c+dx)}\right)}{h(dg-ch)^2}$$
$$+ \frac{b^2pqr^2\,\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{h(bg-ah)^2} - \frac{b^2pqr^2\,\text{PolyLog}\left(2,-\frac{h(c+dx)}{dg-ch}\right)}{h(bg-ah)^2}$$

```
[Out] 2*b*d*p*q*r^2*ln(h*x+g)/h/(-a*h+b*g)/(-c*h+d*g)-1/2*ln(e*(f*(b*x+a)^p*(d*x+
c)^q)^r)^2/h/(h*x+g)^2+b*p*q*r^2*ln(d*x+c)/h/(-a*h+b*g)/(h*x+g)+b^2*p*q*r^2
*ln(-d*(b*x+a)/(-a*d+b*c))*ln(d*x+c)/h/(-a*h+b*g)^2+d^2*p*q*r^2*ln(b*x+a)*l
n(b*(d*x+c)/(-a*d+b*c))/h/(-c*h+d*g)^2+b^2*p^2*r^2*ln(h*x+g)/h/(-a*h+b*g)^2
+d^2*q^2*r^2*ln(h*x+g)/h/(-c*h+d*g)^2+b^2*p^2*r^2*polylog(2,(a*h-b*g)/h/(b*
x+a))/h/(-a*h+b*g)^2+d^2*q^2*r^2*polylog(2,(c*h-d*g)/h/(d*x+c))/h/(-c*h+d*g
)^2-d^2*q*r*ln(d*x+c)*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c
)^q)^r))/h/(-c*h+d*g)^2+b^2*p*r*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a
)^p*(d*x+c)^q)^r))*ln(h*x+g)/h/(-a*h+b*g)^2+d^2*q*r*(p*r*ln(b*x+a)+q*r*ln(d
*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))*ln(h*x+g)/h/(-c*h+d*g)^2-b^2*p^2*r^2
*ln(b*x+a)*ln(1+(-a*h+b*g)/h/(b*x+a))/h/(-a*h+b*g)^2-d^2*q^2*r^2*ln(d*x+c)*
ln(1+(-c*h+d*g)/h/(d*x+c))/h/(-c*h+d*g)^2+d^2*p*q*r^2*polylog(2,-d*(b*x+a)/
(-a*d+b*c))/h/(-c*h+d*g)^2-d^2*p*q*r^2*polylog(2,-h*(b*x+a)/(-a*h+b*g))/h/(
-c*h+d*g)^2-d*q^2*r^2*(d*x+c)*ln(d*x+c)/(-c*h+d*g)^2/(h*x+g)+b^2*p*q*r^2*po
lylog(2,b*(d*x+c)/(-a*d+b*c))/h/(-a*h+b*g)^2-b^2*p*q*r^2*polylog(2,-h*(d*x+
c)/(-c*h+d*g))/h/(-a*h+b*g)^2-b*p^2*r^2*(b*x+a)*ln(b*x+a)/(-a*h+b*g)^2/(h*x
+g)-b*p*r*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/h/(
-a*h+b*g)/(h*x+g)-d*q*r*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x
+c)^q)^r))/h/(-c*h+d*g)/(h*x+g)-b^2*p*r*ln(b*x+a)*(p*r*ln(b*x+a)+q*r*ln(d*x
+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/h/(-a*h+b*g)^2-b*d*p*q*r^2*ln(b*x+a)/h
/(-a*h+b*g)/(-c*h+d*g)-b*d*p*q*r^2*ln(d*x+c)/h/(-a*h+b*g)/(-c*h+d*g)-d^2*p*
q*r^2*ln(b*x+a)*ln(b*(h*x+g)/(-a*h+b*g))/h/(-c*h+d*g)^2-b^2*p*q*r^2*ln(d*x+
c)*ln(d*(h*x+g)/(-c*h+d*g))/h/(-a*h+b*g)^2+d*p*q*r^2*ln(b*x+a)/h/(-c*h+d*g)
/(h*x+g)
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 1304, normalized size of antiderivative = 1.00, number of steps used = 43, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.452$, Rules

used = {2584, 2593, 2458, 2389, 2379, 2438, 2351, 31, 2465, 2441, 2440, 2442, 36, 46}

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^3}\,dx$$

$$= \frac{pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)b^2}{h(bg-ah)^2}$$

$$-\frac{pr\log(a+bx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)b^2}{h(bg-ah)^2}$$

$$+\frac{p^2r^2\log(g+hx)b^2}{h(bg-ah)^2}$$

$$+\frac{pr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right))\log(g+hx)b^2}{h(bg-ah)^2}$$

$$-\frac{pqr^2\log(c+dx)\log\left(\frac{d(g+hx)}{dg-ch}\right)b^2}{h(bg-ah)^2}-\frac{p^2r^2\log(a+bx)\log\left(\frac{bg-ah}{h(a+bx)}+1\right)b^2}{h(bg-ah)^2}$$

$$+\frac{p^2r^2\operatorname{PolyLog}\left(2,-\frac{bg-ah}{h(a+bx)}\right)b^2}{h(bg-ah)^2}+\frac{pqr^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)b^2}{h(bg-ah)^2}$$

$$-\frac{pqr^2\operatorname{PolyLog}\left(2,-\frac{h(c+dx)}{dg-ch}\right)b^2}{h(bg-ah)^2}-\frac{dpqr^2\log(a+bx)b}{h(bg-ah)(dg-ch)}$$

$$-\frac{p^2r^2(a+bx)\log(a+bx)b}{(bg-ah)^2(g+hx)}-\frac{dpqr^2\log(c+dx)b}{h(bg-ah)(dg-ch)}+\frac{pqr^2\log(c+dx)b}{h(bg-ah)(g+hx)}$$

$$-\frac{pr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right))b}{h(bg-ah)(g+hx)}$$

$$+\frac{2dpqr^2\log(g+hx)b}{h(bg-ah)(dg-ch)}-\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h(g+hx)^2}+\frac{dpqr^2\log(a+bx)}{h(dg-ch)(g+hx)}$$

$$-\frac{dq^2r^2(c+dx)\log(c+dx)}{(dg-ch)^2(g+hx)}+\frac{d^2pqr^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{h(dg-ch)^2}$$

$$-\frac{d^2qr\log(c+dx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{h(dg-ch)^2}$$

$$-\frac{dqr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right))}{h(dg-ch)(g+hx)}$$

$$+\frac{d^2q^2r^2\log(g+hx)}{h(dg-ch)^2}$$

$$+\frac{d^2qr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right))\log(g+hx)}{h(dg-ch)^2}$$

$$-\frac{d^2pqr^2\log(a+bx)\log\left(\frac{b(g+hx)}{bg-ah}\right)}{h(dg-ch)^2}$$

$$-\frac{d^2q^2r^2\log(c+dx)\log\left(\frac{dg-ch}{h(c+dx)}+1\right)}{h(dg-ch)^2}+\frac{d^2pqr^2\operatorname{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{h(dg-ch)^2}$$

$$-\frac{d^2pqr^2\operatorname{PolyLog}\left(2,-\frac{h(a+bx)}{bg-ah}\right)}{h(dg-ch)^2}+\frac{d^2q^2r^2\operatorname{PolyLog}\left(2,-\frac{dg-ch}{h(c+dx)}\right)}{h(dg-ch)^2}$$

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(g + h*x)^3,x]
```

```
[Out] -((b*d*p*q*r^2*Log[a + b*x])/(h*(b*g - a*h)*(d*g - c*h))) + (d*p*q*r^2*Log[
a + b*x])/(h*(d*g - c*h)*(g + h*x)) - (b*p^2*r^2*(a + b*x)*Log[a + b*x])/((
b*g - a*h)^2*(g + h*x)) - (b*d*p*q*r^2*Log[c + d*x])/(h*(b*g - a*h)*(d*g -
c*h)) + (b*p*q*r^2*Log[c + d*x])/(h*(b*g - a*h)*(g + h*x)) - (d*q^2*r^2*(c
+ d*x)*Log[c + d*x])/((d*g - c*h)^2*(g + h*x)) + (b^2*p*q*r^2*Log[-((d*(a +
 b*x))/(b*c - a*d))]*Log[c + d*x])/(h*(b*g - a*h)^2) + (d^2*p*q*r^2*Log[a +
 b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(h*(d*g - c*h)^2) - (b*p*r*(p*r*Log[a
 + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(h*(b*g
 - a*h)*(g + h*x)) - (d*q*r*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]))/(h*(d*g - c*h)*(g + h*x)) - (b^2*p*r*Log[a +
 b*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)
^q)^r]))/(h*(b*g - a*h)^2) - (d^2*q*r*Log[c + d*x]*(p*r*Log[a + b*x] + q*r*
Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(h*(d*g - c*h)^2) - L
og[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(2*h*(g + h*x)^2) + (b^2*p^2*r^2*Log[
g + h*x])/(h*(b*g - a*h)^2) + (2*b*d*p*q*r^2*Log[g + h*x])/(h*(b*g - a*h)*(
d*g - c*h)) + (d^2*q^2*r^2*Log[g + h*x])/(h*(d*g - c*h)^2) + (b^2*p*r*(p*r*
Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])*Log
[g + h*x])/(h*(b*g - a*h)^2) + (d^2*q*r*(p*r*Log[a + b*x] + q*r*Log[c + d*x
] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])*Log[g + h*x])/(h*(d*g - c*h)^2) -
 (d^2*p*q*r^2*Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)])/(h*(d*g - c*h)^2
) - (b^2*p*q*r^2*Log[c + d*x]*Log[(d*(g + h*x))/(d*g - c*h)])/(h*(b*g - a*h
)^2) - (b^2*p^2*r^2*Log[a + b*x]*Log[1 + (b*g - a*h)/(h*(a + b*x))])/(h*(b*
g - a*h)^2) - (d^2*q^2*r^2*Log[c + d*x]*Log[1 + (d*g - c*h)/(h*(c + d*x))])
/(h*(d*g - c*h)^2) + (b^2*p^2*r^2*PolyLog[2, -((b*g - a*h)/(h*(a + b*x)))])
/(h*(b*g - a*h)^2) + (d^2*p*q*r^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])
/(h*(d*g - c*h)^2) - (d^2*p*q*r^2*PolyLog[2, -((h*(a + b*x))/(b*g - a*h))])
/(h*(d*g - c*h)^2) + (d^2*q^2*r^2*PolyLog[2, -((d*g - c*h)/(h*(c + d*x)))])
/(h*(d*g - c*h)^2) + (b^2*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(h
*(b*g - a*h)^2) - (b^2*p*q*r^2*PolyLog[2, -((h*(c + d*x))/(d*g - c*h))])/(h
*(b*g - a*h)^2)
```

<u>Rule 31</u>

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

<u>Rule 36</u>

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

<u>Rule 46</u>

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
```

```
NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
```

```
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2584

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1
)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(h*(m + 1))), x] + (-Dist[b*p*r*(
s/(h*(m + 1))), Int[(g + h*x)^(m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1)/(a + b*x)), x], x] - Dist[d*q*r*(s/(h*(m + 1))), Int[(g + h*x)^(m
+ 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(c + d*x)), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2593

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$\text{integral} = -\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h(g+hx)^2} + \frac{(bpr)\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)(g+hx)^2}\,dx}{h} + \frac{(dqr)\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(c+dx)(g+hx)^2}\,dx}{h}$$

$$= -\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h(g+hx)^2} + \frac{(bp^2r^2)\int \frac{\log(a+bx)}{(a+bx)(g+hx)^2}\,dx}{h} + \frac{(bpqr^2)\int \frac{\log(c+dx)}{(a+bx)(g+hx)^2}\,dx}{h} + \frac{(dpqr^2)\int \frac{\log(a+bx)}{(c+dx)(g+hx)^2}\,dx}{h} + \frac{(dq^2r^2)\int \frac{\log(c+dx)}{(c+dx)(g+hx)^2}\,dx}{h} - \frac{(bpr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)))\int \frac{1}{(a+bx)(g+hx)^2}\,dx}{h} - \frac{(dqr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)))\int \frac{1}{(c+dx)(g+hx)^2}\,dx}{h}$$

$$= -\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h(g+hx)^2} + \frac{(p^2r^2)\,\text{Subst}\left(\int \frac{\log(x)}{x\left(\frac{bg-ah}{b}+\frac{hx}{b}\right)^2}\,dx, x, a+bx\right)}{h} + \frac{(bpqr^2)\int \left(\frac{b^2\log(c+dx)}{(bg-ah)^2(a+bx)} - \frac{h\log(c+dx)}{(bg-ah)(g+hx)^2} - \frac{bh\log(c+dx)}{(bg-ah)^2(g+hx)}\right)dx}{h} + \frac{(dpqr^2)\int \left(\frac{d^2\log(a+bx)}{(dg-ch)^2(c+dx)} - \frac{h\log(a+bx)}{(dg-ch)(g+hx)^2} - \frac{dh\log(a+bx)}{(dg-ch)^2(g+hx)}\right)dx}{h} + \frac{(q^2r^2)\,\text{Subst}\left(\int \frac{\log(x)}{x\left(\frac{dg-ch}{d}+\frac{hx}{d}\right)^2}\,dx, x, c+dx\right)}{h} - \frac{(bpr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)))\int \left(\frac{b^2}{(bg-ah)^2(a+bx)} - \frac{h}{(bg-ah)(\cdots}\right.}{h} - \frac{(dqr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)))\int \left(\frac{d^2}{(dg-ch)^2(c+dx)} - \frac{h}{(dg-ch)(\cdots}\right.}{h}$$

$$
\begin{aligned}
&= -\frac{bpr(pr\log(a+bx)+qr\log(c+dx)-\log(e(f(a+bx)^p(c+dx)^q)^r))}{h(bg-ah)(g+hx)} \\
&- \frac{dqr(pr\log(a+bx)+qr\log(c+dx)-\log(e(f(a+bx)^p(c+dx)^q)^r))}{h(dg-ch)(g+hx)} \\
&- \frac{b^2pr\log(a+bx)\left(pr\log(a+bx)+qr\log(c+dx)-\log(e(f(a+bx)^p(c+dx)^q)^r)\right)}{h(bg-ah)^2} \\
&- \frac{d^2qr\log(c+dx)\left(pr\log(a+bx)+qr\log(c+dx)-\log(e(f(a+bx)^p(c+dx)^q)^r)\right)}{h(dg-ch)^2} \\
&- \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h(g+hx)^2} \\
&+ \frac{b^2pr(pr\log(a+bx)+qr\log(c+dx)-\log(e(f(a+bx)^p(c+dx)^q)^r))\log(g+hx)}{h(bg-ah)^2} \\
&+ \frac{d^2qr(pr\log(a+bx)+qr\log(c+dx)-\log(e(f(a+bx)^p(c+dx)^q)^r))\log(g+hx)}{h(dg-ch)^2} \\
&- \frac{(p^2r^2)\,\text{Subst}\left(\int \frac{\log(x)}{\left(\frac{bg-ah}{b}+\frac{hx}{b}\right)^2}\,dx, x, a+bx\right)}{bg-ah} \\
&+ \frac{(bp^2r^2)\,\text{Subst}\left(\int \frac{\log(x)}{x\left(\frac{bg-ah}{b}+\frac{hx}{b}\right)}\,dx, x, a+bx\right)}{h(bg-ah)} - \frac{(b^2pqr^2)\int \frac{\log(c+dx)}{g+hx}\,dx}{(bg-ah)^2} \\
&+ \frac{(b^3pqr^2)\int \frac{\log(c+dx)}{a+bx}\,dx}{h(bg-ah)^2} - \frac{(bpqr^2)\int \frac{\log(c+dx)}{(g+hx)^2}\,dx}{bg-ah} \\
&- \frac{(d^2pqr^2)\int \frac{\log(a+bx)}{g+hx}\,dx}{(dg-ch)^2} + \frac{(d^3pqr^2)\int \frac{\log(a+bx)}{c+dx}\,dx}{h(dg-ch)^2} \\
&- \frac{(dpqr^2)\int \frac{\log(a+bx)}{(g+hx)^2}\,dx}{dg-ch} - \frac{(q^2r^2)\,\text{Subst}\left(\int \frac{\log(x)}{\left(\frac{dg-ch}{d}+\frac{hx}{d}\right)^2}\,dx, x, c+dx\right)}{dg-ch} \\
&+ \frac{(dq^2r^2)\,\text{Subst}\left(\int \frac{\log(x)}{x\left(\frac{dg-ch}{d}+\frac{hx}{d}\right)}\,dx, x, c+dx\right)}{h(dg-ch)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{dpqr^2\log(a+bx)}{h(dg-ch)(g+hx)} - \frac{bp^2r^2(a+bx)\log(a+bx)}{(bg-ah)^2(g+hx)} \\
&+ \frac{bpqr^2\log(c+dx)}{h(bg-ah)(g+hx)} - \frac{dq^2r^2(c+dx)\log(c+dx)}{(dg-ch)^2(g+hx)} \\
&+ \frac{b^2pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{h(bg-ah)^2} + \frac{d^2pqr^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{h(dg-ch)^2} \\
&- \frac{bpr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right))}{h(bg-ah)(g+hx)} \\
&- \frac{dqr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right))}{h(dg-ch)(g+hx)} \\
&- \frac{b^2pr\log(a+bx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{h(bg-ah)^2} \\
&- \frac{d^2qr\log(c+dx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{h(dg-ch)^2} \\
&- \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h(g+hx)^2} \\
&+ \frac{b^2pr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right))\log(g+hx)}{h(bg-ah)^2} \\
&+ \frac{d^2qr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right))\log(g+hx)}{h(dg-ch)^2} \\
&- \frac{d^2pqr^2\log(a+bx)\log\left(\frac{b(g+hx)}{bg-ah}\right)}{h(dg-ch)^2} - \frac{b^2pqr^2\log(c+dx)\log\left(\frac{d(g+hx)}{dg-ch}\right)}{h(bg-ah)^2} \\
&- \frac{b^2p^2r^2\log(a+bx)\log\left(1+\frac{bg-ah}{h(a+bx)}\right)}{h(bg-ah)^2} - \frac{d^2q^2r^2\log(c+dx)\log\left(1+\frac{dg-ch}{h(c+dx)}\right)}{h(dg-ch)^2} \\
&+ \frac{(bp^2r^2)\,\text{Subst}\left(\int \frac{1}{\frac{bg-ah}{b}+\frac{hx}{b}}\,dx, x, a+bx\right)}{(bg-ah)^2} \\
&+ \frac{(b^2p^2r^2)\,\text{Subst}\left(\int \frac{\log\left(1+\frac{bg-ah}{hx}\right)}{x}\,dx, x, a+bx\right)}{h(bg-ah)^2} - \frac{(b^2dpqr^2)\int \frac{\log\left(\frac{d(a+bx)}{-bc+ad}\right)}{c+dx}\,dx}{h(bg-ah)^2} \\
&+ \frac{(b^2dpqr^2)\int \frac{\log\left(\frac{d(g+hx)}{dg-ch}\right)}{c+dx}\,dx}{h(bg-ah)^2} - \frac{(bdpqr^2)\int \frac{1}{(c+dx)(g+hx)}\,dx}{h(bg-ah)} \\
&- \frac{(bd^2pqr^2)\int \frac{\log\left(\frac{b(c+dx)}{bc-ad}\right)}{a+bx}\,dx}{h(dg-ch)^2} + \frac{(bd^2pqr^2)\int \frac{\log\left(\frac{b(g+hx)}{bg-ah}\right)}{a+bx}\,dx}{h(dg-ch)^2} \\
&- \frac{(bdpqr^2)\int \frac{1}{(a+bx)(g+hx)}\,dx}{h(dg-ch)} + \frac{(dq^2r^2)\,\text{Subst}\left(\int \frac{1}{\frac{dg-ch}{d}+\frac{hx}{d}}\,dx, x, c+dx\right)}{(dg-ch)^2} \\
&+ \frac{(d^2q^2r^2)\,\text{Subst}\left(\int \frac{\log\left(1+\frac{dg-ch}{hx}\right)}{x}\,dx, x, c+dx\right)}{h(dg-ch)^2}
\end{aligned}
$$

$$= \frac{dpqr^2\log(a+bx)}{h(dg-ch)(g+hx)} - \frac{bp^2r^2(a+bx)\log(a+bx)}{(bg-ah)^2(g+hx)} + \frac{bpqr^2\log(c+dx)}{h(bg-ah)(g+hx)} - \frac{dq^2r^2(c+dx)\log(c+dx)}{(dg-ch)^2(g+hx)} + \frac{b^2pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{h(bg-ah)^2} + \frac{d^2pqr^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{h(dg-ch)^2} - \frac{bpr(pr\log(a+bx)+qr\log(c+dx)-\log(e(f(a+bx)^p(c+dx)^q)^r))}{h(bg-ah)(g+hx)} - \frac{dqr(pr\log(a+bx)+qr\log(c+dx)-\log(e(f(a+bx)^p(c+dx)^q)^r))}{h(dg-ch)(g+hx)} - \frac{b^2pr\log(a+bx)\left(pr\log(a+bx)+qr\log(c+dx)-\log(e(f(a+bx)^p(c+dx)^q)^r)\right)}{h(bg-ah)^2} - \frac{d^2qr\log(c+dx)\left(pr\log(a+bx)+qr\log(c+dx)-\log(e(f(a+bx)^p(c+dx)^q)^r)\right)}{h(dg-ch)^2} - \frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{2h(g+hx)^2} + \frac{b^2p^2r^2\log(g+hx)}{h(bg-ah)^2} + \frac{d^2q^2r^2\log(g+hx)}{h(dg-ch)^2} + \frac{b^2pr(pr\log(a+bx)+qr\log(c+dx)-\log(e(f(a+bx)^p(c+dx)^q)^r))\log(g+hx)}{h(bg-ah)^2} + \frac{d^2qr(pr\log(a+bx)+qr\log(c+dx)-\log(e(f(a+bx)^p(c+dx)^q)^r))\log(g+hx)}{h(dg-ch)^2} - \frac{d^2pqr^2\log(a+bx)\log\left(\frac{b(g+hx)}{bg-ah}\right)}{h(dg-ch)^2} - \frac{b^2pqr^2\log(c+dx)\log\left(\frac{d(g+hx)}{dg-ch}\right)}{h(bg-ah)^2} - \frac{b^2p^2r^2\log(a+bx)\log\left(1+\frac{bg-ah}{h(a+bx)}\right)}{h(bg-ah)^2} - \frac{d^2q^2r^2\log(c+dx)\log\left(1+\frac{dg-ch}{h(c+dx)}\right)}{h(dg-ch)^2} + \frac{b^2p^2r^2\text{Li}_2\left(-\frac{bg-ah}{h(a+bx)}\right)}{h(bg-ah)^2} + \frac{d^2q^2r^2\text{Li}_2\left(-\frac{dg-ch}{h(c+dx)}\right)}{h(dg-ch)^2} - \frac{(b^2pqr^2)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{bx}{-bc+ad}\right)}{x}\,dx, x, c+dx\right)}{h(bg-ah)^2} + \frac{(b^2pqr^2)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{hx}{dg-ch}\right)}{x}\,dx, x, c+dx\right)}{h(bg-ah)^2} - \frac{(d^2pqr^2)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{dx}{bc-ad}\right)}{x}\,dx, x, a+bx\right)}{h(dg-ch)^2} + \frac{(d^2pqr^2)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{hx}{bg-ah}\right)}{x}\,dx, x, a+bx\right)}{h(dg-ch)^2} + 2\frac{(bdpqr^2)\int\frac{1}{g+hx}\,dx}{(bg-ah)(dg-ch)} - \frac{(b^2dpqr^2)\int\frac{1}{a+bx}\,dx}{h(bg-ah)(dg-ch)} - \frac{(bd^2pqr^2)\int\frac{1}{c+dx}\,dx}{h(bg-ah)(dg-ch)}$$

$$
\begin{aligned}
= & -\frac{bdpqr^2\log(a+bx)}{h(bg-ah)(dg-ch)} + \frac{dpqr^2\log(a+bx)}{h(dg-ch)(g+hx)} - \frac{bp^2r^2(a+bx)\log(a+bx)}{(bg-ah)^2(g+hx)} \\
& - \frac{bdpqr^2\log(c+dx)}{h(bg-ah)(dg-ch)} + \frac{bpqr^2\log(c+dx)}{h(bg-ah)(g+hx)} - \frac{dq^2r^2(c+dx)\log(c+dx)}{(dg-ch)^2(g+hx)} \\
& + \frac{b^2pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{h(bg-ah)^2} + \frac{d^2pqr^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{h(dg-ch)^2} \\
& - \frac{bpr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right))}{h(bg-ah)(g+hx)} \\
& - \frac{dqr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right))}{h(dg-ch)(g+hx)} \\
& - \frac{b^2pr\log(a+bx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{h(bg-ah)^2} \\
& - \frac{d^2qr\log(c+dx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{h(dg-ch)^2} \\
& - \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h(g+hx)^2} + \frac{b^2p^2r^2\log(g+hx)}{h(bg-ah)^2} \\
& + \frac{2bdpqr^2\log(g+hx)}{h(bg-ah)(dg-ch)} + \frac{d^2q^2r^2\log(g+hx)}{h(dg-ch)^2} \\
& + \frac{b^2pr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right))\log(g+hx)}{h(bg-ah)^2} \\
& + \frac{d^2qr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right))\log(g+hx)}{h(dg-ch)^2} \\
& - \frac{d^2pqr^2\log(a+bx)\log\left(\frac{b(g+hx)}{bg-ah}\right)}{h(dg-ch)^2} - \frac{b^2pqr^2\log(c+dx)\log\left(\frac{d(g+hx)}{dg-ch}\right)}{h(bg-ah)^2} \\
& - \frac{b^2p^2r^2\log(a+bx)\log\left(1+\frac{bg-ah}{h(a+bx)}\right)}{h(bg-ah)^2} - \frac{d^2q^2r^2\log(c+dx)\log\left(1+\frac{dg-ch}{h(c+dx)}\right)}{h(dg-ch)^2} \\
& + \frac{b^2p^2r^2\mathrm{Li}_2\left(-\frac{bg-ah}{h(a+bx)}\right)}{h(bg-ah)^2} + \frac{d^2pqr^2\mathrm{Li}_2\left(-\frac{d(a+bx)}{bc-ad}\right)}{h(dg-ch)^2} - \frac{d^2pqr^2\mathrm{Li}_2\left(-\frac{h(a+bx)}{bg-ah}\right)}{h(dg-ch)^2} \\
& + \frac{d^2q^2r^2\mathrm{Li}_2\left(-\frac{dg-ch}{h(c+dx)}\right)}{h(dg-ch)^2} + \frac{b^2pqr^2\mathrm{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)}{h(bg-ah)^2} - \frac{b^2pqr^2\mathrm{Li}_2\left(-\frac{h(c+dx)}{dg-ch}\right)}{h(bg-ah)^2}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 12086 vs. 2(1304) = 2608.

Time = 2.89 (sec) , antiderivative size = 12086, normalized size of antiderivative = 9.27

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^3}\,dx = \text{Result too large to show}$$

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(g + h*x)^3,x]
[Out] Result too large to show
```

Maple [F]

$$\int \frac{\ln\left(e\left(f(bx+a)^p\,(dx+c)^q\right)^r\right)^2}{(hx+g)^3}\,dx$$

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^3,x)
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^3,x)
```

Fricas [F]

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^3}\,dx = \int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{(hx+g)^3}\,dx$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^3,x, algorithm="fricas
")
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(h^3*x^3 + 3*g*h^2*x^2 + 3*
g^2*h*x + g^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^3}\,dx = \text{Timed out}$$

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(h*x+g)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 1857, normalized size of antiderivative = 1.42

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^3,x, algorithm="maxima
")
```

```
[Out] (b^2*f*p*log(b*x + a)/(b^2*g^2 - 2*a*b*g*h + a^2*h^2) + d^2*f*q*log(d*x + c
)/(d^2*g^2 - 2*c*d*g*h + c^2*h^2) + (2*a*b*d^2*f*g*h*q - a^2*d^2*f*h^2*q -
(d^2*f*g^2*(p + q) - 2*c*d*f*g*h*p + c^2*f*h^2*p)*b^2)*log(h*x + g)/((d^2*g
^2*h^2 - 2*c*d*g*h^3 + c^2*h^4)*a^2 - 2*(d^2*g^3*h - 2*c*d*g^2*h^2 + c^2*g*
h^3)*a*b + (d^2*g^4 - 2*c*d*g^3*h + c^2*g^2*h^2)*b^2) + (a*d*f*h*q - (d*f*g
*(p + q) - c*f*h*p)*b)/((d*g^2*h - c*g*h^2)*a - (d*g^3 - c*g^2*h)*b + ((d*g
*h^2 - c*h^3)*a - (d*g^2*h - c*g*h^2)*b)*x))*r*log(((b*x + a)^p*(d*x + c)^q
*f)^r*e)/(f*h) + 1/2*(2*(2*a*b*d^2*f^2*g*h*p*q - a^2*d^2*f^2*h^2*p*q - (2*c
*d*f^2*g*h*p*q - c^2*f^2*h^2*p*q)*b^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c
 - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))/((d*g*h^2 - c*h^3)*a^2 -
2*(d*g^2*h - c*g*h^2)*a*b + (d*g^3 - c*g^2*h)*b^2) - 2*(2*a*b*d^2*f^2*g*h*p
*q - a^2*d^2*f^2*h^2*p*q + (2*c*d*f^2*g*h*p^2 - c^2*f^2*h^2*p^2 - (p^2 + p*
q)*d^2*f^2*g^2)*b^2)*(log(b*x + a)*log((b*h*x + a*h)/(b*g - a*h) + 1) + dil
og(-(b*h*x + a*h)/(b*g - a*h)))/((d*g*h^2 - c*h^3)*a^2 - 2*(d*g^2*h - c*g*h
^2)*a*b + (d*g^3 - c*g^2*h)*b^2) + 2*(2*a*b*d^2*f^2*g*h*q^2 - a^2*d^2*f^2*h
^2*q^2 + (2*c*d*f^2*g*h*p*q - c^2*f^2*h^2*p*q - (p*q + q^2)*d^2*f^2*g^2)*b^
2)*(log(d*x + c)*log((d*h*x + c*h)/(d*g - c*h) + 1) + dilog(-(d*h*x + c*h)/
(d*g - c*h)))/((d^2*g^2*h^2 - 2*c*d*g*h^3 + c^2*h^4)*a^2 - 2*(d^2*g^3*h - 2
*c*d*g^2*h^2 + c^2*g*h^3)*a*b + (d^2*g^4 - 2*c*d*g^3*h + c^2*g^2*h^2)*b^2)
- 2*(a*d^2*f^2*h*q^2 + (c*d*f^2*h*p*q - (p*q + q^2)*d^2*f^2*g)*b)*log(d*x +
 c)/((d^2*g^2*h - 2*c*d*g*h^2 + c^2*h^3)*a - (d^2*g^3 - 2*c*d*g^2*h + c^2*g
*h^2)*b) + 2*(a^2*d^2*f^2*h^2*q^2 + 2*(c*d*f^2*h^2*p*q - (p*q + q^2)*d^2*f^
2*g*h)*a*b + (c^2*f^2*h^2*p^2 + (p^2 + 2*p*q + q^2)*d^2*f^2*g^2 - 2*(p^2 +
p*q)*c*d*f^2*g*h)*b^2)*log(h*x + g)/((d^2*g^2*h^2 - 2*c*d*g*h^3 + c^2*h^4)*
a^2 - 2*(d^2*g^3*h - 2*c*d*g^2*h^2 + c^2*g*h^3)*a*b + (d^2*g^4 - 2*c*d*g^3*
h + c^2*g^2*h^2)*b^2) + ((d^3*f^2*g^3*p^2 - 3*c*d^2*f^2*g^2*h*p^2 + 3*c^2*d
*f^2*g*h^2*p^2 - c^3*f^2*h^3*p^2)*b^2*log(b*x + a)^2 - 2*(b^2*d^2*f^2*g^2*p
*q - 2*a*b*d^2*f^2*g*h*p*q + a^2*d^2*f^2*h^2*p*q)*log(b*x + a)*log(d*x + c)
 - (b^2*d^2*f^2*g^2*q^2 - 2*a*b*d^2*f^2*g*h*q^2 + a^2*d^2*f^2*h^2*q^2)*log(
d*x + c)^2 + 2*((d^2*f^2*g*h*p*q - c*d*f^2*h^2*p*q)*a*b - (c^2*f^2*h^2*p^2
+ (p^2 + p*q)*d^2*f^2*g^2 - (2*p^2 + p*q)*c*d*f^2*g*h)*b^2)*log(b*x + a) -
2*((2*a*b*d^2*f^2*g*h*p*q - a^2*d^2*f^2*h^2*p*q + (2*c*d*f^2*g*h*p^2 - c^2*
f^2*h^2*p^2 - (p^2 + p*q)*d^2*f^2*g^2)*b^2)*log(b*x + a) + (2*a*b*d^2*f^2*g
*h*q^2 - a^2*d^2*f^2*h^2*q^2 + (2*c*d*f^2*g*h*p*q - c^2*f^2*h^2*p*q - (p*q
```

```
+ q^2)*d^2*f^2*g^2)*b^2)*log(d*x + c))*log(h*x + g))/((d^2*g^2*h^2 - 2*c*d*
g*h^3 + c^2*h^4)*a^2 - 2*(d^2*g^3*h - 2*c*d*g^2*h^2 + c^2*g*h^3)*a*b + (d^2
*g^4 - 2*c*d*g^3*h + c^2*g^2*h^2)*b^2))*r^2/(f^2*h) - 1/2*log(((b*x + a)^p*
(d*x + c)^q*f)^r*e)^2/((h*x + g)^2*h)
```

Giac [F]

$$\int \frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{(g+hx)^3}\,dx = \int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{(hx+g)^3}\,dx$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^3,x, algorithm="giac")
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(h*x + g)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{(g+hx)^3}\,dx = \int \frac{\ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)^2}{(g+h\,x)^3}\,dx$$

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(g + h*x)^3,x)
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(g + h*x)^3, x)
```

3.42 $\int \frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{(g+hx)^4}\,dx$

Optimal result 409
Rubi [A] (verified) 411
Mathematica [B] (verified) 419
Maple [F] 419
Fricas [F] 419
Sympy [F(-1)] 419
Maxima [B] (verification not implemented) 420
Giac [F] 422
Mupad [F(-1)] 422

Optimal result

Integrand size = 31, antiderivative size = 1957

$$
\begin{aligned}
&\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^4}\,dx \\
&= -\frac{b^2p^2r^2}{3h(bg-ah)^2(g+hx)} - \frac{2bdpqr^2}{3h(bg-ah)(dg-ch)(g+hx)} \\
&\quad - \frac{d^2q^2r^2}{3h(dg-ch)^2(g+hx)} - \frac{b^3p^2r^2\log(a+bx)}{3h(bg-ah)^3} - \frac{2bd^2pqr^2\log(a+bx)}{3h(bg-ah)(dg-ch)^2} \\
&\quad - \frac{b^2dpqr^2\log(a+bx)}{3h(bg-ah)^2(dg-ch)} + \frac{bp^2r^2\log(a+bx)}{3h(bg-ah)(g+hx)^2} + \frac{dpqr^2\log(a+bx)}{3h(dg-ch)(g+hx)^2} \\
&\quad + \frac{2d^2pqr^2\log(a+bx)}{3h(dg-ch)^2(g+hx)} - \frac{2b^2p^2r^2(a+bx)\log(a+bx)}{3(bg-ah)^3(g+hx)} - \frac{bd^2pqr^2\log(c+dx)}{3h(bg-ah)(dg-ch)^2} \\
&\quad - \frac{2b^2dpqr^2\log(c+dx)}{3h(bg-ah)^2(dg-ch)} - \frac{d^3q^2r^2\log(c+dx)}{3h(dg-ch)^3} + \frac{bpqr^2\log(c+dx)}{3h(bg-ah)(g+hx)^2} \\
&\quad + \frac{dq^2r^2\log(c+dx)}{3h(dg-ch)(g+hx)^2} + \frac{2b^2pqr^2\log(c+dx)}{3h(bg-ah)^2(g+hx)} - \frac{2d^2q^2r^2(c+dx)\log(c+dx)}{3(dg-ch)^3(g+hx)} \\
&\quad + \frac{2b^3pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{3h(bg-ah)^3} + \frac{2d^3pqr^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{3h(dg-ch)^3} \\
&\quad - \frac{bpr\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{3h(bg-ah)(g+hx)^2} \\
&\quad - \frac{dqr\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{3h(dg-ch)(g+hx)^2} \\
&\quad - \frac{2b^2pr\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{3h(bg-ah)^2(g+hx)} \\
&\quad - \frac{2d^2qr\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{3h(dg-ch)^2(g+hx)} \\
&\quad - \frac{2b^3pr\log(a+bx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{3h(bg-ah)^3} \\
&\quad - \frac{2d^3qr\log(c+dx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{3h(dg-ch)^3} \\
&\quad - \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h(g+hx)^3} + \frac{b^3p^2r^2\log(g+hx)}{h(bg-ah)^3} \\
&\quad + \frac{bd^2pqr^2\log(g+hx)}{h(bg-ah)(dg-ch)^2} + \frac{b^2dpqr^2\log(g+hx)}{h(bg-ah)^2(dg-ch)} + \frac{d^3q^2r^2\log(g+hx)}{h(dg-ch)^3} \\
&\quad + \frac{2b^3pr\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)\log(g+hx)}{3h(bg-ah)^3} \\
&\quad + \frac{2d^3qr\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)\log(g+hx)}{3h(dg-ch)^3} \\
&\quad - \frac{2d^3pqr^2\log(a+bx)\log\left(\frac{b(g+hx)}{bg-ah}\right)}{3h(dg-ch)^3} - \frac{2b^3pqr^2\log(c+dx)\log\left(\frac{d(g+hx)}{dg-ch}\right)}{3h(bg-ah)^3} \\
&\quad - \frac{2b^3p^2r^2\log(a+bx)\log\left(1+\frac{bg-ah}{h(a+bx)}\right)}{3h(bg-ah)^3} - \frac{2d^3q^2r^2\log(c+dx)\log\left(1+\frac{dg-ch}{h(c+dx)}\right)}{3h(dg-ch)^3}
\end{aligned}
$$

```
[Out] -1/3*d*q*r*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/h/
(-c*h+d*g)/(h*x+g)^2-2/3*b^2*p*r*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+
a)^p*(d*x+c)^q)^r))/h/(-a*h+b*g)^2/(h*x+g)-2/3*d^2*q*r*(p*r*ln(b*x+a)+q*r*l
n(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/h/(-c*h+d*g)^2/(h*x+g)-2/3*b^3*p*
r*ln(b*x+a)*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/h
/(-a*h+b*g)^3-2/3*d^3*q*r*ln(d*x+c)*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b
*x+a)^p*(d*x+c)^q)^r))/h/(-c*h+d*g)^3+2/3*b^3*p*r*(p*r*ln(b*x+a)+q*r*ln(d*x
+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))*ln(h*x+g)/h/(-a*h+b*g)^3+2/3*d^3*q*r*(
p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))*ln(h*x+g)/h/(-
c*h+d*g)^3-2/3*b^3*p^2*r^2*ln(b*x+a)*ln(1+(-a*h+b*g)/h/(b*x+a))/h/(-a*h+b*g
)^3-2/3*d^3*q^2*r^2*ln(d*x+c)*ln(1+(-c*h+d*g)/h/(d*x+c))/h/(-c*h+d*g)^3+1/3
*b*p^2*r^2*ln(b*x+a)/h/(-a*h+b*g)/(h*x+g)^2-2/3*b^2*p^2*r^2*(b*x+a)*ln(b*x+
a)/(-a*h+b*g)^3/(h*x+g)+1/3*d*q^2*r^2*ln(d*x+c)/h/(-c*h+d*g)/(h*x+g)^2-2/3*
d^2*q^2*r^2*(d*x+c)*ln(d*x+c)/(-c*h+d*g)^3/(h*x+g)-1/3*b*p*r*(p*r*ln(b*x+a)
+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/h/(-a*h+b*g)/(h*x+g)^2+2/3*
d^3*p*q*r^2*polylog(2,-d*(b*x+a)/(-a*d+b*c))/h/(-c*h+d*g)^3-2/3*d^3*p*q*r^2
*polylog(2,-h*(b*x+a)/(-a*h+b*g))/h/(-c*h+d*g)^3+2/3*b^3*p*q*r^2*polylog(2,
b*(d*x+c)/(-a*d+b*c))/h/(-a*h+b*g)^3-2/3*b^3*p*q*r^2*polylog(2,-h*(d*x+c)/(
-c*h+d*g))/h/(-a*h+b*g)^3-2/3*b*d^2*p*q*r^2*ln(b*x+a)/h/(-a*h+b*g)/(-c*h+d*
g)^2-1/3*b^2*d*p*q*r^2*ln(b*x+a)/h/(-a*h+b*g)^2/(-c*h+d*g)-1/3*b*d^2*p*q*r^
2*ln(d*x+c)/h/(-a*h+b*g)/(-c*h+d*g)^2-2/3*b^2*d*p*q*r^2*ln(d*x+c)/h/(-a*h+b
*g)^2/(-c*h+d*g)-2/3*b*d*p*q*r^2/h/(-a*h+b*g)/(-c*h+d*g)/(h*x+g)-1/3*ln(e*(
f*(b*x+a)^p*(d*x+c)^q)^r)^2/h/(h*x+g)^3+b^3*p^2*r^2*ln(h*x+g)/h/(-a*h+b*g)^
3+d^3*q^2*r^2*ln(h*x+g)/h/(-c*h+d*g)^3-2/3*d^3*p*q*r^2*ln(b*x+a)*ln(b*(h*x+
g)/(-a*h+b*g))/h/(-c*h+d*g)^3-2/3*b^3*p*q*r^2*ln(d*x+c)*ln(d*(h*x+g)/(-c*h+
d*g))/h/(-a*h+b*g)^3+1/3*d*p*q*r^2*ln(b*x+a)/h/(-c*h+d*g)/(h*x+g)^2+2/3*d^2
*p*q*r^2*ln(b*x+a)/h/(-c*h+d*g)^2/(h*x+g)+1/3*b*p*q*r^2*ln(d*x+c)/h/(-a*h+b
*g)/(h*x+g)^2+2/3*b^2*p*q*r^2*ln(d*x+c)/h/(-a*h+b*g)^2/(h*x+g)+2/3*b^3*p*q*
r^2*ln(-d*(b*x+a)/(-a*d+b*c))*ln(d*x+c)/h/(-a*h+b*g)^3+2/3*d^3*p*q*r^2*ln(b
*x+a)*ln(b*(d*x+c)/(-a*d+b*c))/h/(-c*h+d*g)^3-1/3*b^3*p^2*r^2*ln(b*x+a)/h/(
-a*h+b*g)^3-1/3*d^3*q^2*r^2*ln(d*x+c)/h/(-c*h+d*g)^3-1/3*b^2*p^2*r^2/h/(-a*
h+b*g)^2/(h*x+g)-1/3*d^2*q^2*r^2/h/(-c*h+d*g)^2/(h*x+g)+2/3*b^3*p^2*r^2*pol
ylog(2,(a*h-b*g)/h/(b*x+a))/h/(-a*h+b*g)^3+2/3*d^3*q^2*r^2*polylog(2,(c*h-d
*g)/h/(d*x+c))/h/(-c*h+d*g)^3+b*d^2*p*q*r^2*ln(h*x+g)/h/(-a*h+b*g)/(-c*h+d*
g)^2+b^2*d*p*q*r^2*ln(h*x+g)/h/(-a*h+b*g)^2/(-c*h+d*g)
```

Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 1957, normalized size of antiderivative = 1.00, number of steps used = 57, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.484, Rules

used = {2584, 2593, 2458, 2389, 2379, 2438, 2351, 31, 2356, 46, 2465, 2441, 2440, 2442, 36}

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^4}\,dx$$
$$= -\frac{p^2r^2\log(a+bx)b^3}{3h(bg-ah)^3} + \frac{2pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)b^3}{3h(bg-ah)^3}$$
$$- \frac{2pr\log(a+bx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)b^3}{3h(bg-ah)^3}$$
$$+ \frac{p^2r^2\log(g+hx)b^3}{h(bg-ah)^3}$$
$$+ \frac{2pr\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)\log(g+hx)b^3}{3h(bg-ah)^3}$$
$$- \frac{2pqr^2\log(c+dx)\log\left(\frac{d(g+hx)}{dg-ch}\right)b^3}{3h(bg-ah)^3} - \frac{2p^2r^2\log(a+bx)\log\left(\frac{bg-ah}{h(a+bx)}+1\right)b^3}{3h(bg-ah)^3}$$
$$+ \frac{2p^2r^2\,\mathrm{PolyLog}\left(2,-\frac{bg-ah}{h(a+bx)}\right)b^3}{3h(bg-ah)^3} + \frac{2pqr^2\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)b^3}{3h(bg-ah)^3}$$
$$- \frac{2pqr^2\,\mathrm{PolyLog}\left(2,-\frac{h(c+dx)}{dg-ch}\right)b^3}{3h(bg-ah)^3} - \frac{dpqr^2\log(a+bx)b^2}{3h(bg-ah)^2(dg-ch)}$$
$$- \frac{2p^2r^2(a+bx)\log(a+bx)b^2}{3(bg-ah)^3(g+hx)} - \frac{2dpqr^2\log(c+dx)b^2}{3h(bg-ah)^2(dg-ch)} + \frac{2pqr^2\log(c+dx)b^2}{3h(bg-ah)^2(g+hx)}$$
$$- \frac{2pr\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)b^2}{3h(bg-ah)^2(g+hx)}$$
$$+ \frac{dpqr^2\log(g+hx)b^2}{h(bg-ah)^2(dg-ch)} - \frac{p^2r^2b^2}{3h(bg-ah)^2(g+hx)} - \frac{2d^2pqr^2\log(a+bx)b}{3h(bg-ah)(dg-ch)^2}$$
$$+ \frac{p^2r^2\log(a+bx)b}{3h(bg-ah)(g+hx)^2} - \frac{d^2pqr^2\log(c+dx)b}{3h(bg-ah)(dg-ch)^2} + \frac{pqr^2\log(c+dx)b}{3h(bg-ah)(g+hx)^2}$$
$$- \frac{pr\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)b}{3h(bg-ah)(g+hx)^2}$$
$$+ \frac{d^2pqr^2\log(g+hx)b}{h(bg-ah)(dg-ch)^2} - \frac{2dpqr^2b}{3h(bg-ah)(dg-ch)(g+hx)}$$
$$- \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h(g+hx)^3} + \frac{2d^2pqr^2\log(a+bx)}{3h(dg-ch)^2(g+hx)}$$
$$+ \frac{dpqr^2\log(a+bx)}{3h(dg-ch)(g+hx)^2} - \frac{d^3q^2r^2\log(c+dx)}{3h(dg-ch)^3} - \frac{2d^2q^2r^2(c+dx)\log(c+dx)}{3(dg-ch)^3(g+hx)}$$
$$+ \frac{dq^2r^2\log(c+dx)}{3h(dg-ch)(g+hx)^2} + \frac{2d^3pqr^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{3h(dg-ch)^3}$$
$$- \frac{2d^3qr\log(c+dx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{3h(dg-ch)^3}$$
$$- \frac{2d^2qr\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{3h(dg-ch)^2(g+hx)}$$
$$- \frac{dqr\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{3h(dg-ch)(g+hx)^2}$$
$$+ \frac{d^3q^2r^2\log(g+hx)}{}$$

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(g + h*x)^4,x]
```

```
[Out] -1/3*(b^2*p^2*r^2)/(h*(b*g - a*h)^2*(g + h*x)) - (2*b*d*p*q*r^2)/(3*h*(b*g
- a*h)*(d*g - c*h)*(g + h*x)) - (d^2*q^2*r^2)/(3*h*(d*g - c*h)^2*(g + h*x))
 - (b^3*p^2*r^2*Log[a + b*x])/(3*h*(b*g - a*h)^3) - (2*b*d^2*p*q*r^2*Log[a
+ b*x])/(3*h*(b*g - a*h)*(d*g - c*h)^2) - (b^2*d*p*q*r^2*Log[a + b*x])/(3*h
*(b*g - a*h)^2*(d*g - c*h)) + (b*p^2*r^2*Log[a + b*x])/(3*h*(b*g - a*h)*(g
+ h*x)^2) + (d*p*q*r^2*Log[a + b*x])/(3*h*(d*g - c*h)*(g + h*x)^2) + (2*d^2
*p*q*r^2*Log[a + b*x])/(3*h*(d*g - c*h)^2*(g + h*x)) - (2*b^2*p^2*r^2*(a +
b*x)*Log[a + b*x])/(3*(b*g - a*h)^3*(g + h*x)) - (b*d^2*p*q*r^2*Log[c + d*x
])/(3*h*(b*g - a*h)*(d*g - c*h)^2) - (2*b^2*d*p*q*r^2*Log[c + d*x])/(3*h*(b
*g - a*h)^2*(d*g - c*h)) - (d^3*q^2*r^2*Log[c + d*x])/(3*h*(d*g - c*h)^3) +
 (b*p*q*r^2*Log[c + d*x])/(3*h*(b*g - a*h)*(g + h*x)^2) + (d*q^2*r^2*Log[c
+ d*x])/(3*h*(d*g - c*h)*(g + h*x)^2) + (2*b^2*p*q*r^2*Log[c + d*x])/(3*h*(
b*g - a*h)^2*(g + h*x)) - (2*d^2*q^2*r^2*(c + d*x)*Log[c + d*x])/(3*(d*g -
c*h)^3*(g + h*x)) + (2*b^3*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c
+ d*x])/(3*h*(b*g - a*h)^3) + (2*d^3*p*q*r^2*Log[a + b*x]*Log[(b*(c + d*x))
/(b*c - a*d)])/(3*h*(d*g - c*h)^3) - (b*p*r*(p*r*Log[a + b*x] + q*r*Log[c +
 d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(3*h*(b*g - a*h)*(g + h*x)^2
) - (d*q*r*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r]))/(3*h*(d*g - c*h)*(g + h*x)^2) - (2*b^2*p*r*(p*r*Log[a + b*x]
+ q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(3*h*(b*g - a*h
)^2*(g + h*x)) - (2*d^2*q*r*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]))/(3*h*(d*g - c*h)^2*(g + h*x)) - (2*b^3*p*r*L
og[a + b*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c
+ d*x)^q)^r]))/(3*h*(b*g - a*h)^3) - (2*d^3*q*r*Log[c + d*x]*(p*r*Log[a + b
*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(3*h*(d*g -
 c*h)^3) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(3*h*(g + h*x)^3) + (b^3*
p^2*r^2*Log[g + h*x])/(h*(b*g - a*h)^3) + (b*d^2*p*q*r^2*Log[g + h*x])/(h*(
b*g - a*h)*(d*g - c*h)^2) + (b^2*d*p*q*r^2*Log[g + h*x])/(h*(b*g - a*h)^2*(
d*g - c*h)) + (d^3*q^2*r^2*Log[g + h*x])/(h*(d*g - c*h)^3) + (2*b^3*p*r*(p*
r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])*L
og[g + h*x])/(3*h*(b*g - a*h)^3) + (2*d^3*q*r*(p*r*Log[a + b*x] + q*r*Log[c
 + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])*Log[g + h*x])/(3*h*(d*g - c
*h)^3) - (2*d^3*p*q*r^2*Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)])/(3*h*(
d*g - c*h)^3) - (2*b^3*p*q*r^2*Log[c + d*x]*Log[(d*(g + h*x))/(d*g - c*h)])
/(3*h*(b*g - a*h)^3) - (2*b^3*p^2*r^2*Log[a + b*x]*Log[1 + (b*g - a*h)/(h*(
a + b*x))])/(3*h*(b*g - a*h)^3) - (2*d^3*q^2*r^2*Log[c + d*x]*Log[1 + (d*g
- c*h)/(h*(c + d*x))])/(3*h*(d*g - c*h)^3) + (2*b^3*p^2*r^2*PolyLog[2, -((b
*g - a*h)/(h*(a + b*x)))])/(3*h*(b*g - a*h)^3) + (2*d^3*p*q*r^2*PolyLog[2,
-((d*(a + b*x))/(b*c - a*d))])/(3*h*(d*g - c*h)^3) - (2*d^3*p*q*r^2*PolyLog
[2, -((h*(a + b*x))/(b*g - a*h))])/(3*h*(d*g - c*h)^3) + (2*d^3*q^2*r^2*Pol
yLog[2, -((d*g - c*h)/(h*(c + d*x)))])/(3*h*(d*g - c*h)^3) + (2*b^3*p*q*r^2
*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(3*h*(b*g - a*h)^3) - (2*b^3*p*q*r^
2*PolyLog[2, -((h*(c + d*x))/(d*g - c*h))])/(3*h*(b*g - a*h)^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
```

```
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2584

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1
)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(h*(m + 1))), x] + (-Dist[b*p*r*(
s/(h*(m + 1))), Int[(g + h*x)^(m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1)/(a + b*x)), x], x] - Dist[d*q*r*(s/(h*(m + 1))), Int[(g + h*x)^(m
+ 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(c + d*x)), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2593

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h(g+hx)^3}+\frac{(2bpr)\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)(g+hx)^3}\,dx}{3h} \\
&\quad +\frac{(2dqr)\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(c+dx)(g+hx)^3}\,dx}{3h} \\
&= -\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h(g+hx)^3} \\
&\quad +\frac{(2bp^2r^2)\int \frac{\log(a+bx)}{(a+bx)(g+hx)^3}\,dx}{3h}+\frac{(2bpqr^2)\int \frac{\log(c+dx)}{(a+bx)(g+hx)^3}\,dx}{3h} \\
&\quad +\frac{(2dpqr^2)\int \frac{\log(a+bx)}{(c+dx)(g+hx)^3}\,dx}{3h}+\frac{(2dq^2r^2)\int \frac{\log(c+dx)}{(c+dx)(g+hx)^3}\,dx}{3h} \\
&\quad -\frac{(2bpr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)))\int \frac{1}{(a+bx)(g+hx)^3}\,dx}{3h} \\
&\quad -\frac{(2dqr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)))\int \frac{1}{(c+dx)(g+hx)^3}\,dx}{3h}
\end{aligned}
$$

$$= -\frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3h(g+hx)^3} + \frac{(2p^2r^2)\,\text{Subst}\left(\int \frac{\log(x)}{x\left(\frac{bg-ah}{b}+\frac{hx}{b}\right)^3}\,dx, x, a+bx\right)}{3h}$$

$$+ \frac{(2bpqr^2)\int \left(\frac{b^3\log(c+dx)}{(bg-ah)^3(a+bx)} - \frac{h\log(c+dx)}{(bg-ah)(g+hx)^3} - \frac{bh\log(c+dx)}{(bg-ah)^2(g+hx)^2} - \frac{b^2h\log(c+dx)}{(bg-ah)^3(g+hx)}\right)dx}{3h}$$

$$+ \frac{(2dpqr^2)\int \left(\frac{d^3\log(a+bx)}{(dg-ch)^3(c+dx)} - \frac{h\log(a+bx)}{(dg-ch)(g+hx)^3} - \frac{dh\log(a+bx)}{(dg-ch)^2(g+hx)^2} - \frac{d^2h\log(a+bx)}{(dg-ch)^3(g+hx)}\right)dx}{3h}$$

$$+ \frac{(2q^2r^2)\,\text{Subst}\left(\int \frac{\log(x)}{x\left(\frac{dg-ch}{d}+\frac{hx}{d}\right)^3}\,dx, x, c+dx\right)}{3h}$$

$$- \frac{\left(2bpr\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\right)\right)\int \left(\frac{b^3}{(bg-ah)^3(a+bx)} - \frac{}{(bg-}\right.}{3h}$$

$$- \frac{\left(2dqr\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\right)\right)\int \left(\frac{d^3}{(dg-ch)^3(c+dx)} - \frac{}{(dg-}\right.}{3h}$$

$$= -\frac{bpr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right))}{3h(bg-ah)(g+hx)^2} - \frac{dqr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right))}{3h(dg-ch)(g+hx)^2} - \frac{2b^2pr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right))}{3h(bg-ah)^2(g+hx)} - \frac{2d^2qr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right))}{3h(dg-ch)^2(g+hx)} - \frac{2b^3pr\log(a+bx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{3h(bg-ah)^3} - \frac{2d^3qr\log(c+dx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)}{3h(dg-ch)^3} - \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h(g+hx)^3} + \frac{2b^3pr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right))\log(g+hx)}{3h(bg-ah)^3} + \frac{2d^3qr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right))\log(g+hx)}{3h(dg-ch)^3} - \frac{(2p^2r^2)\operatorname{Subst}\left(\int\frac{\log(x)}{\left(\frac{bg-ah}{b}+\frac{hx}{b}\right)^3}dx,x,a+bx\right)}{3(bg-ah)} + \frac{(2bp^2r^2)\operatorname{Subst}\left(\int\frac{\log(x)}{x\left(\frac{bg-ah}{b}+\frac{hx}{b}\right)^2}dx,x,a+bx\right)}{3h(bg-ah)} - \frac{(2b^3pqr^2)\int\frac{\log(c+dx)}{g+hx}dx}{3(bg-ah)^3} + \frac{(2b^4pqr^2)\int\frac{\log(c+dx)}{a+bx}dx}{3h(bg-ah)^3} - \frac{(2b^2pqr^2)\int\frac{\log(c+dx)}{(g+hx)^2}dx}{3(bg-ah)^2} - \frac{(2bpqr^2)\int\frac{\log(c+dx)}{(g+hx)^3}dx}{3(bg-ah)} - \frac{(2d^3pqr^2)\int\frac{\log(a+bx)}{g+hx}dx}{3(dg-ch)^3} + \frac{(2d^4pqr^2)\int\frac{\log(a+bx)}{c+dx}dx}{3h(dg-ch)^3} - \frac{(2d^2pqr^2)\int\frac{\log(a+bx)}{(g+hx)^2}dx}{3(dg-ch)^2} - \frac{(2dpqr^2)\int\frac{\log(a+bx)}{(g+hx)^3}dx}{3(dg-ch)} - \frac{(2q^2r^2)\operatorname{Subst}\left(\int\frac{\log(x)}{\left(\frac{dg-ch}{d}+\frac{hx}{d}\right)^3}dx,x,c+dx\right)}{3(dg-ch)} + \frac{(2dq^2r^2)\operatorname{Subst}\left(\int\frac{\log(x)}{x\left(\frac{dg-ch}{d}+\frac{hx}{d}\right)^2}dx,x,c+dx\right)}{3h(dg-ch)}$$

$$= \text{Too large to display}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 47127 vs. 2(1957) = 3914.

Time = 6.30 (sec) , antiderivative size = 47127, normalized size of antiderivative = 24.08

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^4}\,dx=\text{Result too large to show}$$

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(g + h*x)^4,x]
[Out] Result too large to show
```

Maple [F]

$$\int \frac{\ln\left(e\left(f(bx+a)^p\,(dx+c)^q\right)^r\right)^2}{(hx+g)^4}dx$$

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^4,x)
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^4,x)
```

Fricas [F]

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^4}\,dx=\int \frac{\log\left(\left((bx+a)^p(dx+c)^qf\right)^re\right)^2}{(hx+g)^4}\,dx$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^4,x, algorithm="fricas
")
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(h^4*x^4 + 4*g*h^3*x^3 + 6*
g^2*h^2*x^2 + 4*g^3*h*x + g^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^4}\,dx=\text{Timed out}$$

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(h*x+g)**4,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4732 vs. 2(1875) = 3750.

Time = 0.80 (sec) , antiderivative size = 4732, normalized size of antiderivative = 2.42

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^4}\,dx = \text{Too large to display}$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^4,x, algorithm="maxima
")
```

```
[Out] 1/3*(2*b^3*f*p*log(b*x + a)/(b^3*g^3 - 3*a*b^2*g^2*h + 3*a^2*b*g*h^2 - a^3*
h^3) + 2*d^3*f*q*log(d*x + c)/(d^3*g^3 - 3*c*d^2*g^2*h + 3*c^2*d*g*h^2 - c^
3*h^3) - 2*(3*a*b^2*d^3*f*g^2*h*q - 3*a^2*b*d^3*f*g*h^2*q + a^3*d^3*f*h^3*q
 - (d^3*f*g^3*(p + q) - 3*c*d^2*f*g^2*h*p + 3*c^2*d*f*g*h^2*p - c^3*f*h^3*p
)*b^3)*log(h*x + g)/((d^3*g^3*h^3 - 3*c*d^2*g^2*h^4 + 3*c^2*d*g*h^5 - c^3*h
^6)*a^3 - 3*(d^3*g^4*h^2 - 3*c*d^2*g^3*h^3 + 3*c^2*d*g^2*h^4 - c^3*g*h^5)*a
^2*b + 3*(d^3*g^5*h - 3*c*d^2*g^4*h^2 + 3*c^2*d*g^3*h^3 - c^3*g^2*h^4)*a*b^
2 - (d^3*g^6 - 3*c*d^2*g^5*h + 3*c^2*d*g^4*h^2 - c^3*g^3*h^3)*b^3) + ((3*d^
2*f*g*h^2*q - c*d*f*h^3*q)*a^2 - (d^2*f*g^2*h*(p + 6*q) - 2*c*d*f*g*h^2*(p
+ q) + c^2*f*h^3*p)*a*b - (c*d*f*g^2*h*(6*p + q) - 3*d^2*f*g^3*(p + q) - 3*
c^2*f*g*h^2*p)*b^2 - 2*(2*a*b*d^2*f*g*h^2*q - a^2*d^2*f*h^3*q - (d^2*f*g^2*
h*(p + q) - 2*c*d*f*g*h^2*p + c^2*f*h^3*p)*b^2)*x)/((d^2*g^4*h^2 - 2*c*d*g^
3*h^3 + c^2*g^2*h^4)*a^2 - 2*(d^2*g^5*h - 2*c*d*g^4*h^2 + c^2*g^3*h^3)*a*b
+ (d^2*g^6 - 2*c*d*g^5*h + c^2*g^4*h^2)*b^2 + ((d^2*g^2*h^4 - 2*c*d*g*h^5 +
 c^2*h^6)*a^2 - 2*(d^2*g^3*h^3 - 2*c*d*g^2*h^4 + c^2*g*h^5)*a*b + (d^2*g^4*
h^2 - 2*c*d*g^3*h^3 + c^2*g^2*h^4)*b^2)*x^2 + 2*((d^2*g^3*h^3 - 2*c*d*g^2*h
^4 + c^2*g*h^5)*a^2 - 2*(d^2*g^4*h^2 - 2*c*d*g^3*h^3 + c^2*g^2*h^4)*a*b + (
d^2*g^5*h - 2*c*d*g^4*h^2 + c^2*g^3*h^3)*b^2)*x))*r*log(((b*x + a)^p*(d*x +
 c)^q*f)^r*e)/(f*h) + 1/3*(2*(3*a*b^2*d^3*f^2*g^2*h*p*q - 3*a^2*b*d^3*f^2*g
*h^2*p*q + a^3*d^3*f^2*h^3*p*q - (3*c*d^2*f^2*g^2*h*p*q - 3*c^2*d*f^2*g*h^2
*p*q + c^3*f^2*h^3*p*q)*b^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) +
1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))/((d^3*g^3*h^3 - 3*c*d^2*g^2*h^4 + 3
*c^2*d*g*h^5 - c^3*h^6)*a^3 - 3*(d^3*g^4*h^2 - 3*c*d^2*g^3*h^3 + 3*c^2*d*g^
2*h^4 - c^3*g*h^5)*a^2*b + 3*(d^3*g^5*h - 3*c*d^2*g^4*h^2 + 3*c^2*d*g^3*h^3
 - c^3*g^2*h^4)*a*b^2 - (d^3*g^6 - 3*c*d^2*g^5*h + 3*c^2*d*g^4*h^2 - c^3*g^
3*h^3)*b^3) - 2*(3*a*b^2*d^3*f^2*g^2*h*p*q - 3*a^2*b*d^3*f^2*g*h^2*p*q + a^
3*d^3*f^2*h^3*p*q + (3*c*d^2*f^2*g^2*h*p^2 - 3*c^2*d*f^2*g*h^2*p^2 + c^3*f^
2*h^3*p^2 - (p^2 + p*q)*d^3*f^2*g^3)*b^3)*(log(b*x + a)*log((b*h*x + a*h)/(
b*g - a*h) + 1) + dilog(-(b*h*x + a*h)/(b*g - a*h)))/((d^3*g^3*h^3 - 3*c*d^
2*g^2*h^4 + 3*c^2*d*g*h^5 - c^3*h^6)*a^3 - 3*(d^3*g^4*h^2 - 3*c*d^2*g^3*h^3
 + 3*c^2*d*g^2*h^4 - c^3*g*h^5)*a^2*b + 3*(d^3*g^5*h - 3*c*d^2*g^4*h^2 + 3*
c^2*d*g^3*h^3 - c^3*g^2*h^4)*a*b^2 - (d^3*g^6 - 3*c*d^2*g^5*h + 3*c^2*d*g^4
*h^2 - c^3*g^3*h^3)*b^3) - 2*(3*a*b^2*d^3*f^2*g^2*h*q^2 - 3*a^2*b*d^3*f^2*g
*h^2*q^2 + a^3*d^3*f^2*h^3*q^2 + (3*c*d^2*f^2*g^2*h*p*q - 3*c^2*d*f^2*g*h^2
```

```
*p*q + c^3*f^2*h^3*p*q - (p*q + q^2)*d^3*f^2*g^3)*b^3)*(log(d*x + c)*log((d
*h*x + c*h)/(d*g - c*h) + 1) + dilog(-(d*h*x + c*h)/(d*g - c*h)))/((d^3*g^3
*h^3 - 3*c*d^2*g^2*h^4 + 3*c^2*d*g*h^5 - c^3*h^6)*a^3 - 3*(d^3*g^4*h^2 - 3*
c*d^2*g^3*h^3 + 3*c^2*d*g^2*h^4 - c^3*g*h^5)*a^2*b + 3*(d^3*g^5*h - 3*c*d^2
*g^4*h^2 + 3*c^2*d*g^3*h^3 - c^3*g^2*h^4)*a*b^2 - (d^3*g^6 - 3*c*d^2*g^5*h
+ 3*c^2*d*g^4*h^2 - c^3*g^3*h^3)*b^3) - (3*a^2*d^3*f^2*h^2*q^2 + (c*d^2*f^2
*h^2*p*q - (p*q + 6*q^2)*d^3*f^2*g*h)*a*b - (5*c*d^2*f^2*g*h*p*q - 2*c^2*d*
f^2*h^2*p*q - 3*(p*q + q^2)*d^3*f^2*g^2)*b^2)*log(d*x + c)/((d^3*g^3*h^2 -
3*c*d^2*g^2*h^3 + 3*c^2*d*g*h^4 - c^3*h^5)*a^2 - 2*(d^3*g^4*h - 3*c*d^2*g^3
*h^2 + 3*c^2*d*g^2*h^3 - c^3*g*h^4)*a*b + (d^3*g^5 - 3*c*d^2*g^4*h + 3*c^2*
d*g^3*h^2 - c^3*g^2*h^3)*b^2) + 3*(a^3*d^3*f^2*h^3*q^2 + (c*d^2*f^2*h^3*p*q
 - (p*q + 3*q^2)*d^3*f^2*g*h^2)*a^2*b - (4*c*d^2*f^2*g*h^2*p*q - c^2*d*f^2*
h^3*p*q - 3*(p*q + q^2)*d^3*f^2*g^2*h)*a*b^2 + (c^3*f^2*h^3*p^2 - (p^2 + 2*
p*q + q^2)*d^3*f^2*g^3 + 3*(p^2 + p*q)*c*d^2*f^2*g^2*h - (3*p^2 + p*q)*c^2*
d*f^2*g*h^2)*b^3)*log(h*x + g)/((d^3*g^3*h^3 - 3*c*d^2*g^2*h^4 + 3*c^2*d*g*
h^5 - c^3*h^6)*a^3 - 3*(d^3*g^4*h^2 - 3*c*d^2*g^3*h^3 + 3*c^2*d*g^2*h^4 - c
^3*g*h^5)*a^2*b + 3*(d^3*g^5*h - 3*c*d^2*g^4*h^2 + 3*c^2*d*g^3*h^3 - c^3*g^
2*h^4)*a*b^2 - (d^3*g^6 - 3*c*d^2*g^5*h + 3*c^2*d*g^4*h^2 - c^3*g^3*h^3)*b^
3) - ((d^3*f^2*g*h^3*q^2 - c*d^2*f^2*h^4*q^2)*a^3 - (2*c^2*d*f^2*h^4*p*q +
(2*p*q + 3*q^2)*d^3*f^2*g^2*h^2 - (4*p*q + 3*q^2)*c*d^2*f^2*g*h^3)*a^2*b -
(c^3*f^2*h^4*p^2 - (p^2 + 4*p*q + 3*q^2)*d^3*f^2*g^3*h + (3*p^2 + 8*p*q + 3
*q^2)*c*d^2*f^2*g^2*h^2 - (3*p^2 + 4*p*q)*c^2*d*f^2*g*h^3)*a*b^2 + (c^3*f^2
*g*h^3*p^2 - (p^2 + 2*p*q + q^2)*d^3*f^2*g^4 + (3*p^2 + 4*p*q + q^2)*c*d^2*
f^2*g^3*h - (3*p^2 + 2*p*q)*c^2*d*f^2*g^2*h^2)*b^3 - ((d^3*f^2*g^3*h*p^2 -
3*c*d^2*f^2*g^2*h^2*p^2 + 3*c^2*d*f^2*g*h^3*p^2 - c^3*f^2*h^4*p^2)*b^3*x +
(d^3*f^2*g^4*p^2 - 3*c*d^2*f^2*g^3*h*p^2 + 3*c^2*d*f^2*g^2*h^2*p^2 - c^3*f^
2*g*h^3*p^2)*b^3)*log(b*x + a)^2 - 2*(b^3*d^3*f^2*g^4*p*q - 3*a*b^2*d^3*f^2
*g^3*h*p*q + 3*a^2*b*d^3*f^2*g^2*h^2*p*q - a^3*d^3*f^2*g*h^3*p*q + (b^3*d^3
*f^2*g^3*h*p*q - 3*a*b^2*d^3*f^2*g^2*h^2*p*q + 3*a^2*b*d^3*f^2*g*h^3*p*q -
a^3*d^3*f^2*h^4*p*q)*x)*log(b*x + a)*log(d*x + c) - (b^3*d^3*f^2*g^4*q^2 -
3*a*b^2*d^3*f^2*g^3*h*q^2 + 3*a^2*b*d^3*f^2*g^2*h^2*q^2 - a^3*d^3*f^2*g*h^3
*q^2 + (b^3*d^3*f^2*g^3*h*q^2 - 3*a*b^2*d^3*f^2*g^2*h^2*q^2 + 3*a^2*b*d^3*f
^2*g*h^3*q^2 - a^3*d^3*f^2*h^4*q^2)*x)*log(d*x + c)^2 - (2*(d^3*f^2*g^2*h^2
*p*q - c*d^2*f^2*g*h^3*p*q)*a^2*b - (5*d^3*f^2*g^3*h*p*q - 6*c*d^2*f^2*g^2*
h^2*p*q + c^2*d*f^2*g*h^3*p*q)*a*b^2 - (3*c^3*f^2*g*h^3*p^2 - 3*(p^2 + p*q)
*d^3*f^2*g^4 + (9*p^2 + 4*p*q)*c*d^2*f^2*g^3*h - (9*p^2 + p*q)*c^2*d*f^2*g^
2*h^2)*b^3 + (2*(d^3*f^2*g*h^3*p*q - c*d^2*f^2*h^4*p*q)*a^2*b - (5*d^3*f^2*
g^2*h^2*p*q - 6*c*d^2*f^2*g*h^3*p*q + c^2*d*f^2*h^4*p*q)*a*b^2 - (3*c^3*f^2
*h^4*p^2 - 3*(p^2 + p*q)*d^3*f^2*g^3*h + (9*p^2 + 4*p*q)*c*d^2*f^2*g^2*h^2
- (9*p^2 + p*q)*c^2*d*f^2*g*h^3)*b^3)*x)*log(b*x + a) - 2*((3*a*b^2*d^3*f^2
*g^3*h*p*q - 3*a^2*b*d^3*f^2*g^2*h^2*p*q + a^3*d^3*f^2*g*h^3*p*q + (3*c*d^2
*f^2*g^3*h*p^2 - 3*c^2*d*f^2*g^2*h^2*p^2 + c^3*f^2*g*h^3*p^2 - (p^2 + p*q)*
d^3*f^2*g^4)*b^3 + (3*a*b^2*d^3*f^2*g^2*h^2*p*q - 3*a^2*b*d^3*f^2*g*h^3*p*q
 + a^3*d^3*f^2*h^4*p*q + (3*c*d^2*f^2*g^2*h^2*p^2 - 3*c^2*d*f^2*g*h^3*p^2 +
 c^3*f^2*h^4*p^2 - (p^2 + p*q)*d^3*f^2*g^3*h)*b^3)*x)*log(b*x + a) + (3*a*b
```

```
^2*d^3*f^2*g^3*h*q^2 - 3*a^2*b*d^3*f^2*g^2*h^2*q^2 + a^3*d^3*f^2*g*h^3*q^2
+ (3*c*d^2*f^2*g^3*h*p*q - 3*c^2*d*f^2*g^2*h^2*p*q + c^3*f^2*g*h^3*p*q - (p
*q + q^2)*d^3*f^2*g^4)*b^3 + (3*a*b^2*d^3*f^2*g^2*h^2*q^2 - 3*a^2*b*d^3*f^2
*g*h^3*q^2 + a^3*d^3*f^2*h^4*q^2 + (3*c*d^2*f^2*g^2*h^2*p*q - 3*c^2*d*f^2*g
*h^3*p*q + c^3*f^2*h^4*p*q - (p*q + q^2)*d^3*f^2*g^3*h)*b^3)*x)*log(d*x + c
))*log(h*x + g))/((d^3*g^4*h^3 - 3*c*d^2*g^3*h^4 + 3*c^2*d*g^2*h^5 - c^3*g*
h^6)*a^3 - 3*(d^3*g^5*h^2 - 3*c*d^2*g^4*h^3 + 3*c^2*d*g^3*h^4 - c^3*g^2*h^5
)*a^2*b + 3*(d^3*g^6*h - 3*c*d^2*g^5*h^2 + 3*c^2*d*g^4*h^3 - c^3*g^3*h^4)*a
*b^2 - (d^3*g^7 - 3*c*d^2*g^6*h + 3*c^2*d*g^5*h^2 - c^3*g^4*h^3)*b^3 + ((d^
3*g^3*h^4 - 3*c*d^2*g^2*h^5 + 3*c^2*d*g*h^6 - c^3*h^7)*a^3 - 3*(d^3*g^4*h^3
 - 3*c*d^2*g^3*h^4 + 3*c^2*d*g^2*h^5 - c^3*g*h^6)*a^2*b + 3*(d^3*g^5*h^2 -
3*c*d^2*g^4*h^3 + 3*c^2*d*g^3*h^4 - c^3*g^2*h^5)*a*b^2 - (d^3*g^6*h - 3*c*d
^2*g^5*h^2 + 3*c^2*d*g^4*h^3 - c^3*g^3*h^4)*b^3)*x))*r^2/(f^2*h) - 1/3*log(
((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/((h*x + g)^3*h)
```

Giac [F]

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^4}\,dx=\int \frac{\log\left(\left((bx+a)^p(dx+c)^qf\right)^re\right)^2}{(hx+g)^4}\,dx$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^4,x, algorithm="giac")
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(h*x + g)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^4}\,dx=\int \frac{\ln\left(e\left(f\left(a+b\,x\right)^p\left(c+d\,x\right)^q\right)^r\right)^2}{\left(g+h\,x\right)^4}\,dx$$

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(g + h*x)^4,x)
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(g + h*x)^4, x)
```

3.43 $\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$

Optimal result 423
Rubi [A] (verified) 423
Mathematica [A] (verified) 424
Maple [F] 424
Fricas [A] (verification not implemented) 424
Sympy [B] (verification not implemented) 425
Maxima [F] 425
Giac [A] (verification not implemented) 425
Mupad [F(-1)] 426

Optimal result

Integrand size = 40, antiderivative size = 42

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx = -\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^{1+n}}{bc(1+n)}$$

```
[Out] -(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^(1+n)/b/c/(1+n)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2573, 6818}

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx = -\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^{n+1}}{bc(n+1)}$$

```
[In] Int[(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
```

```
[Out] -((a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^(1 + n)/(b*c*(1 + n)))
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] && !Intege
rQ[n]
```

Rule 6818

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{\left(a+b\log\left(\sqrt{\frac{1-cx}{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx, \sqrt{\frac{1-cx}{1+cx}}, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)$$

$$= -\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^{1+n}}{bc(1+n)}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx = -\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^{1+n}}{bc(1+n)}$$

```
[In] Integrate[(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
[Out] -((a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^(1 + n)/(b*c*(1 + n)))
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(\frac{\sqrt{-xc+1}}{\sqrt{xc+1}}\right)\right)^n}{-x^2c^2+1}dx$$

```
[In] int((a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
[Out] int((a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.33

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx = -\frac{\left(b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)\left(b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{bcn+bc}$$

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, algorit
hm="fricas")
[Out] -(b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)*(b*log(sqrt(-c*x + 1)/sqrt(c*x +
 1)) + a)^n/(b*c*n + b*c)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(32) = 64$.

Time = 78.37 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.24

$$\int \frac{\left(a + b \log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1 - c^2x^2}\,dx = \begin{cases} -\frac{a^n \operatorname{atan}\left(\frac{x}{\sqrt{-\frac{1}{c^2}}}\right)}{c^2\sqrt{-\frac{1}{c^2}}} & \text{for } b = 0 \\ a^n x & \text{for } c = 0 \\ -\frac{\begin{cases} \frac{\left(a+b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^{n+1}}{n+1} & \text{for } n \neq -1 \\ \log\left(a + b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right) & \text{otherwise} \end{cases}}{bc} & \text{otherwise} \end{cases}$$

```
[In] integrate((a+b*ln((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**n/(-c**2*x**2+1),x)
```

```
[Out] Piecewise((-a**n*atan(x/sqrt(-1/c**2))/(c**2*sqrt(-1/c**2)), Eq(b, 0)), (a*
*n*x, Eq(c, 0)), (-Piecewise(((a + b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)))**(n
 + 1)/(n + 1), Ne(n, -1)), (log(a + b*log(sqrt(-c*x + 1)/sqrt(c*x + 1))), T
rue))/(b*c), True))
```

Maxima [F]

$$\int \frac{\left(a + b \log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1 - c^2x^2}\,dx = \int -\frac{\left(b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a\right)^n}{c^2x^2 - 1}\,dx$$

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, algorit
hm="maxima")
```

```
[Out] -integrate((b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int \frac{\left(a + b \log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1 - c^2x^2}\,dx = -\frac{\left(-\frac{1}{2}\,b\log\left(cx+1\right) + \frac{1}{2}\,b\log\left(-cx+1\right) + a\right)^{n+1}}{bc(n+1)}$$

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, algorit
hm="giac")
```

```
[Out] -(-1/2*b*log(c*x + 1) + 1/2*b*log(-c*x + 1) + a)^(n + 1)/(b*c*(n + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx = -\int \frac{\left(a+b\,\ln\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^n}{c^2\,x^2-1}\,dx$$

```
[In] int(-(a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1),x)
[Out] -int((a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1), x)
```

3.44 $\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$

Optimal result 427
Rubi [A] (verified) 427
Mathematica [A] (verified) 429
Maple [F] 429
Fricas [B] (verification not implemented) 429
Sympy [B] (verification not implemented) 430
Maxima [B] (verification not implemented) 430
Giac [F] 431
Mupad [F(-1)] 431

Optimal result

Integrand size = 40, antiderivative size = 37

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = -\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc}$$

```
[Out] -1/4*(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^4/b/c
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2573, 2576, 12, 2339, 30}

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = -\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^4}{4bc}$$

```
[In] Int[(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

```
[Out] -1/4*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^4/(b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rule 2576

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*(P2x_)^(m_.), x_Symbol] :> With[{f = Coeff[P2x, x, 0], g = Coef
f[P2x, x, 1], h = Coeff[P2x, x, 2]}, Dist[b*c - a*d, Subst[Int[(b^2*f - a*b
*g + a^2*h - (2*b*d*f - b*c*g - a*d*g + 2*a*c*h)*x + (d^2*f - c*d*g + c^2*h
)*x^2)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(2*(m + 1))), x], x, (a + b*x)/(c
+ d*x)], x]] /; FreeQ[{a, b, c, d, e, A, B, n}, x] && PolyQ[P2x, x, 2] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{\left(a+b\log\left(\sqrt{\frac{1-cx}{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx, \sqrt{\frac{1-cx}{1+cx}}, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\\
&= -\text{Subst}\left((2c)\text{Subst}\left(\int \frac{\left(a+b\log\left(\sqrt{x}\right)\right)^3}{4c^2x}\,dx, x, \frac{1-cx}{1+cx}\right), \sqrt{\frac{1-cx}{1+cx}}, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\\
&= -\text{Subst}\left(\frac{\text{Subst}\left(\int \frac{\left(a+b\log\left(\sqrt{x}\right)\right)^3}{x}\,dx, x, \frac{1-cx}{1+cx}\right)}{2c}, \sqrt{\frac{1-cx}{1+cx}}, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\\
&= -\text{Subst}\left(\frac{\text{Subst}\left(\int x^3\,dx, x, a+b\log\left(\sqrt{\frac{1-cx}{1+cx}}\right)\right)}{bc}, \sqrt{\frac{1-cx}{1+cx}}, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\\
&= -\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx=-\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc}$$

```
[In] Integrate[(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
[Out] -1/4*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^4/(b*c)
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(\frac{\sqrt{-xc+1}}{\sqrt{xc+1}}\right)\right)^3}{-x^2c^2+1}dx$$

```
[In] int((a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x)
[Out] int((a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(31) = 62$.

Time = 0.31 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.73

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = -\frac{b^3\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^4+4\,ab^2\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^3+6\,a^2b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2+4\,a^3\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{4\,c}$$

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, algorit
hm="fricas")
[Out] -1/4*(b^3*log(sqrt(-c*x + 1)/sqrt(c*x + 1))^4 + 4*a*b^2*log(sqrt(-c*x + 1)/
sqrt(c*x + 1))^3 + 6*a^2*b*log(sqrt(-c*x + 1)/sqrt(c*x + 1))^2 + 4*a^3*log(
sqrt(-c*x + 1)/sqrt(c*x + 1)))/c
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(29) = 58$.

Time = 4.51 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.76

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = \begin{cases} -\frac{a^3\operatorname{atan}\left(\frac{x}{\sqrt{-\frac{1}{c^2}}}\right)}{c^2\sqrt{-\frac{1}{c^2}}} & \text{for } b=0 \\ a^3x & \text{for } c=0 \\ -\frac{\left(a+b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^4}{4bc} & \text{otherwise} \end{cases}$$

```
[In] integrate((a+b*ln((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**3/(-c**2*x**2+1),x)
[Out] Piecewise((-a**3*atan(x/sqrt(-1/c**2))/(c**2*sqrt(-1/c**2)), Eq(b, 0)), (a*
*3*x, Eq(c, 0)), (-(a + b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)))**4/(4*b*c), Tr
ue))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 526 vs. $2(31) = 62$.

Time = 0.24 (sec) , antiderivative size = 526, normalized size of antiderivative = 14.22

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = \frac{1}{2}b^3\left(\frac{\log(cx+1)}{c}-\frac{\log(cx-1)}{c}\right)\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^3$$
$$+\frac{3}{2}ab^2\left(\frac{\log(cx+1)}{c}-\frac{\log(cx-1)}{c}\right)\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2$$
$$+\frac{3}{2}a^2b\left(\frac{\log(cx+1)}{c}-\frac{\log(cx-1)}{c}\right)\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)$$
$$+\frac{1}{64}\left(\frac{24\left(\log(cx+1)^2-2\log(cx+1)\log(cx-1)+\log(cx-1)^2\right)\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2}{c}+\frac{8\left(\log(cx+1)^3-\right.}{}\right.$$
$$+\frac{1}{8}ab^2\left(\frac{6\left(\log(cx+1)^2-2\log(cx+1)\log(cx-1)+\log(cx-1)^2\right)\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c}+\frac{\log(cx+1)^3-3\,\mathrm{lo}}{}\right.$$
$$+\frac{1}{2}a^3\left(\frac{\log(cx+1)}{c}-\frac{\log(cx-1)}{c}\right)$$
$$+\frac{3\left(\log(cx+1)^2-2\log(cx+1)\log(cx-1)+\log(cx-1)^2\right)a^2b}{8c}$$

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, algorit
hm="maxima")
```

```
[Out] 1/2*b^3*(log(c*x + 1)/c - log(c*x - 1)/c)*log(sqrt(-c*x + 1)/sqrt(c*x + 1))
^3 + 3/2*a*b^2*(log(c*x + 1)/c - log(c*x - 1)/c)*log(sqrt(-c*x + 1)/sqrt(c*
x + 1))^2 + 3/2*a^2*b*(log(c*x + 1)/c - log(c*x - 1)/c)*log(sqrt(-c*x + 1)/
sqrt(c*x + 1)) + 1/64*(24*(log(c*x + 1)^2 - 2*log(c*x + 1)*log(c*x - 1) + l
og(c*x - 1)^2)*log(sqrt(-c*x + 1)/sqrt(c*x + 1))^2/c + 8*(log(c*x + 1)^3 -
3*log(c*x + 1)^2*log(c*x - 1) + 3*log(c*x + 1)*log(c*x - 1)^2 - log(c*x - 1
)^3)*log(sqrt(-c*x + 1)/sqrt(c*x + 1))/c + (log(c*x + 1)^4 - 4*log(c*x + 1)
^3*log(c*x - 1) + 6*log(c*x + 1)^2*log(c*x - 1)^2 - 4*log(c*x + 1)*log(c*x
- 1)^3 + log(c*x - 1)^4)/c)*b^3 + 1/8*a*b^2*(6*(log(c*x + 1)^2 - 2*log(c*x
+ 1)*log(c*x - 1) + log(c*x - 1)^2)*log(sqrt(-c*x + 1)/sqrt(c*x + 1))/c + (
log(c*x + 1)^3 - 3*log(c*x + 1)^2*log(c*x - 1) + 3*log(c*x + 1)*log(c*x - 1
)^2 - log(c*x - 1)^3)/c) + 1/2*a^3*(log(c*x + 1)/c - log(c*x - 1)/c) + 3/8*
(log(c*x + 1)^2 - 2*log(c*x + 1)*log(c*x - 1) + log(c*x - 1)^2)*a^2*b/c
```

Giac [F]

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx=\int-\frac{\left(b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^3}{c^2x^2-1}\,dx$$

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, algorit
hm="giac")
```

```
[Out] integrate(-(b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^3/(c^2*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx=\int-\frac{\left(a+b\,\ln\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^3}{c^2\,x^2-1}\,dx$$

```
[In] int(-(a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1),x)
```

```
[Out] int(-(a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1), x)
```

3.45 $\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$

Optimal result . . . 432
Rubi [A] (verified) . . . 432
Mathematica [A] (verified) . . . 434
Maple [F] . . . 434
Fricas [B] (verification not implemented) . . . 434
Sympy [B] (verification not implemented) . . . 434
Maxima [B] (verification not implemented) . . . 435
Giac [F] . . . 435
Mupad [F(-1)] . . . 436

Optimal result

Integrand size = 40, antiderivative size = 37

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx=-\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc}$$

```
[Out] -1/3*(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/b/c
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2573, 2576, 12, 2339, 30}

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx=-\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{3bc}$$

```
[In] Int[(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

```
[Out] -1/3*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rule 2576

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*(P2x_)^(m_.), x_Symbol] :> With[{f = Coeff[P2x, x, 0], g = Coef
f[P2x, x, 1], h = Coeff[P2x, x, 2]}, Dist[b*c - a*d, Subst[Int[(b^2*f - a*b
*g + a^2*h - (2*b*d*f - b*c*g - a*d*g + 2*a*c*h)*x + (d^2*f - c*d*g + c^2*h
)*x^2)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(2*(m + 1))), x], x, (a + b*x)/(c
+ d*x)], x]] /; FreeQ[{a, b, c, d, e, A, B, n}, x] && PolyQ[P2x, x, 2] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{\left(a+b\log\left(\sqrt{\frac{1-cx}{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx, \sqrt{\frac{1-cx}{1+cx}}, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\\
&= -\text{Subst}\left((2c)\text{Subst}\left(\int \frac{\left(a+b\log\left(\sqrt{x}\right)\right)^2}{4c^2x}\,dx, x, \frac{1-cx}{1+cx}\right), \sqrt{\frac{1-cx}{1+cx}}, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\\
&= -\text{Subst}\left(\frac{\text{Subst}\left(\int \frac{\left(a+b\log\left(\sqrt{x}\right)\right)^2}{x}\,dx, x, \frac{1-cx}{1+cx}\right)}{2c}, \sqrt{\frac{1-cx}{1+cx}}, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\\
&= -\text{Subst}\left(\frac{\text{Subst}\left(\int x^2\,dx, x, a+b\log\left(\sqrt{\frac{1-cx}{1+cx}}\right)\right)}{bc}, \sqrt{\frac{1-cx}{1+cx}}, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\\
&= -\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx=-\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc}$$

```
[In] Integrate[(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
[Out] -1/3*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(b*c)
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(\frac{\sqrt{-xc+1}}{\sqrt{xc+1}}\right)\right)^2}{-x^2c^2+1}\,dx$$

```
[In] int((a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x)
[Out] int((a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(31) = 62.

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.00

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx=-\frac{b^2\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^3+3\,ab\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2+3\,a^2\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{3\,c}$$

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, algorit
hm="fricas")
[Out] -1/3*(b^2*log(sqrt(-c*x + 1)/sqrt(c*x + 1))^3 + 3*a*b*log(sqrt(-c*x + 1)/sq
rt(c*x + 1))^2 + 3*a^2*log(sqrt(-c*x + 1)/sqrt(c*x + 1)))/c
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(29) = 58.

Time = 3.36 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.76

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx=\begin{cases}-\frac{a^2\operatorname{atan}\left(\frac{x}{\sqrt{-\frac{1}{c^2}}}\right)}{c^2\sqrt{-\frac{1}{c^2}}} & \text{for}\ b=0\\ a^2x & \text{for}\ c=0\\ -\frac{\left(a+b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^3}{3bc} & \text{otherwise}\end{cases}$$

```
[In] integrate((a+b*ln((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2/(-c**2*x**2+1),x)
```

```
[Out] Piecewise((-a**2*atan(x/sqrt(-1/c**2))/(c**2*sqrt(-1/c**2)), Eq(b, 0)), (a*
*2*x, Eq(c, 0)), (-(a + b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)))**3/(3*b*c), Tr
ue))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(31) = 62$.

Time = 0.21 (sec) , antiderivative size = 268, normalized size of antiderivative = 7.24

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = \frac{1}{2}\,b^2\left(\frac{\log\left(cx+1\right)}{c}-\frac{\log\left(cx-1\right)}{c}\right)\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2 + ab\left(\frac{\log\left(cx+1\right)}{c}-\frac{\log\left(cx-1\right)}{c}\right)\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + \frac{1}{24}\,b^2\left(\frac{6\left(\log\left(cx+1\right)^2-2\,\log\left(cx+1\right)\log\left(cx-1\right)+\log\left(cx-1\right)^2\right)\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c}+\frac{\log\left(cx+1\right)^3-3 \right. + \frac{1}{2}\,a^2\left(\frac{\log\left(cx+1\right)}{c}-\frac{\log\left(cx-1\right)}{c}\right) + \frac{\left(\log\left(cx+1\right)^2-2\,\log\left(cx+1\right)\log\left(cx-1\right)+\log\left(cx-1\right)^2\right)ab}{4\,c}$$

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, algorit
hm="maxima")
```

```
[Out] 1/2*b^2*(log(c*x + 1)/c - log(c*x - 1)/c)*log(sqrt(-c*x + 1)/sqrt(c*x + 1))
^2 + a*b*(log(c*x + 1)/c - log(c*x - 1)/c)*log(sqrt(-c*x + 1)/sqrt(c*x + 1)
) + 1/24*b^2*(6*(log(c*x + 1)^2 - 2*log(c*x + 1)*log(c*x - 1) + log(c*x - 1
)^2)*log(sqrt(-c*x + 1)/sqrt(c*x + 1))/c + (log(c*x + 1)^3 - 3*log(c*x + 1)
^2*log(c*x - 1) + 3*log(c*x + 1)*log(c*x - 1)^2 - log(c*x - 1)^3)/c) + 1/2*
a^2*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/4*(log(c*x + 1)^2 - 2*log(c*x + 1
)*log(c*x - 1) + log(c*x - 1)^2)*a*b/c
```

Giac [F]

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = \int -\frac{\left(b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}{c^2x^2-1}\,dx$$

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, algorit
hm="giac")
```

```
[Out] integrate(-(b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2/(c^2*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx=\int -\frac{\left(a+b\,\ln\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^2}{c^2\,x^2-1}\,dx$$

```
[In] int(-(a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1),x)
[Out] int(-(a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1), x)
```

3.46 $\int \frac{a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx$

Optimal result 437
Rubi [A] (verified) 437
Mathematica [A] (verified) 438
Maple [F] 439
Fricas [A] (verification not implemented) 439
Sympy [B] (verification not implemented) 439
Maxima [B] (verification not implemented) 440
Giac [B] (verification not implemented) 440
Mupad [F(-1)] 441

Optimal result

Integrand size = 38, antiderivative size = 37

$$\int \frac{a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx=-\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc}$$

```
[Out] -1/2*(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/b/c
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2573, 2576, 12, 2338}

$$\int \frac{a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx=-\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2bc}$$

```
[In] Int[(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
[Out] -1/2*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rule 2576

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*(P2x_)^(m_.), x_Symbol] :> With[{f = Coeff[P2x, x, 0], g = Coef
f[P2x, x, 1], h = Coeff[P2x, x, 2]}, Dist[b*c - a*d, Subst[Int[(b^2*f - a*b
*g + a^2*h - (2*b*d*f - b*c*g - a*d*g + 2*a*c*h)*x + (d^2*f - c*d*g + c^2*h
)*x^2)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(2*(m + 1))), x], x, (a + b*x)/(c
+ d*x)], x]] /; FreeQ[{a, b, c, d, e, A, B, n}, x] && PolyQ[P2x, x, 2] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{a+b\log\left(\sqrt{\frac{1-cx}{1+cx}}\right)}{1-c^2x^2}\,dx, \sqrt{\frac{1-cx}{1+cx}}, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\\
&= -\text{Subst}\left((2c)\text{Subst}\left(\int \frac{a+b\log\left(\sqrt{x}\right)}{4c^2x}\,dx, x, \frac{1-cx}{1+cx}\right), \sqrt{\frac{1-cx}{1+cx}}, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\\
&= -\text{Subst}\left(\frac{\text{Subst}\left(\int \frac{a+b\log(\sqrt{x})}{x}\,dx, x, \frac{1-cx}{1+cx}\right)}{2c}, \sqrt{\frac{1-cx}{1+cx}}, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\\
&= -\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = -\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc}$$

```
[In] Integrate[(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
[Out] -1/2*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(b*c)
```

Maple [F]

$$\int \frac{a+b\ln\left(\frac{\sqrt{-xc+1}}{\sqrt{xc+1}}\right)}{-x^2c^2+1}dx$$

```
[In] int((a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x)
```

```
[Out] int((a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.27

$$\int \frac{a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = -\frac{b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2+2\,a\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{2\,c}$$

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algorithm
="fricas")
```

```
[Out] -1/2*(b*log(sqrt(-c*x + 1)/sqrt(c*x + 1))^2 + 2*a*log(sqrt(-c*x + 1)/sqrt(c
*x + 1)))/c
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(29) = 58.

Time = 3.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.65

$$\int \frac{a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = \begin{cases} -\frac{a\,\mathrm{atan}\left(\frac{x}{\sqrt{-\frac{1}{c^2}}}\right)}{c^2\sqrt{-\frac{1}{c^2}}} & \text{for } b=0 \\ ax & \text{for } c=0 \\ -\frac{\left(a+b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^2}{2bc} & \text{otherwise} \end{cases}$$

```
[In] integrate((a+b*ln((-c*x+1)**(1/2)/(c*x+1)**(1/2)))/(-c**2*x**2+1),x)
```

```
[Out] Piecewise((-a*atan(x/sqrt(-1/c**2))/(c**2*sqrt(-1/c**2)), Eq(b, 0)), (a*x,
Eq(c, 0)), (-(a + b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)))**2/(2*b*c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(31) = 62$.

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.84

$$\int \frac{a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = \frac{1}{2}b\left(\frac{\log(cx+1)}{c} - \frac{\log(cx-1)}{c}\right)\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + \frac{1}{2}a\left(\frac{\log(cx+1)}{c} - \frac{\log(cx-1)}{c}\right) + \frac{\left(\log(cx+1)^2 - 2\log(cx+1)\log(cx-1) + \log(cx-1)^2\right)b}{8c}$$

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algorithm
="maxima")
```

```
[Out] 1/2*b*(log(c*x + 1)/c - log(c*x - 1)/c)*log(sqrt(-c*x + 1)/sqrt(c*x + 1)) +
 1/2*a*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/8*(log(c*x + 1)^2 - 2*log(c*x
+ 1)*log(c*x - 1) + log(c*x - 1)^2)*b/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(31) = 62$.

Time = 0.32 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.32

$$\int \frac{a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = -\frac{b\log(cx+1)^2}{8c} + \frac{b\log(cx-1)^2}{8c} + \frac{1}{4}\left(\frac{b\log(cx+1)}{c} - \frac{b\log(cx-1)}{c}\right)\log(-cx+1) + \frac{a\log(cx+1)}{2c} - \frac{a\log(cx-1)}{2c}$$

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algorithm
="giac")
```

```
[Out] -1/8*b*log(c*x + 1)^2/c + 1/8*b*log(c*x - 1)^2/c + 1/4*(b*log(c*x + 1)/c -
b*log(c*x - 1)/c)*log(-c*x + 1) + 1/2*a*log(c*x + 1)/c - 1/2*a*log(c*x - 1)
/c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = \int -\frac{a+b\,\ln\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)}{c^2\,x^2-1}\,dx$$

```
[In] int(-(a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1),x)
[Out] int(-(a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1), x)
```

3.47 $\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$

Optimal result 442
Rubi [A] (verified) 442
Mathematica [A] (verified) 443
Maple [F] 443
Fricas [A] (verification not implemented) 443
Sympy [A] (verification not implemented) 444
Maxima [A] (verification not implemented) 444
Giac [A] (verification not implemented) 444
Mupad [F(-1)] 445

Optimal result

Integrand size = 40, antiderivative size = 34

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = -\frac{\log\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{bc}$$

```
[Out] -ln(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/b/c
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2573, 6816}

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = -\frac{\log\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{bc}$$

```
[In] Int[1/((1 - c^2*x^2)*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
[Out] -(Log[a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]]]/(b*c))
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] && !Intege
rQ[n]
```

Rule 6816

```
Int[(u_)/(y_), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Simp[q*L
og[RemoveContent[y, x]], x] /;  !FalseQ[q]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\sqrt{\frac{1-cx}{1+cx}}\right)\right)}\,dx, \sqrt{\frac{1-cx}{1+cx}}, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)$$
$$= -\frac{\log\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{bc}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = -\frac{\log\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{bc}$$

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
[Out] -(Log[a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]]]/(b*c))
```

Maple [F]

$$\int \frac{1}{(-x^2c^2+1)\left(a+b\ln\left(\frac{\sqrt{-xc+1}}{\sqrt{xc+1}}\right)\right)}dx$$

```
[In] int(1/(-c^2*x^2+1)/(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
[Out] int(1/(-c^2*x^2+1)/(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = -\frac{\log\left(b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}{bc}$$

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, algorit
hm="fricas")
[Out] -log(b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(b*c)
```

Sympy [A] (verification not implemented)

Time = 7.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.59

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \begin{cases} \frac{x}{a} & \text{for } b=0 \wedge c=0 \\ \frac{-\frac{\log\left(x-\frac{1}{c}\right)}{2c}+\frac{\log\left(x+\frac{1}{c}\right)}{2c}}{a} & \text{for } b=0 \\ \frac{x}{a} & \text{for } c=0 \\ -\frac{\log\left(\frac{a}{b}+\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)}{bc} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*ln((-c*x+1)**(1/2)/(c*x+1)**(1/2))),x)
[Out] Piecewise((x/a, Eq(b, 0) & Eq(c, 0)), ((-log(x - 1/c)/(2*c) + log(x + 1/c)/
(2*c))/a, Eq(b, 0)), (x/a, Eq(c, 0)), (-log(a/b + log(sqrt(-c*x + 1)/sqrt(c
*x + 1)))/(b*c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = -\frac{\log\left(-\frac{b\log(cx+1)-b\log(-cx+1)-2a}{2b}\right)}{bc}$$

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, algorit
hm="maxima")
[Out] -log(-1/2*(b*log(c*x + 1) - b*log(-c*x + 1) - 2*a)/b)/(b*c)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = -\frac{\log\left(-b\log(cx+1)+b\log(-cx+1)+2a\right)}{bc}$$

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, algorit
hm="giac")
[Out] -log(-b*log(c*x + 1) + b*log(-c*x + 1) + 2*a)/(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = -\int \frac{1}{\left(a+b\,\ln\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)\,(c^2\,x^2-1)}\,dx$$

```
[In] int(-1/((a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)),x)
[Out] -int(1/((a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)), x)
```

3.48 $\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$

Optimal result 446
Rubi [A] (verified) 446
Mathematica [A] (verified) 447
Maple [F] 447
Fricas [A] (verification not implemented) 447
Sympy [B] (verification not implemented) 448
Maxima [A] (verification not implemented) 448
Giac [A] (verification not implemented) 448
Mupad [F(-1)] 449

Optimal result

Integrand size = 40, antiderivative size = 34

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \frac{1}{bc\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}$$

```
[Out] 1/b/c/(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2573, 6818}

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \frac{1}{bc\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}$$

```
[In] Int[1/((1 - c^2*x^2)*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),x]
```

```
[Out] 1/(b*c*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]]))
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] && !Intege
rQ[n]
```

Rule 6818

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\sqrt{\frac{1-cx}{1+cx}}\right)\right)^2}\,dx, \sqrt{\frac{1-cx}{1+cx}}, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)$$
$$= \frac{1}{bc\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \frac{1}{bc\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}$$

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),x]
[Out] 1/(b*c*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]]))
```

Maple [F]

$$\int \frac{1}{(-x^2c^2+1)\left(a+b\ln\left(\frac{\sqrt{-xc+1}}{\sqrt{xc+1}}\right)\right)^2}dx$$

```
[In] int(1/(-c^2*x^2+1)/(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
[Out] int(1/(-c^2*x^2+1)/(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \frac{1}{b^2c\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+abc}$$

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, algor
ithm="fricas")
[Out] 1/(b^2*c*log(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a*b*c)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(26) = 52$.

Time = 74.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.79

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \begin{cases} \frac{x}{a^2} & \text{for } b = 0 \wedge c = 0 \\ \frac{-\frac{\log\left(x-\frac{1}{c}\right)}{2c}+\frac{\log\left(x+\frac{1}{c}\right)}{2c}}{a^2} & \text{for } b = 0 \\ \frac{x}{a^2} & \text{for } c = 0 \\ \frac{1}{abc+b^2c\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*ln((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2,x)
```

```
[Out] Piecewise((x/a**2, Eq(b, 0) & Eq(c, 0)), ((-log(x - 1/c)/(2*c) + log(x + 1/
c)/(2*c))/a**2, Eq(b, 0)), (x/a**2, Eq(c, 0)), (1/(a*b*c + b**2*c*log(sqrt(
-c*x + 1)/sqrt(c*x + 1))), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = -\frac{2}{b^2c\log(cx+1)-b^2c\log(-cx+1)-2\,abc}$$

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, algor
ithm="maxima")
```

```
[Out] -2/(b^2*c*log(c*x + 1) - b^2*c*log(-c*x + 1) - 2*a*b*c)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = -\frac{2}{b^2c\log(cx+1)-b^2c\log(-cx+1)-2\,abc}$$

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, algor
ithm="giac")
```

```
[Out] -2/(b^2*c*log(c*x + 1) - b^2*c*log(-c*x + 1) - 2*a*b*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = -\int \frac{1}{\left(a+b\,\ln\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^2\,(c^2\,x^2-1)}\,dx$$

```
[In] int(-1/((a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)),x)
[Out] -int(1/((a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)), x)
```

3.49 $\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}\,dx$

Optimal result 450
Rubi [A] (verified) 450
Mathematica [A] (verified) 451
Maple [F] 451
Fricas [A] (verification not implemented) 451
Sympy [F(-1)] 452
Maxima [B] (verification not implemented) 452
Giac [B] (verification not implemented) 452
Mupad [F(-1)] 453

Optimal result

Integrand size = 40, antiderivative size = 37

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}\,dx = \frac{1}{2bc\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}$$

```
[Out] 1/2/b/c/(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2573, 6818}

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}\,dx = \frac{1}{2bc\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}$$

```
[In] Int[1/((1 - c^2*x^2)*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3),x]
```

```
[Out] 1/(2*b*c*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2)
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] && !Intege
rQ[n]
```

Rule 6818

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\sqrt{\frac{1-cx}{1+cx}}\right)\right)^3}\,dx, \sqrt{\frac{1-cx}{1+cx}}, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)$$
$$= \frac{1}{2bc\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}\,dx = \frac{1}{2bc\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}$$

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3),x]
[Out] 1/(2*b*c*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2)
```

Maple [F]

$$\int \frac{1}{(-x^2c^2+1)\left(a+b\ln\left(\frac{\sqrt{-xc+1}}{\sqrt{xc+1}}\right)\right)^3}dx$$

```
[In] int(1/(-c^2*x^2+1)/(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3,x)
[Out] int(1/(-c^2*x^2+1)/(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.59

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}\,dx$$
$$= \frac{1}{2\left(b^3c\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2 + 2\,ab^2c\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a^2bc\right)}$$

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3,x, algor
ithm="fricas")
```

```
[Out] 1/2/(b^3*c*log(sqrt(-c*x + 1)/sqrt(c*x + 1))^2 + 2*a*b^2*c*log(sqrt(-c*x +
1)/sqrt(c*x + 1)) + a^2*b*c)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*ln((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**3,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(31) = 62$.

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.16

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}\,dx$$
$$= \frac{2}{b^3c\log(cx+1)^2 + b^3c\log(-cx+1)^2 - 4\,ab^2c\log(cx+1) + 4\,a^2bc - 2\,(b^3c\log(cx+1) - 2\,ab^2c)\log(-cx}$$

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3,x, algor
ithm="maxima")
```

```
[Out] 2/(b^3*c*log(c*x + 1)^2 + b^3*c*log(-c*x + 1)^2 - 4*a*b^2*c*log(c*x + 1) +
4*a^2*b*c - 2*(b^3*c*log(c*x + 1) - 2*a*b^2*c)*log(-c*x + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(31) = 62$.

Time = 0.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.30

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}\,dx$$
$$= \frac{2}{b^3c\log(cx+1)^2 - 2\,b^3c\log(cx+1)\log(-cx+1) + b^3c\log(-cx+1)^2 - 4\,ab^2c\log(cx+1) + 4\,ab^2c\log(-}$$

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3,x, algor
ithm="giac")
```

```
[Out] 2/(b^3*c*log(c*x + 1)^2 - 2*b^3*c*log(c*x + 1)*log(-c*x + 1) + b^3*c*log(-c
*x + 1)^2 - 4*a*b^2*c*log(c*x + 1) + 4*a*b^2*c*log(-c*x + 1) + 4*a^2*b*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}\,dx = -\int \frac{1}{\left(a+b\,\ln\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^3\,(c^2\,x^2-1)}\,dx$$

```
[In] int(-1/((a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3*(c^2*x^2 - 1)),x)
```

```
[Out] -int(1/((a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3*(c^2*x^2 - 1)), x)
```

3.50 $\int \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal result 454
Rubi [A] (verified) 454
Mathematica [A] (verified) 455
Maple [C] (verified) 456
Fricas [A] (verification not implemented) 456
Sympy [B] (verification not implemented) 456
Maxima [B] (verification not implemented) 457
Giac [B] (verification not implemented) 457
Mupad [F(-1)] 457

Optimal result

Integrand size = 34, antiderivative size = 30

$$\int \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\log^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}$$

```
[Out] -1/2*ln((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/a
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2573, 2576, 12, 2338}

$$\int \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\log^2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}$$

```
[In] Int[Log[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] -1/2*Log[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/a
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rule 2576

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*(P2x_)^(m_.), x_Symbol] :> With[{f = Coeff[P2x, x, 0], g = Coef
f[P2x, x, 1], h = Coeff[P2x, x, 2]}, Dist[b*c - a*d, Subst[Int[(b^2*f - a*b
*g + a^2*h - (2*b*d*f - b*c*g - a*d*g + 2*a*c*h)*x + (d^2*f - c*d*g + c^2*h
)*x^2)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(2*(m + 1))), x], x, (a + b*x)/(c
+ d*x)], x]] /; FreeQ[{a, b, c, d, e, A, B, n}, x] && PolyQ[P2x, x, 2] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{\log\left(\sqrt{\frac{1-ax}{1+ax}}\right)}{1-a^2x^2}\,dx, \sqrt{\frac{1-ax}{1+ax}}, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\\
&= -\text{Subst}\left((2a)\text{Subst}\left(\int \frac{\log\left(\sqrt{x}\right)}{4a^2x}\,dx, x, \frac{1-ax}{1+ax}\right), \sqrt{\frac{1-ax}{1+ax}}, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\\
&= -\text{Subst}\left(\frac{\text{Subst}\left(\int \frac{\log(\sqrt{x})}{x}\,dx, x, \frac{1-ax}{1+ax}\right)}{2a}, \sqrt{\frac{1-ax}{1+ax}}, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\\
&= -\frac{\log^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$
\int \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\log^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}
$$

```
[In] Integrate[Log[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
[Out] -1/2*Log[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/a
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.49 (sec) , antiderivative size = 151, normalized size of antiderivative = 5.03

method	result
parts	$\frac{\ln\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)\ln(ax+1)}{2a}-\frac{\ln\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)\ln(ax-1)}{2a}+\frac{\ln(ax-1)^2}{8a}-\frac{\mathrm{dilog}\left(\frac{ax}{2}+\frac{1}{2}\right)}{4a}-\frac{\ln(ax-1)\ln\left(\frac{ax}{2}+\frac{1}{2}\right)}{4a}-\frac{\left(\ln(ax+1)-\ln\left(\frac{ax}{2}+\right.\right.}{2}$

```
[In] int(ln((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln((-a*x+1)^(1/2)/(a*x+1)^(1/2))/a*ln(a*x+1)-1/2*ln((-a*x+1)^(1/2)/(a*x
+1)^(1/2))/a*ln(a*x-1)+1/8/a*ln(a*x-1)^2-1/4/a*dilog(1/2*a*x+1/2)-1/4/a*ln(
a*x-1)*ln(1/2*a*x+1/2)-1/2/a*(1/2*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x+1
/2)-1/2*dilog(1/2*a*x+1/2)-1/4*ln(a*x+1)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=-\frac{\log\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{2\,a}$$

```
[In] integrate(log((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="fric
as")
```

```
[Out] -1/2*log(sqrt(-a*x + 1)/sqrt(a*x + 1))^2/a
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(24) = 48.

Time = 2.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.17

$$\int \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=-\frac{\operatorname{atan}^2\left(\frac{x}{\sqrt{-\frac{1}{a^2}}}\right)}{2a}-\frac{\log\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)\operatorname{atan}\left(\frac{x}{\sqrt{-\frac{1}{a^2}}}\right)}{a^2\sqrt{-\frac{1}{a^2}}}$$

```
[In] integrate(ln((-a*x+1)**(1/2)/(a*x+1)**(1/2))/(-a**2*x**2+1),x)
```

```
[Out] -atan(x/sqrt(-1/a**2))**2/(2*a) - log(sqrt(-a*x + 1)/sqrt(a*x + 1))*atan(x/
sqrt(-1/a**2))/(a**2*sqrt(-1/a**2))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(24) = 48$.

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.77

$$\int \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \frac{1}{2}\left(\frac{\log(ax+1)}{a} - \frac{\log(ax-1)}{a}\right)\log\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right) + \frac{\log(ax-1)^2}{8\,a} + \frac{\log(ax+1)^2 - 2\,\log(ax+1)\log(ax-1)}{8\,a}$$

```
[In] integrate(log((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="maxi
ma")
```

```
[Out] 1/2*(log(a*x + 1)/a - log(a*x - 1)/a)*log(sqrt(-a*x + 1)/sqrt(a*x + 1)) + 1
/8*log(a*x - 1)^2/a + 1/8*(log(a*x + 1)^2 - 2*log(a*x + 1)*log(a*x - 1))/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(24) = 48$.

Time = 0.31 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.93

$$\int \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \frac{1}{4}\left(\frac{\log(ax+1)}{a} - \frac{\log(ax-1)}{a}\right)\log(-ax+1) - \frac{\log(ax+1)^2}{8\,a} + \frac{\log(ax-1)^2}{8\,a}$$

```
[In] integrate(log((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="giac
")
```

```
[Out] 1/4*(log(a*x + 1)/a - log(a*x - 1)/a)*log(-a*x + 1) - 1/8*log(a*x + 1)^2/a
+ 1/8*log(a*x - 1)^2/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{\ln\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)}{a^2\,x^2-1}\,dx$$

```
[In] int(-log((1 - a*x)^(1/2)/(a*x + 1)^(1/2))/(a^2*x^2 - 1),x)
```

```
[Out] -int(log((1 - a*x)^(1/2)/(a*x + 1)^(1/2))/(a^2*x^2 - 1), x)
```

3.51 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)(s+t\log(i(g+hx)^n))^2}{gk+hkx}\,dx$

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Mupad [F(-1)] 466

Optimal result

Integrand size = 48, antiderivative size = 410

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\left(s+t\log\left(i(g+hx)^n\right)\right)^2}{gk+hkx}\,dx = -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log\left(i(g+hx)^n\right)\right)^3}{3hknt}-\frac{qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)\left(s+t\log\left(i(g+hx)^n\right)\right)^3}{3hknt}+\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\left(s+t\log\left(i(g+hx)^n\right)\right)^3}{3hknt}-\frac{pr\left(s+t\log\left(i(g+hx)^n\right)\right)^2\text{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)}{hk}-\frac{qr\left(s+t\log\left(i(g+hx)^n\right)\right)^2\text{PolyLog}\left(2,\frac{d(g+hx)}{dg-ch}\right)}{hk}+\frac{2nprt\left(s+t\log\left(i(g+hx)^n\right)\right)\text{PolyLog}\left(3,\frac{b(g+hx)}{bg-ah}\right)}{hk}+\frac{2nqrt\left(s+t\log\left(i(g+hx)^n\right)\right)\text{PolyLog}\left(3,\frac{d(g+hx)}{dg-ch}\right)}{hk}-\frac{2n^2prt^2\text{PolyLog}\left(4,\frac{b(g+hx)}{bg-ah}\right)}{hk}-\frac{2n^2qrt^2\text{PolyLog}\left(4,\frac{d(g+hx)}{dg-ch}\right)}{hk}$$

```
[Out] -1/3*p*r*ln(-h*(b*x+a)/(-a*h+b*g))*(s+t*ln(i*(h*x+g)^n))^3/h/k/n/t-1/3*q*r*
ln(-h*(d*x+c)/(-c*h+d*g))*(s+t*ln(i*(h*x+g)^n))^3/h/k/n/t+1/3*ln(e*(f*(b*x+
a)^p*(d*x+c)^q)^r)*(s+t*ln(i*(h*x+g)^n))^3/h/k/n/t-p*r*(s+t*ln(i*(h*x+g)^n)
)^2*polylog(2,b*(h*x+g)/(-a*h+b*g))/h/k-q*r*(s+t*ln(i*(h*x+g)^n))^2*polylog
(2,d*(h*x+g)/(-c*h+d*g))/h/k+2*n*p*r*t*(s+t*ln(i*(h*x+g)^n))*polylog(3,b*(h
*x+g)/(-a*h+b*g))/h/k+2*n*q*r*t*(s+t*ln(i*(h*x+g)^n))*polylog(3,d*(h*x+g)/(
```

```
-c*h+d*g))/h/k-2*n^2*p*r*t^2*polylog(4,b*(h*x+g)/(-a*h+b*g))/h/k-2*n^2*q*r*
t^2*polylog(4,d*(h*x+g)/(-c*h+d*g))/h/k
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2585, 2443, 2481, 2421, 2430, 6724}

$$\int \frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\left(s+t\log \left(i(g+hx)^n\right)\right)^2}{gk+hkx}\,dx$$
$$= \frac{\left(t\log \left(i(g+hx)^n\right)+s\right)^3\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3hknt} - \frac{pr\,\text{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)\left(t\log \left(i(g+hx)^n\right)+s\right)^2}{hk} + \frac{2nprt\,\text{PolyLog}\left(3,\frac{b(g+hx)}{bg-ah}\right)\left(t\log \left(i(g+hx)^n\right)+s\right)}{hk} - \frac{pr\log \left(-\frac{h(a+bx)}{bg-ah}\right)\left(t\log \left(i(g+hx)^n\right)+s\right)^3}{3hknt} - \frac{2n^2prt^2\,\text{PolyLog}\left(4,\frac{b(g+hx)}{bg-ah}\right)}{hk} - \frac{qr\,\text{PolyLog}\left(2,\frac{d(g+hx)}{dg-ch}\right)\left(t\log \left(i(g+hx)^n\right)+s\right)^2}{hk} + \frac{2nqrt\,\text{PolyLog}\left(3,\frac{d(g+hx)}{dg-ch}\right)\left(t\log \left(i(g+hx)^n\right)+s\right)}{hk} - \frac{qr\log \left(-\frac{h(c+dx)}{dg-ch}\right)\left(t\log \left(i(g+hx)^n\right)+s\right)^3}{3hknt} - \frac{2n^2qrt^2\,\text{PolyLog}\left(4,\frac{d(g+hx)}{dg-ch}\right)}{hk}$$

```
[In] Int[(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*(s + t*Log[i*(g + h*x)^n])^2)/(g*
k + h*k*x),x]
```

```
[Out] -1/3*(p*r*Log[-((h*(a + b*x))/(b*g - a*h))]*(s + t*Log[i*(g + h*x)^n])^3)/(
h*k*n*t) - (q*r*Log[-((h*(c + d*x))/(d*g - c*h))]*(s + t*Log[i*(g + h*x)^n]
)^3)/(3*h*k*n*t) + (Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*(s + t*Log[i*(g +
h*x)^n])^3)/(3*h*k*n*t) - (p*r*(s + t*Log[i*(g + h*x)^n])^2*PolyLog[2, (b*(
g + h*x))/(b*g - a*h)])/(h*k) - (q*r*(s + t*Log[i*(g + h*x)^n])^2*PolyLog[2
, (d*(g + h*x))/(d*g - c*h)])/(h*k) + (2*n*p*r*t*(s + t*Log[i*(g + h*x)^n])
*PolyLog[3, (b*(g + h*x))/(b*g - a*h)])/(h*k) + (2*n*q*r*t*(s + t*Log[i*(g
+ h*x)^n])*PolyLog[3, (d*(g + h*x))/(d*g - c*h)])/(h*k) - (2*n^2*p*r*t^2*Po
lyLog[4, (b*(g + h*x))/(b*g - a*h)])/(h*k) - (2*n^2*q*r*t^2*PolyLog[4, (d*(
g + h*x))/(d*g - c*h)])/(h*k)
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2585

```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.))^(m_.))/((j_.
) + (k_.)*(x_)), x_Symbol] :> Simp[(s + t*Log[i*(g + h*x)^n])^(m + 1)*(Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(k*n*t*(m + 1))), x] + (-Dist[b*p*(r/(k*n*
t*(m + 1))), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(a + b*x), x], x] - Dis
t[d*q*(r/(k*n*t*(m + 1))), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(c + d*x)
, x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r},
x] && NeQ[b*c - a*d, 0] && EqQ[h*j - g*k, 0] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\left(s+t\log\left(i(g+hx)^n\right)\right)^3}{3hknt} - \frac{(bpr)\int\frac{\left(s+t\log\left(i(g+hx)^n\right)\right)^3}{a+bx}\,dx}{3hknt} - \frac{(dqr)\int\frac{\left(s+t\log\left(i(g+hx)^n\right)\right)^3}{c+dx}\,dx}{3hknt}$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log\left(i(g+hx)^n\right)\right)^3}{3hknt} - \frac{qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)\left(s+t\log\left(i(g+hx)^n\right)\right)^3}{3hknt} + \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\left(s+t\log\left(i(g+hx)^n\right)\right)^3}{3hknt} + \frac{(pr)\int\frac{\log\left(\frac{h(a+bx)}{-bg+ah}\right)\left(s+t\log\left(i(g+hx)^n\right)\right)^2}{g+hx}\,dx}{k} + \frac{(qr)\int\frac{\log\left(\frac{h(c+dx)}{-dg+ch}\right)\left(s+t\log\left(i(g+hx)^n\right)\right)^2}{g+hx}\,dx}{k}$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log\left(i(g+hx)^n\right)\right)^3}{3hknt} - \frac{qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)\left(s+t\log\left(i(g+hx)^n\right)\right)^3}{3hknt} + \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\left(s+t\log\left(i(g+hx)^n\right)\right)^3}{3hknt} + \frac{(pr)\text{Subst}\left(\int\frac{\left(s+t\log\left(ix^n\right)\right)^2\log\left(\frac{h\left(\frac{-bg+ah}{h}+\frac{bx}{h}\right)}{-bg+ah}\right)}{x}\,dx, x, g+hx\right)}{hk} + \frac{(qr)\text{Subst}\left(\int\frac{\left(s+t\log\left(ix^n\right)\right)^2\log\left(\frac{h\left(\frac{-dg+ch}{h}+\frac{dx}{h}\right)}{-dg+ch}\right)}{x}\,dx, x, g+hx\right)}{hk}$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)(s+t\log(i(g+hx)^n))^3}{3hknt} - \frac{qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)(s+t\log(i(g+hx)^n))^3}{3hknt} + \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)(s+t\log(i(g+hx)^n))^3}{3hknt} - \frac{pr(s+t\log(i(g+hx)^n))^2\operatorname{Li}_2\left(\frac{b(g+hx)}{bg-ah}\right)}{hk} - \frac{qr(s+t\log(i(g+hx)^n))^2\operatorname{Li}_2\left(\frac{d(g+hx)}{dg-ch}\right)}{hk} + \frac{(2nprt)\operatorname{Subst}\left(\int\frac{(s+t\log(ix^n))\operatorname{Li}_2\left(-\frac{bx}{-bg+ah}\right)}{x}dx,x,g+hx\right)}{hk} + \frac{(2nqrt)\operatorname{Subst}\left(\int\frac{(s+t\log(ix^n))\operatorname{Li}_2\left(-\frac{dx}{-dg+ch}\right)}{x}dx,x,g+hx\right)}{hk}$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)(s+t\log(i(g+hx)^n))^3}{3hknt} - \frac{qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)(s+t\log(i(g+hx)^n))^3}{3hknt} + \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)(s+t\log(i(g+hx)^n))^3}{3hknt} - \frac{pr(s+t\log(i(g+hx)^n))^2\operatorname{Li}_2\left(\frac{b(g+hx)}{bg-ah}\right)}{hk} - \frac{qr(s+t\log(i(g+hx)^n))^2\operatorname{Li}_2\left(\frac{d(g+hx)}{dg-ch}\right)}{hk} + \frac{2nprt(s+t\log(i(g+hx)^n))\operatorname{Li}_3\left(\frac{b(g+hx)}{bg-ah}\right)}{hk} + \frac{2nqrt(s+t\log(i(g+hx)^n))\operatorname{Li}_3\left(\frac{d(g+hx)}{dg-ch}\right)}{hk} - \frac{(2n^2prt^2)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_3\left(-\frac{bx}{-bg+ah}\right)}{x}dx,x,g+hx\right)}{hk} - \frac{(2n^2qrt^2)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_3\left(-\frac{dx}{-dg+ch}\right)}{x}dx,x,g+hx\right)}{hk}$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)(s+t\log(i(g+hx)^n))^3}{3hknt} - \frac{qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)(s+t\log(i(g+hx)^n))^3}{3hknt} + \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)(s+t\log(i(g+hx)^n))^3}{3hknt} - \frac{pr(s+t\log(i(g+hx)^n))^2\operatorname{Li}_2\left(\frac{b(g+hx)}{bg-ah}\right)}{hk} - \frac{qr(s+t\log(i(g+hx)^n))^2\operatorname{Li}_2\left(\frac{d(g+hx)}{dg-ch}\right)}{hk} + \frac{2nprt(s+t\log(i(g+hx)^n))\operatorname{Li}_3\left(\frac{b(g+hx)}{bg-ah}\right)}{hk} + \frac{2nqrt(s+t\log(i(g+hx)^n))\operatorname{Li}_3\left(\frac{d(g+hx)}{dg-ch}\right)}{hk} - \frac{2n^2prt^2\operatorname{Li}_4\left(\frac{b(g+hx)}{bg-ah}\right)}{hk} - \frac{2n^2qrt^2\operatorname{Li}_4\left(\frac{d(g+hx)}{dg-ch}\right)}{hk}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 958 vs. $2(410) = 820$.

Time = 1.93 (sec) , antiderivative size = 958, normalized size of antiderivative = 2.34

$$\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)(s+t\log(i(g+hx)^n))^2}{gk+hkx}\,dx = -\frac{3prs^2\log\left(\frac{h(a+bx)}{-bg+ah}\right)\log(g+hx) + 3qrs^2\log\left(\frac{h(c+dx)}{-dg+ch}\right)\log(g+hx) - 3s^2\log(e(f(a+bx)^p(c+dx)^q)^r)\log\ldots}{}$$

```
[In] Integrate[(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*(s + t*Log[i*(g + h*x)^n])^
2)/(g*k + h*k*x),x]
```

```
[Out] -1/3*(3*p*r*s^2*Log[(h*(a + b*x))/(-(b*g) + a*h)]*Log[g + h*x] + 3*q*r*s^2*
Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[g + h*x] - 3*s^2*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]*Log[g + h*x] - 3*n*p*r*s*t*Log[(h*(a + b*x))/(-(b*g) + a*h
)]*Log[g + h*x]^2 - 3*n*q*r*s*t*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[g + h
*x]^2 + 3*n*s*t*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[g + h*x]^2 + n^2*p
*r*t^2*Log[(h*(a + b*x))/(-(b*g) + a*h)]*Log[g + h*x]^3 + n^2*q*r*t^2*Log[(
h*(c + d*x))/(-(d*g) + c*h)]*Log[g + h*x]^3 - n^2*t^2*Log[e*(f*(a + b*x)^p*
(c + d*x)^q)^r]*Log[g + h*x]^3 + 6*p*r*s*t*Log[(h*(a + b*x))/(-(b*g) + a*h)
]*Log[g + h*x]*Log[i*(g + h*x)^n] + 6*q*r*s*t*Log[(h*(c + d*x))/(-(d*g) + c
*h)]*Log[g + h*x]*Log[i*(g + h*x)^n] - 6*s*t*Log[e*(f*(a + b*x)^p*(c + d*x)
```

```
^q)^r]*Log[g + h*x]*Log[i*(g + h*x)^n] - 3*n*p*r*t^2*Log[(h*(a + b*x))/(-(b
*g) + a*h)]*Log[g + h*x]^2*Log[i*(g + h*x)^n] - 3*n*q*r*t^2*Log[(h*(c + d*x
))/(-(d*g) + c*h)]*Log[g + h*x]^2*Log[i*(g + h*x)^n] + 3*n*t^2*Log[e*(f*(a
+ b*x)^p*(c + d*x)^q)^r]*Log[g + h*x]^2*Log[i*(g + h*x)^n] + 3*p*r*t^2*Log[
(h*(a + b*x))/(-(b*g) + a*h)]*Log[g + h*x]*Log[i*(g + h*x)^n]^2 + 3*q*r*t^2
*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[g + h*x]*Log[i*(g + h*x)^n]^2 - 3*t^
2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[g + h*x]*Log[i*(g + h*x)^n]^2 +
3*p*r*(s + t*Log[i*(g + h*x)^n])^2*PolyLog[2, (b*(g + h*x))/(b*g - a*h)] +
3*q*r*(s + t*Log[i*(g + h*x)^n])^2*PolyLog[2, (d*(g + h*x))/(d*g - c*h)] -
6*n*p*r*s*t*PolyLog[3, (b*(g + h*x))/(b*g - a*h)] - 6*n*p*r*t^2*Log[i*(g +
h*x)^n]*PolyLog[3, (b*(g + h*x))/(b*g - a*h)] - 6*n*q*r*s*t*PolyLog[3, (d*(
g + h*x))/(d*g - c*h)] - 6*n*q*r*t^2*Log[i*(g + h*x)^n]*PolyLog[3, (d*(g +
h*x))/(d*g - c*h)] + 6*n^2*p*r*t^2*PolyLog[4, (b*(g + h*x))/(b*g - a*h)] +
6*n^2*q*r*t^2*PolyLog[4, (d*(g + h*x))/(d*g - c*h)])/(h*k)
```

Maple [F]

$$\int \frac{\ln \left(e{\left(f{\left(bx+a\right)}^{p} {\left(dx+c\right)}^{q}\right)}^{r}\right) \left(s+t\ln \left(i{\left(hx+g\right)}^{n}\right)\right)^{2}}{hkx+gk}dx$$

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*ln(i*(h*x+g)^n))^2/(h*k*x+g*k),x)
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*ln(i*(h*x+g)^n))^2/(h*k*x+g*k),x)
```

Fricas [F]

$$\int \frac{\log \left(e(f(a+bx)^{p}(c+dx)^{q})^{r}\right) \left(s+t\log \left(i(g+hx)^{n}\right)\right)^{2}}{gk+hkx}\,dx = \int \frac{\left(t\log \left(\left(hx+g\right)^{n}i\right)+s\right)^{2}\log \left(\left(\left(bx+a\right)^{p}(dx+c)^{q}f\right)^{r}e\right)}{hkx+gk}\,dx$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*log(i*(h*x+g)^n))^2/(h*k*x+
g*k),x, algorithm="fricas")
[Out] integral((t^2*log((h*x + g)^n*i)^2 + 2*s*t*log((h*x + g)^n*i) + s^2)*log(((
b*x + a)^p*(d*x + c)^q*f)^r*e)/(h*k*x + g*k), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\left(s+t\log\left(i(g+hx)^n\right)\right)^2}{gk+hkx}\,dx = \text{Timed out}$$

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)*(s+t*ln(i*(h*x+g)**n))**2/(h*k
*x+g*k),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\left(s+t\log\left(i(g+hx)^n\right)\right)^2}{gk+hkx}\,dx$$
$$= \int \frac{\left(t\log\left((hx+g)^n i\right)+s\right)^2\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{hkx+gk}\,dx$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*log(i*(h*x+g)^n))^2/(h*k*x+
g*k),x, algorithm="maxima")
```

```
[Out] 1/3*((n^2*t^2*log(h*x + g)^3 + 3*t^2*log(h*x + g)*log((h*x + g)^n)^2 - 3*(n
*t^2*log(i) + n*s*t)*log(h*x + g)^2 + 3*(t^2*log(i)^2 + 2*s*t*log(i) + s^2)
*log(h*x + g) - 3*(n*t^2*log(h*x + g)^2 - 2*(t^2*log(i) + s*t)*log(h*x + g)
)*log((h*x + g)^n))*log(((b*x + a)^p)^r) + (n^2*t^2*log(h*x + g)^3 + 3*t^2*
log(h*x + g)*log((h*x + g)^n)^2 - 3*(n*t^2*log(i) + n*s*t)*log(h*x + g)^2 +
 3*(t^2*log(i)^2 + 2*s*t*log(i) + s^2)*log(h*x + g) - 3*(n*t^2*log(h*x + g)
^2 - 2*(t^2*log(i) + s*t)*log(h*x + g))*log((h*x + g)^n))*log(((d*x + c)^q)
^r))/(h*k) - integrate(-1/3*(3*((t^2*log(i)^2 + 2*s*t*log(i) + s^2)*h*log(e
) + (r*t^2*log(i)^2 + 2*r*s*t*log(i) + r*s^2)*h*log(f))*b*d*x^2 - ((p*r + q
*r)*b*d*h*n^2*t^2*x^2 + b*c*g*n^2*p*r*t^2 + a*d*g*n^2*q*r*t^2 + (a*d*h*n^2*
q*r*t^2 + (c*h*n^2*p*r*t^2 + (p*r + q*r)*d*g*n^2*t^2)*b)*x)*log(h*x + g)^3
+ 3*((t^2*log(i)^2 + 2*s*t*log(i) + s^2)*h*log(e) + (r*t^2*log(i)^2 + 2*r*s
*t*log(i) + r*s^2)*h*log(f))*a*c + 3*(((p*r + q*r)*n*t^2*log(i) + (p*r*s +
q*r*s)*n*t)*b*d*h*x^2 + (n*p*r*t^2*log(i) + n*p*r*s*t)*b*c*g + (n*q*r*t^2*l
og(i) + n*q*r*s*t)*a*d*g + ((n*q*r*t^2*log(i) + n*q*r*s*t)*a*d*h + (((p*r +
 q*r)*n*t^2*log(i) + (p*r*s + q*r*s)*n*t)*d*g + (n*p*r*t^2*log(i) + n*p*r*s
*t)*c*h)*b)*x)*log(h*x + g)^2 + 3*((h*r*t^2*log(f) + h*t^2*log(e))*b*d*x^2
+ (h*r*t^2*log(f) + h*t^2*log(e))*a*c + ((h*r*t^2*log(f) + h*t^2*log(e))*b*
c + (h*r*t^2*log(f) + h*t^2*log(e))*a*d)*x - ((p*r + q*r)*b*d*h*t^2*x^2 + b
*c*g*p*r*t^2 + a*d*g*q*r*t^2 + (a*d*h*q*r*t^2 + (c*h*p*r*t^2 + (p*r + q*r)*
d*g*t^2)*b)*x)*log(h*x + g))*log((h*x + g)^n)^2 + 3*(((t^2*log(i)^2 + 2*s*t
*log(i) + s^2)*h*log(e) + (r*t^2*log(i)^2 + 2*r*s*t*log(i) + r*s^2)*h*log(f
```

```
))*b*c + ((t^2*log(i)^2 + 2*s*t*log(i) + s^2)*h*log(e) + (r*t^2*log(i)^2 +
2*r*s*t*log(i) + r*s^2)*h*log(f))*a*d)*x - 3*(((p*r + q*r)*t^2*log(i)^2 + p
*r*s^2 + q*r*s^2 + 2*(p*r*s + q*r*s)*t*log(i))*b*d*h*x^2 + (p*r*t^2*log(i)^
2 + 2*p*r*s*t*log(i) + p*r*s^2)*b*c*g + (q*r*t^2*log(i)^2 + 2*q*r*s*t*log(i
) + q*r*s^2)*a*d*g + ((q*r*t^2*log(i)^2 + 2*q*r*s*t*log(i) + q*r*s^2)*a*d*h
 + (((p*r + q*r)*t^2*log(i)^2 + p*r*s^2 + q*r*s^2 + 2*(p*r*s + q*r*s)*t*log
(i))*d*g + (p*r*t^2*log(i)^2 + 2*p*r*s*t*log(i) + p*r*s^2)*c*h)*b)*x)*log(h
*x + g) + 3*(2*((t^2*log(i) + s*t)*h*log(e) + (r*t^2*log(i) + r*s*t)*h*log(
f))*b*d*x^2 + 2*((t^2*log(i) + s*t)*h*log(e) + (r*t^2*log(i) + r*s*t)*h*log
(f))*a*c + ((p*r + q*r)*b*d*h*n*t^2*x^2 + b*c*g*n*p*r*t^2 + a*d*g*n*q*r*t^2
 + (a*d*h*n*q*r*t^2 + (c*h*n*p*r*t^2 + (p*r + q*r)*d*g*n*t^2)*b)*x)*log(h*x
 + g)^2 + 2*(((t^2*log(i) + s*t)*h*log(e) + (r*t^2*log(i) + r*s*t)*h*log(f)
)*b*c + ((t^2*log(i) + s*t)*h*log(e) + (r*t^2*log(i) + r*s*t)*h*log(f))*a*d
)*x - 2*(((p*r + q*r)*t^2*log(i) + (p*r*s + q*r*s)*t)*b*d*h*x^2 + (p*r*t^2*
log(i) + p*r*s*t)*b*c*g + (q*r*t^2*log(i) + q*r*s*t)*a*d*g + ((q*r*t^2*log(
i) + q*r*s*t)*a*d*h + (((p*r + q*r)*t^2*log(i) + (p*r*s + q*r*s)*t)*d*g + (
p*r*t^2*log(i) + p*r*s*t)*c*h)*b)*x)*log(h*x + g))*log((h*x + g)^n))/(b*d*h
^2*k*x^3 + a*c*g*h*k + (a*d*h^2*k + (d*g*h*k + c*h^2*k)*b)*x^2 + (b*c*g*h*k
 + (d*g*h*k + c*h^2*k)*a)*x), x)
```

Giac [F]

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \left(s+t\log \left(i(g+hx)^n\right)\right)^2}{gk+hkx}\,dx = \int \frac{\left(t\log \left((hx+g)^n i\right)+s\right)^2 \log \left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{hkx+gk}\,dx$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*log(i*(h*x+g)^n))^2/(h*k*x+
g*k),x, algorithm="giac")
```

```
[Out] integrate((t*log((h*x + g)^n*i) + s)^2*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)
/(h*k*x + g*k), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \left(s+t\log \left(i(g+hx)^n\right)\right)^2}{gk+hkx}\,dx = \int \frac{\ln \left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)\,\left(s+t\,\ln \left(i\,(g+h\,x)^n\right)\right)^2}{g\,k+h\,k\,x}\,dx$$

```
[In] int((log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(s + t*log(i*(g + h*x)^n))^2)/(g*
k + h*k*x),x)
```

```
[Out] int((log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(s + t*log(i*(g + h*x)^n))^2)/(g*
k + h*k*x), x)
```

3.52 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)(s+t\log(i(g+hx)^n))}{gk+hkx}\,dx$

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Optimal result

Integrand size = 46, antiderivative size = 306

$$\begin{aligned}
&\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\left(s+t\log\left(i(g+hx)^n\right)\right)}{gk+hkx}\,dx \\
&= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log\left(i(g+hx)^n\right)\right)^2}{2hknt} - \frac{qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)\left(s+t\log\left(i(g+hx)^n\right)\right)^2}{2hknt} \\
&\quad + \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\left(s+t\log\left(i(g+hx)^n\right)\right)^2}{2hknt} \\
&\quad - \frac{pr(s+t\log\left(i(g+hx)^n\right))\operatorname{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)}{hk} \\
&\quad - \frac{qr(s+t\log\left(i(g+hx)^n\right))\operatorname{PolyLog}\left(2,\frac{d(g+hx)}{dg-ch}\right)}{hk} \\
&\quad + \frac{nprt\operatorname{PolyLog}\left(3,\frac{b(g+hx)}{bg-ah}\right)}{hk} + \frac{nqrt\operatorname{PolyLog}\left(3,\frac{d(g+hx)}{dg-ch}\right)}{hk}
\end{aligned}$$

```
[Out] -1/2*p*r*ln(-h*(b*x+a)/(-a*h+b*g))*(s+t*ln(i*(h*x+g)^n))^2/h/k/n/t-1/2*q*r*
ln(-h*(d*x+c)/(-c*h+d*g))*(s+t*ln(i*(h*x+g)^n))^2/h/k/n/t+1/2*ln(e*(f*(b*x+
a)^p*(d*x+c)^q)^r)*(s+t*ln(i*(h*x+g)^n))^2/h/k/n/t-p*r*(s+t*ln(i*(h*x+g)^n)
)*polylog(2,b*(h*x+g)/(-a*h+b*g))/h/k-q*r*(s+t*ln(i*(h*x+g)^n))*polylog(2,d
*(h*x+g)/(-c*h+d*g))/h/k+n*p*r*t*polylog(3,b*(h*x+g)/(-a*h+b*g))/h/k+n*q*r*
t*polylog(3,d*(h*x+g)/(-c*h+d*g))/h/k
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.109, Rules used = {2585, 2443, 2481, 2421, 6724}

$$\int \frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\left(s+t\log \left(i(g+hx)^n\right)\right)}{gk+hkx}\,dx$$
$$= \frac{\left(t\log \left(i(g+hx)^n\right)+s\right)^2\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2hknt} - \frac{pr\,\text{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)\left(t\log \left(i(g+hx)^n\right)+s\right)}{hk} - \frac{pr\log \left(-\frac{h(a+bx)}{bg-ah}\right)\left(t\log \left(i(g+hx)^n\right)+s\right)^2}{2hknt} + \frac{nprt\,\text{PolyLog}\left(3,\frac{b(g+hx)}{bg-ah}\right)}{hk} - \frac{qr\,\text{PolyLog}\left(2,\frac{d(g+hx)}{dg-ch}\right)\left(t\log \left(i(g+hx)^n\right)+s\right)}{hk} - \frac{qr\log \left(-\frac{h(c+dx)}{dg-ch}\right)\left(t\log \left(i(g+hx)^n\right)+s\right)^2}{2hknt} + \frac{nqrt\,\text{PolyLog}\left(3,\frac{d(g+hx)}{dg-ch}\right)}{hk}$$

```
[In] Int[(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*(s + t*Log[i*(g + h*x)^n]))/(g*k
+ h*k*x),x]
```

```
[Out] -1/2*(p*r*Log[-((h*(a + b*x))/(b*g - a*h))]*(s + t*Log[i*(g + h*x)^n])^2)/(
h*k*n*t) - (q*r*Log[-((h*(c + d*x))/(d*g - c*h))]*(s + t*Log[i*(g + h*x)^n]
)^2)/(2*h*k*n*t) + (Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*(s + t*Log[i*(g +
h*x)^n])^2)/(2*h*k*n*t) - (p*r*(s + t*Log[i*(g + h*x)^n])*PolyLog[2, (b*(g
+ h*x))/(b*g - a*h)])/(h*k) - (q*r*(s + t*Log[i*(g + h*x)^n])*PolyLog[2, (d
*(g + h*x))/(d*g - c*h)])/(h*k) + (n*p*r*t*PolyLog[3, (b*(g + h*x))/(b*g -
a*h)])/(h*k) + (n*q*r*t*PolyLog[3, (d*(g + h*x))/(d*g - c*h)])/(h*k)
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2585

```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.))^(m_.))/((j_.
) + (k_.)*(x_)), x_Symbol] :> Simp[(s + t*Log[i*(g + h*x)^n])^(m + 1)*(Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(k*n*t*(m + 1))), x] + (-Dist[b*p*(r/(k*n*
t*(m + 1))), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(a + b*x), x], x] - Dis
t[d*q*(r/(k*n*t*(m + 1))), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(c + d*x)
, x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r},
x] && NeQ[b*c - a*d, 0] && EqQ[h*j - g*k, 0] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\left(s+t\log\left(i(g+hx)^n\right)\right)^2}{2hknt} \\
&\quad - \frac{(bpr)\int\frac{(s+t\log(i(g+hx)^n))^2}{a+bx}\,dx}{2hknt} - \frac{(dqr)\int\frac{(s+t\log(i(g+hx)^n))^2}{c+dx}\,dx}{2hknt} \\
&= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log\left(i(g+hx)^n\right)\right)^2}{2hknt} \\
&\quad - \frac{qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)\left(s+t\log\left(i(g+hx)^n\right)\right)^2}{2hknt} \\
&\quad + \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\left(s+t\log\left(i(g+hx)^n\right)\right)^2}{2hknt} \\
&\quad + \frac{(pr)\int\frac{\log\left(\frac{h(a+bx)}{-bg+ah}\right)(s+t\log(i(g+hx)^n))}{g+hx}\,dx}{k} + \frac{(qr)\int\frac{\log\left(\frac{h(c+dx)}{-dg+ch}\right)(s+t\log(i(g+hx)^n))}{g+hx}\,dx}{k}
\end{aligned}
$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)(s+t\log(i(g+hx)^n))^2}{2hknt} - \frac{qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)(s+t\log(i(g+hx)^n))^2}{2hknt} + \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)(s+t\log(i(g+hx)^n))^2}{2hknt} + \frac{(pr)\text{Subst}\left(\int \frac{(s+t\log(ix^n))\log\left(\frac{h\left(\frac{-bg+ah}{h}+\frac{bx}{h}\right)}{-bg+ah}\right)}{x}dx, x, g+hx\right)}{hk} + \frac{(qr)\text{Subst}\left(\int \frac{(s+t\log(ix^n))\log\left(\frac{h\left(\frac{-dg+ch}{h}+\frac{dx}{h}\right)}{-dg+ch}\right)}{x}dx, x, g+hx\right)}{hk}$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)(s+t\log(i(g+hx)^n))^2}{2hknt} - \frac{qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)(s+t\log(i(g+hx)^n))^2}{2hknt} + \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)(s+t\log(i(g+hx)^n))^2}{2hknt} - \frac{pr(s+t\log(i(g+hx)^n))\text{Li}_2\left(\frac{b(g+hx)}{bg-ah}\right)}{hk} - \frac{qr(s+t\log(i(g+hx)^n))\text{Li}_2\left(\frac{d(g+hx)}{dg-ch}\right)}{hk} + \frac{(nprt)\text{Subst}\left(\int \frac{\text{Li}_2\left(-\frac{bx}{-bg+ah}\right)}{x}dx, x, g+hx\right)}{hk} + \frac{(nqrt)\text{Subst}\left(\int \frac{\text{Li}_2\left(-\frac{dx}{-dg+ch}\right)}{x}dx, x, g+hx\right)}{hk}$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log\left(i(g+hx)^n\right)\right)^2}{2hknt} - \frac{qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)\left(s+t\log\left(i(g+hx)^n\right)\right)^2}{2hknt} + \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\left(s+t\log\left(i(g+hx)^n\right)\right)^2}{2hknt} - \frac{pr(s+t\log\left(i(g+hx)^n\right))\operatorname{Li}_2\left(\frac{b(g+hx)}{bg-ah}\right)}{hk} - \frac{qr(s+t\log\left(i(g+hx)^n\right))\operatorname{Li}_2\left(\frac{d(g+hx)}{dg-ch}\right)}{hk} + \frac{nprt\operatorname{Li}_3\left(\frac{b(g+hx)}{bg-ah}\right)}{hk} + \frac{nqrt\operatorname{Li}_3\left(\frac{d(g+hx)}{dg-ch}\right)}{hk}$$

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.42

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\left(s+t\log\left(i(g+hx)^n\right)\right)}{gk+hkx}\,dx$$
$$= \frac{-2prs\log\left(\frac{h(a+bx)}{-bg+ah}\right)\log(g+hx) - 2qrs\log\left(\frac{h(c+dx)}{-dg+ch}\right)\log(g+hx) + 2s\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\log(g+\cdots}{\cdots}$$

```
[In] Integrate[(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*(s + t*Log[i*(g + h*x)^n]))
/(g*k + h*k*x),x]
```

```
[Out] (-2*p*r*s*Log[(h*(a + b*x))/(-(b*g) + a*h)]*Log[g + h*x] - 2*q*r*s*Log[(h*(
c + d*x))/(-(d*g) + c*h)]*Log[g + h*x] + 2*s*Log[e*(f*(a + b*x)^p*(c + d*x)
^q)^r]*Log[g + h*x] + n*p*r*t*Log[(h*(a + b*x))/(-(b*g) + a*h)]*Log[g + h*x
]^2 + n*q*r*t*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[g + h*x]^2 - n*t*Log[e*
(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[g + h*x]^2 - 2*p*r*t*Log[(h*(a + b*x))/(
-(b*g) + a*h)]*Log[g + h*x]*Log[i*(g + h*x)^n] - 2*q*r*t*Log[(h*(c + d*x))/
(-(d*g) + c*h)]*Log[g + h*x]*Log[i*(g + h*x)^n] + 2*t*Log[e*(f*(a + b*x)^p*
(c + d*x)^q)^r]*Log[g + h*x]*Log[i*(g + h*x)^n] - 2*p*r*(s + t*Log[i*(g + h
*x)^n])*PolyLog[2, (b*(g + h*x))/(b*g - a*h)] - 2*q*r*(s + t*Log[i*(g + h*x
)^n])*PolyLog[2, (d*(g + h*x))/(d*g - c*h)] + 2*n*p*r*t*PolyLog[3, (b*(g +
h*x))/(b*g - a*h)] + 2*n*q*r*t*PolyLog[3, (d*(g + h*x))/(d*g - c*h)])/(2*h*
k)
```

Maple [F]

$$\int \frac{\ln \left(e(f(bx+a)^{p} \left(dx+c\right)^{q})^{r}\right) \left(s+t\ln \left(i(hx+g)^{n}\right)\right)}{hkx+gk}dx$$

[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*ln(i*(h*x+g)^n))/(h*k*x+g*k),x)

[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*ln(i*(h*x+g)^n))/(h*k*x+g*k),x)

Fricas [F]

$$\int \frac{\log \left(e(f(a+bx)^{p}(c+dx)^{q})^{r}\right) \left(s+t\log \left(i(g+hx)^{n}\right)\right)}{gk+hkx}\,dx = \int \frac{\left(t\log \left((hx+g)^{n}i\right)+s\right)\log \left(\left(\left(bx+a\right)^{p}(dx+c)^{q}f\right)^{r}e\right)}{hkx+gk}\,dx$$

[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*log(i*(h*x+g)^n))/(h*k*x+g*k),x, algorithm="fricas")

[Out] integral((t*log((h*x + g)^n*i) + s)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(h*k*x + g*k), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(e(f(a+bx)^{p}(c+dx)^{q})^{r}\right) \left(s+t\log \left(i(g+hx)^{n}\right)\right)}{gk+hkx}\,dx = \text{Timed out}$$

[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)*(s+t*ln(i*(h*x+g)**n))/(h*k*x+g*k),x)

[Out] Timed out

Maxima [F]

$$\int \frac{\log \left(e(f(a+bx)^{p}(c+dx)^{q})^{r}\right) \left(s+t\log \left(i(g+hx)^{n}\right)\right)}{gk+hkx}\,dx = \int \frac{\left(t\log \left((hx+g)^{n}i\right)+s\right)\log \left(\left(\left(bx+a\right)^{p}(dx+c)^{q}f\right)^{r}e\right)}{hkx+gk}\,dx$$

[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*log(i*(h*x+g)^n))/(h*k*x+g*k),x, algorithm="maxima")

```
[Out] -1/2*((n*t*log(h*x + g)^2 - 2*t*log(h*x + g)*log((h*x + g)^n) - 2*(t*log(i)
 + s)*log(h*x + g))*log(((b*x + a)^p)^r) + (n*t*log(h*x + g)^2 - 2*t*log(h*
x + g)*log((h*x + g)^n) - 2*(t*log(i) + s)*log(h*x + g))*log(((d*x + c)^q)^
r))/(h*k) - integrate(-1/2*(2*((t*log(i) + s)*h*log(e) + (r*t*log(i) + r*s)
*h*log(f))*b*d*x^2 + 2*((t*log(i) + s)*h*log(e) + (r*t*log(i) + r*s)*h*log(
f))*a*c + ((p*r + q*r)*b*d*h*n*t*x^2 + b*c*g*n*p*r*t + a*d*g*n*q*r*t + (a*d
*h*n*q*r*t + (c*h*n*p*r*t + (p*r + q*r)*d*g*n*t)*b)*x)*log(h*x + g)^2 + 2*(
((t*log(i) + s)*h*log(e) + (r*t*log(i) + r*s)*h*log(f))*b*c + ((t*log(i) +
s)*h*log(e) + (r*t*log(i) + r*s)*h*log(f))*a*d)*x - 2*((p*r*s + q*r*s + (p*
r + q*r)*t*log(i))*b*d*h*x^2 + (p*r*t*log(i) + p*r*s)*b*c*g + (q*r*t*log(i)
 + q*r*s)*a*d*g + ((q*r*t*log(i) + q*r*s)*a*d*h + ((p*r*s + q*r*s + (p*r +
q*r)*t*log(i))*d*g + (p*r*t*log(i) + p*r*s)*c*h)*b)*x)*log(h*x + g) + 2*((h
*r*t*log(f) + h*t*log(e))*b*d*x^2 + (h*r*t*log(f) + h*t*log(e))*a*c + ((h*r
*t*log(f) + h*t*log(e))*b*c + (h*r*t*log(f) + h*t*log(e))*a*d)*x - ((p*r +
q*r)*b*d*h*t*x^2 + b*c*g*p*r*t + a*d*g*q*r*t + (a*d*h*q*r*t + (c*h*p*r*t +
(p*r + q*r)*d*g*t)*b)*x)*log(h*x + g))*log((h*x + g)^n))/(b*d*h^2*k*x^3 + a
*c*g*h*k + (a*d*h^2*k + (d*g*h*k + c*h^2*k)*b)*x^2 + (b*c*g*h*k + (d*g*h*k
+ c*h^2*k)*a)*x), x)
```

Giac [F]

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)\left(s+t\log \left(i(g+hx)^n\right)\right)}{gk+hkx}\,dx = \int \frac{\left(t\log \left((hx+g)^n i\right)+s\right)\log \left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{hkx+gk}\,dx$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*log(i*(h*x+g)^n))/(h*k*x+g*
k),x, algorithm="giac")
```

```
[Out] integrate((t*log((h*x + g)^n*i) + s)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(
h*k*x + g*k), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)\left(s+t\log \left(i(g+hx)^n\right)\right)}{gk+hkx}\,dx = \int \frac{\ln \left(e\left(f\,{\left(a+b\,x\right)}^p\,{\left(c+d\,x\right)}^q\right)^r\right)\,\left(s+t\,\ln \left(i\,{\left(g+h\,x\right)}^n\right)\right)}{g\,k+h\,k\,x}\,dx$$

```
[In] int((log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(s + t*log(i*(g + h*x)^n)))/(g*k
+ h*k*x),x)
```

```
[Out] int((log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(s + t*log(i*(g + h*x)^n)))/(g*k
+ h*k*x), x)
```

3.53 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{gk+hkx}\,dx$

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Optimal result

Integrand size = 32, antiderivative size = 172

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{gk+hkx}\,dx = -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(gk+hkx)}{hk} - \frac{qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log(gk+hkx)}{hk} + \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\log(gk+hkx)}{hk} - \frac{pr\,\mathrm{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)}{hk} - \frac{qr\,\mathrm{PolyLog}\left(2,\frac{d(g+hx)}{dg-ch}\right)}{hk}$$

```
[Out] -p*r*ln(-h*(b*x+a)/(-a*h+b*g))*ln(h*k*x+g*k)/h/k-q*r*ln(-h*(d*x+c)/(-c*h+d*
g))*ln(h*k*x+g*k)/h/k+ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*ln(h*k*x+g*k)/h/k-p*r
*polylog(2,b*(h*x+g)/(-a*h+b*g))/h/k-q*r*polylog(2,d*(h*x+g)/(-c*h+d*g))/h/
k
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used

$= \{2580, 2441, 2440, 2438\}$

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{gk+hkx}\,dx = \frac{\log(gk+hkx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{hk} - \frac{pr\,\text{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)}{hk} - \frac{pr\log(gk+hkx)\log\left(-\frac{h(a+bx)}{bg-ah}\right)}{hk} - \frac{qr\,\text{PolyLog}\left(2,\frac{d(g+hx)}{dg-ch}\right)}{hk} - \frac{qr\log(gk+hkx)\log\left(-\frac{h(c+dx)}{dg-ch}\right)}{hk}$$

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g*k + h*k*x),x]
```

```
[Out] -((p*r*Log[-((h*(a + b*x))/(b*g - a*h))]*Log[g*k + h*k*x])/(h*k)) - (q*r*Lo
g[-((h*(c + d*x))/(d*g - c*h))]*Log[g*k + h*k*x])/(h*k) + (Log[e*(f*(a + b*
x)^p*(c + d*x)^q)^r]*Log[g*k + h*k*x])/(h*k) - (p*r*PolyLog[2, (b*(g + h*x)
)/(b*g - a*h)])/(h*k) - (q*r*PolyLog[2, (d*(g + h*x))/(d*g - c*h)])/(h*k)
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2580

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[Log[g + h*x]*(Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]/h), x] + (-Dist[b*p*(r/h), Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[d*q*(r/h), Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
```

```
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\text{integral} = \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\log(gk+hkx)}{hk} - \frac{(bpr)\int \frac{\log(gk+hkx)}{a+bx}\,dx}{hk} - \frac{(dqr)\int \frac{\log(gk+hkx)}{c+dx}\,dx}{hk}$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(gk+hkx)}{hk} - \frac{qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log(gk+hkx)}{hk} + \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\log(gk+hkx)}{hk} + (pr)\int \frac{\log\left(\frac{hk(a+bx)}{-bgk+ahk}\right)}{gk+hkx}\,dx + (qr)\int \frac{\log\left(\frac{hk(c+dx)}{-dgk+chk}\right)}{gk+hkx}\,dx$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(gk+hkx)}{hk} - \frac{qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log(gk+hkx)}{hk} + \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\log(gk+hkx)}{hk} + \frac{(pr)\text{Subst}\left(\int \frac{\log\left(1+\frac{bx}{-bgk+ahk}\right)}{x}\,dx, x, gk+hkx\right)}{hk} + \frac{(qr)\text{Subst}\left(\int \frac{\log\left(1+\frac{dx}{-dgk+chk}\right)}{x}\,dx, x, gk+hkx\right)}{hk}$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(gk+hkx)}{hk} - \frac{qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log(gk+hkx)}{hk} + \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\log(gk+hkx)}{hk} - \frac{pr\text{Li}_2\left(\frac{b(g+hx)}{bg-ah}\right)}{hk} - \frac{qr\text{Li}_2\left(\frac{d(g+hx)}{dg-ch}\right)}{hk}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.97

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{gk+hkx}\,dx = \frac{-pr\log(a+bx)\log(g+hx) - qr\log(c+dx)\log(g+hx) + \log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\log(g+hx) + p}{}$$

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g*k + h*k*x),x]
```

```
[Out] (-(p*r*Log[a + b*x]*Log[g + h*x]) - q*r*Log[c + d*x]*Log[g + h*x] + Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r]*Log[g + h*x] + p*r*Log[a + b*x]*Log[(b*(g + h
*x))/(b*g - a*h)] + q*r*Log[c + d*x]*Log[(d*(g + h*x))/(d*g - c*h)] + p*r*P
olyLog[2, (h*(a + b*x))/(-(b*g) + a*h)] + q*r*PolyLog[2, (h*(c + d*x))/(-(d
*g) + c*h)])/(h*k)
```

Maple [A] (verified)

Time = 51.53 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.12

method	result
parts	$\frac{\ln(e(f(bx+a)^p(dx+c)^q)^r)\ln(hx+g)}{kh} - \frac{r\left(bph\left(\frac{\operatorname{dilog}\left(\frac{(hx+g)b+ah-bg}{ah-bg}\right)}{b}+\frac{\ln(hx+g)\ln\left(\frac{(hx+g)b+ah-bg}{ah-bg}\right)}{b}\right)+dqh\left(\frac{\operatorname{dilog}\left(\frac{d(hx+g)+ch-c}{ch-dg}\right)}{d}\right.}{k\,h^2}$

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k),x,method=_RETURNVERBOSE)
```

```
[Out] ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/k*ln(h*x+g)/h-1/k/h^2*r*(b*p*h*(dilog(((h*x
+g)*b+a*h-b*g)/(a*h-b*g))/b+ln(h*x+g)*ln(((h*x+g)*b+a*h-b*g)/(a*h-b*g))/b)+
d*q*h*(dilog((d*(h*x+g)+c*h-d*g)/(c*h-d*g))/d+ln(h*x+g)*ln((d*(h*x+g)+c*h-d
*g)/(c*h-d*g))/d))
```

Fricas [F]

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{gk+hkx}\,dx = \int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{hkx+gk}\,dx$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k),x, algorithm="fricas
")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(h*k*x + g*k), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{gk+hkx}\,dx = \text{Timed out}$$

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(h*k*x+g*k),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.19

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{gk+hkx}\,dx = \frac{\left(\frac{\left(\log(bx+a)\log\left(\frac{bhx+ah}{bg-ah}+1\right)+\mathrm{Li}_2\left(-\frac{bhx+ah}{bg-ah}\right)\right)fp}{hk}+\frac{\left(\log(dx+c)\log\left(\frac{dhx+ch}{dg-ch}+1\right)+\mathrm{Li}_2\left(-\frac{dhx+ch}{dg-ch}\right)\right)fq}{hk}\right)r}{f} - \frac{(fp\log(bx+a)+fq\log(dx+c))r\log(hkx+gk)}{fhk} + \frac{\log(hkx+gk)\log\left(\left((bx+a)^p(dx+c)^qf\right)^re\right)}{hk}$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k),x, algorithm="maxima
")
```

```
[Out] ((log(b*x + a)*log((b*h*x + a*h)/(b*g - a*h) + 1) + dilog(-(b*h*x + a*h)/(b
*g - a*h)))*f*p/(h*k) + (log(d*x + c)*log((d*h*x + c*h)/(d*g - c*h) + 1) +
dilog(-(d*h*x + c*h)/(d*g - c*h)))*f*q/(h*k))*r/f - (f*p*log(b*x + a) + f*q
*log(d*x + c))*r*log(h*k*x + g*k)/(f*h*k) + log(h*k*x + g*k)*log(((b*x + a)
^p*(d*x + c)^q*f)^r*e)/(h*k)
```

Giac [F]

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{gk+hkx}\,dx = \int \frac{\log \left(\left((bx+a)^p(dx+c)^qf\right)^re\right)}{hkx+gk}\,dx$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k),x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(h*k*x + g*k), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{gk+hkx}\,dx = \int \frac{\ln\left(e\,{\left(f\,{\left(a+b\,x\right)}^p\,{\left(c+d\,x\right)}^q\right)}^r\right)}{g\,k+h\,k\,x}\,dx$$

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(g*k + h*k*x),x)
```

```
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(g*k + h*k*x), x)
```

3.54 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(gk+hkx)(s+t\log(i(g+hx)^n))}\,dx$

Optimal result 480
Rubi [N/A] 480
Mathematica [N/A] 481
Maple [N/A] 481
Fricas [N/A] 481
Sympy [F(-1)] 482
Maxima [N/A] 482
Giac [N/A] 482
Mupad [N/A] 483

Optimal result

Integrand size = 48, antiderivative size = 48

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)}\,dx=\text{Int}\left(\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)},x\right)$$

```
[Out] Unintegrable(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*ln(i*(h*x+g)^
n)),x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)}\,dx=\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)}\,dx$$

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/((g*k + h*k*x)*(s + t*Log[i*(g + h
*x)^n])),x]
```

```
[Out] Defer[Int][Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/((g*k + h*k*x)*(s + t*Log[i
*(g + h*x)^n])), x]
```

Rubi steps

$$\text{integral}=\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)}\,dx=\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)}\,dx$$

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/((g*k + h*k*x)*(s + t*Log[i*
(g + h*x)^n])),x]
```

```
[Out] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/((g*k + h*k*x)*(s + t*Log[i*
(g + h*x)^n])), x]
```

Maple [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{\ln \left(e(f(bx+a)^p\,(dx+c)^q)^r\right)}{(hkx+gk)\left(s+t\ln\left(i\,(hx+g)^n\right)\right)}dx$$

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*ln(i*(h*x+g)^n)),x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*ln(i*(h*x+g)^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)}\,dx=\int \frac{\log \left(\left((bx+a)^p(dx+c)^qf\right)^r e\right)}{(hkx+gk)(t\log\left((hx+g)^n i\right)+s)}\,dx$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*log(i*(h*x+g)^n
)),x, algorithm="fricas")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(h*k*s*x + g*k*s + (h*k*t*x +
 g*k*t)*log((h*x + g)^n*i)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)}\,dx = \text{Timed out}$$

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(h*k*x+g*k)/(s+t*ln(i*(h*x+g)*
*n)),x)
```

```
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 3.44 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)}\,dx = \int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{(hkx+gk)(t\log\left((hx+g)^n i\right)+s)}\,dx$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*log(i*(h*x+g)^n
)),x, algorithm="maxima")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/((h*k*x + g*k)*(t*log((h*x +
 g)^n*i) + s)), x)
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)}\,dx = \int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{(hkx+gk)(t\log\left((hx+g)^n i\right)+s)}\,dx$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*log(i*(h*x+g)^n
)),x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/((h*k*x + g*k)*(t*log((h*x +
 g)^n*i) + s)), x)
```

Mupad [N/A]

Not integrable

Time = 1.61 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(s+t\log \left(i(g+hx)^n\right)\right)}\,dx = \int \frac{\ln \left(e\left(f\,{\left(a+b\,x\right)}^p\,{\left(c+d\,x\right)}^q\right)^r\right)}{\left(g\,k+h\,k\,x\right)\,\left(s+t\,\ln \left(i\,{\left(g+h\,x\right)}^n\right)\right)}\,dx$$

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/((g*k + h*k*x)*(s + t*log(i*(g + h
*x)^n))),x)
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/((g*k + h*k*x)*(s + t*log(i*(g + h
*x)^n))), x)
```

3.55 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(gk+hkx)(s+t\log(i(g+hx)^n))^2}\,dx$

Optimal result 484
Rubi [N/A] 484
Mathematica [N/A] 485
Maple [N/A] 485
Fricas [N/A] 485
Sympy [F(-1)] 486
Maxima [N/A] 486
Giac [N/A] 486
Mupad [N/A] 487

Optimal result

Integrand size = 48, antiderivative size = 48

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)^2}\,dx=\operatorname{Int}\left(\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)^2},x\right)$$

```
[Out] Unintegrable(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*ln(i*(h*x+g)^
n))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)^2}\,dx=\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)^2}\,dx$$

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/((g*k + h*k*x)*(s + t*Log[i*(g + h
*x)^n])^2),x]
```

```
[Out] Defer[Int][Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/((g*k + h*k*x)*(s + t*Log[i
*(g + h*x)^n])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.50 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(s+t\log \left(i(g+hx)^n\right)\right)^2}\,dx=\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(s+t\log \left(i(g+hx)^n\right)\right)^2}\,dx$$

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/((g*k + h*k*x)*(s + t*Log[i*
(g + h*x)^n])^2),x]
[Out] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/((g*k + h*k*x)*(s + t*Log[i*
(g + h*x)^n])^2), x]
```

Maple [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{\ln \left(e(f(bx+a)^p\,(dx+c)^q)^r\right)}{(hkx+gk)\left(s+t\ln \left(i\,(hx+g)^n\right)\right)^2}dx$$

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*ln(i*(h*x+g)^n))^2,x)
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*ln(i*(h*x+g)^n))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.90

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(s+t\log \left(i(g+hx)^n\right)\right)^2}\,dx=\int \frac{\log \left(\left((bx+a)^p(dx+c)^qf\right)^r e\right)}{(hkx+gk)(t\log \left((hx+g)^n i\right)+s)^2}\,dx$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*log(i*(h*x+g)^n
))^2,x, algorithm="fricas")
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(h*k*s^2*x + g*k*s^2 + (h*k*t
^2*x + g*k*t^2)*log((h*x + g)^n*i)^2 + 2*(h*k*s*t*x + g*k*s*t)*log((h*x + g
)^n*i)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(h*k*x+g*k)/(s+t*ln(i*(h*x+g)*
*n))**2,x)
```

```
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 2.93 (sec) , antiderivative size = 222, normalized size of antiderivative = 4.62

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)^2}\,dx = \int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{(hkx+gk)(t\log\left((hx+g)^n i\right)+s)^2}\,dx$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*log(i*(h*x+g)^n
))^2,x, algorithm="maxima")
```

```
[Out] -(r*log(f) + log(((b*x + a)^p)^r) + log(((d*x + c)^q)^r) + log(e))/(h*k*n*t
^2*log((h*x + g)^n) + (k*n*t^2*log(i) + k*n*s*t)*h) + integrate((b*c*p*r +
a*d*q*r + (p*r + q*r)*b*d*x)/((k*n*t^2*log(i) + k*n*s*t)*b*d*h*x^2 + (k*n*t
^2*log(i) + k*n*s*t)*a*c*h + ((k*n*t^2*log(i) + k*n*s*t)*b*c*h + (k*n*t^2*l
og(i) + k*n*s*t)*a*d*h)*x + (b*d*h*k*n*t^2*x^2 + a*c*h*k*n*t^2 + (b*c*h*k*n
*t^2 + a*d*h*k*n*t^2)*x)*log((h*x + g)^n)), x)
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)^2}\,dx = \int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{(hkx+gk)(t\log\left((hx+g)^n i\right)+s)^2}\,dx$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*log(i*(h*x+g)^n
))^2,x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/((h*k*x + g*k)*(t*log((h*x +
 g)^n*i) + s)^2), x)
```

Mupad [N/A]

Not integrable

Time = 1.75 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(s+t\log \left(i(g+hx)^n\right)\right)^2}\,dx=\int \frac{\ln \left(e\,{\left(f\,{\left(a+b\,x\right)}^p\,{\left(c+d\,x\right)}^q\right)}^r\right)}{\left(g\,k+h\,k\,x\right)\,{\left(s+t\,\ln \left(i\,{\left(g+h\,x\right)}^n\right)\right)}^2}\,dx$$

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/((g*k + h*k*x)*(s + t*log(i*(g + h
*x)^n))^2),x)
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/((g*k + h*k*x)*(s + t*log(i*(g + h
*x)^n))^2), x)
```

3.56 $\int \frac{\log^3\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x}\,dx$

Optimal result 488
Rubi [A] (verified) 489
Mathematica [B] (verified) 493
Maple [F] 495
Fricas [F] 496
Sympy [F(-1)] 496
Maxima [F] 496
Giac [F] 499
Mupad [F(-1)] 499

Optimal result

Integrand size = 39, antiderivative size = 328

$$\begin{aligned}
&\int \frac{\log^3\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x}\,dx \\
&= -\frac{pr\log^4\left(i\left(j(hx)^t\right)^u\right)\log\left(1+\frac{bx}{a}\right)}{4tu}+\frac{\log^4\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4tu} \\
&\quad-\frac{qr\log^4\left(i\left(j(hx)^t\right)^u\right)\log\left(1+\frac{dx}{c}\right)}{4tu}-pr\log^3\left(i\left(j(hx)^t\right)^u\right)\operatorname{PolyLog}\left(2,-\frac{bx}{a}\right) \\
&\quad-qr\log^3\left(i\left(j(hx)^t\right)^u\right)\operatorname{PolyLog}\left(2,-\frac{dx}{c}\right)+3prtu\log^2\left(i\left(j(hx)^t\right)^u\right)\operatorname{PolyLog}\left(3,-\frac{bx}{a}\right) \\
&\quad+3qrtu\log^2\left(i\left(j(hx)^t\right)^u\right)\operatorname{PolyLog}\left(3,-\frac{dx}{c}\right) \\
&\quad-6prt^2u^2\log\left(i\left(j(hx)^t\right)^u\right)\operatorname{PolyLog}\left(4,-\frac{bx}{a}\right) \\
&\quad-6qrt^2u^2\log\left(i\left(j(hx)^t\right)^u\right)\operatorname{PolyLog}\left(4,-\frac{dx}{c}\right) \\
&\quad+6prt^3u^3\operatorname{PolyLog}\left(5,-\frac{bx}{a}\right)+6qrt^3u^3\operatorname{PolyLog}\left(5,-\frac{dx}{c}\right)
\end{aligned}$$

```
[Out] -1/4*p*r*ln(i*(j*(h*x)^t)^u)^4*ln(1+b*x/a)/t/u+1/4*ln(i*(j*(h*x)^t)^u)^4*ln
(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/t/u-1/4*q*r*ln(i*(j*(h*x)^t)^u)^4*ln(1+d*x/c)
/t/u-p*r*ln(i*(j*(h*x)^t)^u)^3*polylog(2,-b*x/a)-q*r*ln(i*(j*(h*x)^t)^u)^3*
polylog(2,-d*x/c)+3*p*r*t*u*ln(i*(j*(h*x)^t)^u)^2*polylog(3,-b*x/a)+3*q*r*t
*u*ln(i*(j*(h*x)^t)^u)^2*polylog(3,-d*x/c)-6*p*r*t^2*u^2*ln(i*(j*(h*x)^t)^u
)*polylog(4,-b*x/a)-6*q*r*t^2*u^2*ln(i*(j*(h*x)^t)^u)*polylog(4,-d*x/c)+6*p
*r*t^3*u^3*polylog(5,-b*x/a)+6*q*r*t^3*u^3*polylog(5,-d*x/c)
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2585, 2354, 2421, 2430, 6724, 2495}

$$\int \frac{\log^3\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx$$
$$= \frac{\log^4\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4tu} - 6prt^2u^2\,\text{PolyLog}\left(4,-\frac{bx}{a}\right)\log\left(i\left(j(hx)^t\right)^u\right) - pr\,\text{PolyLog}\left(2,-\frac{bx}{a}\right)\log^3\left(i\left(j(hx)^t\right)^u\right) + 3prtu\,\text{PolyLog}\left(3,-\frac{bx}{a}\right)\log^2\left(i\left(j(hx)^t\right)^u\right) - \frac{pr\log\left(\frac{bx}{a}+1\right)\log^4\left(i(j(hx)^t)^u\right)}{4tu} + 6prt^3u^3\,\text{PolyLog}\left(5,-\frac{bx}{a}\right) - 6qrt^2u^2\,\text{PolyLog}\left(4,-\frac{dx}{c}\right)\log\left(i\left(j(hx)^t\right)^u\right) - qr\,\text{PolyLog}\left(2,-\frac{dx}{c}\right)\log^3\left(i\left(j(hx)^t\right)^u\right) + 3qrtu\,\text{PolyLog}\left(3,-\frac{dx}{c}\right)\log^2\left(i\left(j(hx)^t\right)^u\right) - \frac{qr\log\left(\frac{dx}{c}+1\right)\log^4\left(i(j(hx)^t)^u\right)}{4tu} + 6qrt^3u^3\,\text{PolyLog}\left(5,-\frac{dx}{c}\right)$$

```
[In] Int[(Log[i*(j*(h*x)^t)^u]^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/x,x]
```

```
[Out] -1/4*(p*r*Log[i*(j*(h*x)^t)^u]^4*Log[1 + (b*x)/a])/(t*u) + (Log[i*(j*(h*x)^
t)^u]^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(4*t*u) - (q*r*Log[i*(j*(h*x)
^t)^u]^4*Log[1 + (d*x)/c])/(4*t*u) - p*r*Log[i*(j*(h*x)^t)^u]^3*PolyLog[2,
-((b*x)/a)] - q*r*Log[i*(j*(h*x)^t)^u]^3*PolyLog[2, -((d*x)/c)] + 3*p*r*t*u
*Log[i*(j*(h*x)^t)^u]^2*PolyLog[3, -((b*x)/a)] + 3*q*r*t*u*Log[i*(j*(h*x)^t
)^u]^2*PolyLog[3, -((d*x)/c)] - 6*p*r*t^2*u^2*Log[i*(j*(h*x)^t)^u]*PolyLog[
4, -((b*x)/a)] - 6*q*r*t^2*u^2*Log[i*(j*(h*x)^t)^u]*PolyLog[4, -((d*x)/c)]
+ 6*p*r*t^3*u^3*PolyLog[5, -((b*x)/a)] + 6*q*r*t^3*u^3*PolyLog[5, -((d*x)/c
)]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
```

```
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rule 2585

```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.))^(m_.))/((j_.
) + (k_.)*(x_)), x_Symbol] :> Simp[(s + t*Log[i*(g + h*x)^n])^(m + 1)*(Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(k*n*t*(m + 1))), x] + (-Dist[b*p*(r/(k*n*
t*(m + 1))), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(a + b*x), x], x] - Dis
t[d*q*(r/(k*n*t*(m + 1))), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(c + d*x)
, x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r},
x] && NeQ[b*c - a*d, 0] && EqQ[h*j - g*k, 0] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{\log^3\left(ij^u(hx)^{tu}\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x}\,dx, ij^u(hx)^{tu}, i\left(j(hx)^t\right)^u\right)$$

$$= \text{Subst}\left(\text{Subst}\left(\int \frac{\log^3\left(h^{tu}ij^ux^{tu}\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x}\,dx, h^{tu}ij^ux^{tu}, ij^u(hx)^{tu}\right), ij^u(hx)^{tu}$$

$$= \frac{\log^4\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4tu}$$
$$-\text{Subst}\left(\text{Subst}\left(\frac{(bpr)\int\frac{\log^4\left(h^{tu}ij^ux^{tu}\right)}{a+bx}\,dx}{4tu}, h^{tu}ij^ux^{tu}, ij^u(hx)^{tu}\right), ij^u(hx)^{tu}, i\left(j(hx)^t\right)^u\right)$$
$$-\text{Subst}\left(\text{Subst}\left(\frac{(dqr)\int\frac{\log^4\left(h^{tu}ij^ux^{tu}\right)}{c+dx}\,dx}{4tu}, h^{tu}ij^ux^{tu}, ij^u(hx)^{tu}\right), ij^u(hx)^{tu}, i\left(j(hx)^t\right)^u\right)$$

$$= -\frac{pr\log^4\left(i(j(hx)^t)^u\right)\log\left(1+\frac{bx}{a}\right)}{4tu}$$
$$+\frac{\log^4\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4tu} - \frac{qr\log^4\left(i(j(hx)^t)^u\right)\log\left(1+\frac{dx}{c}\right)}{4tu}$$
$$+\text{Subst}\left(\text{Subst}\left((pr)\int\frac{\log^3\left(h^{tu}ij^ux^{tu}\right)\log\left(1+\frac{bx}{a}\right)}{x}\,dx, h^{tu}ij^ux^{tu}, ij^u(hx)^{tu}\right), ij^u(hx)^{tu}, i\left(j(hx)\right.\right.$$
$$+\text{Subst}\left(\text{Subst}\left((qr)\int\frac{\log^3\left(h^{tu}ij^ux^{tu}\right)\log\left(1+\frac{dx}{c}\right)}{x}\,dx, h^{tu}ij^ux^{tu}, ij^u(hx)^{tu}\right), ij^u(hx)^{tu}, i\left(j(hx)\right.\right.$$

$$= -\frac{pr\log^4\left(i(j(hx)^t)^u\right)\log\left(1+\frac{bx}{a}\right)}{4tu} + \frac{\log^4\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4tu}$$
$$-\frac{qr\log^4\left(i(j(hx)^t)^u\right)\log\left(1+\frac{dx}{c}\right)}{4tu}$$
$$-pr\log^3\left(i\left(j(hx)^t\right)^u\right)\text{Li}_2\left(-\frac{bx}{a}\right) - qr\log^3\left(i\left(j(hx)^t\right)^u\right)\text{Li}_2\left(-\frac{dx}{c}\right)$$
$$+\text{Subst}\left(\text{Subst}\left((3prtu)\int\frac{\log^2\left(h^{tu}ij^ux^{tu}\right)\text{Li}_2\left(-\frac{bx}{a}\right)}{x}\,dx, h^{tu}ij^ux^{tu}, ij^u(hx)^{tu}\right), ij^u(hx)^{tu}, i\left(j(hx\right.\right.$$
$$+\text{Subst}\left(\text{Subst}\left((3qrtu)\int\frac{\log^2\left(h^{tu}ij^ux^{tu}\right)\text{Li}_2\left(-\frac{dx}{c}\right)}{x}\,dx, h^{tu}ij^ux^{tu}, ij^u(hx)^{tu}\right), ij^u(hx)^{tu}, i\left(j(hx\right.\right.$$

$$= -\frac{pr\log^4\left(i(j(hx)^t)^u\right)\log\left(1+\frac{bx}{a}\right)}{4tu} + \frac{\log^4\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4tu}$$
$$-\frac{qr\log^4\left(i(j(hx)^t)^u\right)\log\left(1+\frac{dx}{c}\right)}{4tu} - pr\log^3\left(i\left(j(hx)^t\right)^u\right)\text{Li}_2\left(-\frac{bx}{a}\right)$$
$$-qr\log^3\left(i\left(j(hx)^t\right)^u\right)\text{Li}_2\left(-\frac{dx}{c}\right) + 3prtu\log^2\left(i\left(j(hx)^t\right)^u\right)\text{Li}_3\left(-\frac{bx}{a}\right)$$
$$+3qrtu\log^2\left(i\left(j(hx)^t\right)^u\right)\text{Li}_3\left(-\frac{dx}{c}\right)$$
$$-\text{Subst}\left(\text{Subst}\left((6prt^2u^2)\int\frac{\log\left(h^{tu}ij^ux^{tu}\right)\text{Li}_3\left(-\frac{bx}{a}\right)}{x}\,dx, h^{tu}ij^ux^{tu}, ij^u(hx)^{tu}\right), ij^u(hx)^{tu}, i\left(j(h\right.\right.$$
$$-\text{Subst}\left(\text{Subst}\left((6qrt^2u^2)\int\frac{\log\left(h^{tu}ij^ux^{tu}\right)\text{Li}_3\left(-\frac{dx}{c}\right)}{x}\,dx, h^{tu}ij^ux^{tu}, ij^u(hx)^{tu}\right), ij^u(hx)^{tu}, i\left(j(h\right.\right.$$

$$= -\frac{pr\log^4\left(i(j(hx)^t)^u\right)\log\left(1+\frac{bx}{a}\right)}{4tu} + \frac{\log^4\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4tu}$$
$$- \frac{qr\log^4\left(i(j(hx)^t)^u\right)\log\left(1+\frac{dx}{c}\right)}{4tu} - pr\log^3\left(i\left(j(hx)^t\right)^u\right)\text{Li}_2\left(-\frac{bx}{a}\right)$$
$$- qr\log^3\left(i\left(j(hx)^t\right)^u\right)\text{Li}_2\left(-\frac{dx}{c}\right) + 3prtu\log^2\left(i\left(j(hx)^t\right)^u\right)\text{Li}_3\left(-\frac{bx}{a}\right)$$
$$+ 3qrtu\log^2\left(i\left(j(hx)^t\right)^u\right)\text{Li}_3\left(-\frac{dx}{c}\right)$$
$$- 6prt^2u^2\log\left(i\left(j(hx)^t\right)^u\right)\text{Li}_4\left(-\frac{bx}{a}\right) - 6qrt^2u^2\log\left(i\left(j(hx)^t\right)^u\right)\text{Li}_4\left(-\frac{dx}{c}\right)$$
$$+\text{Subst}\left(\text{Subst}\left(\left(6prt^3u^3\right)\int\frac{\text{Li}_4\left(-\frac{bx}{a}\right)}{x}\,dx, h^{tu}ij^ux^{tu}, ij^u(hx)^{tu}\right), ij^u(hx)^{tu}, i\left(j(hx)^t\right)^u\right)$$
$$+\text{Subst}\left(\text{Subst}\left(\left(6qrt^3u^3\right)\int\frac{\text{Li}_4\left(-\frac{dx}{c}\right)}{x}\,dx, h^{tu}ij^ux^{tu}, ij^u(hx)^{tu}\right), ij^u(hx)^{tu}, i\left(j(hx)^t\right)^u\right)$$

$$= -\frac{pr\log^4\left(i(j(hx)^t)^u\right)\log\left(1+\frac{bx}{a}\right)}{4tu} + \frac{\log^4\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4tu}$$
$$- \frac{qr\log^4\left(i(j(hx)^t)^u\right)\log\left(1+\frac{dx}{c}\right)}{4tu} - pr\log^3\left(i\left(j(hx)^t\right)^u\right)\text{Li}_2\left(-\frac{bx}{a}\right)$$
$$- qr\log^3\left(i\left(j(hx)^t\right)^u\right)\text{Li}_2\left(-\frac{dx}{c}\right) + 3prtu\log^2\left(i\left(j(hx)^t\right)^u\right)\text{Li}_3\left(-\frac{bx}{a}\right)$$
$$+ 3qrtu\log^2\left(i\left(j(hx)^t\right)^u\right)\text{Li}_3\left(-\frac{dx}{c}\right) - 6prt^2u^2\log\left(i\left(j(hx)^t\right)^u\right)\text{Li}_4\left(-\frac{bx}{a}\right)$$
$$- 6qrt^2u^2\log\left(i\left(j(hx)^t\right)^u\right)\text{Li}_4\left(-\frac{dx}{c}\right) + 6prt^3u^3\text{Li}_5\left(-\frac{bx}{a}\right) + 6qrt^3u^3\text{Li}_5\left(-\frac{dx}{c}\right)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1241 vs. $2(328) = 656$.

Time = 1.09 (sec) , antiderivative size = 1241, normalized size of antiderivative = 3.78

$$\int \frac{\log^3\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x}\,dx$$

$$\begin{aligned}
&= prt^3u^3\log(x)\log^3(hx)\log(a+bx) - prt^3u^3\log^4(hx)\log(a+bx) \\
&\quad - 3prt^2u^2\log(x)\log^2(hx)\log\left(i\left(j(hx)^t\right)^u\right)\log(a+bx) \\
&\quad + 3prt^2u^2\log^3(hx)\log\left(i\left(j(hx)^t\right)^u\right)\log(a+bx) \\
&\quad + 3prtu\log(x)\log(hx)\log^2\left(i\left(j(hx)^t\right)^u\right)\log(a+bx) \\
&\quad - 3prtu\log^2(hx)\log^2\left(i\left(j(hx)^t\right)^u\right)\log(a+bx) - pr\log(x)\log^3\left(i\left(j(hx)^t\right)^u\right)\log(a+bx) \\
&\quad + pr\log(hx)\log^3\left(i\left(j(hx)^t\right)^u\right)\log(a+bx) + \frac{1}{4}prt^3u^3\log^4(hx)\log\left(1+\frac{bx}{a}\right) \\
&\quad - prt^2u^2\log^3(hx)\log\left(i\left(j(hx)^t\right)^u\right)\log\left(1+\frac{bx}{a}\right) \\
&\quad + \frac{3}{2}prtu\log^2(hx)\log^2\left(i\left(j(hx)^t\right)^u\right)\log\left(1+\frac{bx}{a}\right) \\
&\quad - pr\log(hx)\log^3\left(i\left(j(hx)^t\right)^u\right)\log\left(1+\frac{bx}{a}\right) + qrt^3u^3\log(x)\log^3(hx)\log(c+dx) \\
&\quad - qrt^3u^3\log^4(hx)\log(c+dx) - 3qrt^2u^2\log(x)\log^2(hx)\log\left(i\left(j(hx)^t\right)^u\right)\log(c+dx) \\
&\quad + 3qrt^2u^2\log^3(hx)\log\left(i\left(j(hx)^t\right)^u\right)\log(c+dx) \\
&\quad + 3qrtu\log(x)\log(hx)\log^2\left(i\left(j(hx)^t\right)^u\right)\log(c+dx) \\
&\quad - 3qrtu\log^2(hx)\log^2\left(i\left(j(hx)^t\right)^u\right)\log(c+dx) \\
&\quad - qr\log(x)\log^3\left(i\left(j(hx)^t\right)^u\right)\log(c+dx) + qr\log(hx)\log^3\left(i\left(j(hx)^t\right)^u\right)\log(c+dx) \\
&\quad - t^3u^3\log(x)\log^3(hx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) \\
&\quad + \frac{3}{4}t^3u^3\log^4(hx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) \\
&\quad + 3t^2u^2\log(x)\log^2(hx)\log\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) \\
&\quad - 2t^2u^2\log^3(hx)\log\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) \\
&\quad - 3tu\log(x)\log(hx)\log^2\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) \\
&\quad + \frac{3}{2}tu\log^2(hx)\log^2\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) \\
&\quad + \log(x)\log^3\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) \\
&\quad + \frac{1}{4}qrt^3u^3\log^4(hx)\log\left(1+\frac{dx}{c}\right) - qrt^2u^2\log^3(hx)\log\left(i\left(j(hx)^t\right)^u\right)\log\left(1+\frac{dx}{c}\right) \\
&\quad + \frac{3}{2}qrtu\log^2(hx)\log^2\left(i\left(j(hx)^t\right)^u\right)\log\left(1+\frac{dx}{c}\right) \\
&\quad - qr\log(hx)\log^3\left(i\left(j(hx)^t\right)^u\right)\log\left(1+\frac{dx}{c}\right) - pr\log^3\left(i\left(j(hx)^t\right)^u\right)\text{PolyLog}\left(2,-\frac{bx}{a}\right) \\
&\quad - qr\log^3\left(i\left(j(hx)^t\right)^u\right)\text{PolyLog}\left(2,-\frac{dx}{c}\right) + 3prtu\log^2\left(i\left(j(hx)^t\right)^u\right)\text{PolyLog}\left(3,-\frac{bx}{a}\right) \\
&\quad + 3qrtu\log^2\left(i\left(j(hx)^t\right)^u\right)\text{PolyLog}\left(3,-\frac{dx}{c}\right) \\
&\quad - 6prt^2u^2\log\left(i\left(j(hx)^t\right)^u\right)\text{PolyLog}\left(4,-\frac{bx}{a}\right) \\
&\quad - 6qrt^2u^2\log\left(i\left(j(hx)^t\right)^u\right)\text{PolyLog}\left(4,-\frac{dx}{c}\right)
\end{aligned}$$

```
[In] Integrate[(Log[i*(j*(h*x)^t)^u]^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/x,x
]
```

```
[Out] p*r*t^3*u^3*Log[x]*Log[h*x]^3*Log[a + b*x] - p*r*t^3*u^3*Log[h*x]^4*Log[a +
 b*x] - 3*p*r*t^2*u^2*Log[x]*Log[h*x]^2*Log[i*(j*(h*x)^t)^u]*Log[a + b*x] +
 3*p*r*t^2*u^2*Log[h*x]^3*Log[i*(j*(h*x)^t)^u]*Log[a + b*x] + 3*p*r*t*u*Log
[x]*Log[h*x]*Log[i*(j*(h*x)^t)^u]^2*Log[a + b*x] - 3*p*r*t*u*Log[h*x]^2*Log
[i*(j*(h*x)^t)^u]^2*Log[a + b*x] - p*r*Log[x]*Log[i*(j*(h*x)^t)^u]^3*Log[a
+ b*x] + p*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]^3*Log[a + b*x] + (p*r*t^3*u^3*Lo
g[h*x]^4*Log[1 + (b*x)/a])/4 - p*r*t^2*u^2*Log[h*x]^3*Log[i*(j*(h*x)^t)^u]*
Log[1 + (b*x)/a] + (3*p*r*t*u*Log[h*x]^2*Log[i*(j*(h*x)^t)^u]^2*Log[1 + (b*
x)/a])/2 - p*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]^3*Log[1 + (b*x)/a] + q*r*t^3*u
^3*Log[x]*Log[h*x]^3*Log[c + d*x] - q*r*t^3*u^3*Log[h*x]^4*Log[c + d*x] - 3
*q*r*t^2*u^2*Log[x]*Log[h*x]^2*Log[i*(j*(h*x)^t)^u]*Log[c + d*x] + 3*q*r*t^
2*u^2*Log[h*x]^3*Log[i*(j*(h*x)^t)^u]*Log[c + d*x] + 3*q*r*t*u*Log[x]*Log[h
*x]*Log[i*(j*(h*x)^t)^u]^2*Log[c + d*x] - 3*q*r*t*u*Log[h*x]^2*Log[i*(j*(h*
x)^t)^u]^2*Log[c + d*x] - q*r*Log[x]*Log[i*(j*(h*x)^t)^u]^3*Log[c + d*x] +
q*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]^3*Log[c + d*x] - t^3*u^3*Log[x]*Log[h*x]^
3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (3*t^3*u^3*Log[h*x]^4*Log[e*(f*(a
+ b*x)^p*(c + d*x)^q)^r])/4 + 3*t^2*u^2*Log[x]*Log[h*x]^2*Log[i*(j*(h*x)^t)
^u]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 2*t^2*u^2*Log[h*x]^3*Log[i*(j*(h
*x)^t)^u]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 3*t*u*Log[x]*Log[h*x]*Log[
i*(j*(h*x)^t)^u]^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (3*t*u*Log[h*x]^2
*Log[i*(j*(h*x)^t)^u]^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/2 + Log[x]*Lo
g[i*(j*(h*x)^t)^u]^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (q*r*t^3*u^3*Lo
g[h*x]^4*Log[1 + (d*x)/c])/4 - q*r*t^2*u^2*Log[h*x]^3*Log[i*(j*(h*x)^t)^u]*
Log[1 + (d*x)/c] + (3*q*r*t*u*Log[h*x]^2*Log[i*(j*(h*x)^t)^u]^2*Log[1 + (d*
x)/c])/2 - q*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]^3*Log[1 + (d*x)/c] - p*r*Log[i
*(j*(h*x)^t)^u]^3*PolyLog[2, -((b*x)/a)] - q*r*Log[i*(j*(h*x)^t)^u]^3*PolyL
og[2, -((d*x)/c)] + 3*p*r*t*u*Log[i*(j*(h*x)^t)^u]^2*PolyLog[3, -((b*x)/a)]
 + 3*q*r*t*u*Log[i*(j*(h*x)^t)^u]^2*PolyLog[3, -((d*x)/c)] - 6*p*r*t^2*u^2*
Log[i*(j*(h*x)^t)^u]*PolyLog[4, -((b*x)/a)] - 6*q*r*t^2*u^2*Log[i*(j*(h*x)^
t)^u]*PolyLog[4, -((d*x)/c)] + 6*p*r*t^3*u^3*PolyLog[5, -((b*x)/a)] + 6*q*r
*t^3*u^3*PolyLog[5, -((d*x)/c)]
```

Maple [F]

$$\int \frac{\ln \left(i\left(j(hx)^{t}\right)^{u}\right)^{3} \ln \left(e\left(f(bx+a)^{p}\left(dx+c\right)^{q}\right)^{r}\right)}{x}dx$$

```
[In] int(ln(i*(j*(h*x)^t)^u)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x)
```

```
[Out] int(ln(i*(j*(h*x)^t)^u)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x)
```

Fricas [F]

$$\int \frac{\log^3\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx = \int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)\log\left(\left((hx)^t j\right)^u i\right)^3}{x}\,dx$$

```
[In] integrate(log(i*(j*(h*x)^t)^u)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algo
rithm="fricas")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)*log(((h*x)^t*j)^u*i)^3/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log^3\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx = \text{Timed out}$$

```
[In] integrate(ln(i*(j*(h*x)**t)**u)**3*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/x,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\log^3\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx = \int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)\log\left(\left((hx)^t j\right)^u i\right)^3}{x}\,dx$$

```
[In] integrate(log(i*(j*(h*x)^t)^u)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algo
rithm="maxima")
```

```
[Out] -1/4*(t^3*u^3*log(x)^4 - 4*(t^3*u^3*log(h) + t^2*u^3*log(j) + t^2*u^2*log(i
))*log(x)^3 - 4*log(x)*log((x^t)^u)^3 + 6*(t^3*u^3*log(h)^2 + t*u^3*log(j)^
2 + 2*t*u^2*log(i)*log(j) + t*u*log(i)^2 + 2*(t^2*u^3*log(j) + t^2*u^2*log(
i))*log(h))*log(x)^2 + 6*(t*u*log(x)^2 - 2*(t*u*log(h) + u*log(j) + log(i))
*log(x))*log((x^t)^u)^2 - 4*(t^3*u^3*log(h)^3 + u^3*log(j)^3 + 3*u^2*log(i)
*log(j)^2 + 3*u*log(i)^2*log(j) + 3*(t^2*u^3*log(j) + t^2*u^2*log(i))*log(h
)^2 + log(i)^3 + 3*(t*u^3*log(j)^2 + 2*t*u^2*log(i)*log(j) + t*u*log(i)^2)*
log(h))*log(x) - 4*(t^2*u^2*log(x)^3 - 3*(t^2*u^2*log(h) + t*u^2*log(j) + t
```

```
*u*log(i))*log(x)^2 + 3*(t^2*u^2*log(h)^2 + u^2*log(j)^2 + 2*u*log(i)*log(j
) + 2*(t*u^2*log(j) + t*u*log(i))*log(h) + log(i)^2)*log(x))*log((x^t)^u))*
log(((b*x + a)^p)^r) - 1/4*(t^3*u^3*log(x)^4 - 4*(t^3*u^3*log(h) + t^2*u^3*
log(j) + t^2*u^2*log(i))*log(x)^3 - 4*log(x)*log((x^t)^u)^3 + 6*(t^3*u^3*lo
g(h)^2 + t*u^3*log(j)^2 + 2*t*u^2*log(i)*log(j) + t*u*log(i)^2 + 2*(t^2*u^3
*log(j) + t^2*u^2*log(i))*log(h))*log(x)^2 + 6*(t*u*log(x)^2 - 2*(t*u*log(h
) + u*log(j) + log(i))*log(x))*log((x^t)^u)^2 - 4*(t^3*u^3*log(h)^3 + u^3*l
og(j)^3 + 3*u^2*log(i)*log(j)^2 + 3*u*log(i)^2*log(j) + 3*(t^2*u^3*log(j) +
 t^2*u^2*log(i))*log(h)^2 + log(i)^3 + 3*(t*u^3*log(j)^2 + 2*t*u^2*log(i)*l
og(j) + t*u*log(i)^2)*log(h))*log(x) - 4*(t^2*u^2*log(x)^3 - 3*(t^2*u^2*log
(h) + t*u^2*log(j) + t*u*log(i))*log(x)^2 + 3*(t^2*u^2*log(h)^2 + u^2*log(j
)^2 + 2*u*log(i)*log(j) + 2*(t*u^2*log(j) + t*u*log(i))*log(h) + log(i)^2)*
log(x))*log((x^t)^u))*log(((d*x + c)^q)^r) - integrate(-1/4*(4*((t^3*u^3*lo
g(h)^3 + u^3*log(j)^3 + 3*u^2*log(i)*log(j)^2 + 3*u*log(i)^2*log(j) + 3*(t^
2*u^3*log(j) + t^2*u^2*log(i))*log(h)^2 + log(i)^3 + 3*(t*u^3*log(j)^2 + 2*
t*u^2*log(i)*log(j) + t*u*log(i)^2)*log(h))*log(e) + (r*t^3*u^3*log(h)^3 +
r*u^3*log(j)^3 + 3*r*u^2*log(i)*log(j)^2 + 3*r*u*log(i)^2*log(j) + r*log(i)
^3 + 3*(r*t^2*u^3*log(j) + r*t^2*u^2*log(i))*log(h)^2 + 3*(r*t*u^3*log(j)^2
 + 2*r*t*u^2*log(i)*log(j) + r*t*u*log(i)^2)*log(h))*log(f))*b*d*x^2 + ((p*
r*t^3*u^3 + q*r*t^3*u^3)*b*d*x^2 + (b*c*p*r*t^3*u^3 + a*d*q*r*t^3*u^3)*x)*l
og(x)^4 + 4*((r*log(f) + log(e))*b*d*x^2 + (r*log(f) + log(e))*a*c + ((r*lo
g(f) + log(e))*b*c + (r*log(f) + log(e))*a*d)*x - ((p*r + q*r)*b*d*x^2 + (b
*c*p*r + a*d*q*r)*x)*log(x))*log((x^t)^u)^3 - 4*(((p*r*t^3*u^3 + q*r*t^3*u^
3)*log(h) + (p*r*t^2*u^2 + q*r*t^2*u^2)*log(i) + (p*r*t^2*u^3 + q*r*t^2*u^3
)*log(j))*b*d*x^2 + ((p*r*t^3*u^3*log(h) + p*r*t^2*u^3*log(j) + p*r*t^2*u^2
*log(i))*b*c + (q*r*t^3*u^3*log(h) + q*r*t^2*u^3*log(j) + q*r*t^2*u^2*log(i
))*a*d)*x)*log(x)^3 + 4*((t^3*u^3*log(h)^3 + u^3*log(j)^3 + 3*u^2*log(i)*lo
g(j)^2 + 3*u*log(i)^2*log(j) + 3*(t^2*u^3*log(j) + t^2*u^2*log(i))*log(h)^2
 + log(i)^3 + 3*(t*u^3*log(j)^2 + 2*t*u^2*log(i)*log(j) + t*u*log(i)^2)*log
(h))*log(e) + (r*t^3*u^3*log(h)^3 + r*u^3*log(j)^3 + 3*r*u^2*log(i)*log(j)^
2 + 3*r*u*log(i)^2*log(j) + r*log(i)^3 + 3*(r*t^2*u^3*log(j) + r*t^2*u^2*lo
g(i))*log(h)^2 + 3*(r*t*u^3*log(j)^2 + 2*r*t*u^2*log(i)*log(j) + r*t*u*log(
i)^2)*log(h))*log(f))*a*c + 6*(((p*r*t^3*u^3 + q*r*t^3*u^3)*log(h)^2 + (p*r
*t*u + q*r*t*u)*log(i)^2 + 2*(p*r*t*u^2 + q*r*t*u^2)*log(i)*log(j) + (p*r*t
*u^3 + q*r*t*u^3)*log(j)^2 + 2*((p*r*t^2*u^2 + q*r*t^2*u^2)*log(i) + (p*r*t
^2*u^3 + q*r*t^2*u^3)*log(j))*log(h))*b*d*x^2 + ((p*r*t^3*u^3*log(h)^2 + p*
r*t*u^3*log(j)^2 + 2*p*r*t*u^2*log(i)*log(j) + p*r*t*u*log(i)^2 + 2*(p*r*t^
2*u^3*log(j) + p*r*t^2*u^2*log(i))*log(h))*b*c + (q*r*t^3*u^3*log(h)^2 + q*
r*t*u^3*log(j)^2 + 2*q*r*t*u^2*log(i)*log(j) + q*r*t*u*log(i)^2 + 2*(q*r*t^
2*u^3*log(j) + q*r*t^2*u^2*log(i))*log(h))*a*d)*x)*log(x)^2 + 6*(2*((t*u*lo
g(h) + u*log(j) + log(i))*log(e) + (r*t*u*log(h) + r*u*log(j) + r*log(i))*l
og(f))*b*d*x^2 + 2*((t*u*log(h) + u*log(j) + log(i))*log(e) + (r*t*u*log(h)
 + r*u*log(j) + r*log(i))*log(f))*a*c + ((p*r*t*u + q*r*t*u)*b*d*x^2 + (b*c
*p*r*t*u + a*d*q*r*t*u)*x)*log(x)^2 + 2*(((t*u*log(h) + u*log(j) + log(i))*
log(e) + (r*t*u*log(h) + r*u*log(j) + r*log(i))*log(f))*b*c + ((t*u*log(h)
```

```
+ u*log(j) + log(i))*log(e) + (r*t*u*log(h) + r*u*log(j) + r*log(i))*log(f)
)*a*d)*x - 2*(((p*r*t*u + q*r*t*u)*log(h) + (p*r + q*r)*log(i) + (p*r*u + q
*r*u)*log(j))*b*d*x^2 + ((p*r*t*u*log(h) + p*r*u*log(j) + p*r*log(i))*b*c +
 (q*r*t*u*log(h) + q*r*u*log(j) + q*r*log(i))*a*d)*x)*log(x))*log((x^t)^u)^
2 + 4*(((t^3*u^3*log(h)^3 + u^3*log(j)^3 + 3*u^2*log(i)*log(j)^2 + 3*u*log(
i)^2*log(j) + 3*(t^2*u^3*log(j) + t^2*u^2*log(i))*log(h)^2 + log(i)^3 + 3*(
t*u^3*log(j)^2 + 2*t*u^2*log(i)*log(j) + t*u*log(i)^2)*log(h))*log(e) + (r*
t^3*u^3*log(h)^3 + r*u^3*log(j)^3 + 3*r*u^2*log(i)*log(j)^2 + 3*r*u*log(i)^
2*log(j) + r*log(i)^3 + 3*(r*t^2*u^3*log(j) + r*t^2*u^2*log(i))*log(h)^2 +
3*(r*t*u^3*log(j)^2 + 2*r*t*u^2*log(i)*log(j) + r*t*u*log(i)^2)*log(h))*log
(f))*b*c + ((t^3*u^3*log(h)^3 + u^3*log(j)^3 + 3*u^2*log(i)*log(j)^2 + 3*u*
log(i)^2*log(j) + 3*(t^2*u^3*log(j) + t^2*u^2*log(i))*log(h)^2 + log(i)^3 +
 3*(t*u^3*log(j)^2 + 2*t*u^2*log(i)*log(j) + t*u*log(i)^2)*log(h))*log(e) +
 (r*t^3*u^3*log(h)^3 + r*u^3*log(j)^3 + 3*r*u^2*log(i)*log(j)^2 + 3*r*u*log
(i)^2*log(j) + r*log(i)^3 + 3*(r*t^2*u^3*log(j) + r*t^2*u^2*log(i))*log(h)^
2 + 3*(r*t*u^3*log(j)^2 + 2*r*t*u^2*log(i)*log(j) + r*t*u*log(i)^2)*log(h))
*log(f))*a*d)*x - 4*(((p*r*t^3*u^3 + q*r*t^3*u^3)*log(h)^3 + (p*r + q*r)*lo
g(i)^3 + 3*(p*r*u + q*r*u)*log(i)^2*log(j) + 3*(p*r*u^2 + q*r*u^2)*log(i)*l
og(j)^2 + (p*r*u^3 + q*r*u^3)*log(j)^3 + 3*((p*r*t^2*u^2 + q*r*t^2*u^2)*log
(i) + (p*r*t^2*u^3 + q*r*t^2*u^3)*log(j))*log(h)^2 + 3*((p*r*t*u + q*r*t*u)
*log(i)^2 + 2*(p*r*t*u^2 + q*r*t*u^2)*log(i)*log(j) + (p*r*t*u^3 + q*r*t*u^
3)*log(j)^2)*log(h))*b*d*x^2 + ((p*r*t^3*u^3*log(h)^3 + p*r*u^3*log(j)^3 +
3*p*r*u^2*log(i)*log(j)^2 + 3*p*r*u*log(i)^2*log(j) + p*r*log(i)^3 + 3*(p*r
*t^2*u^3*log(j) + p*r*t^2*u^2*log(i))*log(h)^2 + 3*(p*r*t*u^3*log(j)^2 + 2*
p*r*t*u^2*log(i)*log(j) + p*r*t*u*log(i)^2)*log(h))*b*c + (q*r*t^3*u^3*log(
h)^3 + q*r*u^3*log(j)^3 + 3*q*r*u^2*log(i)*log(j)^2 + 3*q*r*u*log(i)^2*log(
j) + q*r*log(i)^3 + 3*(q*r*t^2*u^3*log(j) + q*r*t^2*u^2*log(i))*log(h)^2 +
3*(q*r*t*u^3*log(j)^2 + 2*q*r*t*u^2*log(i)*log(j) + q*r*t*u*log(i)^2)*log(h
))*a*d)*x)*log(x) + 4*(3*((t^2*u^2*log(h)^2 + u^2*log(j)^2 + 2*u*log(i)*log
(j) + 2*(t*u^2*log(j) + t*u*log(i))*log(h) + log(i)^2)*log(e) + (r*t^2*u^2*
log(h)^2 + r*u^2*log(j)^2 + 2*r*u*log(i)*log(j) + r*log(i)^2 + 2*(r*t*u^2*l
og(j) + r*t*u*log(i))*log(h))*log(f))*b*d*x^2 - ((p*r*t^2*u^2 + q*r*t^2*u^2
)*b*d*x^2 + (b*c*p*r*t^2*u^2 + a*d*q*r*t^2*u^2)*x)*log(x)^3 + 3*((t^2*u^2*l
og(h)^2 + u^2*log(j)^2 + 2*u*log(i)*log(j) + 2*(t*u^2*log(j) + t*u*log(i))*
log(h) + log(i)^2)*log(e) + (r*t^2*u^2*log(h)^2 + r*u^2*log(j)^2 + 2*r*u*lo
g(i)*log(j) + r*log(i)^2 + 2*(r*t*u^2*log(j) + r*t*u*log(i))*log(h))*log(f)
)*a*c + 3*(((p*r*t^2*u^2 + q*r*t^2*u^2)*log(h) + (p*r*t*u + q*r*t*u)*log(i)
 + (p*r*t*u^2 + q*r*t*u^2)*log(j))*b*d*x^2 + ((p*r*t^2*u^2*log(h) + p*r*t*u
^2*log(j) + p*r*t*u*log(i))*b*c + (q*r*t^2*u^2*log(h) + q*r*t*u^2*log(j) +
q*r*t*u*log(i))*a*d)*x)*log(x)^2 + 3*(((t^2*u^2*log(h)^2 + u^2*log(j)^2 + 2
*u*log(i)*log(j) + 2*(t*u^2*log(j) + t*u*log(i))*log(h) + log(i)^2)*log(e)
+ (r*t^2*u^2*log(h)^2 + r*u^2*log(j)^2 + 2*r*u*log(i)*log(j) + r*log(i)^2 +
 2*(r*t*u^2*log(j) + r*t*u*log(i))*log(h))*log(f))*b*c + ((t^2*u^2*log(h)^2
 + u^2*log(j)^2 + 2*u*log(i)*log(j) + 2*(t*u^2*log(j) + t*u*log(i))*log(h)
+ log(i)^2)*log(e) + (r*t^2*u^2*log(h)^2 + r*u^2*log(j)^2 + 2*r*u*log(i)*lo
```

```
g(j) + r*log(i)^2 + 2*(r*t*u^2*log(j) + r*t*u*log(i))*log(h))*log(f))*a*d)*
x - 3*(((p*r*t^2*u^2 + q*r*t^2*u^2)*log(h)^2 + (p*r + q*r)*log(i)^2 + 2*(p*
r*u + q*r*u)*log(i)*log(j) + (p*r*u^2 + q*r*u^2)*log(j)^2 + 2*((p*r*t*u + q
*r*t*u)*log(i) + (p*r*t*u^2 + q*r*t*u^2)*log(j))*log(h))*b*d*x^2 + ((p*r*t^
2*u^2*log(h)^2 + p*r*u^2*log(j)^2 + 2*p*r*u*log(i)*log(j) + p*r*log(i)^2 +
2*(p*r*t*u^2*log(j) + p*r*t*u*log(i))*log(h))*b*c + (q*r*t^2*u^2*log(h)^2 +
 q*r*u^2*log(j)^2 + 2*q*r*u*log(i)*log(j) + q*r*log(i)^2 + 2*(q*r*t*u^2*log
(j) + q*r*t*u*log(i))*log(h))*a*d)*x)*log(x))*log((x^t)^u))/(b*d*x^3 + a*c*
x + (b*c + a*d)*x^2), x)
```

Giac [F]

$$\int \frac{\log^3\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx = \int \frac{\log\left(((bx+a)^p(dx+c)^q f)^r e\right)\log\left(\left((hx)^t j\right)^u i\right)^3}{x}\,dx$$

```
[In] integrate(log(i*(j*(h*x)^t)^u)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algo
rithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)*log(((h*x)^t*j)^u*i)^3/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^3\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx = \int \frac{\ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)\,\ln\left(i\,\left(j\,(h\,x)^t\right)^u\right)^3}{x}\,dx$$

```
[In] int((log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*log(i*(j*(h*x)^t)^u)^3)/x,x)
```

```
[Out] int((log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*log(i*(j*(h*x)^t)^u)^3)/x, x)
```

3.57 $\int \frac{\log^2\left(i\left(j(hx)^t\right)^u\right)\log(e(f(a+bx)^p(c+dx)^q)^r)}{x}\,dx$

Optimal result 500
Rubi [A] (verified) 501
Mathematica [B] (verified) 504
Maple [F] 505
Fricas [F] 505
Sympy [F(-1)] 506
Maxima [F] 506
Giac [F] 507
Mupad [F(-1)] 508

Optimal result

Integrand size = 39, antiderivative size = 262

$$\begin{aligned}
&\int \frac{\log^2\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x}\,dx \\
&= -\frac{pr\log^3\left(i\left(j(hx)^t\right)^u\right)\log\left(1+\frac{bx}{a}\right)}{3tu} + \frac{\log^3\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3tu} \\
&\quad - \frac{qr\log^3\left(i\left(j(hx)^t\right)^u\right)\log\left(1+\frac{dx}{c}\right)}{3tu} - pr\log^2\left(i\left(j(hx)^t\right)^u\right)\text{PolyLog}\left(2,-\frac{bx}{a}\right) \\
&\quad - qr\log^2\left(i\left(j(hx)^t\right)^u\right)\text{PolyLog}\left(2,-\frac{dx}{c}\right) + 2prtu\log\left(i\left(j(hx)^t\right)^u\right)\text{PolyLog}\left(3,-\frac{bx}{a}\right) \\
&\quad + 2qrtu\log\left(i\left(j(hx)^t\right)^u\right)\text{PolyLog}\left(3,-\frac{dx}{c}\right) \\
&\quad - 2prt^2u^2\,\text{PolyLog}\left(4,-\frac{bx}{a}\right) - 2qrt^2u^2\,\text{PolyLog}\left(4,-\frac{dx}{c}\right)
\end{aligned}$$

```
[Out] -1/3*p*r*ln(i*(j*(h*x)^t)^u)^3*ln(1+b*x/a)/t/u+1/3*ln(i*(j*(h*x)^t)^u)^3*ln
(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/t/u-1/3*q*r*ln(i*(j*(h*x)^t)^u)^3*ln(1+d*x/c)
/t/u-p*r*ln(i*(j*(h*x)^t)^u)^2*polylog(2,-b*x/a)-q*r*ln(i*(j*(h*x)^t)^u)^2*
polylog(2,-d*x/c)+2*p*r*t*u*ln(i*(j*(h*x)^t)^u)*polylog(3,-b*x/a)+2*q*r*t*u
*ln(i*(j*(h*x)^t)^u)*polylog(3,-d*x/c)-2*p*r*t^2*u^2*polylog(4,-b*x/a)-2*q*
r*t^2*u^2*polylog(4,-d*x/c)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2585, 2354, 2421, 2430, 6724, 2495}

$$\begin{aligned}
&\int \frac{\log^2\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx\\
&=\frac{\log^3\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3tu}\\
&\quad-pr\,\text{PolyLog}\left(2,-\frac{bx}{a}\right)\log^2\left(i(j(hx)^t)^u\right)+2prtu\,\text{PolyLog}\left(3,-\frac{bx}{a}\right)\log\left(i(j(hx)^t)^u\right)\\
&\quad-\frac{pr\log\left(\frac{bx}{a}+1\right)\log^3\left(i(j(hx)^t)^u\right)}{3tu}-2prt^2u^2\,\text{PolyLog}\left(4,-\frac{bx}{a}\right)\\
&\quad-qr\,\text{PolyLog}\left(2,-\frac{dx}{c}\right)\log^2\left(i(j(hx)^t)^u\right)+2qrtu\,\text{PolyLog}\left(3,-\frac{dx}{c}\right)\log\left(i(j(hx)^t)^u\right)\\
&\quad-\frac{qr\log\left(\frac{dx}{c}+1\right)\log^3\left(i(j(hx)^t)^u\right)}{3tu}-2qrt^2u^2\,\text{PolyLog}\left(4,-\frac{dx}{c}\right)
\end{aligned}$$

```
[In] Int[(Log[i*(j*(h*x)^t)^u]^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/x,x]
```

```
[Out] -1/3*(p*r*Log[i*(j*(h*x)^t)^u]^3*Log[1 + (b*x)/a])/(t*u) + (Log[i*(j*(h*x)^
t)^u]^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(3*t*u) - (q*r*Log[i*(j*(h*x)
^t)^u]^3*Log[1 + (d*x)/c])/(3*t*u) - p*r*Log[i*(j*(h*x)^t)^u]^2*PolyLog[2,
-((b*x)/a)] - q*r*Log[i*(j*(h*x)^t)^u]^2*PolyLog[2, -((d*x)/c)] + 2*p*r*t*u
*Log[i*(j*(h*x)^t)^u]*PolyLog[3, -((b*x)/a)] + 2*q*r*t*u*Log[i*(j*(h*x)^t)^
u]*PolyLog[3, -((d*x)/c)] - 2*p*r*t^2*u^2*PolyLog[4, -((b*x)/a)] - 2*q*r*t^
2*u^2*PolyLog[4, -((d*x)/c)]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rule 2585

```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.))^(m_.))/((j_.
) + (k_.)*(x_)), x_Symbol] :> Simp[(s + t*Log[i*(g + h*x)^n])^(m + 1)*(Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(k*n*t*(m + 1))), x] + (-Dist[b*p*(r/(k*n*
t*(m + 1))), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(a + b*x), x], x] - Dis
t[d*q*(r/(k*n*t*(m + 1))), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(c + d*x)
, x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r},
x] && NeQ[b*c - a*d, 0] && EqQ[h*j - g*k, 0] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{\log^2\left(ij^u(hx)^{tu}\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x}\,dx, ij^u(hx)^{tu}, i\left(j(hx)^t\right)^u\right)$$

$$= \text{Subst}\left(\text{Subst}\left(\int \frac{\log^2\left(h^{tu}ij^ux^{tu}\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x}\,dx, h^{tu}ij^ux^{tu}, ij^u(hx)^{tu}\right), ij^u(hx)^{tu}$$

$$= \frac{\log^3\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3tu}$$
$$-\text{Subst}\left(\text{Subst}\left(\frac{(bpr)\int \frac{\log^3\left(h^{tu}ij^ux^{tu}\right)}{a+bx}\,dx}{3tu}, h^{tu}ij^ux^{tu}, ij^u(hx)^{tu}\right), ij^u(hx)^{tu}, i\left(j(hx)^t\right)^u\right)$$
$$-\text{Subst}\left(\text{Subst}\left(\frac{(dqr)\int \frac{\log^3\left(h^{tu}ij^ux^{tu}\right)}{c+dx}\,dx}{3tu}, h^{tu}ij^ux^{tu}, ij^u(hx)^{tu}\right), ij^u(hx)^{tu}, i\left(j(hx)^t\right)^u\right)$$

$$= -\frac{pr\log^3\left(i(j(hx)^t)^u\right)\log\left(1+\frac{bx}{a}\right)}{3tu} + \frac{\log^3\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3tu} - \frac{qr\log^3\left(i(j(hx)^t)^u\right)\log\left(1+\frac{dx}{c}\right)}{3tu} + \text{Subst}\left(\text{Subst}\left((pr)\int\frac{\log^2\left(h^{tu}ij^ux^{tu}\right)\log\left(1+\frac{bx}{a}\right)}{x}\,dx, h^{tu}ij^ux^{tu}, ij^u(hx)^{tu}\right), ij^u(hx)^{tu}, i\left(j(hx\right.\right.$$

$$+\text{Subst}\left(\text{Subst}\left((qr)\int\frac{\log^2\left(h^{tu}ij^ux^{tu}\right)\log\left(1+\frac{dx}{c}\right)}{x}\,dx, h^{tu}ij^ux^{tu}, ij^u(hx)^{tu}\right), ij^u(hx)^{tu}, i\left(j(hx\right.\right.$$

$$= -\frac{pr\log^3\left(i(j(hx)^t)^u\right)\log\left(1+\frac{bx}{a}\right)}{3tu} + \frac{\log^3\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3tu} - \frac{qr\log^3\left(i(j(hx)^t)^u\right)\log\left(1+\frac{dx}{c}\right)}{3tu} - pr\log^2\left(i\left(j(hx)^t\right)^u\right)\text{Li}_2\left(-\frac{bx}{a}\right) - qr\log^2\left(i\left(j(hx)^t\right)^u\right)\text{Li}_2\left(-\frac{dx}{c}\right) + \text{Subst}\left(\text{Subst}\left((2prtu)\int\frac{\log\left(h^{tu}ij^ux^{tu}\right)\text{Li}_2\left(-\frac{bx}{a}\right)}{x}\,dx, h^{tu}ij^ux^{tu}, ij^u(hx)^{tu}\right), ij^u(hx)^{tu}, i\left(j(hx\right.\right.$$

$$+\text{Subst}\left(\text{Subst}\left((2qrtu)\int\frac{\log\left(h^{tu}ij^ux^{tu}\right)\text{Li}_2\left(-\frac{dx}{c}\right)}{x}\,dx, h^{tu}ij^ux^{tu}, ij^u(hx)^{tu}\right), ij^u(hx)^{tu}, i\left(j(hx\right.\right.$$

$$= -\frac{pr\log^3\left(i(j(hx)^t)^u\right)\log\left(1+\frac{bx}{a}\right)}{3tu} + \frac{\log^3\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3tu} - \frac{qr\log^3\left(i(j(hx)^t)^u\right)\log\left(1+\frac{dx}{c}\right)}{3tu} - pr\log^2\left(i\left(j(hx)^t\right)^u\right)\text{Li}_2\left(-\frac{bx}{a}\right) - qr\log^2\left(i\left(j(hx)^t\right)^u\right)\text{Li}_2\left(-\frac{dx}{c}\right) + 2prtu\log\left(i\left(j(hx)^t\right)^u\right)\text{Li}_3\left(-\frac{bx}{a}\right) + 2qrtu\log\left(i\left(j(hx)^t\right)^u\right)\text{Li}_3\left(-\frac{dx}{c}\right) - \text{Subst}\left(\text{Subst}\left(\left(2prt^2u^2\right)\int\frac{\text{Li}_3\left(-\frac{bx}{a}\right)}{x}\,dx, h^{tu}ij^ux^{tu}, ij^u(hx)^{tu}\right), ij^u(hx)^{tu}, i\left(j(hx)^t\right)^u\right) - \text{Subst}\left(\text{Subst}\left(\left(2qrt^2u^2\right)\int\frac{\text{Li}_3\left(-\frac{dx}{c}\right)}{x}\,dx, h^{tu}ij^ux^{tu}, ij^u(hx)^{tu}\right), ij^u(hx)^{tu}, i\left(j(hx)^t\right)^u\right)$$

$$= -\frac{pr\log^3\left(i(j(hx)^t)^u\right)\log\left(1+\frac{bx}{a}\right)}{3tu} + \frac{\log^3\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3tu} - \frac{qr\log^3\left(i(j(hx)^t)^u\right)\log\left(1+\frac{dx}{c}\right)}{3tu} - pr\log^2\left(i\left(j(hx)^t\right)^u\right)\text{Li}_2\left(-\frac{bx}{a}\right) - qr\log^2\left(i\left(j(hx)^t\right)^u\right)\text{Li}_2\left(-\frac{dx}{c}\right) + 2prtu\log\left(i\left(j(hx)^t\right)^u\right)\text{Li}_3\left(-\frac{bx}{a}\right) + 2qrtu\log\left(i\left(j(hx)^t\right)^u\right)\text{Li}_3\left(-\frac{dx}{c}\right) - 2prt^2u^2\text{Li}_4\left(-\frac{bx}{a}\right) - 2qrt^2u^2\text{Li}_4\left(-\frac{dx}{c}\right)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 839 vs. $2(262) = 524$.

Time = 0.55 (sec) , antiderivative size = 839, normalized size of antiderivative = 3.20

$$
\begin{aligned}
&\int \frac{\log^2\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx \\
&= -prt^2u^2\log(x)\log^2(hx)\log(a+bx)+prt^2u^2\log^3(hx)\log(a+bx) \\
&\quad +2prtu\log(x)\log(hx)\log\left(i\left(j(hx)^t\right)^u\right)\log(a+bx) \\
&\quad -2prtu\log^2(hx)\log\left(i\left(j(hx)^t\right)^u\right)\log(a+bx)-pr\log(x)\log^2\left(i\left(j(hx)^t\right)^u\right)\log(a+bx) \\
&\quad +pr\log(hx)\log^2\left(i\left(j(hx)^t\right)^u\right)\log(a+bx)-\frac{1}{3}prt^2u^2\log^3(hx)\log\left(1+\frac{bx}{a}\right) \\
&\quad +prtu\log^2(hx)\log\left(i\left(j(hx)^t\right)^u\right)\log\left(1+\frac{bx}{a}\right) \\
&\quad -pr\log(hx)\log^2\left(i\left(j(hx)^t\right)^u\right)\log\left(1+\frac{bx}{a}\right)-qrt^2u^2\log(x)\log^2(hx)\log(c+dx) \\
&\quad +qrt^2u^2\log^3(hx)\log(c+dx)+2qrtu\log(x)\log(hx)\log\left(i\left(j(hx)^t\right)^u\right)\log(c+dx) \\
&\quad -2qrtu\log^2(hx)\log\left(i\left(j(hx)^t\right)^u\right)\log(c+dx) \\
&\quad -qr\log(x)\log^2\left(i\left(j(hx)^t\right)^u\right)\log(c+dx)+qr\log(hx)\log^2\left(i\left(j(hx)^t\right)^u\right)\log(c+dx) \\
&\quad +t^2u^2\log(x)\log^2(hx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right) \\
&\quad -\frac{2}{3}t^2u^2\log^3(hx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right) \\
&\quad -2tu\log(x)\log(hx)\log\left(i\left(j(hx)^t\right)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right) \\
&\quad +tu\log^2(hx)\log\left(i\left(j(hx)^t\right)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right) \\
&\quad +\log(x)\log^2\left(i\left(j(hx)^t\right)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right) \\
&\quad -\frac{1}{3}qrt^2u^2\log^3(hx)\log\left(1+\frac{dx}{c}\right)+qrtu\log^2(hx)\log\left(i\left(j(hx)^t\right)^u\right)\log\left(1+\frac{dx}{c}\right) \\
&\quad -qr\log(hx)\log^2\left(i\left(j(hx)^t\right)^u\right)\log\left(1+\frac{dx}{c}\right)-pr\log^2\left(i\left(j(hx)^t\right)^u\right)\operatorname{PolyLog}\left(2,-\frac{bx}{a}\right) \\
&\quad -qr\log^2\left(i\left(j(hx)^t\right)^u\right)\operatorname{PolyLog}\left(2,-\frac{dx}{c}\right)+2prtu\log\left(i\left(j(hx)^t\right)^u\right)\operatorname{PolyLog}\left(3,-\frac{bx}{a}\right) \\
&\quad +2qrtu\log\left(i\left(j(hx)^t\right)^u\right)\operatorname{PolyLog}\left(3,-\frac{dx}{c}\right) \\
&\quad -2prt^2u^2\operatorname{PolyLog}\left(4,-\frac{bx}{a}\right)-2qrt^2u^2\operatorname{PolyLog}\left(4,-\frac{dx}{c}\right)
\end{aligned}
$$

```
[In] Integrate[(Log[i*(j*(h*x)^t)^u]^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/x,x
]
```

```
[Out] -(p*r*t^2*u^2*Log[x]*Log[h*x]^2*Log[a + b*x]) + p*r*t^2*u^2*Log[h*x]^3*Log[
a + b*x] + 2*p*r*t*u*Log[x]*Log[h*x]*Log[i*(j*(h*x)^t)^u]*Log[a + b*x] - 2*
```

```
p*r*t*u*Log[h*x]^2*Log[i*(j*(h*x)^t)^u]*Log[a + b*x] - p*r*Log[x]*Log[i*(j*
(h*x)^t)^u]^2*Log[a + b*x] + p*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]^2*Log[a + b*
x] - (p*r*t^2*u^2*Log[h*x]^3*Log[1 + (b*x)/a])/3 + p*r*t*u*Log[h*x]^2*Log[i
*(j*(h*x)^t)^u]*Log[1 + (b*x)/a] - p*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]^2*Log[
1 + (b*x)/a] - q*r*t^2*u^2*Log[x]*Log[h*x]^2*Log[c + d*x] + q*r*t^2*u^2*Log
[h*x]^3*Log[c + d*x] + 2*q*r*t*u*Log[x]*Log[h*x]*Log[i*(j*(h*x)^t)^u]*Log[c
 + d*x] - 2*q*r*t*u*Log[h*x]^2*Log[i*(j*(h*x)^t)^u]*Log[c + d*x] - q*r*Log[
x]*Log[i*(j*(h*x)^t)^u]^2*Log[c + d*x] + q*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]^
2*Log[c + d*x] + t^2*u^2*Log[x]*Log[h*x]^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q
)^r] - (2*t^2*u^2*Log[h*x]^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/3 - 2*t*
u*Log[x]*Log[h*x]*Log[i*(j*(h*x)^t)^u]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
 + t*u*Log[h*x]^2*Log[i*(j*(h*x)^t)^u]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
 + Log[x]*Log[i*(j*(h*x)^t)^u]^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - (q*
r*t^2*u^2*Log[h*x]^3*Log[1 + (d*x)/c])/3 + q*r*t*u*Log[h*x]^2*Log[i*(j*(h*x
)^t)^u]*Log[1 + (d*x)/c] - q*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]^2*Log[1 + (d*x
)/c] - p*r*Log[i*(j*(h*x)^t)^u]^2*PolyLog[2, -((b*x)/a)] - q*r*Log[i*(j*(h*
x)^t)^u]^2*PolyLog[2, -((d*x)/c)] + 2*p*r*t*u*Log[i*(j*(h*x)^t)^u]*PolyLog[
3, -((b*x)/a)] + 2*q*r*t*u*Log[i*(j*(h*x)^t)^u]*PolyLog[3, -((d*x)/c)] - 2*
p*r*t^2*u^2*PolyLog[4, -((b*x)/a)] - 2*q*r*t^2*u^2*PolyLog[4, -((d*x)/c)]
```

Maple [F]

$$\int \frac{\ln\left(i\left(j\,(hx)^{t}\right)^{u}\right)^{2} \ln\left(e\left(f\,(bx+a)^{p}\,(dx+c)^{q}\right)^{r}\right)}{x}dx$$

```
[In] int(ln(i*(j*(h*x)^t)^u)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x)
[Out] int(ln(i*(j*(h*x)^t)^u)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x)
```

Fricas [F]

$$\int \frac{\log^2\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx = \int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)\log\left(\left((hx)^t j\right)^u i\right)^2}{x}\,dx$$

```
[In] integrate(log(i*(j*(h*x)^t)^u)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algo
rithm="fricas")
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)*log(((h*x)^t*j)^u*i)^2/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log^2\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx = \text{Timed out}$$

```
[In] integrate(ln(i*(j*(h*x)**t)**u)**2*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/x,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\log^2\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx = \int \frac{\log\left(((bx+a)^p(dx+c)^q f)^r e\right)\log\left(((hx)^t j)^u i\right)^2}{x}\,dx$$

```
[In] integrate(log(i*(j*(h*x)^t)^u)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algo
rithm="maxima")
[Out] 1/3*(t^2*u^2*log(x)^3 - 3*(t^2*u^2*log(h) + t*u^2*log(j) + t*u*log(i))*log(
x)^2 + 3*log(x)*log((x^t)^u)^2 + 3*(t^2*u^2*log(h)^2 + u^2*log(j)^2 + 2*u*l
og(i)*log(j) + 2*(t*u^2*log(j) + t*u*log(i))*log(h) + log(i)^2)*log(x) - 3*
(t*u*log(x)^2 - 2*(t*u*log(h) + u*log(j) + log(i))*log(x))*log((x^t)^u))*lo
g(((b*x + a)^p)^r) + 1/3*(t^2*u^2*log(x)^3 - 3*(t^2*u^2*log(h) + t*u^2*log(
j) + t*u*log(i))*log(x)^2 + 3*log(x)*log((x^t)^u)^2 + 3*(t^2*u^2*log(h)^2 +
 u^2*log(j)^2 + 2*u*log(i)*log(j) + 2*(t*u^2*log(j) + t*u*log(i))*log(h) +
log(i)^2)*log(x) - 3*(t*u*log(x)^2 - 2*(t*u*log(h) + u*log(j) + log(i))*log
(x))*log((x^t)^u))*log(((d*x + c)^q)^r) - integrate(-1/3*(3*((t^2*u^2*log(h
)^2 + u^2*log(j)^2 + 2*u*log(i)*log(j) + 2*(t*u^2*log(j) + t*u*log(i))*log(
h) + log(i)^2)*log(e) + (r*t^2*u^2*log(h)^2 + r*u^2*log(j)^2 + 2*r*u*log(i)
*log(j) + r*log(i)^2 + 2*(r*t*u^2*log(j) + r*t*u*log(i))*log(h))*log(f))*b*
d*x^2 - ((p*r*t^2*u^2 + q*r*t^2*u^2)*b*d*x^2 + (b*c*p*r*t^2*u^2 + a*d*q*r*t
^2*u^2)*x)*log(x)^3 + 3*((t^2*u^2*log(h)^2 + u^2*log(j)^2 + 2*u*log(i)*log(
j) + 2*(t*u^2*log(j) + t*u*log(i))*log(h) + log(i)^2)*log(e) + (r*t^2*u^2*l
og(h)^2 + r*u^2*log(j)^2 + 2*r*u*log(i)*log(j) + r*log(i)^2 + 2*(r*t*u^2*lo
g(j) + r*t*u*log(i))*log(h))*log(f))*a*c + 3*((r*log(f) + log(e))*b*d*x^2 +
 (r*log(f) + log(e))*a*c + ((r*log(f) + log(e))*b*c + (r*log(f) + log(e))*a
*d)*x - ((p*r + q*r)*b*d*x^2 + (b*c*p*r + a*d*q*r)*x)*log(x))*log((x^t)^u)^
2 + 3*(((p*r*t^2*u^2 + q*r*t^2*u^2)*log(h) + (p*r*t*u + q*r*t*u)*log(i) + (
p*r*t*u^2 + q*r*t*u^2)*log(j))*b*d*x^2 + ((p*r*t^2*u^2*log(h) + p*r*t*u^2*l
og(j) + p*r*t*u*log(i))*b*c + (q*r*t^2*u^2*log(h) + q*r*t*u^2*log(j) + q*r*
```

```
t*u*log(i))*a*d)*x)*log(x)^2 + 3*(((t^2*u^2*log(h)^2 + u^2*log(j)^2 + 2*u*l
og(i)*log(j) + 2*(t*u^2*log(j) + t*u*log(i))*log(h) + log(i)^2)*log(e) + (r
*t^2*u^2*log(h)^2 + r*u^2*log(j)^2 + 2*r*u*log(i)*log(j) + r*log(i)^2 + 2*(
r*t*u^2*log(j) + r*t*u*log(i))*log(h))*log(f))*b*c + ((t^2*u^2*log(h)^2 + u
^2*log(j)^2 + 2*u*log(i)*log(j) + 2*(t*u^2*log(j) + t*u*log(i))*log(h) + lo
g(i)^2)*log(e) + (r*t^2*u^2*log(h)^2 + r*u^2*log(j)^2 + 2*r*u*log(i)*log(j)
 + r*log(i)^2 + 2*(r*t*u^2*log(j) + r*t*u*log(i))*log(h))*log(f))*a*d)*x -
3*(((p*r*t^2*u^2 + q*r*t^2*u^2)*log(h)^2 + (p*r + q*r)*log(i)^2 + 2*(p*r*u
+ q*r*u)*log(i)*log(j) + (p*r*u^2 + q*r*u^2)*log(j)^2 + 2*((p*r*t*u + q*r*t
*u)*log(i) + (p*r*t*u^2 + q*r*t*u^2)*log(j))*log(h))*b*d*x^2 + ((p*r*t^2*u^
2*log(h)^2 + p*r*u^2*log(j)^2 + 2*p*r*u*log(i)*log(j) + p*r*log(i)^2 + 2*(p
*r*t*u^2*log(j) + p*r*t*u*log(i))*log(h))*b*c + (q*r*t^2*u^2*log(h)^2 + q*r
*u^2*log(j)^2 + 2*q*r*u*log(i)*log(j) + q*r*log(i)^2 + 2*(q*r*t*u^2*log(j)
+ q*r*t*u*log(i))*log(h))*a*d)*x)*log(x) + 3*(2*((t*u*log(h) + u*log(j) + l
og(i))*log(e) + (r*t*u*log(h) + r*u*log(j) + r*log(i))*log(f))*b*d*x^2 + 2*
((t*u*log(h) + u*log(j) + log(i))*log(e) + (r*t*u*log(h) + r*u*log(j) + r*l
og(i))*log(f))*a*c + ((p*r*t*u + q*r*t*u)*b*d*x^2 + (b*c*p*r*t*u + a*d*q*r*
t*u)*x)*log(x)^2 + 2*(((t*u*log(h) + u*log(j) + log(i))*log(e) + (r*t*u*log
(h) + r*u*log(j) + r*log(i))*log(f))*b*c + ((t*u*log(h) + u*log(j) + log(i)
)*log(e) + (r*t*u*log(h) + r*u*log(j) + r*log(i))*log(f))*a*d)*x - 2*(((p*r
*t*u + q*r*t*u)*log(h) + (p*r + q*r)*log(i) + (p*r*u + q*r*u)*log(j))*b*d*x
^2 + ((p*r*t*u*log(h) + p*r*u*log(j) + p*r*log(i))*b*c + (q*r*t*u*log(h) +
q*r*u*log(j) + q*r*log(i))*a*d)*x)*log(x))*log((x^t)^u))/(b*d*x^3 + a*c*x +
 (b*c + a*d)*x^2), x)
```

Giac [F]

$$\int \frac{\log^2\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx = \int \frac{\log\left(((bx+a)^p(dx+c)^qf)^r e\right)\log\left(\left((hx)^t j\right)^u i\right)^2}{x}\,dx$$

```
[In] integrate(log(i*(j*(h*x)^t)^u)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algo
rithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)*log(((h*x)^t*j)^u*i)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx$$
$$=\int \frac{\ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)\,\ln\left(i\,\left(j\,(h\,x)^t\right)^u\right)^2}{x}\,dx$$

```
[In] int((log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*log(i*(j*(h*x)^t)^u)^2)/x,x)
[Out] int((log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*log(i*(j*(h*x)^t)^u)^2)/x, x)
```

3.58 $\int \frac{\log\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x}\,dx$

Optimal result 509
Rubi [A] (verified) 509
Mathematica [B] (verified) 512
Maple [F] 513
Fricas [F] 513
Sympy [F(-1)] 514
Maxima [F] 514
Giac [F] 515
Mupad [F(-1)] 515

Optimal result

Integrand size = 37, antiderivative size = 194

$$\begin{aligned}\int \frac{\log\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx &= -\frac{pr\log^2\left(i(j(hx)^t)^u\right)\log\left(1+\frac{bx}{a}\right)}{2tu}+\frac{\log^2\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2tu}\\&\quad-\frac{qr\log^2\left(i(j(hx)^t)^u\right)\log\left(1+\frac{dx}{c}\right)}{2tu}-pr\log\left(i\left(j(hx)^t\right)^u\right)\operatorname{PolyLog}\left(2,-\frac{bx}{a}\right)\\&\quad-qr\log\left(i\left(j(hx)^t\right)^u\right)\operatorname{PolyLog}\left(2,-\frac{dx}{c}\right)\\&\quad+prtu\operatorname{PolyLog}\left(3,-\frac{bx}{a}\right)+qrtu\operatorname{PolyLog}\left(3,-\frac{dx}{c}\right)\end{aligned}$$

```
[Out] -1/2*p*r*ln(i*(j*(h*x)^t)^u)^2*ln(1+b*x/a)/t/u+1/2*ln(i*(j*(h*x)^t)^u)^2*ln
(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/t/u-1/2*q*r*ln(i*(j*(h*x)^t)^u)^2*ln(1+d*x/c)
/t/u-p*r*ln(i*(j*(h*x)^t)^u)*polylog(2,-b*x/a)-q*r*ln(i*(j*(h*x)^t)^u)*poly
log(2,-d*x/c)+p*r*t*u*polylog(3,-b*x/a)+q*r*t*u*polylog(3,-d*x/c)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used

$= \{2585, 2354, 2421, 6724, 2495\}$

$$\int \frac{\log\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx$$
$$= \frac{\log^2\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2tu}$$
$$- pr\,\text{PolyLog}\left(2,-\frac{bx}{a}\right)\log\left(i\left(j(hx)^t\right)^u\right) - \frac{pr\log\left(\frac{bx}{a}+1\right)\log^2\left(i(j(hx)^t)^u\right)}{2tu}$$
$$+ prtu\,\text{PolyLog}\left(3,-\frac{bx}{a}\right) - qr\,\text{PolyLog}\left(2,-\frac{dx}{c}\right)\log\left(i\left(j(hx)^t\right)^u\right)$$
$$- \frac{qr\log\left(\frac{dx}{c}+1\right)\log^2\left(i(j(hx)^t)^u\right)}{2tu} + qrtu\,\text{PolyLog}\left(3,-\frac{dx}{c}\right)$$

```
[In] Int[(Log[i*(j*(h*x)^t)^u]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/x,x]
```

```
[Out] -1/2*(p*r*Log[i*(j*(h*x)^t)^u]^2*Log[1 + (b*x)/a])/(t*u) + (Log[i*(j*(h*x)^
t)^u]^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*t*u) - (q*r*Log[i*(j*(h*x)
^t)^u]^2*Log[1 + (d*x)/c])/(2*t*u) - p*r*Log[i*(j*(h*x)^t)^u]*PolyLog[2, -(
(b*x)/a)] - q*r*Log[i*(j*(h*x)^t)^u]*PolyLog[2, -((d*x)/c)] + p*r*t*u*PolyL
og[3, -((b*x)/a)] + q*r*t*u*PolyLog[3, -((d*x)/c)]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rule 2585

```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.))^(m_.))/((j_.
) + (k_.)*(x_)), x_Symbol] :> Simp[(s + t*Log[i*(g + h*x)^n])^(m + 1)*(Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(k*n*t*(m + 1))), x] + (-Dist[b*p*(r/(k*n*
t*(m + 1))), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(a + b*x), x], x] - Dis
t[d*q*(r/(k*n*t*(m + 1))), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(c + d*x)
, x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r},
x] && NeQ[b*c - a*d, 0] && EqQ[h*j - g*k, 0] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{\log\left(ij^u(hx)^{tu}\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx, ij^u(hx)^{tu}, i\left(j(hx)^t\right)^u\right)$$

$$= \text{Subst}\left(\text{Subst}\left(\int \frac{\log\left(h^{tu}ij^ux^{tu}\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx, h^{tu}ij^ux^{tu}, ij^u(hx)^{tu}\right), ij^u(hx)^{t}$$

$$= \frac{\log^2\left(i\left(j(hx)^t\right)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2tu}$$
$$-\text{Subst}\left(\text{Subst}\left(\frac{(bpr)\int \frac{\log^2\left(h^{tu}ij^ux^{tu}\right)}{a+bx}\,dx}{2tu}, h^{tu}ij^ux^{tu}, ij^u(hx)^{tu}\right), ij^u(hx)^{tu}, i\left(j(hx)^t\right)^u\right)$$
$$-\text{Subst}\left(\text{Subst}\left(\frac{(dqr)\int \frac{\log^2\left(h^{tu}ij^ux^{tu}\right)}{c+dx}\,dx}{2tu}, h^{tu}ij^ux^{tu}, ij^u(hx)^{tu}\right), ij^u(hx)^{tu}, i\left(j(hx)^t\right)^u\right)$$

$$= -\frac{pr\log^2\left(i\left(j(hx)^t\right)^u\right)\log\left(1+\frac{bx}{a}\right)}{2tu}$$
$$+\frac{\log^2\left(i\left(j(hx)^t\right)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2tu} - \frac{qr\log^2\left(i\left(j(hx)^t\right)^u\right)\log\left(1+\frac{dx}{c}\right)}{2tu}$$
$$+\text{Subst}\left(\text{Subst}\left((pr)\int \frac{\log\left(h^{tu}ij^ux^{tu}\right)\log\left(1+\frac{bx}{a}\right)}{x}\,dx, h^{tu}ij^ux^{tu}, ij^u(hx)^{tu}\right), ij^u(hx)^{tu}, i\left(j(hx)^t\right.$$
$$+\text{Subst}\left(\text{Subst}\left((qr)\int \frac{\log\left(h^{tu}ij^ux^{tu}\right)\log\left(1+\frac{dx}{c}\right)}{x}\,dx, h^{tu}ij^ux^{tu}, ij^u(hx)^{tu}\right), ij^u(hx)^{tu}, i\left(j(hx)^t\right.$$

$$= -\frac{pr\log^2\left(i(j(hx)^t)^u\right)\log\left(1+\frac{bx}{a}\right)}{2tu} + \frac{\log^2\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2tu} - \frac{qr\log^2\left(i(j(hx)^t)^u\right)\log\left(1+\frac{dx}{c}\right)}{2tu} - pr\log\left(i\left(j(hx)^t\right)^u\right)\text{Li}_2\left(-\frac{bx}{a}\right) - qr\log\left(i\left(j(hx)^t\right)^u\right)\text{Li}_2\left(-\frac{dx}{c}\right) + \text{Subst}\left(\text{Subst}\left((prtu)\int\frac{\text{Li}_2\left(-\frac{bx}{a}\right)}{x}\,dx, h^{tu}ij^ux^{tu}, ij^u(hx)^{tu}\right), ij^u(hx)^{tu}, i\left(j(hx)^t\right)^u\right) + \text{Subst}\left(\text{Subst}\left((qrtu)\int\frac{\text{Li}_2\left(-\frac{dx}{c}\right)}{x}\,dx, h^{tu}ij^ux^{tu}, ij^u(hx)^{tu}\right), ij^u(hx)^{tu}, i\left(j(hx)^t\right)^u\right)$$

$$= -\frac{pr\log^2\left(i(j(hx)^t)^u\right)\log\left(1+\frac{bx}{a}\right)}{2tu} + \frac{\log^2\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2tu} - \frac{qr\log^2\left(i(j(hx)^t)^u\right)\log\left(1+\frac{dx}{c}\right)}{2tu} - pr\log\left(i\left(j(hx)^t\right)^u\right)\text{Li}_2\left(-\frac{bx}{a}\right) - qr\log\left(i\left(j(hx)^t\right)^u\right)\text{Li}_2\left(-\frac{dx}{c}\right) + prtu\text{Li}_3\left(-\frac{bx}{a}\right) + qrtu\text{Li}_3\left(-\frac{dx}{c}\right)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 451 vs. $2(194) = 388$.

Time = 0.27 (sec) , antiderivative size = 451, normalized size of antiderivative = 2.32

$$\int \frac{\log\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx$$

$$= prtu\log(x)\log(hx)\log(a+bx) - prtu\log^2(hx)\log(a+bx) - pr\log(x)\log\left(i\left(j(hx)^t\right)^u\right)\log(a+bx) + pr\log(hx)\log\left(i\left(j(hx)^t\right)^u\right)\log(a+bx) + \frac{1}{2}prtu\log^2(hx)\log\left(1+\frac{bx}{a}\right) - pr\log(hx)\log\left(i\left(j(hx)^t\right)^u\right)\log\left(1+\frac{bx}{a}\right) + qrtu\log(x)\log(hx)\log(c+dx) - qrtu\log^2(hx)\log(c+dx) - qr\log(x)\log\left(i\left(j(hx)^t\right)^u\right)\log(c+dx) + qr\log(hx)\log\left(i\left(j(hx)^t\right)^u\right)\log(c+dx) - tu\log(x)\log(hx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right) + \frac{1}{2}tu\log^2(hx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right) + \log(x)\log\left(i\left(j(hx)^t\right)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right) + \frac{1}{2}qrtu\log^2(hx)\log\left(1+\frac{dx}{c}\right) - qr\log(hx)\log\left(i\left(j(hx)^t\right)^u\right)\log\left(1+\frac{dx}{c}\right) - pr\log\left(i\left(j(hx)^t\right)^u\right)\text{PolyLog}\left(2,-\frac{bx}{a}\right) - qr\log\left(i\left(j(hx)^t\right)^u\right)\text{PolyLog}\left(2,-\frac{dx}{c}\right) + prtu\,\text{PolyLog}\left(3,-\frac{bx}{a}\right) + qrtu\,\text{PolyLog}\left(3,-\frac{dx}{c}\right)$$

[In] Integrate[(Log[i*(j*(h*x)^t)^u]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/x,x]

[Out] p*r*t*u*Log[x]*Log[h*x]*Log[a + b*x] - p*r*t*u*Log[h*x]^2*Log[a + b*x] - p*r*Log[x]*Log[i*(j*(h*x)^t)^u]*Log[a + b*x] + p*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]*Log[a + b*x] + (p*r*t*u*Log[h*x]^2*Log[1 + (b*x)/a])/2 - p*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]*Log[1 + (b*x)/a] + q*r*t*u*Log[x]*Log[h*x]*Log[c + d*x] - q*r*t*u*Log[h*x]^2*Log[c + d*x] - q*r*Log[x]*Log[i*(j*(h*x)^t)^u]*Log[c + d*x] + q*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]*Log[c + d*x] - t*u*Log[x]*Log[h*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (t*u*Log[h*x]^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/2 + Log[x]*Log[i*(j*(h*x)^t)^u]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (q*r*t*u*Log[h*x]^2*Log[1 + (d*x)/c])/2 - q*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]*Log[1 + (d*x)/c] - p*r*Log[i*(j*(h*x)^t)^u]*PolyLog[2, -((b*x)/a)] - q*r*Log[i*(j*(h*x)^t)^u]*PolyLog[2, -((d*x)/c)] + p*r*t*u*PolyLog[3, -((b*x)/a)] + q*r*t*u*PolyLog[3, -((d*x)/c)]

Maple [F]

$$\int \frac{\ln \left(i\left(j(hx)^t\right)^u\right) \ln \left(e\left(f(bx+a)^p (dx+c)^q\right)^r\right)}{x}dx$$

[In] int(ln(i*(j*(h*x)^t)^u)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x)

[Out] int(ln(i*(j*(h*x)^t)^u)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x)

Fricas [F]

$$\int \frac{\log \left(i(j(hx)^t)^u\right) \log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x} \, dx = \int \frac{\log \left(\left((bx+a)^p(dx+c)^q f\right)^r e\right) \log \left(\left((hx)^t j\right)^u i\right)}{x} \, dx$$

[In] integrate(log(i*(j*(h*x)^t)^u)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algorithm="fricas")

[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)*log(((h*x)^t*j)^u*i)/x, x)

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx = \text{Timed out}$$

```
[In] integrate(ln(i*(j*(h*x)**t)**u)*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/x,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\log\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx$$
$$= \int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)\log\left(\left((hx)^t j\right)^u i\right)}{x}\,dx$$

```
[In] integrate(log(i*(j*(h*x)^t)^u)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algori
thm="maxima")
```

```
[Out] -1/2*(t*u*log(x)^2 - 2*(t*u*log(h) + u*log(j) + log(i))*log(x) - 2*log(x)*l
og((x^t)^u))*log(((b*x + a)^p)^r) - 1/2*(t*u*log(x)^2 - 2*(t*u*log(h) + u*l
og(j) + log(i))*log(x) - 2*log(x)*log((x^t)^u))*log(((d*x + c)^q)^r) - inte
grate(-1/2*(2*((t*u*log(h) + u*log(j) + log(i))*log(e) + (r*t*u*log(h) + r*
u*log(j) + r*log(i))*log(f))*b*d*x^2 + 2*((t*u*log(h) + u*log(j) + log(i))*
log(e) + (r*t*u*log(h) + r*u*log(j) + r*log(i))*log(f))*a*c + ((p*r*t*u + q
*r*t*u)*b*d*x^2 + (b*c*p*r*t*u + a*d*q*r*t*u)*x)*log(x)^2 + 2*(((t*u*log(h)
 + u*log(j) + log(i))*log(e) + (r*t*u*log(h) + r*u*log(j) + r*log(i))*log(f
))*b*c + ((t*u*log(h) + u*log(j) + log(i))*log(e) + (r*t*u*log(h) + r*u*log
(j) + r*log(i))*log(f))*a*d)*x + 2*((r*log(f) + log(e))*b*d*x^2 + (r*log(f)
 + log(e))*a*c + ((r*log(f) + log(e))*b*c + (r*log(f) + log(e))*a*d)*x - ((
p*r + q*r)*b*d*x^2 + (b*c*p*r + a*d*q*r)*x)*log(x))*log((x^t)^u) - 2*(((p*r
*t*u + q*r*t*u)*log(h) + (p*r + q*r)*log(i) + (p*r*u + q*r*u)*log(j))*b*d*x
^2 + ((p*r*t*u*log(h) + p*r*u*log(j) + p*r*log(i))*b*c + (q*r*t*u*log(h) +
q*r*u*log(j) + q*r*log(i))*a*d)*x)*log(x))/(b*d*x^3 + a*c*x + (b*c + a*d)*x
^2), x)
```

Giac [F]

$$\int \frac{\log\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx = \int \frac{\log\left(((bx+a)^p(dx+c)^q f)^r e\right)\log\left(\left((hx)^t j\right)^u i\right)}{x}\,dx$$

```
[In] integrate(log(i*(j*(h*x)^t)^u)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algori
thm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)*log(((h*x)^t*j)^u*i)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx = \int \frac{\ln\left(e\,\left(f\,{\left(a+b\,x\right)}^p\,{\left(c+d\,x\right)}^q\right)^r\right)\,\ln\left(i\,{\left(j\,{\left(h\,x\right)}^t\right)}^u\right)}{x}\,dx$$

```
[In] int((log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*log(i*(j*(h*x)^t)^u))/x,x)
```

```
[Out] int((log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*log(i*(j*(h*x)^t)^u))/x, x)
```

3.59 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{x}\,dx$

Optimal result 516
Rubi [A] (verified) 516
Mathematica [A] (verified) 518
Maple [A] (verified) 518
Fricas [F] 518
Sympy [F] 519
Maxima [A] (verification not implemented) 519
Giac [F] 519
Mupad [F(-1)] 520

Optimal result

Integrand size = 25, antiderivative size = 81

$$\begin{aligned}\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx = {} & -pr\log(x)\log\left(1+\frac{bx}{a}\right) \\ & + \log(x)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right) \\ & - qr\log(x)\log\left(1+\frac{dx}{c}\right) \\ & - pr\,\text{PolyLog}\left(2,-\frac{bx}{a}\right) - qr\,\text{PolyLog}\left(2,-\frac{dx}{c}\right)\end{aligned}$$

```
[Out] -p*r*ln(x)*ln(1+b*x/a)+ln(x)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)-q*r*ln(x)*ln(1
+d*x/c)-p*r*polylog(2,-b*x/a)-q*r*polylog(2,-d*x/c)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2580, 2354, 2438}

$$\begin{aligned}\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx = {} & \log(x)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right) \\ & - pr\,\text{PolyLog}\left(2,-\frac{bx}{a}\right) - pr\log(x)\log\left(\frac{bx}{a}+1\right) \\ & - qr\,\text{PolyLog}\left(2,-\frac{dx}{c}\right) - qr\log(x)\log\left(\frac{dx}{c}+1\right)\end{aligned}$$

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/x,x]
```

```
[Out] -(p*r*Log[x]*Log[1 + (b*x)/a]) + Log[x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r
] - q*r*Log[x]*Log[1 + (d*x)/c] - p*r*PolyLog[2, -((b*x)/a)] - q*r*PolyLog[
2, -((d*x)/c)]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2580

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[Log[g + h*x]*(Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]/h), x] + (-Dist[b*p*(r/h), Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[d*q*(r/h), Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \log(x)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)-(bpr)\int\frac{\log(x)}{a+bx}\,dx-(dqr)\int\frac{\log(x)}{c+dx}\,dx\\
&= -pr\log(x)\log\left(1+\frac{bx}{a}\right)+\log(x)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\\
&\quad -qr\log(x)\log\left(1+\frac{dx}{c}\right)+(pr)\int\frac{\log\left(1+\frac{bx}{a}\right)}{x}\,dx+(qr)\int\frac{\log\left(1+\frac{dx}{c}\right)}{x}\,dx\\
&= -pr\log(x)\log\left(1+\frac{bx}{a}\right)+\log(x)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\\
&\quad -qr\log(x)\log\left(1+\frac{dx}{c}\right)-pr\text{Li}_2\left(-\frac{bx}{a}\right)-qr\text{Li}_2\left(-\frac{dx}{c}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx = \log(x)\left(-pr\log\left(1+\frac{bx}{a}\right)+\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)-qr\log\left(1+\frac{dx}{c}\right)\right)-pr\,\text{PolyLog}\left(2,-\frac{bx}{a}\right)-qr\,\text{PolyLog}\left(2,-\frac{dx}{c}\right)$$

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/x,x]
```

```
[Out] Log[x]*(-(p*r*Log[1 + (b*x)/a]) + Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - q*
r*Log[1 + (d*x)/c]) - p*r*PolyLog[2, -((b*x)/a)] - q*r*PolyLog[2, -((d*x)/c
)]
```

Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.27

method	result
parts	$\ln(x)\ln\left(e(f(bx+a)^p(dx+c)^q)^r\right)-\frac{r\left(bfp\left(\frac{\text{dilog}\left(\frac{bx+a}{a}\right)}{b}+\frac{\ln(x)\ln\left(\frac{bx+a}{a}\right)}{b}\right)+dfq\left(\frac{\text{dilog}\left(\frac{dx+c}{c}\right)}{d}+\frac{\ln(x)\ln\left(\frac{dx+c}{c}\right)}{d}\right)\right)}{f}$

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)-r/f*(b*f*p*(dilog((b*x+a)/a)/b+ln(x)*
ln((b*x+a)/a)/b)+d*f*q*(dilog((d*x+c)/c)/d+ln(x)*ln((d*x+c)/c)/d))
```

Fricas [F]

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx = \int \frac{\log \left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{x}\,dx$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algorithm="fricas")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/x, x)
```

Sympy [F]

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx = \int \frac{\log\left(e\left(f(a+bx)^p\left(c+dx\right)^q\right)^r\right)}{x}\,dx$$

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/x,x)
[Out] Integral(log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.56

$$\begin{aligned}
&\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx \\
&= -\frac{(fp\log(bx+a)+fq\log(dx+c))r\log(x)}{f} + \log\left(\left((bx+a)^p(dx+c)^qf\right)^re\right)\log(x) \\
&\quad + \frac{\left(\left(\log(bx+a)\log\left(-\frac{bx+a}{a}+1\right)+\mathrm{Li}_2\left(\frac{bx+a}{a}\right)\right)fp+\left(\log(dx+c)\log\left(-\frac{dx+c}{c}+1\right)+\mathrm{Li}_2\left(\frac{dx+c}{c}\right)\right)fq\right)r}{f}
\end{aligned}$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algorithm="maxima")
[Out] -(f*p*log(b*x + a) + f*q*log(d*x + c))*r*log(x)/f + log(((b*x + a)^p*(d*x +
 c)^q*f)^r*e)*log(x) + ((log(b*x + a)*log(-(b*x + a)/a + 1) + dilog((b*x +
a)/a))*f*p + (log(d*x + c)*log(-(d*x + c)/c + 1) + dilog((d*x + c)/c))*f*q)
*r/f
```

Giac [F]

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx = \int \frac{\log\left(\left((bx+a)^p(dx+c)^qf\right)^re\right)}{x}\,dx$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algorithm="giac")
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx = \int \frac{\ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)}{x}\,dx$$

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/x,x)
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/x, x)
```

3.60 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{x\log(i(j(hx)^t)^u)}\,dx$

Optimal result 521
Rubi [N/A] 521
Mathematica [N/A] 522
Maple [N/A] 522
Fricas [N/A] 522
Sympy [F(-1)] 523
Maxima [N/A] 523
Giac [N/A] 523
Mupad [N/A] 524

Optimal result

Integrand size = 39, antiderivative size = 39

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log\left(i\left(j(hx)^t\right)^u\right)}\,dx=\text{Int}\left(\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log\left(i\left(j(hx)^t\right)^u\right)},x\right)$$

```
[Out] CannotIntegrate(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/ln(i*(j*(h*x)^t)^u),x)
```

Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log\left(i\left(j(hx)^t\right)^u\right)}\,dx=\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log\left(i\left(j(hx)^t\right)^u\right)}\,dx$$

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(x*Log[i*(j*(h*x)^t)^u]),x]
```

```
[Out] Defer[Int][Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(x*Log[i*(j*(h*x)^t)^u]), x
]
```

Rubi steps

$$\text{integral}=\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log\left(i\left(j(hx)^t\right)^u\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log \left(i\left(j(hx)^t\right)^u\right)}\,dx=\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log \left(i\left(j(hx)^t\right)^u\right)}\,dx$$

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(x*Log[i*(j*(h*x)^t)^u]),x]
[Out] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(x*Log[i*(j*(h*x)^t)^u]), x]
```

Maple [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{\ln \left(e(f(bx+a)^p\,(dx+c)^q)^r\right)}{x\ln \left(i\left(j\,(hx)^t\right)^u\right)}dx$$

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/ln(i*(j*(h*x)^t)^u),x)
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/ln(i*(j*(h*x)^t)^u),x)
```

Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log \left(i\left(j(hx)^t\right)^u\right)}\,dx=\int \frac{\log \left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{x\log \left(\left((hx)^t j\right)^u i\right)}\,dx$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/log(i*(j*(h*x)^t)^u),x, algori
thm="fricas")
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(x*log(((h*x)^t*j)^u*i)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log (e(f(a+bx)^p(c+dx)^q)^r)}{x\log (i(j(hx)^t)^u)}\,dx = \text{Timed out}$$

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/x/ln(i*(j*(h*x)**t)**u),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 3.37 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{\log (e(f(a+bx)^p(c+dx)^q)^r)}{x\log (i(j(hx)^t)^u)}\,dx = \int \frac{\log \left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{x\log \left(\left((hx)^t j\right)^u i\right)}\,dx$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/log(i*(j*(h*x)^t)^u),x, algori
thm="maxima")
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(x*log(((h*x)^t*j)^u*i)), x)
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{\log (e(f(a+bx)^p(c+dx)^q)^r)}{x\log (i(j(hx)^t)^u)}\,dx = \int \frac{\log \left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{x\log \left(\left((hx)^t j\right)^u i\right)}\,dx$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/log(i*(j*(h*x)^t)^u),x, algori
thm="giac")
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(x*log(((h*x)^t*j)^u*i)), x)
```

Mupad [N/A]

Not integrable

Time = 1.48 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{\log (e(f(a+bx)^p(c+dx)^q)^r)}{x\log (i\,(j(hx)^t)^u)}\,dx = \int \frac{\ln \left(e\,\left(f\,{\left(a+b\,x\right)}^p\,{\left(c+d\,x\right)}^q\right)^r\right)}{x\,\ln \left(i\,{\left(j\,{\left(h\,x\right)}^t\right)}^u\right)}\,dx$$

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(x*log(i*(j*(h*x)^t)^u)),x)
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(x*log(i*(j*(h*x)^t)^u)), x)
```

3.61 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{x\log^2\left(i(j(hx)^t)^u\right)}\,dx$

Optimal result 525
Rubi [N/A] 525
Mathematica [N/A] 526
Maple [N/A] 526
Fricas [N/A] 526
Sympy [F(-1)] 527
Maxima [N/A] 527
Giac [N/A] 527
Mupad [N/A] 528

Optimal result

Integrand size = 39, antiderivative size = 39

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log^2\left(i\left(j(hx)^t\right)^u\right)}\,dx=\operatorname{Int}\left(\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log^2\left(i\left(j(hx)^t\right)^u\right)},x\right)$$

```
[Out] CannotIntegrate(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/ln(i*(j*(h*x)^t)^u)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log^2\left(i\left(j(hx)^t\right)^u\right)}\,dx=\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log^2\left(i\left(j(hx)^t\right)^u\right)}\,dx$$

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(x*Log[i*(j*(h*x)^t)^u]^2),x]
```

```
[Out] Defer[Int][Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(x*Log[i*(j*(h*x)^t)^u]^2),
 x]
```

Rubi steps

$$\text{integral}=\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log^2\left(i\left(j(hx)^t\right)^u\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.61 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log^2\left(i\left(j(hx)^t\right)^u\right)}\,dx=\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log^2\left(i\left(j(hx)^t\right)^u\right)}\,dx$$

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(x*Log[i*(j*(h*x)^t)^u]^2),x
]
[Out] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(x*Log[i*(j*(h*x)^t)^u]^2),
x]
```

Maple [N/A]

Not integrable

Time = 1.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{\ln \left(e(f(bx+a)^p\,(dx+c)^q)^r\right)}{x\ln \left(i\left(j\,(hx)^t\right)^u\right)^2}dx$$

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/ln(i*(j*(h*x)^t)^u)^2,x)
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/ln(i*(j*(h*x)^t)^u)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log^2\left(i\left(j(hx)^t\right)^u\right)}\,dx=\int \frac{\log \left(((bx+a)^p(dx+c)^q f)^r\,e\right)}{x\log \left(\left((hx)^t\,j\right)^u\,i\right)^2}\,dx$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/log(i*(j*(h*x)^t)^u)^2,x, algo
rithm="fricas")
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(x*log(((h*x)^t*j)^u*i)^2), x
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log^2\left(i\left(j(hx)^t\right)^u\right)}\,dx = \text{Timed out}$$

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/x/ln(i*(j*(h*x)**t)**u)**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 3.12 (sec) , antiderivative size = 231, normalized size of antiderivative = 5.92

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log^2\left(i\left(j(hx)^t\right)^u\right)}\,dx = \int \frac{\log \left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{x\log \left(\left((hx)^t j\right)^u i\right)^2}\,dx$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/log(i*(j*(h*x)^t)^u)^2,x, algo
rithm="maxima")
[Out] -(r*log(f) + log(((b*x + a)^p)^r) + log(((d*x + c)^q)^r) + log(e))/(t^2*u^2
*log(h) + t*u^2*log(j) + t*u*log(i) + t*u*log((x^t)^u)) + integrate((b*c*p*
r + a*d*q*r + (p*r + q*r)*b*d*x)/((t^2*u^2*log(h) + t*u^2*log(j) + t*u*log(
i))*b*d*x^2 + (t^2*u^2*log(h) + t*u^2*log(j) + t*u*log(i))*a*c + ((t^2*u^2*
log(h) + t*u^2*log(j) + t*u*log(i))*b*c + (t^2*u^2*log(h) + t*u^2*log(j) +
t*u*log(i))*a*d)*x + (b*d*t*u*x^2 + a*c*t*u + (b*c*t*u + a*d*t*u)*x)*log((x
^t)^u)), x)
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log^2\left(i\left(j(hx)^t\right)^u\right)}\,dx = \int \frac{\log \left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{x\log \left(\left((hx)^t j\right)^u i\right)^2}\,dx$$

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/log(i*(j*(h*x)^t)^u)^2,x, algo
rithm="giac")
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(x*log(((h*x)^t*j)^u*i)^2),
x)
```

Mupad [N/A]

Not integrable

Time = 1.51 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log^2\left(i\left(j(hx)^t\right)^u\right)}\,dx=\int \frac{\ln \left(e\,\left(f\,{\left(a+b\,x\right)}^p\,{\left(c+d\,x\right)}^q\right)^r\right)}{x\,{\ln \left(i\,{\left(j\,{\left(h\,x\right)}^t\right)}^u\right)}^2}\,dx$$

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(x*log(i*(j*(h*x)^t)^u)^2),x)
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(x*log(i*(j*(h*x)^t)^u)^2), x)
```

3.62 $\int \frac{\log(x)\log^3\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx$

Optimal result 529
Rubi [N/A] 529
Mathematica [N/A] 530
Maple [N/A] 530
Fricas [N/A] 530
Sympy [N/A] 531
Maxima [N/A] 531
Giac [N/A] 531
Mupad [N/A] 532

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\log(x)\log^3\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = \text{Int}\left(\frac{\log(x)\log^3\left(\frac{a+bx}{(bc-ad)x}\right)}{x},x\right)$$

```
[Out] Unintegrable(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)^3/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log(x)\log^3\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = \int \frac{\log(x)\log^3\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx$$

```
[In] Int[(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)]^3)/x,x]
```

```
[Out] Defer[Int][(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)]^3)/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\log(x)\log^3\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log(x)\log^3\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = \int \frac{\log(x)\log^3\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx$$

```
[In] Integrate[(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)]^3)/x,x]
[Out] Integrate[(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)]^3)/x, x]
```

Maple [N/A]

Not integrable

Time = 1.82 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\ln(x)\ln\left(\frac{bx+a}{(-ad+cb)x}\right)^3}{x}dx$$

```
[In] int(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)^3/x,x)
[Out] int(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)^3/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log(x)\log^3\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = \int \frac{\log(x)\log\left(\frac{bx+a}{(bc-ad)x}\right)^3}{x}\,dx$$

```
[In] integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)^3/x,x, algorithm="fricas")
[Out] integral(log(x)*log((b*x + a)/((b*c - a*d)*x))^3/x, x)
```

Sympy [N/A]

Not integrable

Time = 4.15 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.54

$$\int \frac{\log(x)\log^3\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = \frac{3a\int \frac{\log(x)^2\log\left(\frac{a}{-adx+bcx}+\frac{bx}{-adx+bcx}\right)^2}{ax+bx^2}\,dx}{2} + \frac{\log(x)^2\log\left(\frac{a+bx}{x(-ad+bc)}\right)^3}{2}$$

```
[In] integrate(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)**3/x,x)
```

```
[Out] 3*a*Integral(log(x)**2*log(a/(-a*d*x + b*c*x) + b*x/(-a*d*x + b*c*x))**2/(a
*x + b*x**2), x)/2 + log(x)**2*log((a + b*x)/(x*(-a*d + b*c)))**3/2
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 272, normalized size of antiderivative = 9.71

$$\int \frac{\log(x)\log^3\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = \int \frac{\log(x)\log\left(\frac{bx+a}{(bc-ad)x}\right)^3}{x}\,dx$$

```
[In] integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)^3/x,x, algorithm="maxima")
```

```
[Out] 1/2*log(b*x + a)^3*log(x)^2 - integrate(1/2*(2*(b*x + a)*log(x)^4 + 6*(b*x*
log(b*c - a*d) + a*log(b*c - a*d))*log(x)^3 + 3*((3*b*x + 2*a)*log(x)^2 + 2
*(b*x*log(b*c - a*d) + a*log(b*c - a*d))*log(x))*log(b*x + a)^2 + 6*(b*x*lo
g(b*c - a*d)^2 + a*log(b*c - a*d)^2)*log(x)^2 - 6*((b*x + a)*log(x)^3 + 2*(
b*x*log(b*c - a*d) + a*log(b*c - a*d))*log(x)^2 + (b*x*log(b*c - a*d)^2 + a
*log(b*c - a*d)^2)*log(x))*log(b*x + a) + 2*(b*x*log(b*c - a*d)^3 + a*log(b
*c - a*d)^3)*log(x))/(b*x^2 + a*x), x)
```

Giac [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log(x)\log^3\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = \int \frac{\log(x)\log\left(\frac{bx+a}{(bc-ad)x}\right)^3}{x}\,dx$$

```
[In] integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)^3/x,x, algorithm="giac")
```

```
[Out] integrate(log(x)*log((b*x + a)/((b*c - a*d)*x))^3/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.36 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{\log(x)\log^3\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx=\int \frac{\ln\left(-\frac{a+b\,x}{x\,(a\,d-b\,c)}\right)^3\,\ln(x)}{x}\,dx$$

```
[In] int((log(-(a + b*x)/(x*(a*d - b*c)))^3*log(x))/x,x)
[Out] int((log(-(a + b*x)/(x*(a*d - b*c)))^3*log(x))/x, x)
```

3.63 $\int \frac{\log(x)\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx$

Optimal result 533
Rubi [N/A] 533
Mathematica [N/A] 534
Maple [N/A] 534
Fricas [N/A] 534
Sympy [N/A] 535
Maxima [N/A] 535
Giac [N/A] 535
Mupad [N/A] 536

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\log(x)\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = \text{Int}\left(\frac{\log(x)\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}{x},x\right)$$

```
[Out] Unintegrable(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)^2/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log(x)\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = \int \frac{\log(x)\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx$$

```
[In] Int[(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)]^2)/x,x]
```

```
[Out] Defer[Int][(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)]^2)/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\log(x)\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log(x)\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = \int \frac{\log(x)\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx$$

```
[In] Integrate[(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)]^2)/x,x]
[Out] Integrate[(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)]^2)/x, x]
```

Maple [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\ln\left(x\right)\ln\left(\frac{bx+a}{(-ad+cb)x}\right)^{2}}{x}dx$$

```
[In] int(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)^2/x,x)
[Out] int(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)^2/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log(x)\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = \int \frac{\log\left(x\right)\log\left(\frac{bx+a}{(bc-ad)x}\right)^{2}}{x}\,dx$$

```
[In] integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)^2/x,x, algorithm="fricas")
[Out] integral(log(x)*log((b*x + a)/((b*c - a*d)*x))^2/x, x)
```

Sympy [N/A]

Not integrable

Time = 6.48 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.36

$$\int \frac{\log(x)\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = a\int \frac{\log(x)^2\log\left(\frac{a}{-adx+bcx}+\frac{bx}{-adx+bcx}\right)}{ax+bx^2}\,dx + \frac{\log(x)^2\log\left(\frac{a+bx}{x(-ad+bc)}\right)^2}{2}$$

```
[In] integrate(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)**2/x,x)
```

```
[Out] a*Integral(log(x)**2*log(a/(-a*d*x + b*c*x) + b*x/(-a*d*x + b*c*x))/(a*x +
b*x**2), x) + log(x)**2*log((a + b*x)/(x*(-a*d + b*c)))**2/2
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 154, normalized size of antiderivative = 5.50

$$\int \frac{\log(x)\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = \int \frac{\log(x)\log\left(\frac{bx+a}{(bc-ad)x}\right)^2}{x}\,dx$$

```
[In] integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)^2/x,x, algorithm="maxima")
```

```
[Out] 1/2*log(b*x + a)^2*log(x)^2 - integrate(-((b*x + a)*log(x)^3 + 2*(b*x*log(b
*c - a*d) + a*log(b*c - a*d))*log(x)^2 - ((3*b*x + 2*a)*log(x)^2 + 2*(b*x*l
og(b*c - a*d) + a*log(b*c - a*d))*log(x))*log(b*x + a) + (b*x*log(b*c - a*d
)^2 + a*log(b*c - a*d)^2)*log(x))/(b*x^2 + a*x), x)
```

Giac [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log(x)\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = \int \frac{\log(x)\log\left(\frac{bx+a}{(bc-ad)x}\right)^2}{x}\,dx$$

```
[In] integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)^2/x,x, algorithm="giac")
```

```
[Out] integrate(log(x)*log((b*x + a)/((b*c - a*d)*x))^2/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{\log(x)\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = \int \frac{\ln\left(-\frac{a+b\,x}{x\,(a\,d-b\,c)}\right)^2\,\ln\left(x\right)}{x}\,dx$$

```
[In] int((log(-(a + b*x)/(x*(a*d - b*c)))^2*log(x))/x,x)
[Out] int((log(-(a + b*x)/(x*(a*d - b*c)))^2*log(x))/x, x)
```

3.64 $\int \frac{\log(x)\log\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx$

Optimal result . . . 537
Rubi [A] (verified) . . . 537
Mathematica [A] (verified) . . . 539
Maple [A] (verified) . . . 539
Fricas [F] . . . 540
Sympy [F] . . . 540
Maxima [F(-2)] . . . 540
Giac [F] . . . 540
Mupad [F(-1)] . . . 541

Optimal result

Integrand size = 26, antiderivative size = 82

$$\int \frac{\log(x)\log\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = -\frac{1}{2}\log\left(1+\frac{a}{bx}\right)\log^2(x)+\frac{1}{2}\log\left(\frac{b}{bc-ad}+\frac{a}{(bc-ad)x}\right)\log^2(x) + \log(x)\operatorname{PolyLog}\left(2,-\frac{a}{bx}\right)+\operatorname{PolyLog}\left(3,-\frac{a}{bx}\right)$$

```
[Out] -1/2*ln(1+a/b/x)*ln(x)^2+1/2*ln(b/(-a*d+b*c)+a/(-a*d+b*c)/x)*ln(x)^2+ln(x)*
polylog(2,-a/b/x)+polylog(3,-a/b/x)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2427, 2422, 2375, 2421, 6724}

$$\int \frac{\log(x)\log\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = \frac{1}{2}\log^2(x)\log\left(\frac{a}{x(bc-ad)}+\frac{b}{bc-ad}\right)+\operatorname{PolyLog}\left(3,-\frac{a}{bx}\right) + \log(x)\operatorname{PolyLog}\left(2,-\frac{a}{bx}\right)-\frac{1}{2}\log^2(x)\log\left(\frac{a}{bx}+1\right)$$

```
[In] Int[(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)])/x,x]
```

```
[Out] -1/2*(Log[1 + a/(b*x)]*Log[x]^2) + (Log[b/(b*c - a*d) + a/((b*c - a*d)*x)]*
Log[x]^2)/2 + Log[x]*PolyLog[2, -(a/(b*x))] + PolyLog[3, -(a/(b*x))]
```

Rule 2375

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Dist[b*f^m*n*(p/(e*r)), Int[Log[1 + e*(x^r/d)]*((a + b
*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2422

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[f*m*(r/(b*n*(p + 1))), Int[x^(m -
 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2427

```
Int[Log[(d_.)*(u_)^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((g_.
)*(x_))^(q_.), x_Symbol] :> Int[(g*x)^q*Log[d*ExpandToSum[u, x]^r]*(a + b*L
og[c*x^n])^p, x] /; FreeQ[{a, b, c, d, g, r, n, p, q}, x] && BinomialQ[u, x
] &&  !BinomialMatchQ[u, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \int \frac{\log\left(\frac{b}{bc-ad}+\frac{a}{(bc-ad)x}\right)\log(x)}{x}\,dx$$

$$= \frac{1}{2}\log\left(\frac{b}{bc-ad}+\frac{a}{(bc-ad)x}\right)\log^2(x)+\frac{a\int\frac{\log^2(x)}{\left(\frac{b}{bc-ad}+\frac{a}{(bc-ad)x}\right)x^2}\,dx}{2(bc-ad)}$$

$$= -\frac{1}{2}\log\left(1+\frac{a}{bx}\right)\log^2(x)+\frac{1}{2}\log\left(\frac{b}{bc-ad}+\frac{a}{(bc-ad)x}\right)\log^2(x)+\int\frac{\log\left(1+\frac{a}{bx}\right)\log(x)}{x}\,dx$$

$$= -\frac{1}{2}\log\left(1+\frac{a}{bx}\right)\log^2(x)+\frac{1}{2}\log\left(\frac{b}{bc-ad}+\frac{a}{(bc-ad)x}\right)\log^2(x) + \log(x)\text{Li}_2\left(-\frac{a}{bx}\right)-\int\frac{\text{Li}_2\left(-\frac{a}{bx}\right)}{x}dx$$

$$= -\frac{1}{2}\log\left(1+\frac{a}{bx}\right)\log^2(x)+\frac{1}{2}\log\left(\frac{b}{bc-ad}+\frac{a}{(bc-ad)x}\right)\log^2(x) + \log(x)\text{Li}_2\left(-\frac{a}{bx}\right)+\text{Li}_3\left(-\frac{a}{bx}\right)$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.80

$$\int\frac{\log(x)\log\left(\frac{a+bx}{(bc-ad)x}\right)}{x}dx=\frac{1}{6}\log^2(x)\left(\log(x)-3\log\left(1+\frac{bx}{a}\right)+3\log\left(\frac{a+bx}{bcx-adx}\right)\right) - \log(x)\,\text{PolyLog}\left(2,-\frac{bx}{a}\right)+\text{PolyLog}\left(3,-\frac{bx}{a}\right)$$

```
[In] Integrate[(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)])/x,x]
[Out] (Log[x]^2*(Log[x] - 3*Log[1 + (b*x)/a] + 3*Log[(a + b*x)/(b*c*x - a*d*x)]))
/6 - Log[x]*PolyLog[2, -((b*x)/a)] + PolyLog[3, -((b*x)/a)]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.15

method	result
default	$\frac{\ln\left(-\frac{bx+a}{(ad-cb)x}\right)\ln(x)^2}{2}+\frac{\left(-\frac{ad}{2}+\frac{cb}{2}\right)\left(-\frac{\ln(x)^3}{3}+\ln(x)^2\ln\left(1+\frac{xb}{a}\right)+2\ln(x)\,\text{Li}_2\left(-\frac{xb}{a}\right)-2\,\text{Li}_3\left(-\frac{xb}{a}\right)\right)}{ad-cb}$
risch	$\frac{\ln(x)^2\ln(bx+a)}{2}-\frac{\ln(x)^3}{3}-\frac{\left(2i\pi\,\text{csgn}\left(\frac{i(bx+a)}{x(ad-cb)}\right)^2+i\pi\,\text{csgn}(i(bx+a))\,\text{csgn}\left(\frac{i}{ad-cb}\right)\text{csgn}\left(\frac{i(bx+a)}{ad-cb}\right)-i\pi\,\text{csgn}(i(bx+a))\,\text{csgn}\left(\frac{i(b}{ad}\right.\right.}{}$

```
[In] int(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)/x,x,method=_RETURNVERBOSE)
[Out] 1/2*ln(-(b*x+a)/(a*d-b*c)/x)*ln(x)^2+(-1/2*a*d+1/2*c*b)/(a*d-b*c)*(-1/3*ln(
x)^3+ln(x)^2*ln(1+x/a*b)+2*ln(x)*polylog(2,-x/a*b)-2*polylog(3,-x/a*b))
```

Fricas [F]

$$\int \frac{\log(x)\log\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = \int \frac{\log\left(x\right)\log\left(\frac{bx+a}{(bc-ad)x}\right)}{x}\,dx$$

[In] integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)/x,x, algorithm="fricas")

[Out] integral(log(x)*log((b*x + a)/((b*c - a*d)*x))/x, x)

Sympy [F]

$$\int \frac{\log(x)\log\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = \frac{a\int \frac{\log\left(x\right)^2}{ax+bx^2}\,dx}{2} + \frac{\log\left(x\right)^2\log\left(\frac{a+bx}{x(-ad+bc)}\right)}{2}$$

[In] integrate(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)/x,x)

[Out] a*Integral(log(x)**2/(a*x + b*x**2), x)/2 + log(x)**2*log((a + b*x)/(x*(-a*d + b*c)))/2

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log(x)\log\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = \text{Exception raised: TypeError}$$

[In] integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)/x,x, algorithm="maxima")

[Out] Exception raised: TypeError >> unable to make sense of Maxima expression 'li[2]' in Sage

Giac [F]

$$\int \frac{\log(x)\log\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = \int \frac{\log\left(x\right)\log\left(\frac{bx+a}{(bc-ad)x}\right)}{x}\,dx$$

[In] integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)/x,x, algorithm="giac")

[Out] integrate(log(x)*log((b*x + a)/((b*c - a*d)*x))/x, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(x)\log\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = \int \frac{\ln\left(-\frac{a+b\,x}{x\,(a\,d-b\,c)}\right)\,\ln\left(x\right)}{x}\,dx$$

```
[In] int((log(-(a + b*x)/(x*(a*d - b*c)))*log(x))/x,x)
[Out] int((log(-(a + b*x)/(x*(a*d - b*c)))*log(x))/x, x)
```

3.65 $\int \frac{\log(x)}{x\log\left(\frac{a+bx}{(bc-ad)x}\right)}\,dx$

Optimal result 542
Rubi [N/A] 542
Mathematica [N/A] 543
Maple [N/A] 543
Fricas [N/A] 543
Sympy [N/A] 544
Maxima [N/A] 544
Giac [N/A] 544
Mupad [N/A] 545

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\log(x)}{x\log\left(\frac{a+bx}{(bc-ad)x}\right)}\,dx=\text{Int}\left(\frac{\log(x)}{x\log\left(\frac{a+bx}{(bc-ad)x}\right)},x\right)$$

```
[Out] Unintegrable(ln(x)/x/ln((b*x+a)/(-a*d+b*c)/x),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log(x)}{x\log\left(\frac{a+bx}{(bc-ad)x}\right)}\,dx=\int \frac{\log(x)}{x\log\left(\frac{a+bx}{(bc-ad)x}\right)}\,dx$$

```
[In] Int[Log[x]/(x*Log[(a + b*x)/((b*c - a*d)*x)]),x]
```

```
[Out] Defer[Int][Log[x]/(x*Log[(a + b*x)/((b*c - a*d)*x)]), x]
```

Rubi steps

$$\text{integral}=\int \frac{\log(x)}{x\log\left(\frac{a+bx}{(bc-ad)x}\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log(x)}{x\log\left(\frac{a+bx}{(bc-ad)x}\right)}\,dx=\int \frac{\log(x)}{x\log\left(\frac{a+bx}{(bc-ad)x}\right)}\,dx$$

```
[In] Integrate[Log[x]/(x*Log[(a + b*x)/((b*c - a*d)*x)]),x]
[Out] Integrate[Log[x]/(x*Log[(a + b*x)/((b*c - a*d)*x)]), x]
```

Maple [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\ln(x)}{x\ln\left(\frac{bx+a}{(-ad+cb)x}\right)}dx$$

```
[In] int(ln(x)/x/ln((b*x+a)/(-a*d+b*c)/x),x)
[Out] int(ln(x)/x/ln((b*x+a)/(-a*d+b*c)/x),x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log(x)}{x\log\left(\frac{a+bx}{(bc-ad)x}\right)}\,dx=\int \frac{\log(x)}{x\log\left(\frac{bx+a}{(bc-ad)x}\right)}\,dx$$

```
[In] integrate(log(x)/x/log((b*x+a)/(-a*d+b*c)/x),x, algorithm="fricas")
[Out] integral(log(x)/(x*log((b*x + a)/((b*c - a*d)*x))), x)
```

Sympy [N/A]

Not integrable

Time = 27.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \frac{\log(x)}{x \log\left(\frac{a+bx}{(bc-ad)x}\right)} dx = \int \frac{\log (x)}{x \log \left(\frac{a}{-adx+bcx} + \frac{bx}{-adx+bcx}\right)} dx$$

```
[In] integrate(ln(x)/x/ln((b*x+a)/(-a*d+b*c)/x),x)
[Out] Integral(log(x)/(x*log(a/(-a*d*x + b*c*x) + b*x/(-a*d*x + b*c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log(x)}{x \log\left(\frac{a+bx}{(bc-ad)x}\right)} dx = \int \frac{\log (x)}{x \log \left(\frac{bx+a}{(bc-ad)x}\right)} dx$$

```
[In] integrate(log(x)/x/log((b*x+a)/(-a*d+b*c)/x),x, algorithm="maxima")
[Out] integrate(log(x)/(x*log((b*x + a)/((b*c - a*d)*x))), x)
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log(x)}{x \log\left(\frac{a+bx}{(bc-ad)x}\right)} dx = \int \frac{\log (x)}{x \log \left(\frac{bx+a}{(bc-ad)x}\right)} dx$$

```
[In] integrate(log(x)/x/log((b*x+a)/(-a*d+b*c)/x),x, algorithm="giac")
[Out] integrate(log(x)/(x*log((b*x + a)/((b*c - a*d)*x))), x)
```

Mupad [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{\log(x)}{x\log\left(\frac{a+bx}{(bc-ad)x}\right)}\,dx=\int \frac{\ln\left(x\right)}{x\,\ln\left(-\frac{a+b\,x}{x\,\left(a\,d-b\,c\right)}\right)}\,dx$$

```
[In] int(log(x)/(x*log(-(a + b*x)/(x*(a*d - b*c)))),x)
[Out] int(log(x)/(x*log(-(a + b*x)/(x*(a*d - b*c)))), x)
```

3.66 $\int \frac{\log(x)}{x \log^2\left(\frac{a+bx}{(bc-ad)x}\right)} dx$

Optimal result . . . 546
Rubi [N/A] . . . 546
Mathematica [N/A] . . . 547
Maple [N/A] . . . 547
Fricas [N/A] . . . 547
Sympy [N/A] . . . 548
Maxima [N/A] . . . 548
Giac [N/A] . . . 549
Mupad [N/A] . . . 549

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\log(x)}{x \log^2\left(\frac{a+bx}{(bc-ad)x}\right)} dx = \text{Int}\left(\frac{\log(x)}{x \log^2\left(\frac{a+bx}{(bc-ad)x}\right)}, x\right)$$

```
[Out] Unintegrable(ln(x)/x/ln((b*x+a)/(-a*d+b*c)/x)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log(x)}{x \log^2\left(\frac{a+bx}{(bc-ad)x}\right)} dx = \int \frac{\log(x)}{x \log^2\left(\frac{a+bx}{(bc-ad)x}\right)} dx$$

```
[In] Int[Log[x]/(x*Log[(a + b*x)/((b*c - a*d)*x)]^2),x]
```

```
[Out] Defer[Int][Log[x]/(x*Log[(a + b*x)/((b*c - a*d)*x)]^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{\log(x)}{x \log^2\left(\frac{a+bx}{(bc-ad)x}\right)} dx$$

Mathematica [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log(x)}{x\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}\,dx=\int \frac{\log(x)}{x\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}\,dx$$

```
[In] Integrate[Log[x]/(x*Log[(a + b*x)/((b*c - a*d)*x)]^2),x]
[Out] Integrate[Log[x]/(x*Log[(a + b*x)/((b*c - a*d)*x)]^2), x]
```

Maple [N/A]

Not integrable

Time = 33.56 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\ln\left(x\right)}{x\ln\left(\frac{bx+a}{(-ad+cb)x}\right)^{2}}dx$$

```
[In] int(ln(x)/x/ln((b*x+a)/(-a*d+b*c)/x)^2,x)
[Out] int(ln(x)/x/ln((b*x+a)/(-a*d+b*c)/x)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log(x)}{x\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}\,dx=\int \frac{\log\left(x\right)}{x\log\left(\frac{bx+a}{(bc-ad)x}\right)^{2}}\,dx$$

```
[In] integrate(log(x)/x/log((b*x+a)/(-a*d+b*c)/x)^2,x, algorithm="fricas")
[Out] integral(log(x)/(x*log((b*x + a)/((b*c - a*d)*x))^2), x)
```

Sympy [N/A]

Not integrable

Time = 41.30 (sec) , antiderivative size = 126, normalized size of antiderivative = 4.50

$$\int \frac{\log(x)}{x\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}\,dx = \frac{a\log{(x)} + bx\log{(x)}}{a\log\left(\frac{a+bx}{x(-ad+bc)}\right)} - \frac{\int \frac{b}{\log\left(\frac{a}{-adx+bcx}+\frac{bx}{-adx+bcx}\right)}\,dx + \int \frac{a}{x\log\left(\frac{a}{-adx+bcx}+\frac{bx}{-adx+bcx}\right)}\,dx + \int \frac{b\log{(x)}}{\log\left(\frac{a}{-adx+bcx}+\frac{bx}{-adx+bcx}\right)}\,dx}{a}$$

```
[In] integrate(ln(x)/x/ln((b*x+a)/(-a*d+b*c)/x)**2,x)
```

```
[Out] (a*log(x) + b*x*log(x))/(a*log((a + b*x)/(x*(-a*d + b*c)))) - (Integral(b/l
og(a/(-a*d*x + b*c*x) + b*x/(-a*d*x + b*c*x)), x) + Integral(a/(x*log(a/(-a
*d*x + b*c*x) + b*x/(-a*d*x + b*c*x))), x) + Integral(b*log(x)/log(a/(-a*d*
x + b*c*x) + b*x/(-a*d*x + b*c*x)), x))/a
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.96

$$\int \frac{\log(x)}{x\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}\,dx = \int \frac{\log{(x)}}{x\log\left(\frac{bx+a}{(bc-ad)x}\right)^2}\,dx$$

```
[In] integrate(log(x)/x/log((b*x+a)/(-a*d+b*c)/x)^2,x, algorithm="maxima")
```

```
[Out] -(b*x + a)*log(x)/(a*log(b*c - a*d) - a*log(b*x + a) + a*log(x)) - integrat
e(-(b*x*log(x) + b*x + a)/(a*x*log(b*c - a*d) - a*x*log(b*x + a) + a*x*log(
x)), x)
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\log(x)}{x\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}\,dx = \int \frac{\log(x)}{x\log\left(\frac{bx+a}{(bc-ad)x}\right)^2}\,dx$$

```
[In] integrate(log(x)/x/log((b*x+a)/(-a*d+b*c)/x)^2,x, algorithm="giac")
[Out] integrate(log(x)/(x*log((b*x + a)/((b*c - a*d)*x))^2), x)
```

Mupad [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{\log(x)}{x\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}\,dx = \int \frac{\ln(x)}{x\ln\left(-\frac{a+b\,x}{x\,(a\,d-b\,c)}\right)^2}\,dx$$

```
[In] int(log(x)/(x*log(-(a + b*x)/(x*(a*d - b*c)))^2),x)
[Out] int(log(x)/(x*log(-(a + b*x)/(x*(a*d - b*c)))^2), x)
```

3.67 $\int \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log(h(f+gx)^m)}{(a+bx)(c+dx)}\,dx$

Optimal result 551
Rubi [A] (verified) 552
Mathematica [F] 556
Maple [F] 557
Fricas [F] 557
Sympy [F(-1)] 557
Maxima [F] 557
Giac [F] 560
Mupad [F(-1)] 560

Optimal result

Integrand size = 45, antiderivative size = 620

$$\int \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx = \frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{4(bc-ad)n} + \frac{\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{4(bc-ad)n} - \frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{4(bc-ad)n} + \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bc-ad} - \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad} - \frac{3mn\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{bc-ad} + \frac{3mn\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{PolyLog}\left(3,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad} + \frac{6mn^2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{PolyLog}\left(4,\frac{d(a+bx)}{b(c+dx)}\right)}{bc-ad} - \frac{6mn^2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{PolyLog}\left(4,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad} - \frac{6mn^3\operatorname{PolyLog}\left(5,\frac{d(a+bx)}{b(c+dx)}\right)}{bc-ad} + \frac{6mn^3\operatorname{PolyLog}\left(5,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad}$$

```
[Out] 1/4*m*ln(e*((b*x+a)/(d*x+c))^n)^4*ln((-a*d+b*c)/b/(d*x+c))/(-a*d+b*c)/n+1/4
*ln(e*((b*x+a)/(d*x+c))^n)^4*ln(h*(g*x+f)^m)/(-a*d+b*c)/n-1/4*m*ln(e*((b*x+
a)/(d*x+c))^n)^4*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*d+b*c)/n+m
*ln(e*((b*x+a)/(d*x+c))^n)^3*polylog(2,d*(b*x+a)/b/(d*x+c))/(-a*d+b*c)-m*ln
(e*((b*x+a)/(d*x+c))^n)^3*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/
(-a*d+b*c)-3*m*n*ln(e*((b*x+a)/(d*x+c))^n)^2*polylog(3,d*(b*x+a)/b/(d*x+c))
/(-a*d+b*c)+3*m*n*ln(e*((b*x+a)/(d*x+c))^n)^2*polylog(3,(-c*g+d*f)*(b*x+a)/
(-a*g+b*f)/(d*x+c))/(-a*d+b*c)+6*m*n^2*ln(e*((b*x+a)/(d*x+c))^n)*polylog(4,
d*(b*x+a)/b/(d*x+c))/(-a*d+b*c)-6*m*n^2*ln(e*((b*x+a)/(d*x+c))^n)*polylog(4
,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*d+b*c)-6*m*n^3*polylog(5,d*(b*x
+a)/b/(d*x+c))/(-a*d+b*c)+6*m*n^3*polylog(5,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(
d*x+c))/(-a*d+b*c)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 620, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {2589, 2553, 2404, 2354, 2421, 2430, 6724}

$$\int \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx = \frac{\log\left(h(f+gx)^m\right)\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4n(bc-ad)} - \frac{6mn^2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{PolyLog}\left(4,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad} - \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad} + \frac{3mn\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{PolyLog}\left(3,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad} - \frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{4n(bc-ad)} + \frac{6mn^2\text{PolyLog}\left(4,\frac{d(a+bx)}{b(c+dx)}\right)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bc-ad} + \frac{m\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bc-ad} - \frac{3mn\,\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bc-ad} + \frac{m\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4n(bc-ad)} + \frac{6mn^3\text{PolyLog}\left(5,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad} - \frac{6mn^3\text{PolyLog}\left(5,\frac{d(a+bx)}{b(c+dx)}\right)}{bc-ad}$$

```
[In] Int[(Log[e*((a + b*x)/(c + d*x))^n]^3*Log[h*(f + g*x)^m])/((a + b*x)*(c + d
*x)),x]
```

```
[Out] (m*Log[e*((a + b*x)/(c + d*x))^n]^4*Log[(b*c - a*d)/(b*(c + d*x))])/(4*(b*c
 - a*d)*n) + (Log[e*((a + b*x)/(c + d*x))^n]^4*Log[h*(f + g*x)^m])/(4*(b*c
- a*d)*n) - (m*Log[e*((a + b*x)/(c + d*x))^n]^4*Log[1 - ((d*f - c*g)*(a + b
*x))/((b*f - a*g)*(c + d*x))])/(4*(b*c - a*d)*n) + (m*Log[e*((a + b*x)/(c +
 d*x))^n]^3*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b*c - a*d) - (m*Log[e
*((a + b*x)/(c + d*x))^n]^3*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)
```

```
*(c + d*x))])/(b*c - a*d) - (3*m*n*Log[e*((a + b*x)/(c + d*x))^n]^2*PolyLog
[3, (d*(a + b*x))/(b*(c + d*x))])/(b*c - a*d) + (3*m*n*Log[e*((a + b*x)/(c
+ d*x))^n]^2*PolyLog[3, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/(
b*c - a*d) + (6*m*n^2*Log[e*((a + b*x)/(c + d*x))^n]*PolyLog[4, (d*(a + b*x
))/(b*(c + d*x))])/(b*c - a*d) - (6*m*n^2*Log[e*((a + b*x)/(c + d*x))^n]*Po
lyLog[4, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/(b*c - a*d) - (6
*m*n^3*PolyLog[5, (d*(a + b*x))/(b*(c + d*x))])/(b*c - a*d) + (6*m*n^3*Poly
Log[5, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/(b*c - a*d)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rule 2589

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*Log[(i_.)*((j_.)*((g_.) + (h_.)*(x_))^(t_.))^(u_.)]*(v_), x_S
ymbol] :> With[{k = Simplify[v*(a + b*x)*(c + d*x)]}, Simp[k*Log[i*(j*(g +
h*x)^t)^u]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s + 1)/(p*r*(s + 1)*(b*c
- a*d))), x] - Dist[k*h*t*(u/(p*r*(s + 1)*(b*c - a*d))), Int[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^(s + 1)/(g + h*x), x], x] /; FreeQ[k, x]] /; FreeQ[{
a, b, c, d, e, f, g, h, i, j, p, q, r, s, t, u}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[p + q, 0] && NeQ[s, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{4(bc-ad)n} - \frac{(gm)\int\frac{\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx}\,dx}{4(bc-ad)n}$$

$$= \frac{\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{4(bc-ad)n} - \frac{(gm)\text{Subst}\left(\int\frac{\log^4(ex^n)}{(b-dx)(bf-ag-(df-cg)x)}\,dx,x,\frac{a+bx}{c+dx}\right)}{4n}$$

$$= \frac{\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{4(bc-ad)n} - \frac{(gm)\text{Subst}\left(\int\left(\frac{d\log^4(ex^n)}{(bc-ad)g(b-dx)}+\frac{(-df+cg)\log^4(ex^n)}{(bc-ad)g(bf-ag-(df-cg)x)}\right)dx,x,\frac{a+bx}{c+dx}\right)}{4n}$$

$$= \frac{\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{4(bc-ad)n} - \frac{(dm)\text{Subst}\left(\int\frac{\log^4(ex^n)}{b-dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{4(bc-ad)n} + \frac{((df-cg)m)\text{Subst}\left(\int\frac{\log^4(ex^n)}{bf-ag+(-df+cg)x}\,dx,x,\frac{a+bx}{c+dx}\right)}{4(bc-ad)n}$$

$$= \frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{4(bc-ad)n} + \frac{\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{4(bc-ad)n}$$
$$- \frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{4(bc-ad)n}$$
$$- \frac{m\text{Subst}\left(\int \frac{\log^3(ex^n)\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{bc-ad}$$
$$+ \frac{m\text{Subst}\left(\int \frac{\log^3(ex^n)\log\left(1+\frac{(-df+cg)x}{bf-ag}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{bc-ad}$$

$$= \frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{4(bc-ad)n} + \frac{\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{4(bc-ad)n}$$
$$- \frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{4(bc-ad)n}$$
$$+ \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bc-ad} - \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{Li}_2\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad}$$
$$- \frac{(3mn)\text{Subst}\left(\int \frac{\log^2(ex^n)\text{Li}_2\left(\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{bc-ad}$$
$$+ \frac{(3mn)\text{Subst}\left(\int \frac{\log^2(ex^n)\text{Li}_2\left(-\frac{(-df+cg)x}{bf-ag}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{bc-ad}$$

$$= \frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{4(bc-ad)n} + \frac{\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{4(bc-ad)n}$$
$$- \frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{4(bc-ad)n}$$
$$+ \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bc-ad} - \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{Li}_2\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad}$$
$$- \frac{3mn\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bc-ad} + \frac{3mn\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{Li}_3\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad}$$
$$+ \frac{(6mn^2)\,\text{Subst}\left(\int \frac{\log(ex^n)\text{Li}_3\left(\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{bc-ad}$$
$$- \frac{(6mn^2)\,\text{Subst}\left(\int \frac{\log(ex^n)\text{Li}_3\left(-\frac{(-df+cg)x}{bf-ag}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{bc-ad}$$

$$= \frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{4(bc-ad)n} + \frac{\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{4(bc-ad)n} - \frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{4(bc-ad)n} + \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bc-ad} - \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{Li}_2\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad} - \frac{3mn\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bc-ad} + \frac{3mn\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{Li}_3\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad} + \frac{6mn^2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{Li}_4\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bc-ad} - \frac{6mn^2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{Li}_4\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad} - \frac{(6mn^3)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_4\left(\frac{dx}{b}\right)}{x}dx, x, \frac{a+bx}{c+dx}\right)}{bc-ad} + \frac{(6mn^3)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_4\left(-\frac{(-df+cg)x}{bf-ag}\right)}{x}dx, x, \frac{a+bx}{c+dx}\right)}{bc-ad}$$

$$= \frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{4(bc-ad)n} + \frac{\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{4(bc-ad)n} - \frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{4(bc-ad)n} + \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bc-ad} - \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{Li}_2\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad} - \frac{3mn\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bc-ad} + \frac{3mn\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{Li}_3\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad} + \frac{6mn^2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{Li}_4\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bc-ad} - \frac{6mn^2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{Li}_4\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad} - \frac{6mn^3\operatorname{Li}_5\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bc-ad} + \frac{6mn^3\operatorname{Li}_5\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad}$$

Mathematica [F]

$$\int\frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}dx = \int\frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}dx$$

```
[In] Integrate[(Log[e*((a + b*x)/(c + d*x))^n]^3*Log[h*(f + g*x)^m])/((a + b*x)*
(c + d*x)),x]
```

```
[Out] Integrate[(Log[e*((a + b*x)/(c + d*x))^n]^3*Log[h*(f + g*x)^m])/((a + b*x)*
(c + d*x)), x]
```

Maple [F]

$$\int \frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)^{3} \ln\left(h(gx+f)^{m}\right)}{(bx+a)\,(dx+c)}dx$$

```
[In] int(ln(e*((b*x+a)/(d*x+c))^n)^3*ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x)
[Out] int(ln(e*((b*x+a)/(d*x+c))^n)^3*ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x)
```

Fricas [F]

$$\int \frac{\log^{3}\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\log\left(h(f+gx)^{m}\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\log\left((gx+f)^{m}h\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)^{3}}{(bx+a)(dx+c)}\,dx$$

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^3*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x,
algorithm="fricas")
[Out] integral(log((g*x + f)^m*h)*log(e*((b*x + a)/(d*x + c))^n)^3/(b*d*x^2 + a*c
 + (b*c + a*d)*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log^{3}\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\log\left(h(f+gx)^{m}\right)}{(a+bx)(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate(ln(e*((b*x+a)/(d*x+c))**n)**3*ln(h*(g*x+f)**m)/(b*x+a)/(d*x+c),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\log^{3}\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\log\left(h(f+gx)^{m}\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\log\left((gx+f)^{m}h\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)^{3}}{(bx+a)(dx+c)}\,dx$$

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^3*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x,
algorithm="maxima")
[Out] -1/4*(n^3*log(b*x + a)^4 + n^3*log(d*x + c)^4 - 4*n^2*log(b*x + a)^3*log(e)
 + 6*n*log(b*x + a)^2*log(e)^2 - 4*(n^3*log(b*x + a) - n^2*log(e))*log(d*x
+ c)^3 - 4*(log(b*x + a) - log(d*x + c))*log((b*x + a)^n)^3 + 4*(log(b*x +
a) - log(d*x + c))*log((d*x + c)^n)^3 - 4*log(b*x + a)*log(e)^3 + 6*(n^3*lo
```

g(b*x + a)^2 - 2*n^2*log(b*x + a)*log(e) + n*log(e)^2)*log(d*x + c)^2 + 6*(
n*log(b*x + a)^2 + n*log(d*x + c)^2 - 2*(n*log(b*x + a) - log(e))*log(d*x +
 c) - 2*log(b*x + a)*log(e))*log((b*x + a)^n)^2 + 6*(n*log(b*x + a)^2 + n*l
og(d*x + c)^2 - 2*(n*log(b*x + a) - log(e))*log(d*x + c) - 2*(log(b*x + a)
 - log(d*x + c))*log((b*x + a)^n) - 2*log(b*x + a)*log(e))*log((d*x + c)^n)^
2 - 4*(n^3*log(b*x + a)^3 - 3*n^2*log(b*x + a)^2*log(e) + 3*n*log(b*x + a)*
log(e)^2 - log(e)^3)*log(d*x + c) - 4*(n^2*log(b*x + a)^3 - n^2*log(d*x + c
)^3 - 3*n*log(b*x + a)^2*log(e) + 3*(n^2*log(b*x + a) - n*log(e))*log(d*x +
 c)^2 + 3*log(b*x + a)*log(e)^2 - 3*(n^2*log(b*x + a)^2 - 2*n*log(b*x + a)*
log(e) + log(e)^2)*log(d*x + c))*log((b*x + a)^n) + 4*(n^2*log(b*x + a)^3 -
 n^2*log(d*x + c)^3 - 3*n*log(b*x + a)^2*log(e) + 3*(n^2*log(b*x + a) - n*l
og(e))*log(d*x + c)^2 + 3*(log(b*x + a) - log(d*x + c))*log((b*x + a)^n)^2
 + 3*log(b*x + a)*log(e)^2 - 3*(n^2*log(b*x + a)^2 - 2*n*log(b*x + a)*log(e)
 + log(e)^2)*log(d*x + c) - 3*(n*log(b*x + a)^2 + n*log(d*x + c)^2 - 2*(n*l
og(b*x + a) - log(e))*log(d*x + c) - 2*log(b*x + a)*log(e))*log((b*x + a)^n
))*log((d*x + c)^n))*log((g*x + f)^m)/(b*c - a*d) + integrate(1/4*(4*b*c*f*
log(e)^3*log(h) - 4*a*d*f*log(e)^3*log(h) + (b*d*g*m*n^3*x^2 + a*c*g*m*n^3
 + (b*c*g*m*n^3 + a*d*g*m*n^3)*x)*log(b*x + a)^4 + (b*d*g*m*n^3*x^2 + a*c*g*
m*n^3 + (b*c*g*m*n^3 + a*d*g*m*n^3)*x)*log(d*x + c)^4 - 4*(b*d*g*m*n^2*x^2*
log(e) + a*c*g*m*n^2*log(e) + (b*c*g*m*n^2*log(e) + a*d*g*m*n^2*log(e))*x)*
log(b*x + a)^3 + 4*(b*d*g*m*n^2*x^2*log(e) + a*c*g*m*n^2*log(e) + (b*c*g*m*
n^2*log(e) + a*d*g*m*n^2*log(e))*x - (b*d*g*m*n^3*x^2 + a*c*g*m*n^3 + (b*c*
g*m*n^3 + a*d*g*m*n^3)*x)*log(b*x + a))*log(d*x + c)^3 + 4*(b*c*f*log(h) -
a*d*f*log(h) + (b*c*g*log(h) - a*d*g*log(h))*x - (b*d*g*m*x^2 + a*c*g*m + (
b*c*g*m + a*d*g*m)*x)*log(b*x + a) + (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*
d*g*m)*x)*log(d*x + c))*log((b*x + a)^n)^3 - 4*(b*c*f*log(h) - a*d*f*log(h)
 + (b*c*g*log(h) - a*d*g*log(h))*x - (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*
d*g*m)*x)*log(b*x + a) + (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g*m)*x)*lo
g(d*x + c))*log((d*x + c)^n)^3 + 6*(b*d*g*m*n*x^2*log(e)^2 + a*c*g*m*n*log(
e)^2 + (b*c*g*m*n*log(e)^2 + a*d*g*m*n*log(e)^2)*x)*log(b*x + a)^2 + 6*(b*d
*g*m*n*x^2*log(e)^2 + a*c*g*m*n*log(e)^2 + (b*d*g*m*n^3*x^2 + a*c*g*m*n^3 +
 (b*c*g*m*n^3 + a*d*g*m*n^3)*x)*log(b*x + a)^2 + (b*c*g*m*n*log(e)^2 + a*d*
g*m*n*log(e)^2)*x - 2*(b*d*g*m*n^2*x^2*log(e) + a*c*g*m*n^2*log(e) + (b*c*g
*m*n^2*log(e) + a*d*g*m*n^2*log(e))*x)*log(b*x + a))*log(d*x + c)^2 + 6*(2*
b*c*f*log(e)*log(h) - 2*a*d*f*log(e)*log(h) + (b*d*g*m*n*x^2 + a*c*g*m*n +
(b*c*g*m*n + a*d*g*m*n)*x)*log(b*x + a)^2 + (b*d*g*m*n*x^2 + a*c*g*m*n + (b
*c*g*m*n + a*d*g*m*n)*x)*log(d*x + c)^2 + 2*(b*c*g*log(e)*log(h) - a*d*g*lo
g(e)*log(h))*x - 2*(b*d*g*m*x^2*log(e) + a*c*g*m*log(e) + (b*c*g*m*log(e) +
 a*d*g*m*log(e))*x)*log(b*x + a) + 2*(b*d*g*m*x^2*log(e) + a*c*g*m*log(e) +
 (b*c*g*m*log(e) + a*d*g*m*log(e))*x - (b*d*g*m*n*x^2 + a*c*g*m*n + (b*c*g*
m*n + a*d*g*m*n)*x)*log(b*x + a))*log(d*x + c))*log((b*x + a)^n)^2 + 6*(2*b
*c*f*log(e)*log(h) - 2*a*d*f*log(e)*log(h) + (b*d*g*m*n*x^2 + a*c*g*m*n + (
b*c*g*m*n + a*d*g*m*n)*x)*log(b*x + a)^2 + (b*d*g*m*n*x^2 + a*c*g*m*n + (b*
c*g*m*n + a*d*g*m*n)*x)*log(d*x + c)^2 + 2*(b*c*g*log(e)*log(h) - a*d*g*log
(e)*log(h))*x - 2*(b*d*g*m*x^2*log(e) + a*c*g*m*log(e) + (b*c*g*m*log(e) +

```
a*d*g*m*log(e))*x)*log(b*x + a) + 2*(b*d*g*m*x^2*log(e) + a*c*g*m*log(e) +
(b*c*g*m*log(e) + a*d*g*m*log(e))*x - (b*d*g*m*n*x^2 + a*c*g*m*n + (b*c*g*m
*n + a*d*g*m*n)*x)*log(b*x + a))*log(d*x + c) + 2*(b*c*f*log(h) - a*d*f*log
(h) + (b*c*g*log(h) - a*d*g*log(h))*x - (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m +
 a*d*g*m)*x)*log(b*x + a) + (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g*m)*x)
*log(d*x + c))*log((b*x + a)^n))*log((d*x + c)^n)^2 + 4*(b*c*g*log(e)^3*log
(h) - a*d*g*log(e)^3*log(h))*x - 4*(b*d*g*m*x^2*log(e)^3 + a*c*g*m*log(e)^3
 + (b*c*g*m*log(e)^3 + a*d*g*m*log(e)^3)*x)*log(b*x + a) + 4*(b*d*g*m*x^2*l
og(e)^3 + a*c*g*m*log(e)^3 - (b*d*g*m*n^3*x^2 + a*c*g*m*n^3 + (b*c*g*m*n^3
+ a*d*g*m*n^3)*x)*log(b*x + a)^3 + 3*(b*d*g*m*n^2*x^2*log(e) + a*c*g*m*n^2*
log(e) + (b*c*g*m*n^2*log(e) + a*d*g*m*n^2*log(e))*x)*log(b*x + a)^2 + (b*c
*g*m*log(e)^3 + a*d*g*m*log(e)^3)*x - 3*(b*d*g*m*n*x^2*log(e)^2 + a*c*g*m*n
*log(e)^2 + (b*c*g*m*n*log(e)^2 + a*d*g*m*n*log(e)^2)*x)*log(b*x + a))*log(
d*x + c) + 4*(3*b*c*f*log(e)^2*log(h) - 3*a*d*f*log(e)^2*log(h) - (b*d*g*m*
n^2*x^2 + a*c*g*m*n^2 + (b*c*g*m*n^2 + a*d*g*m*n^2)*x)*log(b*x + a)^3 + (b*
d*g*m*n^2*x^2 + a*c*g*m*n^2 + (b*c*g*m*n^2 + a*d*g*m*n^2)*x)*log(d*x + c)^3
 + 3*(b*d*g*m*n*x^2*log(e) + a*c*g*m*n*log(e) + (b*c*g*m*n*log(e) + a*d*g*m
*n*log(e))*x)*log(b*x + a)^2 + 3*(b*d*g*m*n*x^2*log(e) + a*c*g*m*n*log(e) +
 (b*c*g*m*n*log(e) + a*d*g*m*n*log(e))*x - (b*d*g*m*n^2*x^2 + a*c*g*m*n^2 +
 (b*c*g*m*n^2 + a*d*g*m*n^2)*x)*log(b*x + a))*log(d*x + c)^2 + 3*(b*c*g*log
(e)^2*log(h) - a*d*g*log(e)^2*log(h))*x - 3*(b*d*g*m*x^2*log(e)^2 + a*c*g*m
*log(e)^2 + (b*c*g*m*log(e)^2 + a*d*g*m*log(e)^2)*x)*log(b*x + a) + 3*(b*d*
g*m*x^2*log(e)^2 + a*c*g*m*log(e)^2 + (b*d*g*m*n^2*x^2 + a*c*g*m*n^2 + (b*c
*g*m*n^2 + a*d*g*m*n^2)*x)*log(b*x + a)^2 + (b*c*g*m*log(e)^2 + a*d*g*m*log
(e)^2)*x - 2*(b*d*g*m*n*x^2*log(e) + a*c*g*m*n*log(e) + (b*c*g*m*n*log(e) +
 a*d*g*m*n*log(e))*x)*log(b*x + a))*log(d*x + c))*log((b*x + a)^n) - 4*(3*b
*c*f*log(e)^2*log(h) - 3*a*d*f*log(e)^2*log(h) - (b*d*g*m*n^2*x^2 + a*c*g*m
*n^2 + (b*c*g*m*n^2 + a*d*g*m*n^2)*x)*log(b*x + a)^3 + (b*d*g*m*n^2*x^2 + a
*c*g*m*n^2 + (b*c*g*m*n^2 + a*d*g*m*n^2)*x)*log(d*x + c)^3 + 3*(b*d*g*m*n*x
^2*log(e) + a*c*g*m*n*log(e) + (b*c*g*m*n*log(e) + a*d*g*m*n*log(e))*x)*log
(b*x + a)^2 + 3*(b*d*g*m*n*x^2*log(e) + a*c*g*m*n*log(e) + (b*c*g*m*n*log(e
) + a*d*g*m*n*log(e))*x - (b*d*g*m*n^2*x^2 + a*c*g*m*n^2 + (b*c*g*m*n^2 + a
*d*g*m*n^2)*x)*log(b*x + a))*log(d*x + c)^2 + 3*(b*c*f*log(h) - a*d*f*log(h
) + (b*c*g*log(h) - a*d*g*log(h))*x - (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a
*d*g*m)*x)*log(b*x + a) + (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g*m)*x)*l
og(d*x + c))*log((b*x + a)^n)^2 + 3*(b*c*g*log(e)^2*log(h) - a*d*g*log(e)^2
*log(h))*x - 3*(b*d*g*m*x^2*log(e)^2 + a*c*g*m*log(e)^2 + (b*c*g*m*log(e)^2
 + a*d*g*m*log(e)^2)*x)*log(b*x + a) + 3*(b*d*g*m*x^2*log(e)^2 + a*c*g*m*lo
g(e)^2 + (b*d*g*m*n^2*x^2 + a*c*g*m*n^2 + (b*c*g*m*n^2 + a*d*g*m*n^2)*x)*lo
g(b*x + a)^2 + (b*c*g*m*log(e)^2 + a*d*g*m*log(e)^2)*x - 2*(b*d*g*m*n*x^2*l
og(e) + a*c*g*m*n*log(e) + (b*c*g*m*n*log(e) + a*d*g*m*n*log(e))*x)*log(b*x
 + a))*log(d*x + c) + 3*(2*b*c*f*log(e)*log(h) - 2*a*d*f*log(e)*log(h) + (b
*d*g*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(b*x + a)^2 + (b*d
*g*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(d*x + c)^2 + 2*(b*c
*g*log(e)*log(h) - a*d*g*log(e)*log(h))*x - 2*(b*d*g*m*x^2*log(e) + a*c*g*m
```

```
*log(e) + (b*c*g*m*log(e) + a*d*g*m*log(e))*x)*log(b*x + a) + 2*(b*d*g*m*x^
2*log(e) + a*c*g*m*log(e) + (b*c*g*m*log(e) + a*d*g*m*log(e))*x - (b*d*g*m*
n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(b*x + a))*log(d*x + c))*
log((b*x + a)^n))*log((d*x + c)^n))/(a*b*c^2*f - a^2*c*d*f + (b^2*c*d*g - a
*b*d^2*g)*x^3 - (a*b*d^2*f + a^2*d^2*g - (c*d*f + c^2*g)*b^2)*x^2 + (b^2*c^
2*f + a*b*c^2*g - (d^2*f + c*d*g)*a^2)*x), x)
```

Giac [F]

$$\int \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx=\int \frac{\log\left((gx+f)^m h\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^3}{(bx+a)(dx+c)}\,dx$$

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^3*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x,
algorithm="giac")
```

```
[Out] integrate(log((g*x + f)^m*h)*log(e*((b*x + a)/(d*x + c))^n)^3/((b*x + a)*(d
*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx=\int \frac{\ln\left(h\,{\left(f+g\,x\right)}^m\right)\,{\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}^3}{\left(a+b\,x\right)\,\left(c+d\,x\right)}\,dx$$

```
[In] int((log(h*(f + g*x)^m)*log(e*((a + b*x)/(c + d*x))^n)^3)/((a + b*x)*(c + d
*x)),x)
```

```
[Out] int((log(h*(f + g*x)^m)*log(e*((a + b*x)/(c + d*x))^n)^3)/((a + b*x)*(c + d
*x)), x)
```

3.68 $\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log(h(f+gx)^m)}{(a+bx)(c+dx)}\,dx$

Optimal result 561
Rubi [A] (verified) 562
Mathematica [B] (verified) 566
Maple [F] 566
Fricas [F] 567
Sympy [F(-1)] 567
Maxima [F] 567
Giac [F] 568
Mupad [F(-1)] 569

Optimal result

Integrand size = 45, antiderivative size = 496

$$\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log(h(f+gx)^m)}{(a+bx)(c+dx)}\,dx = \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{3(bc-ad)n} + \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log(h(f+gx)^m)}{3(bc-ad)n} - \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{3(bc-ad)n} + \frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bc-ad} - \frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad} - \frac{2mn\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{bc-ad} + \frac{2mn\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{PolyLog}\left(3,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad} + \frac{2mn^2\operatorname{PolyLog}\left(4,\frac{d(a+bx)}{b(c+dx)}\right)}{bc-ad} - \frac{2mn^2\operatorname{PolyLog}\left(4,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad}$$

```
[Out] 1/3*m*ln(e*((b*x+a)/(d*x+c))^n)^3*ln((-a*d+b*c)/b/(d*x+c))/(-a*d+b*c)/n+1/3
*ln(e*((b*x+a)/(d*x+c))^n)^3*ln(h*(g*x+f)^m)/(-a*d+b*c)/n-1/3*m*ln(e*((b*x+
```

```
a)/(d*x+c))^n)^3*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*d+b*c)/n+m
*ln(e*((b*x+a)/(d*x+c))^n)^2*polylog(2,d*(b*x+a)/b/(d*x+c))/(-a*d+b*c)-m*ln
(e*((b*x+a)/(d*x+c))^n)^2*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/
(-a*d+b*c)-2*m*n*ln(e*((b*x+a)/(d*x+c))^n)*polylog(3,d*(b*x+a)/b/(d*x+c))/(
-a*d+b*c)+2*m*n*ln(e*((b*x+a)/(d*x+c))^n)*polylog(3,(-c*g+d*f)*(b*x+a)/(-a*
g+b*f)/(d*x+c))/(-a*d+b*c)+2*m*n^2*polylog(4,d*(b*x+a)/b/(d*x+c))/(-a*d+b*c
)-2*m*n^2*polylog(4,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*d+b*c)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2589, 2553, 2404, 2354, 2421, 2430, 6724}

$$\begin{aligned}\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx = {} & \frac{\log\left(h(f+gx)^m\right)\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3n(bc-ad)} \\ & -\frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad} \\ & +\frac{2mn\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{PolyLog}\left(3,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad} \\ & -\frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{3n(bc-ad)} \\ & +\frac{m\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bc-ad} \\ & -\frac{2mn\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bc-ad} \\ & +\frac{m\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3n(bc-ad)} \\ & -\frac{2mn^2\operatorname{PolyLog}\left(4,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad} \\ & +\frac{2mn^2\operatorname{PolyLog}\left(4,\frac{d(a+bx)}{b(c+dx)}\right)}{bc-ad}\end{aligned}$$

```
[In] Int[(Log[e*((a + b*x)/(c + d*x))^n]^2*Log[h*(f + g*x)^m])/((a + b*x)*(c + d
*x)),x]
```

```
[Out] (m*Log[e*((a + b*x)/(c + d*x))^n]^3*Log[(b*c - a*d)/(b*(c + d*x))])/(3*(b*c
 - a*d)*n) + (Log[e*((a + b*x)/(c + d*x))^n]^3*Log[h*(f + g*x)^m])/(3*(b*c
- a*d)*n) - (m*Log[e*((a + b*x)/(c + d*x))^n]^3*Log[1 - ((d*f - c*g)*(a + b
```

```
*x))/((b*f - a*g)*(c + d*x))])/(3*(b*c - a*d)*n) + (m*Log[e*((a + b*x)/(c +
 d*x))^n]^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b*c - a*d) - (m*Log[e
*((a + b*x)/(c + d*x))^n]^2*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)
*(c + d*x))])/(b*c - a*d) - (2*m*n*Log[e*((a + b*x)/(c + d*x))^n]*PolyLog[3
, (d*(a + b*x))/(b*(c + d*x))])/(b*c - a*d) + (2*m*n*Log[e*((a + b*x)/(c +
d*x))^n]*PolyLog[3, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/(b*c
- a*d) + (2*m*n^2*PolyLog[4, (d*(a + b*x))/(b*(c + d*x))])/(b*c - a*d) - (2
*m*n^2*PolyLog[4, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/(b*c -
a*d)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rule 2589

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*Log[(i_.)*((j_.)*((g_.) + (h_.)*(x_))^(t_.))^(u_.)]*(v_), x_S
ymbol] :> With[{k = Simplify[v*(a + b*x)*(c + d*x)]}, Simp[k*Log[i*(j*(g +
h*x)^t)^u]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s + 1)/(p*r*(s + 1)*(b*c
- a*d))), x] - Dist[k*h*t*(u/(p*r*(s + 1)*(b*c - a*d))), Int[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^(s + 1)/(g + h*x), x], x] /; FreeQ[k, x]] /; FreeQ[{
a, b, c, d, e, f, g, h, i, j, p, q, r, s, t, u}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[p + q, 0] && NeQ[s, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{3(bc-ad)n} - \frac{(gm)\int \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx}\,dx}{3(bc-ad)n}$$

$$= \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{3(bc-ad)n} - \frac{(gm)\text{Subst}\left(\int \frac{\log^3(ex^n)}{(b-dx)(bf-ag-(df-cg)x)}\,dx, x, \frac{a+bx}{c+dx}\right)}{3n}$$

$$= \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{3(bc-ad)n} - \frac{(gm)\text{Subst}\left(\int \left(\frac{d\log^3(ex^n)}{(bc-ad)g(b-dx)} + \frac{(-df+cg)\log^3(ex^n)}{(bc-ad)g(bf-ag-(df-cg)x)}\right)dx, x, \frac{a+bx}{c+dx}\right)}{3n}$$

$$= \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{3(bc-ad)n} - \frac{(dm)\text{Subst}\left(\int \frac{\log^3(ex^n)}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{3(bc-ad)n} + \frac{((df-cg)m)\text{Subst}\left(\int \frac{\log^3(ex^n)}{bf-ag+(-df+cg)x}\,dx, x, \frac{a+bx}{c+dx}\right)}{3(bc-ad)n}$$

$$= \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{3(bc-ad)n} + \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{3(bc-ad)n} - \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{3(bc-ad)n} - \frac{m\text{Subst}\left(\int \frac{\log^2(ex^n)\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{bc-ad} + \frac{m\text{Subst}\left(\int \frac{\log^2(ex^n)\log\left(1+\frac{(-df+cg)x}{bf-ag}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{bc-ad}$$

$$= \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{3(bc-ad)n} + \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{3(bc-ad)n} - \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{3(bc-ad)n} + \frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bc-ad} - \frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{Li}_2\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad} - \frac{(2mn)\text{Subst}\left(\int \frac{\log(ex^n)\text{Li}_2\left(\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{bc-ad} + \frac{(2mn)\text{Subst}\left(\int \frac{\log(ex^n)\text{Li}_2\left(-\frac{(-df+cg)x}{bf-ag}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{bc-ad}$$

$$= \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{3(bc-ad)n} + \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{3(bc-ad)n} - \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{3(bc-ad)n} + \frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bc-ad} - \frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{Li}_2\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad} - \frac{2mn\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bc-ad} + \frac{2mn\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{Li}_3\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad} + \frac{(2mn^2)\,\text{Subst}\left(\int \frac{\text{Li}_3\left(\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{bc-ad} - \frac{(2mn^2)\,\text{Subst}\left(\int \frac{\text{Li}_3\left(-\frac{(-df+cg)x}{bf-ag}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{bc-ad}$$

$$
\begin{aligned}
&= \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{3(bc-ad)n} + \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{3(bc-ad)n} \\
&\quad - \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{3(bc-ad)n} \\
&\quad + \frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bc-ad} - \frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{Li}_2\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad} \\
&\quad - \frac{2mn\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bc-ad} + \frac{2mn\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{Li}_3\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad} \\
&\quad + \frac{2mn^2\operatorname{Li}_4\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bc-ad} - \frac{2mn^2\operatorname{Li}_4\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 25557 vs. $2(496) = 992$.

Time = 7.15 (sec) , antiderivative size = 25557, normalized size of antiderivative = 51.53

$$
\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx = \text{Result too large to show}
$$

```
[In] Integrate[(Log[e*((a + b*x)/(c + d*x))^n]^2*Log[h*(f + g*x)^m])/((a + b*x)*
(c + d*x)),x]
```

```
[Out] Result too large to show
```

Maple [F]

$$
\int \frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2\ln\left(h(gx+f)^m\right)}{(bx+a)\,(dx+c)}dx
$$

```
[In] int(ln(e*((b*x+a)/(d*x+c))^n)^2*ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x)
```

```
[Out] int(ln(e*((b*x+a)/(d*x+c))^n)^2*ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x)
```

Fricas [F]

$$\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx=\int \frac{\log\left((gx+f)^m h\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2}{(bx+a)(dx+c)}\,dx$$

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^2*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x,
algorithm="fricas")
```

```
[Out] integral(log((g*x + f)^m*h)*log(e*((b*x + a)/(d*x + c))^n)^2/(b*d*x^2 + a*c
 + (b*c + a*d)*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx=\text{Timed out}$$

```
[In] integrate(ln(e*((b*x+a)/(d*x+c))**n)**2*ln(h*(g*x+f)**m)/(b*x+a)/(d*x+c),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx=\int \frac{\log\left((gx+f)^m h\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2}{(bx+a)(dx+c)}\,dx$$

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^2*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x,
algorithm="maxima")
```

```
[Out] 1/3*(n^2*log(b*x + a)^3 - n^2*log(d*x + c)^3 - 3*n*log(b*x + a)^2*log(e) +
3*(n^2*log(b*x + a) - n*log(e))*log(d*x + c)^2 + 3*(log(b*x + a) - log(d*x
+ c))*log((b*x + a)^n)^2 + 3*(log(b*x + a) - log(d*x + c))*log((d*x + c)^n)
^2 + 3*log(b*x + a)*log(e)^2 - 3*(n^2*log(b*x + a)^2 - 2*n*log(b*x + a)*log
(e) + log(e)^2)*log(d*x + c) - 3*(n*log(b*x + a)^2 + n*log(d*x + c)^2 - 2*(
n*log(b*x + a) - log(e))*log(d*x + c) - 2*log(b*x + a)*log(e))*log((b*x + a
)^n) + 3*(n*log(b*x + a)^2 + n*log(d*x + c)^2 - 2*(n*log(b*x + a) - log(e))
*log(d*x + c) - 2*(log(b*x + a) - log(d*x + c))*log((b*x + a)^n) - 2*log(b*
x + a)*log(e))*log((d*x + c)^n))*log((g*x + f)^m)/(b*c - a*d) - integrate(-
1/3*(3*b*c*f*log(e)^2*log(h) - 3*a*d*f*log(e)^2*log(h) - (b*d*g*m*n^2*x^2 +
 a*c*g*m*n^2 + (b*c*g*m*n^2 + a*d*g*m*n^2)*x)*log(b*x + a)^3 + (b*d*g*m*n^2
*x^2 + a*c*g*m*n^2 + (b*c*g*m*n^2 + a*d*g*m*n^2)*x)*log(d*x + c)^3 + 3*(b*d
*g*m*n*x^2*log(e) + a*c*g*m*n*log(e) + (b*c*g*m*n*log(e) + a*d*g*m*n*log(e)
```

```
)*x)*log(b*x + a)^2 + 3*(b*d*g*m*n*x^2*log(e) + a*c*g*m*n*log(e) + (b*c*g*m
*n*log(e) + a*d*g*m*n*log(e))*x - (b*d*g*m*n^2*x^2 + a*c*g*m*n^2 + (b*c*g*m
*n^2 + a*d*g*m*n^2)*x)*log(b*x + a))*log(d*x + c)^2 + 3*(b*c*f*log(h) - a*d
*f*log(h) + (b*c*g*log(h) - a*d*g*log(h))*x - (b*d*g*m*x^2 + a*c*g*m + (b*c
*g*m + a*d*g*m)*x)*log(b*x + a) + (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g
*m)*x)*log(d*x + c))*log((b*x + a)^n)^2 + 3*(b*c*f*log(h) - a*d*f*log(h) +
(b*c*g*log(h) - a*d*g*log(h))*x - (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g
*m)*x)*log(b*x + a) + (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g*m)*x)*log(d
*x + c))*log((d*x + c)^n)^2 + 3*(b*c*g*log(e)^2*log(h) - a*d*g*log(e)^2*log
(h))*x - 3*(b*d*g*m*x^2*log(e)^2 + a*c*g*m*log(e)^2 + (b*c*g*m*log(e)^2 + a
*d*g*m*log(e)^2)*x)*log(b*x + a) + 3*(b*d*g*m*x^2*log(e)^2 + a*c*g*m*log(e)
^2 + (b*d*g*m*n^2*x^2 + a*c*g*m*n^2 + (b*c*g*m*n^2 + a*d*g*m*n^2)*x)*log(b*
x + a)^2 + (b*c*g*m*log(e)^2 + a*d*g*m*log(e)^2)*x - 2*(b*d*g*m*n*x^2*log(e
) + a*c*g*m*n*log(e) + (b*c*g*m*n*log(e) + a*d*g*m*n*log(e))*x)*log(b*x + a
))*log(d*x + c) + 3*(2*b*c*f*log(e)*log(h) - 2*a*d*f*log(e)*log(h) + (b*d*g
*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(b*x + a)^2 + (b*d*g*m
*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(d*x + c)^2 + 2*(b*c*g*l
og(e)*log(h) - a*d*g*log(e)*log(h))*x - 2*(b*d*g*m*x^2*log(e) + a*c*g*m*log
(e) + (b*c*g*m*log(e) + a*d*g*m*log(e))*x)*log(b*x + a) + 2*(b*d*g*m*x^2*lo
g(e) + a*c*g*m*log(e) + (b*c*g*m*log(e) + a*d*g*m*log(e))*x - (b*d*g*m*n*x^
2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(b*x + a))*log(d*x + c))*log(
(b*x + a)^n) - 3*(2*b*c*f*log(e)*log(h) - 2*a*d*f*log(e)*log(h) + (b*d*g*m*
n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(b*x + a)^2 + (b*d*g*m*n*
x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(d*x + c)^2 + 2*(b*c*g*log(
e)*log(h) - a*d*g*log(e)*log(h))*x - 2*(b*d*g*m*x^2*log(e) + a*c*g*m*log(e)
 + (b*c*g*m*log(e) + a*d*g*m*log(e))*x)*log(b*x + a) + 2*(b*d*g*m*x^2*log(e
) + a*c*g*m*log(e) + (b*c*g*m*log(e) + a*d*g*m*log(e))*x - (b*d*g*m*n*x^2 +
 a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(b*x + a))*log(d*x + c) + 2*(b*c
*f*log(h) - a*d*f*log(h) + (b*c*g*log(h) - a*d*g*log(h))*x - (b*d*g*m*x^2 +
 a*c*g*m + (b*c*g*m + a*d*g*m)*x)*log(b*x + a) + (b*d*g*m*x^2 + a*c*g*m + (
b*c*g*m + a*d*g*m)*x)*log(d*x + c))*log((b*x + a)^n))*log((d*x + c)^n))/(a*
b*c^2*f - a^2*c*d*f + (b^2*c*d*g - a*b*d^2*g)*x^3 - (a*b*d^2*f + a^2*d^2*g
- (c*d*f + c^2*g)*b^2)*x^2 + (b^2*c^2*f + a*b*c^2*g - (d^2*f + c*d*g)*a^2)*
x), x)
```

Giac [F]

$$\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\log\left((gx+f)^m h\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2}{(bx+a)(dx+c)}\,dx$$

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^2*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x,
algorithm="giac")
```

```
[Out] integrate(log((g*x + f)^m*h)*log(e*((b*x + a)/(d*x + c))^n)^2/((b*x + a)*(d
*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx=\int \frac{\ln\left(h\,{\left(f+g\,x\right)}^m\right)\,{\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}^2}{\left(a+b\,x\right)\,\left(c+d\,x\right)}\,dx$$

```
[In] int((log(h*(f + g*x)^m)*log(e*((a + b*x)/(c + d*x))^n)^2)/((a + b*x)*(c + d
*x)),x)
```

```
[Out] int((log(h*(f + g*x)^m)*log(e*((a + b*x)/(c + d*x))^n)^2)/((a + b*x)*(c + d
*x)), x)
```

3.69 $\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log(h(f+gx)^m)}{(a+bx)(c+dx)}\,dx$

Optimal result 570
Rubi [A] (verified) 571
Mathematica [B] (verified) 574
Maple [F] 575
Fricas [F] 575
Sympy [F(-1)] 576
Maxima [F] 576
Giac [F] 577
Mupad [F(-1)] 577

Optimal result

Integrand size = 43, antiderivative size = 371

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx = \frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{2(bc-ad)n} + \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{2(bc-ad)n} - \frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{2(bc-ad)n} + \frac{m\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bc-ad} - \frac{m\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad} - \frac{mn\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{bc-ad} + \frac{mn\operatorname{PolyLog}\left(3,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad}$$

```
[Out] 1/2*m*ln(e*((b*x+a)/(d*x+c))^n)^2*ln((-a*d+b*c)/b/(d*x+c))/(-a*d+b*c)/n+1/2
*ln(e*((b*x+a)/(d*x+c))^n)^2*ln(h*(g*x+f)^m)/(-a*d+b*c)/n-1/2*m*ln(e*((b*x+
a)/(d*x+c))^n)^2*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*d+b*c)/n+m
*ln(e*((b*x+a)/(d*x+c))^n)*polylog(2,d*(b*x+a)/b/(d*x+c))/(-a*d+b*c)-m*ln(e
*((b*x+a)/(d*x+c))^n)*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*
d+b*c)-m*n*polylog(3,d*(b*x+a)/b/(d*x+c))/(-a*d+b*c)+m*n*polylog(3,(-c*g+d*
f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*d+b*c)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.140, Rules used = {2589, 2553, 2404, 2354, 2421, 6724}

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx = \frac{\log\left(h(f+gx)^m\right)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2n(bc-ad)} - \frac{m\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{PolyLog}\left(2,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad} - \frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{2n(bc-ad)} + \frac{m\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bc-ad} + \frac{m\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2n(bc-ad)} + \frac{mn\operatorname{PolyLog}\left(3,\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad} - \frac{mn\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{bc-ad}$$

```
[In] Int[(Log[e*((a + b*x)/(c + d*x))^n]*Log[h*(f + g*x)^m])/((a + b*x)*(c + d*x
)),x]
```

```
[Out] (m*Log[e*((a + b*x)/(c + d*x))^n]^2*Log[(b*c - a*d)/(b*(c + d*x))])/(2*(b*c
 - a*d)*n) + (Log[e*((a + b*x)/(c + d*x))^n]^2*Log[h*(f + g*x)^m])/(2*(b*c
- a*d)*n) - (m*Log[e*((a + b*x)/(c + d*x))^n]^2*Log[1 - ((d*f - c*g)*(a + b
*x))/((b*f - a*g)*(c + d*x))])/(2*(b*c - a*d)*n) + (m*Log[e*((a + b*x)/(c +
 d*x))^n]*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b*c - a*d) - (m*Log[e*(
(a + b*x)/(c + d*x))^n]*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c
+ d*x))])/(b*c - a*d) - (m*n*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/(b*c
- a*d) + (m*n*PolyLog[3, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/
(b*c - a*d)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rule 2589

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*Log[(i_.)*((j_.)*((g_.) + (h_.)*(x_))^(t_.))^(u_.)]*(v_), x_S
ymbol] :> With[{k = Simplify[v*(a + b*x)*(c + d*x)]}, Simp[k*Log[i*(j*(g +
h*x)^t)^u]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s + 1)/(p*r*(s + 1)*(b*c
- a*d))), x] - Dist[k*h*t*(u/(p*r*(s + 1)*(b*c - a*d))), Int[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^(s + 1)/(g + h*x), x], x] /; FreeQ[k, x]] /; FreeQ[{
a, b, c, d, e, f, g, h, i, j, p, q, r, s, t, u}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[p + q, 0] && NeQ[s, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{2(bc-ad)n} - \frac{(gm)\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx}\,dx}{2(bc-ad)n}$$

$$= \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{2(bc-ad)n} - \frac{(gm)\text{Subst}\left(\int \frac{\log^2(ex^n)}{(b-dx)(bf-ag-(df-cg)x)}\,dx, x, \frac{a+bx}{c+dx}\right)}{2n}$$

$$= \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{2(bc-ad)n} - \frac{(gm)\text{Subst}\left(\int\left(\frac{d\log^2(ex^n)}{(bc-ad)g(b-dx)} + \frac{(-df+cg)\log^2(ex^n)}{(bc-ad)g(bf-ag-(df-cg)x)}\right)dx, x, \frac{a+bx}{c+dx}\right)}{2n}$$

$$= \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{2(bc-ad)n} - \frac{(dm)\text{Subst}\left(\int\frac{\log^2(ex^n)}{b-dx}dx, x, \frac{a+bx}{c+dx}\right)}{2(bc-ad)n} + \frac{((df-cg)m)\text{Subst}\left(\int\frac{\log^2(ex^n)}{bf-ag+(-df+cg)x}dx, x, \frac{a+bx}{c+dx}\right)}{2(bc-ad)n}$$

$$= \frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{2(bc-ad)n} + \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{2(bc-ad)n} - \frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{2(bc-ad)n} - \frac{m\text{Subst}\left(\int\frac{\log(ex^n)\log\left(1-\frac{dx}{b}\right)}{x}dx, x, \frac{a+bx}{c+dx}\right)}{bc-ad} + \frac{m\text{Subst}\left(\int\frac{\log(ex^n)\log\left(1+\frac{(-df+cg)x}{bf-ag}\right)}{x}dx, x, \frac{a+bx}{c+dx}\right)}{bc-ad}$$

$$= \frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{2(bc-ad)n} + \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{2(bc-ad)n} - \frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{2(bc-ad)n} + \frac{m\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bc-ad} - \frac{m\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{Li}_2\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad} - \frac{(mn)\text{Subst}\left(\int\frac{\text{Li}_2\left(\frac{dx}{b}\right)}{x}dx, x, \frac{a+bx}{c+dx}\right)}{bc-ad} + \frac{(mn)\text{Subst}\left(\int\frac{\text{Li}_2\left(-\frac{(-df+cg)x}{bf-ag}\right)}{x}dx, x, \frac{a+bx}{c+dx}\right)}{bc-ad}$$

$$= \frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{2(bc-ad)n} + \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{2(bc-ad)n} - \frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{2(bc-ad)n} + \frac{m\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bc-ad} - \frac{m\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{Li}_2\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad} - \frac{mn\text{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bc-ad} + \frac{mn\text{Li}_3\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{bc-ad}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1842 vs. 2(371) = 742.

Time = 0.88 (sec) , antiderivative size = 1842, normalized size of antiderivative = 4.96

$$\int \frac{\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log \left(h(f+gx)^m\right)}{(a+bx)(c+dx)} \, dx = \frac{mn \log \left(\frac{-bc+ad}{d(a+bx)}\right) \log^2 \left(\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right) - 2mn \log^2 \left(\frac{a}{b}+x\right) \log(f+gx) + 2mn \log \left(\frac{a}{b}+x\right) \log \left(\frac{c}{d}+x\right) \log(f+$$

```
[In] Integrate[(Log[e*((a + b*x)/(c + d*x))^n]*Log[h*(f + g*x)^m])/((a + b*x)*(c
 + d*x)),x]
```

```
[Out] (m*n*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*f - a*g)*(c + d*x))/((d*f -
c*g)*(a + b*x))]^2 - 2*m*n*Log[a/b + x]^2*Log[f + g*x] + 2*m*n*Log[a/b + x]
*Log[c/d + x]*Log[f + g*x] - 2*m*n*Log[c/d + x]^2*Log[f + g*x] + 2*m*n*Log[
a/b + x]*Log[a + b*x]*Log[f + g*x] - 2*m*n*Log[c/d + x]*Log[a + b*x]*Log[f
+ g*x] + 2*m*n*Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[f + g*x]
+ 2*m*Log[a/b + x]*Log[e*((a + b*x)/(c + d*x))^n]*Log[f + g*x] - 2*m*Log[c/
d + x]*Log[e*((a + b*x)/(c + d*x))^n]*Log[f + g*x] - 2*m*Log[a + b*x]*Log[e
*((a + b*x)/(c + d*x))^n]*Log[f + g*x] - 2*m*n*Log[a/b + x]*Log[c + d*x]*Lo
g[f + g*x] + 2*m*n*Log[c/d + x]*Log[c + d*x]*Log[f + g*x] + 2*m*Log[e*((a +
 b*x)/(c + d*x))^n]*Log[c + d*x]*Log[f + g*x] + 2*m*n*Log[a/b + x]*Log[(b*(
c + d*x))/(b*c - a*d)]*Log[f + g*x] + m*n*Log[a/b + x]^2*Log[(b*(f + g*x))/
(b*f - a*g)] - 2*m*Log[a/b + x]*Log[e*((a + b*x)/(c + d*x))^n]*Log[(b*(f +
g*x))/(b*f - a*g)] - 2*m*n*Log[a/b + x]*Log[(g*(c + d*x))/(-(d*f) + c*g)]*L
og[(b*(f + g*x))/(b*f - a*g)] + m*n*Log[(g*(c + d*x))/(-(d*f) + c*g)]^2*Log
[(b*(f + g*x))/(b*f - a*g)] - 2*m*n*Log[(g*(c + d*x))/(-(d*f) + c*g)]*Log[(
(b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))]*Log[(b*(f + g*x))/(b*f - a*
g)] + m*n*Log[((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))]^2*Log[(b*(f
+ g*x))/(b*f - a*g)] - 2*m*n*Log[a/b + x]*Log[c/d + x]*Log[(d*(f + g*x))/(d
*f - c*g)] + m*n*Log[c/d + x]^2*Log[(d*(f + g*x))/(d*f - c*g)] + 2*m*Log[c/
d + x]*Log[e*((a + b*x)/(c + d*x))^n]*Log[(d*(f + g*x))/(d*f - c*g)] + 2*m*
n*Log[a/b + x]*Log[(g*(c + d*x))/(-(d*f) + c*g)]*Log[(d*(f + g*x))/(d*f - c
*g)] - m*n*Log[(g*(c + d*x))/(-(d*f) + c*g)]^2*Log[(d*(f + g*x))/(d*f - c*g
)] + 2*m*n*Log[(g*(c + d*x))/(-(d*f) + c*g)]*Log[((b*f - a*g)*(c + d*x))/((
d*f - c*g)*(a + b*x))]*Log[(d*(f + g*x))/(d*f - c*g)] - m*n*Log[((b*f - a*g
)*(c + d*x))/((d*f - c*g)*(a + b*x))]^2*Log[((-(b*c) + a*d)*(f + g*x))/((d*
f - c*g)*(a + b*x))] + n*Log[a/b + x]^2*Log[h*(f + g*x)^m] + n*Log[c/d + x]
^2*Log[h*(f + g*x)^m] - 2*n*Log[a/b + x]*Log[a + b*x]*Log[h*(f + g*x)^m] +
2*n*Log[c/d + x]*Log[a + b*x]*Log[h*(f + g*x)^m] - 2*n*Log[c/d + x]*Log[(d*
(a + b*x))/(-(b*c) + a*d)]*Log[h*(f + g*x)^m] + 2*Log[a + b*x]*Log[e*((a +
b*x)/(c + d*x))^n]*Log[h*(f + g*x)^m] + 2*n*Log[a/b + x]*Log[c + d*x]*Log[h
*(f + g*x)^m] - 2*n*Log[c/d + x]*Log[c + d*x]*Log[h*(f + g*x)^m] - 2*Log[e*
```

```
((a + b*x)/(c + d*x))^n]*Log[c + d*x]*Log[h*(f + g*x)^m] - 2*n*Log[a/b + x]
*Log[(b*(c + d*x))/(b*c - a*d)]*Log[h*(f + g*x)^m] + 2*n*(m*Log[f + g*x] -
Log[h*(f + g*x)^m])*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] - 2*m*(Log[e*(
(a + b*x)/(c + d*x))^n] + n*Log[((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b
*x))])*PolyLog[2, (g*(a + b*x))/(-(b*f) + a*g)] + 2*m*n*Log[f + g*x]*PolyLo
g[2, (b*(c + d*x))/(b*c - a*d)] - 2*n*Log[h*(f + g*x)^m]*PolyLog[2, (b*(c +
 d*x))/(b*c - a*d)] + 2*m*Log[e*((a + b*x)/(c + d*x))^n]*PolyLog[2, (g*(c +
 d*x))/(-(d*f) + c*g)] + 2*m*n*Log[((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a
+ b*x))]*PolyLog[2, (g*(c + d*x))/(-(d*f) + c*g)] + 2*m*n*Log[((b*f - a*g)*
(c + d*x))/((d*f - c*g)*(a + b*x))]*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))]
 - 2*m*n*Log[((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))]*PolyLog[2, ((
b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))] - 2*m*n*PolyLog[3, (b*(c + d
*x))/(d*(a + b*x))] + 2*m*n*PolyLog[3, ((b*f - a*g)*(c + d*x))/((d*f - c*g)
*(a + b*x))])/(2*b*c - 2*a*d)
```

Maple [F]

$$\int \frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\ln\left(h(gx+f)^{m}\right)}{(bx+a)\,(dx+c)}dx$$

```
[In] int(ln(e*((b*x+a)/(d*x+c))^n)*ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x)
[Out] int(ln(e*((b*x+a)/(d*x+c))^n)*ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x)
```

Fricas [F]

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\log\left(h(f+gx)^{m}\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\log\left((gx+f)^{m}h\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{(bx+a)(dx+c)}\,dx$$

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x, al
gorithm="fricas")
[Out] integral(log((g*x + f)^m*h)*log(e*((b*x + a)/(d*x + c))^n)/(b*d*x^2 + a*c +
 (b*c + a*d)*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate(ln(e*((b*x+a)/(d*x+c))**n)*ln(h*(g*x+f)**m)/(b*x+a)/(d*x+c),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\log\left((gx+f)^m h\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(bx+a)(dx+c)}\,dx$$

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x, al
gorithm="maxima")
[Out] -1/2*(n*log(b*x + a)^2 + n*log(d*x + c)^2 - 2*(n*log(b*x + a) - log(e))*log
(d*x + c) - 2*(log(b*x + a) - log(d*x + c))*log((b*x + a)^n) + 2*(log(b*x +
 a) - log(d*x + c))*log((d*x + c)^n) - 2*log(b*x + a)*log(e))*log((g*x + f)
^m)/(b*c - a*d) + integrate(1/2*(2*b*c*f*log(e)*log(h) - 2*a*d*f*log(e)*log
(h) + (b*d*g*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(b*x + a)^
2 + (b*d*g*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(d*x + c)^2
+ 2*(b*c*g*log(e)*log(h) - a*d*g*log(e)*log(h))*x - 2*(b*d*g*m*x^2*log(e) +
 a*c*g*m*log(e) + (b*c*g*m*log(e) + a*d*g*m*log(e))*x)*log(b*x + a) + 2*(b*
d*g*m*x^2*log(e) + a*c*g*m*log(e) + (b*c*g*m*log(e) + a*d*g*m*log(e))*x - (
b*d*g*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(b*x + a))*log(d*
x + c) + 2*(b*c*f*log(h) - a*d*f*log(h) + (b*c*g*log(h) - a*d*g*log(h))*x -
 (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g*m)*x)*log(b*x + a) + (b*d*g*m*x^
2 + a*c*g*m + (b*c*g*m + a*d*g*m)*x)*log(d*x + c))*log((b*x + a)^n) - 2*(b*
c*f*log(h) - a*d*f*log(h) + (b*c*g*log(h) - a*d*g*log(h))*x - (b*d*g*m*x^2
+ a*c*g*m + (b*c*g*m + a*d*g*m)*x)*log(b*x + a) + (b*d*g*m*x^2 + a*c*g*m +
(b*c*g*m + a*d*g*m)*x)*log(d*x + c))*log((d*x + c)^n))/(a*b*c^2*f - a^2*c*d
*f + (b^2*c*d*g - a*b*d^2*g)*x^3 - (a*b*d^2*f + a^2*d^2*g - (c*d*f + c^2*g)
*b^2)*x^2 + (b^2*c^2*f + a*b*c^2*g - (d^2*f + c*d*g)*a^2)*x), x)
```

Giac [F]

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\log\left((gx+f)^m h\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(bx+a)(dx+c)}\,dx$$

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x, al
gorithm="giac")
```

```
[Out] integrate(log((g*x + f)^m*h)*log(e*((b*x + a)/(d*x + c))^n)/((b*x + a)*(d*x
 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\ln\left(h\,{\left(f+g\,x\right)}^m\right)\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{\left(a+b\,x\right)\,\left(c+d\,x\right)}\,dx$$

```
[In] int((log(h*(f + g*x)^m)*log(e*((a + b*x)/(c + d*x))^n))/((a + b*x)*(c + d*x
)),x)
```

```
[Out] int((log(h*(f + g*x)^m)*log(e*((a + b*x)/(c + d*x))^n))/((a + b*x)*(c + d*x
)), x)
```

3.70 $\int \frac{\log(h(f+gx)^m)}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

Optimal result 578
Rubi [N/A] 578
Mathematica [N/A] 579
Maple [N/A] 579
Fricas [N/A] 579
Sympy [F(-1)] 580
Maxima [N/A] 580
Giac [N/A] 580
Mupad [N/A] 581

Optimal result

Integrand size = 45, antiderivative size = 45

$$\int \frac{\log(h(f+gx)^m)}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{b\text{Int}\left(\frac{\log(h(f+gx)^m)}{(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)},x\right)}{bc-ad} - \frac{d\text{Int}\left(\frac{\log(h(f+gx)^m)}{(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)},x\right)}{bc-ad}$$

```
[Out] b*Unintegrable(ln(h*(g*x+f)^m)/(b*x+a)/ln(e*((b*x+a)/(d*x+c))^n),x)/(-a*d+b
*c)-d*Unintegrable(ln(h*(g*x+f)^m)/(d*x+c)/ln(e*((b*x+a)/(d*x+c))^n),x)/(-a
*d+b*c)
```

Rubi [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log(h(f+gx)^m)}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{\log(h(f+gx)^m)}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

```
[In] Int[Log[h*(f + g*x)^m]/((a + b*x)*(c + d*x)*Log[e*((a + b*x)/(c + d*x))^n])
,x]
```

```
[Out] (b*Defer[Int][Log[h*(f + g*x)^m]/((a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])
, x])/(b*c - a*d) - (d*Defer[Int][Log[h*(f + g*x)^m]/((c + d*x)*Log[e*((a +
 b*x)/(c + d*x))^n]), x])/(b*c - a*d)
```

Rubi steps

$$\text{integral} = \int \left(\frac{b\log(h(f+gx)^m)}{(bc-ad)(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} - \frac{d\log(h(f+gx)^m)}{(bc-ad)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\right)dx$$

$$= \frac{b\int \frac{\log(h(f+gx)^m)}{(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx}{bc-ad} - \frac{d\int \frac{\log(h(f+gx)^m)}{(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx}{bc-ad}$$

Mathematica [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int \frac{\log(h(f+gx)^m)}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{\log(h(f+gx)^m)}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

```
[In] Integrate[Log[h*(f + g*x)^m]/((a + b*x)*(c + d*x)*Log[e*((a + b*x)/(c + d*x
))^n]),x]
[Out] Integrate[Log[h*(f + g*x)^m]/((a + b*x)*(c + d*x)*Log[e*((a + b*x)/(c + d*x
))^n]), x]
```

Maple [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{\ln(h(gx+f)^m)}{(bx+a)(dx+c)\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}dx$$

```
[In] int(ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/ln(e*((b*x+a)/(d*x+c))^n),x)
[Out] int(ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/ln(e*((b*x+a)/(d*x+c))^n),x)
```

Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.20

$$\int \frac{\log(h(f+gx)^m)}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{\log((gx+f)^m h)}{(bx+a)(dx+c)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

```
[In] integrate(log(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/log(e*((b*x+a)/(d*x+c))^n),x, al
gorithm="fricas")
[Out] integral(log((g*x + f)^m*h)/((b*d*x^2 + a*c + (b*c + a*d)*x)*log(e*((b*x +
a)/(d*x + c))^n)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(h(f+gx)^m\right)}{(a+bx)(c+dx)\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\text{Timed out}$$

```
[In] integrate(ln(h*(g*x+f)**m)/(b*x+a)/(d*x+c)/ln(e*((b*x+a)/(d*x+c))**n),x)
```

```
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 1.72 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int \frac{\log \left(h(f+gx)^m\right)}{(a+bx)(c+dx)\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{\log \left((gx+f)^m h\right)}{(bx+a)(dx+c)\log \left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

```
[In] integrate(log(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/log(e*((b*x+a)/(d*x+c))^n),x, al
gorithm="maxima")
```

```
[Out] integrate(log((g*x + f)^m*h)/((b*x + a)*(d*x + c)*log(e*((b*x + a)/(d*x + c
))^n)), x)
```

Giac [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int \frac{\log \left(h(f+gx)^m\right)}{(a+bx)(c+dx)\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{\log \left((gx+f)^m h\right)}{(bx+a)(dx+c)\log \left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

```
[In] integrate(log(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/log(e*((b*x+a)/(d*x+c))^n),x, al
gorithm="giac")
```

```
[Out] integrate(log((g*x + f)^m*h)/((b*x + a)*(d*x + c)*log(e*((b*x + a)/(d*x + c
))^n)), x)
```

Mupad [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int \frac{\log \left(h(f+gx)^m\right)}{(a+bx)(c+dx)\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{\ln \left(h\,{\left(f+g\,x\right)}^m\right)}{\ln \left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\,\left(a+b\,x\right)\,\left(c+d\,x\right)} \,d x$$

```
[In] int(log(h*(f + g*x)^m)/(log(e*((a + b*x)/(c + d*x))^n)*(a + b*x)*(c + d*x))
,x)
```

```
[Out] int(log(h*(f + g*x)^m)/(log(e*((a + b*x)/(c + d*x))^n)*(a + b*x)*(c + d*x))
, x)
```

3.71 $\int \frac{\log(h(f+gx)^m)}{(a+bx)(c+dx)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

Optimal result 582
Rubi [N/A] 582
Mathematica [N/A] 583
Maple [N/A] 583
Fricas [N/A] 583
Sympy [F(-1)] 584
Maxima [N/A] 584
Giac [N/A] 584
Mupad [N/A] 585

Optimal result

Integrand size = 45, antiderivative size = 45

$$\int \frac{\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = -\frac{\log\left(h(f+gx)^m\right)}{(bc-ad)n\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} + \frac{gm\text{Int}\left(\frac{1}{(f+gx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)},x\right)}{(bc-ad)n}$$

```
[Out] -ln(h*(g*x+f)^m)/(-a*d+b*c)/n/ln(e*((b*x+a)/(d*x+c))^n)+g*m*Unintegrable(1/
(g*x+f)/ln(e*((b*x+a)/(d*x+c))^n),x)/(-a*d+b*c)/n
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

```
[In] Int[Log[h*(f + g*x)^m]/((a + b*x)*(c + d*x)*Log[e*((a + b*x)/(c + d*x))^n]^
2),x]
```

```
[Out] -(Log[h*(f + g*x)^m]/((b*c - a*d)*n*Log[e*((a + b*x)/(c + d*x))^n])) + (g*m
*Defer[Int][1/((f + g*x)*Log[e*((a + b*x)/(c + d*x))^n]), x])/((b*c - a*d)*
n)
```

Rubi steps

$$\text{integral} = -\frac{\log\left(h(f+gx)^m\right)}{(bc-ad)n\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} + \frac{(gm)\int\frac{1}{(f+gx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx}{(bc-ad)n}$$

Mathematica [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int\frac{\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int\frac{\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

```
[In] Integrate[Log[h*(f + g*x)^m]/((a + b*x)*(c + d*x)*Log[e*((a + b*x)/(c + d*x
))^n]^2),x]
```

```
[Out] Integrate[Log[h*(f + g*x)^m]/((a + b*x)*(c + d*x)*Log[e*((a + b*x)/(c + d*x
))^n]^2), x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int\frac{\ln\left(h(gx+f)^m\right)}{(bx+a)\,(dx+c)\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2}dx$$

```
[In] int(ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/ln(e*((b*x+a)/(d*x+c))^n)^2,x)
```

```
[Out] int(ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/ln(e*((b*x+a)/(d*x+c))^n)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.20

$$\int\frac{\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int\frac{\log\left((gx+f)^m h\right)}{(bx+a)(dx+c)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2}\,dx$$

```
[In] integrate(log(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/log(e*((b*x+a)/(d*x+c))^n)^2,x,
algorithm="fricas")
```

```
[Out] integral(log((g*x + f)^m*h)/((b*d*x^2 + a*c + (b*c + a*d)*x)*log(e*((b*x +
a)/(d*x + c))^n)^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(h(f+gx)^m\right)}{(a+bx)(c+dx)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \text{Timed out}$$

```
[In] integrate(ln(h*(g*x+f)**m)/(b*x+a)/(d*x+c)/ln(e*((b*x+a)/(d*x+c))**n)**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 181, normalized size of antiderivative = 4.02

$$\int \frac{\log \left(h(f+gx)^m\right)}{(a+bx)(c+dx)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{\log \left((gx+f)^m h\right)}{(bx+a)(dx+c)\log \left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2}\,dx$$

```
[In] integrate(log(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/log(e*((b*x+a)/(d*x+c))^n)^2,x,
algorithm="maxima")
[Out] g*m*integrate(1/(b*c*f*n*log(e) - a*d*f*n*log(e) + (b*c*g*n*log(e) - a*d*g*
n*log(e))*x + (b*c*f*n - a*d*f*n + (b*c*g*n - a*d*g*n)*x)*log((b*x + a)^n)
- (b*c*f*n - a*d*f*n + (b*c*g*n - a*d*g*n)*x)*log((d*x + c)^n)), x) - (log(
(g*x + f)^m) + log(h))/(b*c*n*log(e) - a*d*n*log(e) + (b*c*n - a*d*n)*log((
b*x + a)^n) - (b*c*n - a*d*n)*log((d*x + c)^n))
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int \frac{\log \left(h(f+gx)^m\right)}{(a+bx)(c+dx)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{\log \left((gx+f)^m h\right)}{(bx+a)(dx+c)\log \left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2}\,dx$$

```
[In] integrate(log(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/log(e*((b*x+a)/(d*x+c))^n)^2,x,
algorithm="giac")
[Out] integrate(log((g*x + f)^m*h)/((b*x + a)*(d*x + c)*log(e*((b*x + a)/(d*x + c
))^n)^2), x)
```

Mupad [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int \frac{\log(h(f+gx)^m)}{(a+bx)(c+dx)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{\ln\left(h\left(f+g\,x\right)^m\right)}{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2\,\left(a+b\,x\right)\,\left(c+d\,x\right)}\,dx$$

```
[In] int(log(h*(f + g*x)^m)/(log(e*((a + b*x)/(c + d*x))^n)^2*(a + b*x)*(c + d*x
)),x)
```

```
[Out] int(log(h*(f + g*x)^m)/(log(e*((a + b*x)/(c + d*x))^n)^2*(a + b*x)*(c + d*x
)), x)
```

3.72 $\int \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx$

Optimal result 586
Rubi [N/A] 586
Mathematica [N/A] 587
Maple [N/A] 587
Fricas [N/A] 587
Sympy [F(-2)] 588
Maxima [N/A] 588
Giac [N/A] 588
Mupad [N/A] 589

Optimal result

Integrand size = 48, antiderivative size = 48

$$\int \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx=\frac{b\text{Int}\left(\frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)\log^2\left(\frac{a+bx}{c+dx}\right)},x\right)}{bc-ad}-\frac{d\text{Int}\left(\frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)},x\right)}{bc-ad}$$

```
[Out] b*CannotIntegrate(ln(1+(-b*x-a)/(d*x+c))/(b*x+a)/ln((b*x+a)/(d*x+c))^2,x)/(
-a*d+b*c)-d*CannotIntegrate(ln(1+(-b*x-a)/(d*x+c))/(d*x+c)/ln((b*x+a)/(d*x+
c))^2,x)/(-a*d+b*c)
```

Rubi [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx=\int \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx$$

```
[In] Int[Log[1 - (a + b*x)/(c + d*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(c + d*
x)]^2),x]
```

```
[Out] (b*Defer[Int][Log[1 - (a + b*x)/(c + d*x)]/((a + b*x)*Log[(a + b*x)/(c + d*
x)]^2), x])/(b*c - a*d) - (d*Defer[Int][Log[1 - (a + b*x)/(c + d*x)]/((c +
d*x)*Log[(a + b*x)/(c + d*x)]^2), x])/(b*c - a*d)
```

Rubi steps

$$\text{integral} = \int \left(\frac{b \log \left(1 - \frac{a+bx}{c+dx}\right)}{(bc-ad)(a+bx)\log^2\left(\frac{a+bx}{c+dx}\right)} - \frac{d \log \left(1 - \frac{a+bx}{c+dx}\right)}{(bc-ad)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)} \right) dx$$

$$= \frac{b \int \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)\log^2\left(\frac{a+bx}{c+dx}\right)} dx}{bc-ad} - \frac{d \int \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)} dx}{bc-ad}$$

Mathematica [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{\log \left(1 - \frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)} dx = \int \frac{\log \left(1 - \frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)} dx$$

```
[In] Integrate[Log[1 - (a + b*x)/(c + d*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(
c + d*x)]^2),x]
[Out] Integrate[Log[1 - (a + b*x)/(c + d*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(
c + d*x)]^2), x]
```

Maple [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{\ln \left(1 + \frac{-bx-a}{dx+c}\right)}{(bx+a)(dx+c)\ln\left(\frac{bx+a}{dx+c}\right)^2} dx$$

```
[In] int(ln(1+(-b*x-a)/(d*x+c))/(b*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))^2,x)
[Out] int(ln(1+(-b*x-a)/(d*x+c))/(b*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.29

$$\int \frac{\log \left(1 - \frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)} dx = \int \frac{\log \left(-\frac{bx+a}{dx+c} + 1\right)}{(bx+a)(dx+c)\log\left(\frac{bx+a}{dx+c}\right)^2} dx$$

```
[In] integrate(log(1+(-b*x-a)/(d*x+c))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x,
 algorithm="fricas")
[Out] integral(log(-((b - d)*x + a - c)/(d*x + c))/((b*d*x^2 + a*c + (b*c + a*d)*
x)*log((b*x + a)/(d*x + c))^2), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(ln(1+(-b*x-a)/(d*x+c))/(b*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))**2,x)
```

```
[Out] Exception raised: TypeError >> '>' not supported between instances of 'Poly
' and 'int'
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 154, normalized size of antiderivative = 3.21

$$\int \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx = \int \frac{\log\left(-\frac{bx+a}{dx+c}+1\right)}{(bx+a)(dx+c)\log\left(\frac{bx+a}{dx+c}\right)^2}\,dx$$

```
[In] integrate(log(1+(-b*x-a)/(d*x+c))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x,
 algorithm="maxima")
```

```
[Out] -(log(-(b - d)*x - a + c) - log(b*x + a))/((b*c - a*d)*log(b*x + a) - (b*c
- a*d)*log(d*x + c)) - integrate(-1/(((b*d - d^2)*x^2 + a*c - c^2 + (b*c +
a*d - 2*c*d)*x)*log(b*x + a) - ((b*d - d^2)*x^2 + a*c - c^2 + (b*c + a*d -
2*c*d)*x)*log(d*x + c)), x)
```

Giac [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx = \int \frac{\log\left(-\frac{bx+a}{dx+c}+1\right)}{(bx+a)(dx+c)\log\left(\frac{bx+a}{dx+c}\right)^2}\,dx$$

```
[In] integrate(log(1+(-b*x-a)/(d*x+c))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x,
 algorithm="giac")
```

```
[Out] integrate(log(-(b*x + a)/(d*x + c) + 1)/((b*x + a)*(d*x + c)*log((b*x + a)/
(d*x + c))^2), x)
```

Mupad [N/A]

Not integrable

Time = 1.94 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{\log \left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx=\int \frac{\ln \left(1-\frac{a+b\,x}{c+d\,x}\right)}{\ln \left(\frac{a+b\,x}{c+d\,x}\right)^2\,\left(a+b\,x\right)\,\left(c+d\,x\right)}\,dx$$

```
[In] int(log(1 - (a + b*x)/(c + d*x))/(log((a + b*x)/(c + d*x))^2*(a + b*x)*(c +
 d*x)),x)
[Out] int(log(1 - (a + b*x)/(c + d*x))/(log((a + b*x)/(c + d*x))^2*(a + b*x)*(c +
 d*x)), x)
```

3.73 $\int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx$

Optimal result . . . 590
Rubi [N/A] . . . 590
Mathematica [N/A] . . . 591
Maple [N/A] . . . 591
Fricas [N/A] . . . 591
Sympy [F(-2)] . . . 592
Maxima [N/A] . . . 592
Giac [N/A] . . . 592
Mupad [N/A] . . . 593

Optimal result

Integrand size = 48, antiderivative size = 48

$$\int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx = \frac{b\text{Int}\left(\frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)\log^2\left(\frac{a+bx}{c+dx}\right)},x\right)}{bc-ad} - \frac{d\text{Int}\left(\frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)},x\right)}{bc-ad}$$

```
[Out] b*CannotIntegrate(ln(1+(-d*x-c)/(b*x+a))/(b*x+a)/ln((b*x+a)/(d*x+c))^2,x)/(
-a*d+b*c)-d*CannotIntegrate(ln(1+(-d*x-c)/(b*x+a))/(d*x+c)/ln((b*x+a)/(d*x+
c))^2,x)/(-a*d+b*c)
```

Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx = \int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx$$

```
[In] Int[Log[1 - (c + d*x)/(a + b*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(c + d*
x)]^2),x]
```

```
[Out] (b*Defer[Int][Log[1 - (c + d*x)/(a + b*x)]/((a + b*x)*Log[(a + b*x)/(c + d*
x)]^2), x])/(b*c - a*d) - (d*Defer[Int][Log[1 - (c + d*x)/(a + b*x)]/((c +
d*x)*Log[(a + b*x)/(c + d*x)]^2), x])/(b*c - a*d)
```

Rubi steps

$$\text{integral} = \int \left(\frac{b\log\left(1-\frac{c+dx}{a+bx}\right)}{(bc-ad)(a+bx)\log^2\left(\frac{a+bx}{c+dx}\right)} - \frac{d\log\left(1-\frac{c+dx}{a+bx}\right)}{(bc-ad)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)dx$$

$$= \frac{b\int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx}{bc-ad} - \frac{d\int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx}{bc-ad}$$

Mathematica [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx = \int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx$$

```
[In] Integrate[Log[1 - (c + d*x)/(a + b*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(
c + d*x)]^2),x]
[Out] Integrate[Log[1 - (c + d*x)/(a + b*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(
c + d*x)]^2), x]
```

Maple [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{\ln\left(1+\frac{-dx-c}{bx+a}\right)}{(bx+a)\,(dx+c)\ln\left(\frac{bx+a}{dx+c}\right)^2}dx$$

```
[In] int(ln(1+(-d*x-c)/(b*x+a))/(b*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))^2,x)
[Out] int(ln(1+(-d*x-c)/(b*x+a))/(b*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.27

$$\int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx = \int \frac{\log\left(-\frac{dx+c}{bx+a}+1\right)}{(bx+a)(dx+c)\log\left(\frac{bx+a}{dx+c}\right)^2}\,dx$$

```
[In] integrate(log(1+(-d*x-c)/(b*x+a))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x,
 algorithm="fricas")
[Out] integral(log(((b - d)*x + a - c)/(b*x + a))/((b*d*x^2 + a*c + (b*c + a*d)*x
)*log((b*x + a)/(d*x + c))^2), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(ln(1+(-d*x-c)/(b*x+a))/(b*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))**2,x)
```

```
[Out] Exception raised: TypeError >> '>' not supported between instances of 'Poly
' and 'int'
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 155, normalized size of antiderivative = 3.23

$$\int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx = \int \frac{\log\left(-\frac{dx+c}{bx+a}+1\right)}{(bx+a)(dx+c)\log\left(\frac{bx+a}{dx+c}\right)^2}\,dx$$

```
[In] integrate(log(1+(-d*x-c)/(b*x+a))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x,
 algorithm="maxima")
```

```
[Out] -(log((b - d)*x + a - c) - log(b*x + a))/((b*c - a*d)*log(b*x + a) - (b*c -
 a*d)*log(d*x + c)) - integrate(-1/(((b^2 - b*d)*x^2 + a^2 - a*c + (a*(2*b
- d) - b*c)*x)*log(b*x + a) - ((b^2 - b*d)*x^2 + a^2 - a*c + (a*(2*b - d) -
 b*c)*x)*log(d*x + c)), x)
```

Giac [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx = \int \frac{\log\left(-\frac{dx+c}{bx+a}+1\right)}{(bx+a)(dx+c)\log\left(\frac{bx+a}{dx+c}\right)^2}\,dx$$

```
[In] integrate(log(1+(-d*x-c)/(b*x+a))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x,
 algorithm="giac")
```

```
[Out] integrate(log(-(d*x + c)/(b*x + a) + 1)/((b*x + a)*(d*x + c)*log((b*x + a)/
(d*x + c))^2), x)
```

Mupad [N/A]

Not integrable

Time = 1.77 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{\log \left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx=\int \frac{\ln \left(1-\frac{c+d\,x}{a+b\,x}\right)}{{\ln \left(\frac{a+b\,x}{c+d\,x}\right)}^2\,\left(a+b\,x\right)\,\left(c+d\,x\right)} \,d x$$

```
[In] int(log(1 - (c + d*x)/(a + b*x))/(log((a + b*x)/(c + d*x))^2*(a + b*x)*(c +
 d*x)),x)
```

```
[Out] int(log(1 - (c + d*x)/(a + b*x))/(log((a + b*x)/(c + d*x))^2*(a + b*x)*(c +
 d*x)), x)
```

3.74 $\int \left(\frac{1}{(c+dx)(-a+c+(-b+d)x)\log\left(\frac{a+bx}{c+dx}\right)}+\frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)$

Optimal result 594
Rubi [F] 594
Mathematica [A] (verified) 595
Maple [B] (verified) 595
Fricas [A] (verification not implemented) 596
Sympy [F(-2)] 596
Maxima [A] (verification not implemented) 597
Giac [F] 597
Mupad [B] (verification not implemented) 597

Optimal result

Integrand size = 87, antiderivative size = 45

$$\int \left(\frac{1}{(c+dx)(-a+c+(-b+d)x)\log\left(\frac{a+bx}{c+dx}\right)}+\frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)dx = -\frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(bc-ad)\log\left(\frac{a+bx}{c+dx}\right)}$$

```
[Out] -ln(1+(-b*x-a)/(d*x+c))/(-a*d+b*c)/ln((b*x+a)/(d*x+c))
```

Rubi [F]

$$\int \left(\frac{1}{(c+dx)(-a+c+(-b+d)x)\log\left(\frac{a+bx}{c+dx}\right)}+\frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)dx = \int \left(\frac{1}{(c+dx)(-a+c+(-b+d)x)\log\left(\frac{a+bx}{c+dx}\right)}+\frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)dx$$

```
[In] Int[1/((c + d*x)*(-a + c + (-b + d)*x)*Log[(a + b*x)/(c + d*x)]) + Log[1 -
(a + b*x)/(c + d*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(c + d*x)]^2),x]
```

```
[Out] Defer[Int][1/((c + d*x)*(-a + c + (-b + d)*x)*Log[(a + b*x)/(c + d*x)]), x]
 + (b*Defer[Int][Log[1 - (a + b*x)/(c + d*x)]/((a + b*x)*Log[(a + b*x)/(c +
 d*x)]^2), x])/(b*c - a*d) - (d*Defer[Int][Log[1 - (a + b*x)/(c + d*x)]/((c
 + d*x)*Log[(a + b*x)/(c + d*x)]^2), x])/(b*c - a*d)
```

Rubi steps

$$\text{integral} = \int \frac{1}{(c+dx)(-a+c+(-b+d)x)\log\left(\frac{a+bx}{c+dx}\right)}\,dx + \int \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx$$

$$= \int \frac{1}{(c+dx)(-a+c+(-b+d)x)\log\left(\frac{a+bx}{c+dx}\right)}\,dx + \int \left(\frac{b\log\left(1-\frac{a+bx}{c+dx}\right)}{(bc-ad)(a+bx)\log^2\left(\frac{a+bx}{c+dx}\right)} - \frac{d\log\left(1-\frac{a+bx}{c+dx}\right)}{(bc-ad)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)dx$$

$$= \frac{b\int \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx}{bc-ad} - \frac{d\int \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx}{bc-ad} + \int \frac{1}{(c+dx)(-a+c+(-b+d)x)\log\left(\frac{a+bx}{c+dx}\right)}\,dx$$

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \left(\frac{1}{(c+dx)(-a+c+(-b+d)x)\log\left(\frac{a+bx}{c+dx}\right)} + \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)dx = \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(-bc+ad)\log\left(\frac{a+bx}{c+dx}\right)}$$

```
[In] Integrate[1/((c + d*x)*(-a + c + (-b + d)*x)*Log[(a + b*x)/(c + d*x)]) + Lo
g[1 - (a + b*x)/(c + d*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(c + d*x)]^2)
,x]
[Out] Log[1 - (a + b*x)/(c + d*x)]/((-(b*c) + a*d)*Log[(a + b*x)/(c + d*x)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(47) = 94.

Time = 218.22 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.96

method	result	size
parallelrisch	$-\frac{-\ln\left(-\frac{bx-dx+a-c}{dx+c}\right)b^2d^4+2\ln\left(-\frac{bx-dx+a-c}{dx+c}\right)b^3d^3-\ln\left(-\frac{bx-dx+a-c}{dx+c}\right)b^4d^2}{\ln\left(\frac{bx+a}{dx+c}\right)(b-d)^2d^2(ad-cb)b^2}$	133

```
[In] int(1/(d*x+c)/(-a+c+(-b+d)*x)/ln((b*x+a)/(d*x+c))+ln(1+(-b*x-a)/(d*x+c))/(b
*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(-ln(-(b*x-d*x+a-c)/(d*x+c))*b^2*d^4+2*ln(-(b*x-d*x+a-c)/(d*x+c))*b^3*d^3-
ln(-(b*x-d*x+a-c)/(d*x+c))*b^4*d^2)/ln((b*x+a)/(d*x+c))/(b-d)^2/d^2/(a*d-b*
c)/b^2
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int \left(\frac{1}{(c+dx)(-a+c+(-b+d)x)\log\left(\frac{a+bx}{c+dx}\right)}+\frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)dx$$
$$=-\frac{\log\left(-\frac{(b-d)x+a-c}{dx+c}\right)}{(bc-ad)\log\left(\frac{bx+a}{dx+c}\right)}$$

```
[In] integrate(1/(d*x+c)/(-a+c+(-b+d)*x)/log((b*x+a)/(d*x+c))+log(1+(-b*x-a)/(d*
x+c))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -log(-((b - d)*x + a - c)/(d*x + c))/((b*c - a*d)*log((b*x + a)/(d*x + c)))
```

Sympy [F(-2)]

Exception generated.

$$\int \left(\frac{1}{(c+dx)(-a+c+(-b+d)x)\log\left(\frac{a+bx}{c+dx}\right)}+\frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)dx$$
$$=\text{Exception raised: TypeError}$$

```
[In] integrate(1/(d*x+c)/(-a+c+(-b+d)*x)/ln((b*x+a)/(d*x+c))+ln(1+(-b*x-a)/(d*x+
c))/(b*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))**2,x)
```

```
[Out] Exception raised: TypeError >> '>' not supported between instances of 'Poly
' and 'int'
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.31

$$\int \left(\frac{1}{(c+dx)(-a+c+(-b+d)x)\log\left(\frac{a+bx}{c+dx}\right)}+\frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)dx$$
$$=-\frac{\log(-(b-d)x-a+c)-\log(bx+a)}{(bc-ad)\log(bx+a)-(bc-ad)\log(dx+c)}$$

```
[In] integrate(1/(d*x+c)/(-a+c+(-b+d)*x)/log((b*x+a)/(d*x+c))+log(1+(-b*x-a)/(d*
x+c))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(log(-(b - d)*x - a + c) - log(b*x + a))/((b*c - a*d)*log(b*x + a) - (b*c
- a*d)*log(d*x + c))
```

Giac [F]

$$\int \left(\frac{1}{(c+dx)(-a+c+(-b+d)x)\log\left(\frac{a+bx}{c+dx}\right)}+\frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)dx$$
$$=\int -\frac{1}{((b-d)x+a-c)(dx+c)\log\left(\frac{bx+a}{dx+c}\right)}+\frac{\log\left(-\frac{bx+a}{dx+c}+1\right)}{(bx+a)(dx+c)\log\left(\frac{bx+a}{dx+c}\right)^2}\,dx$$

```
[In] integrate(1/(d*x+c)/(-a+c+(-b+d)*x)/log((b*x+a)/(d*x+c))+log(1+(-b*x-a)/(d*
x+c))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(-1/(((b - d)*x + a - c)*(d*x + c)*log((b*x + a)/(d*x + c))) + log
(-(b*x + a)/(d*x + c) + 1)/((b*x + a)*(d*x + c)*log((b*x + a)/(d*x + c))^2)
, x)
```

Mupad [B] (verification not implemented)

Time = 1.76 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \left(\frac{1}{(c+dx)(-a+c+(-b+d)x)\log\left(\frac{a+bx}{c+dx}\right)}+\frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)dx$$
$$=\frac{\ln\left(1-\frac{a+b\,x}{c+d\,x}\right)}{\ln\left(\frac{a+b\,x}{c+d\,x}\right)\,(a\,d-b\,c)}$$

```
[In] int(log(1 - (a + b*x)/(c + d*x))/(log((a + b*x)/(c + d*x))^2*(a + b*x)*(c +
 d*x)) - 1/(log((a + b*x)/(c + d*x))*(c + d*x)*(a - c + x*(b - d))),x)
```

```
[Out] log(1 - (a + b*x)/(c + d*x))/(log((a + b*x)/(c + d*x))*(a*d - b*c))
```

3.75 $\int \left(-\frac{1}{(a+bx)(a-c+(b-d)x)\log\left(\frac{a+bx}{c+dx}\right)}+\frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)$

Optimal result 598
Rubi [F] 598
Mathematica [A] (verified) 599
Maple [B] (verified) 599
Fricas [A] (verification not implemented) 600
Sympy [F(-2)] 600
Maxima [A] (verification not implemented) 601
Giac [F] 601
Mupad [B] (verification not implemented) 601

Optimal result

Integrand size = 88, antiderivative size = 45

$$\int \left(-\frac{1}{(a+bx)(a-c+(b-d)x)\log\left(\frac{a+bx}{c+dx}\right)}+\frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)dx = -\frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(bc-ad)\log\left(\frac{a+bx}{c+dx}\right)}$$

```
[Out] -ln(1+(-d*x-c)/(b*x+a))/(-a*d+b*c)/ln((b*x+a)/(d*x+c))
```

Rubi [F]

$$\int \left(-\frac{1}{(a+bx)(a-c+(b-d)x)\log\left(\frac{a+bx}{c+dx}\right)}+\frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)dx = \int \left(-\frac{1}{(a+bx)(a-c+(b-d)x)\log\left(\frac{a+bx}{c+dx}\right)}+\frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)dx$$

```
[In] Int[-(1/((a + b*x)*(a - c + (b - d)*x)*Log[(a + b*x)/(c + d*x)])) + Log[1 -
 (c + d*x)/(a + b*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(c + d*x)]^2),x]
```

```
[Out] -Defer[Int][1/((a + b*x)*(a - c + (b - d)*x)*Log[(a + b*x)/(c + d*x)]), x]
+ (b*Defer[Int][Log[1 - (c + d*x)/(a + b*x)]/((a + b*x)*Log[(a + b*x)/(c +
d*x)]^2), x])/(b*c - a*d) - (d*Defer[Int][Log[1 - (c + d*x)/(a + b*x)]/((c
+ d*x)*Log[(a + b*x)/(c + d*x)]^2), x])/(b*c - a*d)
```

Rubi steps

$$\text{integral} = -\int \frac{1}{(a+bx)(a-c+(b-d)x)\log\left(\frac{a+bx}{c+dx}\right)}\,dx + \int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx$$
$$= -\int \frac{1}{(a+bx)(a-c+(b-d)x)\log\left(\frac{a+bx}{c+dx}\right)}\,dx + \int \left(\frac{b\log\left(1-\frac{c+dx}{a+bx}\right)}{(bc-ad)(a+bx)\log^2\left(\frac{a+bx}{c+dx}\right)} - \frac{d\log\left(1-\frac{c+dx}{a+bx}\right)}{(bc-ad)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)dx$$
$$= \frac{b\int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx}{bc-ad} - \frac{d\int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx}{bc-ad} - \int \frac{1}{(a+bx)(a-c+(b-d)x)\log\left(\frac{a+bx}{c+dx}\right)}\,dx$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \left(-\frac{1}{(a+bx)(a-c+(b-d)x)\log\left(\frac{a+bx}{c+dx}\right)} + \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)dx = -\frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(bc-ad)\log\left(\frac{a+bx}{c+dx}\right)}$$

```
[In] Integrate[-(1/((a + b*x)*(a - c + (b - d)*x)*Log[(a + b*x)/(c + d*x)])) + L
og[1 - (c + d*x)/(a + b*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(c + d*x)]^2
),x]
[Out] -(Log[1 - (c + d*x)/(a + b*x)]/((b*c - a*d)*Log[(a + b*x)/(c + d*x)]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(47) = 94.

Time = 220.25 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.89

method	result
parallelrisch	$-\frac{2\ln\left(\frac{bx-dx+a-c}{bx+a}\right)b^3d^3-\ln\left(\frac{bx-dx+a-c}{bx+a}\right)b^4d^2-\ln\left(\frac{bx-dx+a-c}{bx+a}\right)b^2d^4}{\ln\left(\frac{bx+a}{dx+c}\right)(b-d)^2(ad-cb)b^2d^2}$
risch	$\frac{2i\ln(bx-dx+a-c)}{(ad-cb)\left(\pi\ \text{csgn}\left(\frac{i(bx+a)}{dx+c}\right)\text{csgn}(i(bx+a))\,\text{csgn}\left(\frac{i}{dx+c}\right)-\pi\ \text{csgn}\left(\frac{i(bx+a)}{dx+c}\right)^2\text{csgn}(i(bx+a))-\pi\ \text{csgn}\left(\frac{i(bx+a)}{dx+c}\right)^2\text{csgn}\left(\frac{i}{dx+c}\right)+\pi\right.}$

```
[In] int(-1/(b*x+a)/(a-c+(b-d)*x)/ln((b*x+a)/(d*x+c))+ln(1+(-d*x-c)/(b*x+a))/(b*
x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(2*ln((b*x-d*x+a-c)/(b*x+a))*b^3*d^3-ln((b*x-d*x+a-c)/(b*x+a))*b^4*d^2-ln(
(b*x-d*x+a-c)/(b*x+a))*b^2*d^4)/ln((b*x+a)/(d*x+c))/(b-d)^2/(a*d-b*c)/b^2/d
^2
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.09

$$\int \left(-\frac{1}{(a+bx)(a-c+(b-d)x)\log\left(\frac{a+bx}{c+dx}\right)}+\frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)dx$$
$$=-\frac{\log\left(\frac{(b-d)x+a-c}{bx+a}\right)}{(bc-ad)\log\left(\frac{bx+a}{dx+c}\right)}$$

```
[In] integrate(-1/(b*x+a)/(a-c+(b-d)*x)/log((b*x+a)/(d*x+c))+log(1+(-d*x-c)/(b*x
+a))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -log(((b - d)*x + a - c)/(b*x + a))/((b*c - a*d)*log((b*x + a)/(d*x + c)))
```

Sympy [F(-2)]

Exception generated.

$$\int \left(-\frac{1}{(a+bx)(a-c+(b-d)x)\log\left(\frac{a+bx}{c+dx}\right)}+\frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)dx$$
$$=\text{Exception raised: TypeError}$$

```
[In] integrate(-1/(b*x+a)/(a-c+(b-d)*x)/ln((b*x+a)/(d*x+c))+ln(1+(-d*x-c)/(b*x+a
))/(b*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))**2,x)
```

```
[Out] Exception raised: TypeError >> '>' not supported between instances of 'Poly
' and 'int'
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.29

$$\int \left(-\frac{1}{(a+bx)(a-c+(b-d)x)\log\left(\frac{a+bx}{c+dx}\right)}+\frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)dx = -\frac{\log((b-d)x+a-c)-\log(bx+a)}{(bc-ad)\log(bx+a)-(bc-ad)\log(dx+c)}$$

```
[In] integrate(-1/(b*x+a)/(a-c+(b-d)*x)/log((b*x+a)/(d*x+c))+log(1+(-d*x-c)/(b*x
+a))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(log((b - d)*x + a - c) - log(b*x + a))/((b*c - a*d)*log(b*x + a) - (b*c -
 a*d)*log(d*x + c))
```

Giac [F]

$$\int \left(-\frac{1}{(a+bx)(a-c+(b-d)x)\log\left(\frac{a+bx}{c+dx}\right)}+\frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)dx = \int -\frac{1}{((b-d)x+a-c)(bx+a)\log\left(\frac{bx+a}{dx+c}\right)}+\frac{\log\left(-\frac{dx+c}{bx+a}+1\right)}{(bx+a)(dx+c)\log\left(\frac{bx+a}{dx+c}\right)^2}\,dx$$

```
[In] integrate(-1/(b*x+a)/(a-c+(b-d)*x)/log((b*x+a)/(d*x+c))+log(1+(-d*x-c)/(b*x
+a))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(-1/(((b - d)*x + a - c)*(b*x + a)*log((b*x + a)/(d*x + c))) + log
(-(d*x + c)/(b*x + a) + 1)/((b*x + a)*(d*x + c)*log((b*x + a)/(d*x + c))^2)
, x)
```

Mupad [B] (verification not implemented)

Time = 1.57 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \left(-\frac{1}{(a+bx)(a-c+(b-d)x)\log\left(\frac{a+bx}{c+dx}\right)}+\frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)dx = \frac{\ln\left(1-\frac{c+d\,x}{a+b\,x}\right)}{\ln\left(\frac{a+b\,x}{c+d\,x}\right)\,\left(a\,d-b\,c\right)}$$

```
[In] int(log(1 - (c + d*x)/(a + b*x))/(log((a + b*x)/(c + d*x))^2*(a + b*x)*(c +
 d*x)) - 1/(log((a + b*x)/(c + d*x))*(a + b*x)*(a - c + x*(b - d))),x)
```

```
[Out] log(1 - (c + d*x)/(a + b*x))/(log((a + b*x)/(c + d*x))*(a*d - b*c))
```

3.76 $\int \frac{x^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2} dx$

Optimal result 602
Rubi [A] (verified) 603
Mathematica [A] (verified) 608
Maple [A] (verified) 608
Fricas [F] 609
Sympy [F(-1)] 610
Maxima [F] 610
Giac [F] 610
Mupad [F(-1)] 610

Optimal result

Integrand size = 32, antiderivative size = 560

$$\begin{aligned}\int \frac{x^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2} dx &= -\frac{anx}{2bg} + \frac{cnx}{2dg} + \frac{a^2 n \log(a+bx)}{2b^2 g} - \frac{nx^2 \log(a+bx)}{2g} \\ &\quad - \frac{c^2 n \log(c+dx)}{2d^2 g} + \frac{nx^2 \log(c+dx)}{2g} \\ &\quad + \frac{x^2\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{2g} \\ &\quad - \frac{fn\log(a+bx)\log\left(\frac{b(\sqrt{f}-\sqrt{g}x)}{b\sqrt{f}+a\sqrt{g}}\right)}{2g^2} \\ &\quad + \frac{fn\log(c+dx)\log\left(\frac{d(\sqrt{f}-\sqrt{g}x)}{d\sqrt{f}+c\sqrt{g}}\right)}{2g^2} \\ &\quad - \frac{fn\log(a+bx)\log\left(\frac{b(\sqrt{f}+\sqrt{g}x)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g^2} \\ &\quad + \frac{fn\log(c+dx)\log\left(\frac{d(\sqrt{f}+\sqrt{g}x)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g^2} \\ &\quad + \frac{f\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)\log\left(f-gx^2\right)}{2g^2} \\ &\quad - \frac{fn\operatorname{PolyLog}\left(2, -\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g^2} - \frac{fn\operatorname{PolyLog}\left(2, \frac{\sqrt{g}(a+bx)}{b\sqrt{f}+a\sqrt{g}}\right)}{2g^2} \\ &\quad + \frac{fn\operatorname{PolyLog}\left(2, -\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g^2} + \frac{fn\operatorname{PolyLog}\left(2, \frac{\sqrt{g}(c+dx)}{d\sqrt{f}+c\sqrt{g}}\right)}{2g^2}\end{aligned}$$

```
[Out] -1/2*a*n*x/b/g+1/2*c*n*x/d/g+1/2*a^2*n*ln(b*x+a)/b^2/g-1/2*n*x^2*ln(b*x+a)/
g-1/2*c^2*n*ln(d*x+c)/d^2/g+1/2*n*x^2*ln(d*x+c)/g+1/2*x^2*(n*ln(b*x+a)-ln(e
*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))/g+1/2*f*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+
c))^n)-n*ln(d*x+c))*ln(-g*x^2+f)/g^2-1/2*f*n*ln(b*x+a)*ln(b*(f^(1/2)-x*g^(1
/2))/(b*f^(1/2)+a*g^(1/2)))/g^2+1/2*f*n*ln(d*x+c)*ln(d*(f^(1/2)-x*g^(1/2))/
(d*f^(1/2)+c*g^(1/2)))/g^2-1/2*f*n*ln(b*x+a)*ln(b*(f^(1/2)+x*g^(1/2))/(b*f^
(1/2)-a*g^(1/2)))/g^2+1/2*f*n*ln(d*x+c)*ln(d*(f^(1/2)+x*g^(1/2))/(d*f^(1/2)
-c*g^(1/2)))/g^2-1/2*f*n*polylog(2,-(b*x+a)*g^(1/2)/(b*f^(1/2)-a*g^(1/2)))/
g^2-1/2*f*n*polylog(2,(b*x+a)*g^(1/2)/(b*f^(1/2)+a*g^(1/2)))/g^2+1/2*f*n*po
lylog(2,-(d*x+c)*g^(1/2)/(d*f^(1/2)-c*g^(1/2)))/g^2+1/2*f*n*polylog(2,(d*x+
c)*g^(1/2)/(d*f^(1/2)+c*g^(1/2)))/g^2
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 560, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2593, 272, 45, 2463, 2442, 266, 2441, 2440, 2438}

$$\int \frac{x^3 \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2} \, dx = \frac{a^2 n \log(a+bx)}{2b^2 g} + \frac{f \log\left(f-gx^2\right)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)}{2g^2} + \frac{x^2\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)}{2g} - \frac{fn \operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g^2} - \frac{fn \operatorname{PolyLog}\left(2,\frac{\sqrt{g}(a+bx)}{\sqrt{g}a+b\sqrt{f}}\right)}{2g^2} - \frac{fn \log(a+bx)\log\left(\frac{b(\sqrt{f}-\sqrt{g}x)}{a\sqrt{g}+b\sqrt{f}}\right)}{2g^2} - \frac{fn \log(a+bx)\log\left(\frac{b(\sqrt{f}+\sqrt{g}x)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g^2} - \frac{nx^2\log(a+bx)}{2g} - \frac{anx}{2bg} - \frac{c^2 n\log(c+dx)}{2d^2 g} + \frac{fn \operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g^2} + \frac{fn \operatorname{PolyLog}\left(2,\frac{\sqrt{g}(c+dx)}{\sqrt{g}c+d\sqrt{f}}\right)}{2g^2} + \frac{fn\log(c+dx)\log\left(\frac{d(\sqrt{f}-\sqrt{g}x)}{c\sqrt{g}+d\sqrt{f}}\right)}{2g^2} + \frac{fn\log(c+dx)\log\left(\frac{d(\sqrt{f}+\sqrt{g}x)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g^2} + \frac{nx^2\log(c+dx)}{2g} + \frac{cnx}{2dg}$$

```
[In] Int[(x^3*Log[e*((a + b*x)/(c + d*x))^n])/(f - g*x^2),x]
```

```
[Out] -1/2*(a*n*x)/(b*g) + (c*n*x)/(2*d*g) + (a^2*n*Log[a + b*x])/(2*b^2*g) - (n*
x^2*Log[a + b*x])/(2*g) - (c^2*n*Log[c + d*x])/(2*d^2*g) + (n*x^2*Log[c + d
*x])/(2*g) + (x^2*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[
```

```
c + d*x]))/(2*g) - (f*n*Log[a + b*x]*Log[(b*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[
f] + a*Sqrt[g])])/(2*g^2) + (f*n*Log[c + d*x]*Log[(d*(Sqrt[f] - Sqrt[g]*x))
/(d*Sqrt[f] + c*Sqrt[g])])/(2*g^2) - (f*n*Log[a + b*x]*Log[(b*(Sqrt[f] + Sq
rt[g]*x))/(b*Sqrt[f] - a*Sqrt[g])])/(2*g^2) + (f*n*Log[c + d*x]*Log[(d*(Sqr
t[f] + Sqrt[g]*x))/(d*Sqrt[f] - c*Sqrt[g])])/(2*g^2) + (f*(n*Log[a + b*x] -
 Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])*Log[f - g*x^2])/(2*g^2) -
 (f*n*PolyLog[2, -((Sqrt[g]*(a + b*x))/(b*Sqrt[f] - a*Sqrt[g]))])/(2*g^2) -
 (f*n*PolyLog[2, (Sqrt[g]*(a + b*x))/(b*Sqrt[f] + a*Sqrt[g])])/(2*g^2) + (f
*n*PolyLog[2, -((Sqrt[g]*(c + d*x))/(d*Sqrt[f] - c*Sqrt[g]))])/(2*g^2) + (f
*n*PolyLog[2, (Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])])/(2*g^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2593

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= n\int \frac{x^3\log(a+bx)}{f-gx^2}\,dx - n\int \frac{x^3\log(c+dx)}{f-gx^2}\,dx \\
&\quad - \left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\int \frac{x^3}{f-gx^2}\,dx \\
&= n\int \left(-\frac{x\log(a+bx)}{g}+\frac{fx\log(a+bx)}{g\left(f-gx^2\right)}\right)\,dx - n\int \left(-\frac{x\log(c+dx)}{g}+\frac{fx\log(c+dx)}{g\left(f-gx^2\right)}\right)\,dx \\
&\quad - \frac{1}{2}\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\text{Subst}\left(\int \frac{x}{f-gx}\,dx,x,x^2\right) \\
&= -\frac{n\int x\log(a+bx)\,dx}{g}+\frac{n\int x\log(c+dx)\,dx}{g}+\frac{(fn)\int \frac{x\log(a+bx)}{f-gx^2}\,dx}{g} \\
&\quad - \frac{(fn)\int \frac{x\log(c+dx)}{f-gx^2}\,dx}{g}-\frac{1}{2}\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right. \\
&\quad \left. -n\log(c+dx)\right)\text{Subst}\left(\int \left(-\frac{1}{g}-\frac{f}{g(-f+gx)}\right)\,dx,x,x^2\right)
\end{aligned}$$

$$= -\frac{nx^2\log(a+bx)}{2g} + \frac{nx^2\log(c+dx)}{2g} + \frac{x^2\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{2g} + \frac{f\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)\log\left(f-gx^2\right)}{2g^2} + \frac{(bn)\int\frac{x^2}{a+bx}\,dx}{2g} - \frac{(dn)\int\frac{x^2}{c+dx}\,dx}{2g} + \frac{(fn)\int\left(\frac{\log(a+bx)}{2\sqrt{g}(\sqrt{f}-\sqrt{g}x)} - \frac{\log(a+bx)}{2\sqrt{g}(\sqrt{f}+\sqrt{g}x)}\right)dx}{g} - \frac{(fn)\int\left(\frac{\log(c+dx)}{2\sqrt{g}(\sqrt{f}-\sqrt{g}x)} - \frac{\log(c+dx)}{2\sqrt{g}(\sqrt{f}+\sqrt{g}x)}\right)dx}{g}$$

$$= -\frac{nx^2\log(a+bx)}{2g} + \frac{nx^2\log(c+dx)}{2g} + \frac{x^2\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{2g} + \frac{f\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)\log\left(f-gx^2\right)}{2g^2} + \frac{(fn)\int\frac{\log(a+bx)}{\sqrt{f}-\sqrt{g}x}\,dx}{2g^{3/2}} - \frac{(fn)\int\frac{\log(a+bx)}{\sqrt{f}+\sqrt{g}x}\,dx}{2g^{3/2}} - \frac{(fn)\int\frac{\log(c+dx)}{\sqrt{f}-\sqrt{g}x}\,dx}{2g^{3/2}} + \frac{(fn)\int\frac{\log(c+dx)}{\sqrt{f}+\sqrt{g}x}\,dx}{2g^{3/2}} + \frac{(bn)\int\left(-\frac{a}{b^2} + \frac{x}{b} + \frac{a^2}{b^2(a+bx)}\right)dx}{2g} - \frac{(dn)\int\left(-\frac{c}{d^2} + \frac{x}{d} + \frac{c^2}{d^2(c+dx)}\right)dx}{2g}$$

$$= -\frac{anx}{2bg} + \frac{cnx}{2dg} + \frac{a^2n\log(a+bx)}{2b^2g} - \frac{nx^2\log(a+bx)}{2g} - \frac{c^2n\log(c+dx)}{2d^2g} + \frac{nx^2\log(c+dx)}{2g} + \frac{x^2\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{2g} - \frac{fn\log(a+bx)\log\left(\frac{b(\sqrt{f}-\sqrt{g}x)}{b\sqrt{f}+a\sqrt{g}}\right)}{2g^2} + \frac{fn\log(c+dx)\log\left(\frac{d(\sqrt{f}-\sqrt{g}x)}{d\sqrt{f}+c\sqrt{g}}\right)}{2g^2} - \frac{fn\log(a+bx)\log\left(\frac{b(\sqrt{f}+\sqrt{g}x)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g^2} + \frac{fn\log(c+dx)\log\left(\frac{d(\sqrt{f}+\sqrt{g}x)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g^2} + \frac{f\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)\log\left(f-gx^2\right)}{2g^2} + \frac{(bfn)\int\frac{\log\left(\frac{b(\sqrt{f}-\sqrt{g}x)}{b\sqrt{f}+a\sqrt{g}}\right)}{a+bx}\,dx}{2g^2} + \frac{(bfn)\int\frac{\log\left(\frac{b(\sqrt{f}+\sqrt{g}x)}{b\sqrt{f}-a\sqrt{g}}\right)}{a+bx}\,dx}{2g^2} - \frac{(dfn)\int\frac{\log\left(\frac{d(\sqrt{f}-\sqrt{g}x)}{d\sqrt{f}+c\sqrt{g}}\right)}{c+dx}\,dx}{2g^2} - \frac{(dfn)\int\frac{\log\left(\frac{d(\sqrt{f}+\sqrt{g}x)}{d\sqrt{f}-c\sqrt{g}}\right)}{c+dx}\,dx}{2g^2}$$

$$= -\frac{anx}{2bg} + \frac{cnx}{2dg} + \frac{a^2 n \log(a+bx)}{2b^2 g} - \frac{nx^2 \log(a+bx)}{2g} - \frac{c^2 n \log(c+dx)}{2d^2 g} + \frac{nx^2 \log(c+dx)}{2g} + \frac{x^2 \left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{2g} - \frac{fn \log(a+bx) \log\left(\frac{b(\sqrt{f}-\sqrt{g}x)}{b\sqrt{f}+a\sqrt{g}}\right)}{2g^2} + \frac{fn \log(c+dx) \log\left(\frac{d(\sqrt{f}-\sqrt{g}x)}{d\sqrt{f}+c\sqrt{g}}\right)}{2g^2} - \frac{fn \log(a+bx) \log\left(\frac{b(\sqrt{f}+\sqrt{g}x)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g^2} + \frac{fn \log(c+dx) \log\left(\frac{d(\sqrt{f}+\sqrt{g}x)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g^2} + \frac{f\left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right) \log(f-gx^2)}{2g^2} + \frac{(fn)\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{g}x}{b\sqrt{f}-a\sqrt{g}}\right)}{x} dx, x, a+bx\right)}{2g^2} + \frac{(fn)\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{g}x}{b\sqrt{f}+a\sqrt{g}}\right)}{x} dx, x, a+bx\right)}{2g^2} - \frac{(fn)\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{g}x}{d\sqrt{f}-c\sqrt{g}}\right)}{x} dx, x, c+dx\right)}{2g^2} - \frac{(fn)\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{g}x}{d\sqrt{f}+c\sqrt{g}}\right)}{x} dx, x, c+dx\right)}{2g^2}$$

$$= -\frac{anx}{2bg} + \frac{cnx}{2dg} + \frac{a^2 n \log(a+bx)}{2b^2 g} - \frac{nx^2 \log(a+bx)}{2g} - \frac{c^2 n \log(c+dx)}{2d^2 g} + \frac{nx^2 \log(c+dx)}{2g} + \frac{x^2 \left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{2g} - \frac{fn \log(a+bx) \log\left(\frac{b(\sqrt{f}-\sqrt{g}x)}{b\sqrt{f}+a\sqrt{g}}\right)}{2g^2} + \frac{fn \log(c+dx) \log\left(\frac{d(\sqrt{f}-\sqrt{g}x)}{d\sqrt{f}+c\sqrt{g}}\right)}{2g^2} - \frac{fn \log(a+bx) \log\left(\frac{b(\sqrt{f}+\sqrt{g}x)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g^2} + \frac{fn \log(c+dx) \log\left(\frac{d(\sqrt{f}+\sqrt{g}x)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g^2} + \frac{f\left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right) \log(f-gx^2)}{2g^2} - \frac{fn\text{Li}_2\left(-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g^2} - \frac{fn\text{Li}_2\left(\frac{\sqrt{g}(a+bx)}{b\sqrt{f}+a\sqrt{g}}\right)}{2g^2} + \frac{fn\text{Li}_2\left(-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g^2} + \frac{fn\text{Li}_2\left(\frac{\sqrt{g}(c+dx)}{d\sqrt{f}+c\sqrt{g}}\right)}{2g^2}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 461, normalized size of antiderivative = 0.82

$$\int \frac{x^3 \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx$$
$$= \frac{-gx^2 \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + \frac{gn(a^2d^2\log(a+bx)-b(d(-bc+ad)x+bc^2\log(c+dx)))}{b^2d^2} - f\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\sqrt{f}-\sqrt{g}x\right) - f\log}{}$$

```
[In] Integrate[(x^3*Log[e*((a + b*x)/(c + d*x))^n])/(f - g*x^2),x]
```

```
[Out] (-(g*x^2*Log[e*((a + b*x)/(c + d*x))^n]) + (g*n*(a^2*d^2*Log[a + b*x] - b*(
d*(-(b*c) + a*d)*x + b*c^2*Log[c + d*x])))/(b^2*d^2) - f*Log[e*((a + b*x)/(
c + d*x))^n]*Log[Sqrt[f] - Sqrt[g]*x] - f*Log[e*((a + b*x)/(c + d*x))^n]*Lo
g[Sqrt[f] + Sqrt[g]*x] + f*n*((Log[(Sqrt[g]*(a + b*x))/(b*Sqrt[f] + a*Sqrt[
g])] - Log[(Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])])*Log[Sqrt[f] - Sqrt
[g]*x] + PolyLog[2, (b*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])] - Po
lyLog[2, (d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g])]) + f*n*((Log[-(
(Sqrt[g]*(a + b*x))/(b*Sqrt[f] - a*Sqrt[g]))] - Log[-((Sqrt[g]*(c + d*x))/(
d*Sqrt[f] - c*Sqrt[g]))])*Log[Sqrt[f] + Sqrt[g]*x] + PolyLog[2, (b*(Sqrt[f]
 + Sqrt[g]*x))/(b*Sqrt[f] - a*Sqrt[g])] - PolyLog[2, (d*(Sqrt[f] + Sqrt[g]*
x))/(d*Sqrt[f] - c*Sqrt[g])]))/(2*g^2)
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 538, normalized size of antiderivative = 0.96

method	result
parts	$-\frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)x^{2}}{2g}-\frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)f\ln\left(-g\,x^{2}+f\right)}{2g^{2}}-\frac{n\left(\frac{(ad-cb)\left(\frac{x}{bd}+\frac{c^{2}\ln(dx+c)}{d^{2}(ad-cb)}-\frac{a^{2}\ln(bx+a)}{b^{2}(ad-cb)}\right)}{g}+\frac{f(ad-cb)\left(\left(\frac{\ln(dx+c)\ln\left(-g\right.}{d}\right.\right.}{}\right.}{}$

[In] int(x^3*ln(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x,method=_RETURNVERBOSE)

[Out] -1/2*ln(e*((b*x+a)/(d*x+c))^n)*x^2/g-1/2*ln(e*((b*x+a)/(d*x+c))^n)*f/g^2*ln(-g*x^2+f)-1/2*n*((a*d-b*c)/g*(x/b/d+1/d^2*c^2/(a*d-b*c)*ln(d*x+c)-1/b^2*a^2/(a*d-b*c)*ln(b*x+a))+f*(a*d-b*c)/g^2*((ln(d*x+c)/d*ln(-g*x^2+f)+2/d*g*(-1/2*ln(d*x+c)*(ln((d*(f*g)^(1/2)-(d*x+c)*g+c*g)/(d*(f*g)^(1/2)+c*g))+ln((d*(f*g)^(1/2)+(d*x+c)*g-c*g)/(d*(f*g)^(1/2)-c*g)))/g-1/2*(dilog((d*(f*g)^(1/2)-(d*x+c)*g+c*g)/(d*(f*g)^(1/2)+c*g))+dilog((d*(f*g)^(1/2)+(d*x+c)*g-c*g)/(d*(f*g)^(1/2)-c*g)))/g))*d/(a*d-b*c)-(ln(b*x+a)/b*ln(-g*x^2+f)+2/b*g*(-1/2*ln(b*x+a)*(ln((b*(f*g)^(1/2)-g*(b*x+a)+a*g)/(b*(f*g)^(1/2)+a*g))+ln((b*(f*g)^(1/2)+g*(b*x+a)-a*g)/(b*(f*g)^(1/2)-a*g)))/g-1/2*(dilog((b*(f*g)^(1/2)-g*(b*x+a)+a*g)/(b*(f*g)^(1/2)+a*g))+dilog((b*(f*g)^(1/2)+g*(b*x+a)-a*g)/(b*(f*g)^(1/2)-a*g)))/g))*b/(a*d-b*c)))

Fricas [F]

$$\int \frac{x^3\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx=\int -\frac{x^3\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{gx^2-f}\,dx$$

[In] integrate(x^3*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="fricas")

[Out] integral(-x^3*log(e*((b*x + a)/(d*x + c))^n)/(g*x^2 - f), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3 \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{f - gx^2} \, dx = \text{Timed out}$$

```
[In] integrate(x**3*ln(e*((b*x+a)/(d*x+c))**n)/(-g*x**2+f),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^3 \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{f - gx^2} \, dx = \int -\frac{x^3 \log \left(e \left(\frac{bx+a}{dx+c}\right)^n\right)}{gx^2 - f} \, dx$$

```
[In] integrate(x^3*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="maxima")
[Out] -integrate(x^3*log(e*((b*x + a)/(d*x + c))^n)/(g*x^2 - f), x)
```

Giac [F]

$$\int \frac{x^3 \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{f - gx^2} \, dx = \int -\frac{x^3 \log \left(e \left(\frac{bx+a}{dx+c}\right)^n\right)}{gx^2 - f} \, dx$$

```
[In] integrate(x^3*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="giac")
[Out] integrate(-x^3*log(e*((b*x + a)/(d*x + c))^n)/(g*x^2 - f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{f - gx^2} \, dx = \int \frac{x^3 \ln \left(e \left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{f - g\,x^2} \, dx$$

```
[In] int((x^3*log(e*((a + b*x)/(c + d*x))^n))/(f - g*x^2),x)
[Out] int((x^3*log(e*((a + b*x)/(c + d*x))^n))/(f - g*x^2), x)
```

3.77 $\int \frac{x^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx$

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Rubi [A] (verified) . . . 612
Mathematica [A] (verified) . . . 617
Maple [F] . . . 618
Fricas [F] . . . 618
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Maxima [B] (verification not implemented) . . . 618
Giac [F] . . . 619
Mupad [F(-1)] . . . 619

Optimal result

Integrand size = 32, antiderivative size = 550

$$\begin{aligned}
&\int \frac{x^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx \\
&= -\frac{n(a+bx)\log(a+bx)}{bg}+\frac{n(c+dx)\log(c+dx)}{dg} \\
&+\frac{x\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{g} \\
&-\frac{\sqrt{f}\operatorname{arctanh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{g^{3/2}} \\
&-\frac{\sqrt{f}n\log(a+bx)\log\left(\frac{b(\sqrt{f}-\sqrt{g}x)}{b\sqrt{f}+a\sqrt{g}}\right)}{2g^{3/2}}+\frac{\sqrt{f}n\log(c+dx)\log\left(\frac{d(\sqrt{f}-\sqrt{g}x)}{d\sqrt{f}+c\sqrt{g}}\right)}{2g^{3/2}} \\
&+\frac{\sqrt{f}n\log(a+bx)\log\left(\frac{b(\sqrt{f}+\sqrt{g}x)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g^{3/2}}-\frac{\sqrt{f}n\log(c+dx)\log\left(\frac{d(\sqrt{f}+\sqrt{g}x)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g^{3/2}} \\
&+\frac{\sqrt{f}n\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g^{3/2}}-\frac{\sqrt{f}n\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(a+bx)}{b\sqrt{f}+a\sqrt{g}}\right)}{2g^{3/2}} \\
&-\frac{\sqrt{f}n\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g^{3/2}}+\frac{\sqrt{f}n\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(c+dx)}{d\sqrt{f}+c\sqrt{g}}\right)}{2g^{3/2}}
\end{aligned}$$

```
[Out] -n*(b*x+a)*ln(b*x+a)/b/g+n*(d*x+c)*ln(d*x+c)/d/g+x*(n*ln(b*x+a)-ln(e*((b*x+
a)/(d*x+c))^n)-n*ln(d*x+c))/g-arctanh(x*g^(1/2)/f^(1/2))*(n*ln(b*x+a)-ln(e*
((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))*f^(1/2)/g^(3/2)-1/2*n*ln(b*x+a)*ln(b*(f^(
1/2)-x*g^(1/2))/(b*f^(1/2)+a*g^(1/2)))*f^(1/2)/g^(3/2)+1/2*n*ln(d*x+c)*ln(d
*(f^(1/2)-x*g^(1/2))/(d*f^(1/2)+c*g^(1/2)))*f^(1/2)/g^(3/2)+1/2*n*ln(b*x+a)
```

```
*ln(b*(f^(1/2)+x*g^(1/2))/(b*f^(1/2)-a*g^(1/2)))*f^(1/2)/g^(3/2)-1/2*n*ln(d
*x+c)*ln(d*(f^(1/2)+x*g^(1/2))/(d*f^(1/2)-c*g^(1/2)))*f^(1/2)/g^(3/2)+1/2*n
*polylog(2,-(b*x+a)*g^(1/2)/(b*f^(1/2)-a*g^(1/2)))*f^(1/2)/g^(3/2)-1/2*n*po
lylog(2,(b*x+a)*g^(1/2)/(b*f^(1/2)+a*g^(1/2)))*f^(1/2)/g^(3/2)-1/2*n*polylo
g(2,-(d*x+c)*g^(1/2)/(d*f^(1/2)-c*g^(1/2)))*f^(1/2)/g^(3/2)+1/2*n*polylog(2
,(d*x+c)*g^(1/2)/(d*f^(1/2)+c*g^(1/2)))*f^(1/2)/g^(3/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 550, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2593, 327, 214, 2463, 2436, 2332, 2456, 2441, 2440, 2438}

$$\begin{aligned}
&\int \frac{x^2 \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2} \, dx \\
&= -\frac{\sqrt{f}\text{arctanh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\left(-\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)}{g^{3/2}} \\
&+\frac{x\left(-\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)}{g} \\
&+\frac{\sqrt{f}n\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g^{3/2}}-\frac{\sqrt{f}n\,\text{PolyLog}\left(2,\frac{\sqrt{g}(a+bx)}{\sqrt{g}a+b\sqrt{f}}\right)}{2g^{3/2}} \\
&-\frac{\sqrt{f}n\log(a+bx)\log\left(\frac{b(\sqrt{f}-\sqrt{g}x)}{a\sqrt{g}+b\sqrt{f}}\right)}{2g^{3/2}}+\frac{\sqrt{f}n\log(a+bx)\log\left(\frac{b(\sqrt{f}+\sqrt{g}x)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g^{3/2}} \\
&-\frac{n(a+bx)\log(a+bx)}{bg}-\frac{\sqrt{f}n\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g^{3/2}} \\
&+\frac{\sqrt{f}n\,\text{PolyLog}\left(2,\frac{\sqrt{g}(c+dx)}{\sqrt{g}c+d\sqrt{f}}\right)}{2g^{3/2}}+\frac{\sqrt{f}n\log(c+dx)\log\left(\frac{d(\sqrt{f}-\sqrt{g}x)}{c\sqrt{g}+d\sqrt{f}}\right)}{2g^{3/2}} \\
&-\frac{\sqrt{f}n\log(c+dx)\log\left(\frac{d(\sqrt{f}+\sqrt{g}x)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g^{3/2}}+\frac{n(c+dx)\log(c+dx)}{dg}
\end{aligned}$$

```
[In] Int[(x^2*Log[e*((a + b*x)/(c + d*x))^n])/(f - g*x^2),x]
```

```
[Out] -((n*(a + b*x)*Log[a + b*x])/(b*g)) + (n*(c + d*x)*Log[c + d*x])/(d*g) + (x
*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/g - (S
qrt[f]*ArcTanh[(Sqrt[g]*x)/Sqrt[f]]*(n*Log[a + b*x] - Log[e*((a + b*x)/(c +
 d*x))^n] - n*Log[c + d*x]))/g^(3/2) - (Sqrt[f]*n*Log[a + b*x]*Log[(b*(Sqrt
[f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])])/(2*g^(3/2)) + (Sqrt[f]*n*Log[c
+ d*x]*Log[(d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g])])/(2*g^(3/2))
+ (Sqrt[f]*n*Log[a + b*x]*Log[(b*(Sqrt[f] + Sqrt[g]*x))/(b*Sqrt[f] - a*Sqrt
[g])])/(2*g^(3/2)) - (Sqrt[f]*n*Log[c + d*x]*Log[(d*(Sqrt[f] + Sqrt[g]*x))/
```

```
(d*Sqrt[f] - c*Sqrt[g])])/(2*g^(3/2)) + (Sqrt[f]*n*PolyLog[2, -((Sqrt[g]*(a
 + b*x))/(b*Sqrt[f] - a*Sqrt[g]))])/(2*g^(3/2)) - (Sqrt[f]*n*PolyLog[2, (Sq
rt[g]*(a + b*x))/(b*Sqrt[f] + a*Sqrt[g])])/(2*g^(3/2)) - (Sqrt[f]*n*PolyLog
[2, -((Sqrt[g]*(c + d*x))/(d*Sqrt[f] - c*Sqrt[g]))])/(2*g^(3/2)) + (Sqrt[f]
*n*PolyLog[2, (Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])])/(2*g^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2593

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= n\int \frac{x^2\log(a+bx)}{f-gx^2}\,dx - n\int \frac{x^2\log(c+dx)}{f-gx^2}\,dx \\
&\quad - \left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)\int \frac{x^2}{f-gx^2}\,dx \\
&= \frac{x\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{g} \\
&\quad + n\int \left(-\frac{\log(a+bx)}{g} + \frac{f\log(a+bx)}{g\left(f-gx^2\right)}\right)dx \\
&\quad - n\int \left(-\frac{\log(c+dx)}{g} + \frac{f\log(c+dx)}{g\left(f-gx^2\right)}\right)dx \\
&\quad - \frac{\left(f\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)\right)\int \frac{1}{f-gx^2}\,dx}{g}
\end{aligned}$$

$$= \frac{x\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{g} - \frac{\sqrt{f}\tanh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{g^{3/2}} - \frac{n\int\log(a+bx)\,dx}{g} + \frac{n\int\log(c+dx)\,dx}{g} + \frac{(fn)\int\frac{\log(a+bx)}{f-gx^2}\,dx}{g} - \frac{(fn)\int\frac{\log(c+dx)}{f-gx^2}\,dx}{g}$$

$$= \frac{x\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{g} - \frac{\sqrt{f}\tanh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{g^{3/2}} - \frac{n\text{Subst}(\int\log(x)\,dx,x,a+bx)}{bg} + \frac{n\text{Subst}(\int\log(x)\,dx,x,c+dx)}{dg} + \frac{(fn)\int\left(\frac{\log(a+bx)}{2\sqrt{f}(\sqrt{f}-\sqrt{g}x)}+\frac{\log(a+bx)}{2\sqrt{f}(\sqrt{f}+\sqrt{g}x)}\right)dx}{g} - \frac{(fn)\int\left(\frac{\log(c+dx)}{2\sqrt{f}(\sqrt{f}-\sqrt{g}x)}+\frac{\log(c+dx)}{2\sqrt{f}(\sqrt{f}+\sqrt{g}x)}\right)dx}{g}$$

$$= -\frac{n(a+bx)\log(a+bx)}{bg} + \frac{n(c+dx)\log(c+dx)}{dg} + \frac{x\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{g} - \frac{\sqrt{f}\tanh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{g^{3/2}} + \frac{(\sqrt{f}n)\int\frac{\log(a+bx)}{\sqrt{f}-\sqrt{g}x}\,dx}{2g} + \frac{(\sqrt{f}n)\int\frac{\log(a+bx)}{\sqrt{f}+\sqrt{g}x}\,dx}{2g} - \frac{(\sqrt{f}n)\int\frac{\log(c+dx)}{\sqrt{f}-\sqrt{g}x}\,dx}{2g} - \frac{(\sqrt{f}n)\int\frac{\log(c+dx)}{\sqrt{f}+\sqrt{g}x}\,dx}{2g}$$

$$= -\frac{n(a+bx)\log(a+bx)}{bg} + \frac{n(c+dx)\log(c+dx)}{dg} + \frac{x\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{g} - \frac{\sqrt{f}\tanh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{g^{3/2}} - \frac{\sqrt{f}n\log(a+bx)\log\left(\frac{b(\sqrt{f}-\sqrt{g}x)}{b\sqrt{f}+a\sqrt{g}}\right)}{2g^{3/2}} + \frac{\sqrt{f}n\log(c+dx)\log\left(\frac{d(\sqrt{f}-\sqrt{g}x)}{d\sqrt{f}+c\sqrt{g}}\right)}{2g^{3/2}} + \frac{\sqrt{f}n\log(a+bx)\log\left(\frac{b(\sqrt{f}+\sqrt{g}x)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g^{3/2}} - \frac{\sqrt{f}n\log(c+dx)\log\left(\frac{d(\sqrt{f}+\sqrt{g}x)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g^{3/2}} + \frac{\left(b\sqrt{f}n\right)\int \frac{\log\left(\frac{b(\sqrt{f}-\sqrt{g}x)}{b\sqrt{f}+a\sqrt{g}}\right)}{a+bx}dx}{2g^{3/2}} - \frac{\left(b\sqrt{f}n\right)\int \frac{\log\left(\frac{b(\sqrt{f}+\sqrt{g}x)}{b\sqrt{f}-a\sqrt{g}}\right)}{a+bx}dx}{2g^{3/2}} - \frac{\left(d\sqrt{f}n\right)\int \frac{\log\left(\frac{d(\sqrt{f}-\sqrt{g}x)}{d\sqrt{f}+c\sqrt{g}}\right)}{c+dx}dx}{2g^{3/2}} + \frac{\left(d\sqrt{f}n\right)\int \frac{\log\left(\frac{d(\sqrt{f}+\sqrt{g}x)}{d\sqrt{f}-c\sqrt{g}}\right)}{c+dx}dx}{2g^{3/2}}$$

$$= -\frac{n(a+bx)\log(a+bx)}{bg} + \frac{n(c+dx)\log(c+dx)}{dg} + \frac{x\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{g} - \frac{\sqrt{f}\tanh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{g^{3/2}} - \frac{\sqrt{f}n\log(a+bx)\log\left(\frac{b(\sqrt{f}-\sqrt{g}x)}{b\sqrt{f}+a\sqrt{g}}\right)}{2g^{3/2}} + \frac{\sqrt{f}n\log(c+dx)\log\left(\frac{d(\sqrt{f}-\sqrt{g}x)}{d\sqrt{f}+c\sqrt{g}}\right)}{2g^{3/2}} + \frac{\sqrt{f}n\log(a+bx)\log\left(\frac{b(\sqrt{f}+\sqrt{g}x)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g^{3/2}} - \frac{\sqrt{f}n\log(c+dx)\log\left(\frac{d(\sqrt{f}+\sqrt{g}x)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g^{3/2}} - \frac{\left(\sqrt{f}n\right)\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{g}x}{b\sqrt{f}-a\sqrt{g}}\right)}{x}dx, x, a+bx\right)}{2g^{3/2}} + \frac{\left(\sqrt{f}n\right)\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{g}x}{b\sqrt{f}+a\sqrt{g}}\right)}{x}dx, x, a+bx\right)}{2g^{3/2}} + \frac{\left(\sqrt{f}n\right)\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{g}x}{d\sqrt{f}-c\sqrt{g}}\right)}{x}dx, x, c+dx\right)}{2g^{3/2}} - \frac{\left(\sqrt{f}n\right)\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{g}x}{d\sqrt{f}+c\sqrt{g}}\right)}{x}dx, x, c+dx\right)}{2g^{3/2}}$$

$$
\begin{aligned}
&= -\frac{n(a+bx)\log(a+bx)}{bg} + \frac{n(c+dx)\log(c+dx)}{dg} \\
&\quad + \frac{x\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{g} \\
&\quad - \frac{\sqrt{f}\tanh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{g^{3/2}} \\
&\quad - \frac{\sqrt{f}n\log(a+bx)\log\left(\frac{b(\sqrt{f}-\sqrt{g}x)}{b\sqrt{f}+a\sqrt{g}}\right)}{2g^{3/2}} + \frac{\sqrt{f}n\log(c+dx)\log\left(\frac{d(\sqrt{f}-\sqrt{g}x)}{d\sqrt{f}+c\sqrt{g}}\right)}{2g^{3/2}} \\
&\quad + \frac{\sqrt{f}n\log(a+bx)\log\left(\frac{b(\sqrt{f}+\sqrt{g}x)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g^{3/2}} - \frac{\sqrt{f}n\log(c+dx)\log\left(\frac{d(\sqrt{f}+\sqrt{g}x)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g^{3/2}} \\
&\quad + \frac{\sqrt{f}n\mathrm{Li}_2\left(-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g^{3/2}} - \frac{\sqrt{f}n\mathrm{Li}_2\left(\frac{\sqrt{g}(a+bx)}{b\sqrt{f}+a\sqrt{g}}\right)}{2g^{3/2}} \\
&\quad - \frac{\sqrt{f}n\mathrm{Li}_2\left(-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g^{3/2}} + \frac{\sqrt{f}n\mathrm{Li}_2\left(\frac{\sqrt{g}(c+dx)}{d\sqrt{f}+c\sqrt{g}}\right)}{2g^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 467, normalized size of antiderivative = 0.85

$$
\int \frac{x^2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx
$$

$$
= \frac{-\frac{2\sqrt{g}(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b} + \frac{2(bc-ad)\sqrt{g}n\log(c+dx)}{bd} - \sqrt{f}\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\sqrt{f}-\sqrt{g}x\right) + \sqrt{f}\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\,\mathrm{l}\ldots}{\ldots}
$$

```
[In] Integrate[(x^2*Log[e*((a + b*x)/(c + d*x))^n])/(f - g*x^2),x]
```

```
[Out] ((-2*Sqrt[g]*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b + (2*(b*c - a*d)*S
qrt[g]*n*Log[c + d*x])/(b*d) - Sqrt[f]*Log[e*((a + b*x)/(c + d*x))^n]*Log[S
qrt[f] - Sqrt[g]*x] + Sqrt[f]*Log[e*((a + b*x)/(c + d*x))^n]*Log[Sqrt[f] +
Sqrt[g]*x] + Sqrt[f]*n*((Log[(Sqrt[g]*(a + b*x))/(b*Sqrt[f] + a*Sqrt[g])] -
 Log[(Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])])*Log[Sqrt[f] - Sqrt[g]*x]
 + PolyLog[2, (b*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])] - PolyLog[
2, (d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g])]) - Sqrt[f]*n*((Log[-(
(Sqrt[g]*(a + b*x))/(b*Sqrt[f] - a*Sqrt[g]))] - Log[-((Sqrt[g]*(c + d*x))/(
d*Sqrt[f] - c*Sqrt[g]))])*Log[Sqrt[f] + Sqrt[g]*x] + PolyLog[2, (b*(Sqrt[f]
 + Sqrt[g]*x))/(b*Sqrt[f] - a*Sqrt[g])] - PolyLog[2, (d*(Sqrt[f] + Sqrt[g]*
x))/(d*Sqrt[f] - c*Sqrt[g])]))/(2*g^(3/2))
```

Maple [F]

$$\int \frac{x^2 \ln \left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{-g\,x^2+f}dx$$

[In] int(x^2*ln(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x)

[Out] int(x^2*ln(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x)

Fricas [F]

$$\int \frac{x^2 \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx = \int -\frac{x^2 \log \left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{gx^2-f}\,dx$$

[In] integrate(x^2*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="fricas")

[Out] integral(-x^2*log(e*((b*x + a)/(d*x + c))^n)/(g*x^2 - f), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2 \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx = \text{Timed out}$$

[In] integrate(x**2*ln(e*((b*x+a)/(d*x+c))**n)/(-g*x**2+f),x)

[Out] Timed out

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1047 vs. $2(438) = 876$.

Time $= 0.40$ (sec) , antiderivative size $= 1047$, normalized size of antiderivative $= 1.90$

$$\int \frac{x^2 \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx = \text{Too large to display}$$

[In] integrate(x^2*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="maxima")

[Out] -1/2*(2*b*c*(c^2/((b*c*d^3 - a*d^4)*g*x + (b*c^2*d^2 - a*c*d^3)*g) + a^2*lo
g(b*x + a)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g) + (b*c^2 - 2*a*c*d)*log(
d*x + c)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*g))*d - 2*(c^3/((b*c*d^4 -
a*d^5)*g*x + (b*c^2*d^3 - a*c*d^4)*g) + a^3*log(b*x + a)/((b^4*c^2 - 2*a*b^
3*c*d + a^2*b^2*d^2)*g) + (2*b*c^3 - 3*a*c^2*d)*log(d*x + c)/((b^2*c^2*d^3
- 2*a*b*c*d^4 + a^2*d^5)*g) - x/(b*d^2*g))*b*d^2 + 2*a*(c^2/((b*c*d^3 - a*d

```
^4)*g*x + (b*c^2*d^2 - a*c*d^3)*g) + a^2*log(b*x + a)/((b^3*c^2 - 2*a*b^2*c
*d + a^2*b*d^2)*g) + (b*c^2 - 2*a*c*d)*log(d*x + c)/((b^2*c^2*d^2 - 2*a*b*c
*d^3 + a^2*d^4)*g))*d^2 - 2*a*c*d*(c/((b*c*d^2 - a*d^3)*g*x + (b*c^2*d - a*
c*d^2)*g) + a*log(b*x + a)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*g) - a*log(d*x
+ c)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*g)) - 2*b*d*(a^2*log(b*x + a)/((b^3*c
 - a*b^2*d)*g) - c^2*log(d*x + c)/((b*c*d^2 - a*d^3)*g) + x/(b*d*g)) + 2*b*
c*(a*log(b*x + a)/((b^2*c - a*b*d)*g) - c*log(d*x + c)/((b*c*d - a*d^2)*g))
 - (log(sqrt(g)*x - sqrt(f))*log((b*sqrt(g)*x - b*sqrt(f))/(b*sqrt(f) + a*s
qrt(g)) + 1) + dilog(-(b*sqrt(g)*x - b*sqrt(f))/(b*sqrt(f) + a*sqrt(g))))*s
qrt(f)/g^(3/2) + (log(sqrt(g)*x + sqrt(f))*log(-(b*sqrt(g)*x + b*sqrt(f))/(
b*sqrt(f) - a*sqrt(g)) + 1) + dilog((b*sqrt(g)*x + b*sqrt(f))/(b*sqrt(f) -
a*sqrt(g))))*sqrt(f)/g^(3/2) + (log(sqrt(g)*x - sqrt(f))*log((d*sqrt(g)*x -
 d*sqrt(f))/(d*sqrt(f) + c*sqrt(g)) + 1) + dilog(-(d*sqrt(g)*x - d*sqrt(f))
/(d*sqrt(f) + c*sqrt(g))))*sqrt(f)/g^(3/2) - (log(sqrt(g)*x + sqrt(f))*log(
-(d*sqrt(g)*x + d*sqrt(f))/(d*sqrt(f) - c*sqrt(g)) + 1) + dilog((d*sqrt(g)*
x + d*sqrt(f))/(d*sqrt(f) - c*sqrt(g))))*sqrt(f)/g^(3/2))*n - 1/2*(f*log((g
*x - sqrt(f*g))/(g*x + sqrt(f*g)))/(sqrt(f*g)*g) + 2*x/g)*log(e*((b*x + a)/
(d*x + c))^n)
```

Giac [F]

$$\int \frac{x^2 \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx = \int -\frac{x^2 \log \left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{gx^2-f}\,dx$$

```
[In] integrate(x^2*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="giac")
[Out] integrate(-x^2*log(e*((b*x + a)/(d*x + c))^n)/(g*x^2 - f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx = \int \frac{x^2\,\ln \left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{f-g\,x^2}\,dx$$

```
[In] int((x^2*log(e*((a + b*x)/(c + d*x))^n))/(f - g*x^2),x)
[Out] int((x^2*log(e*((a + b*x)/(c + d*x))^n))/(f - g*x^2), x)
```

3.78 $\int \frac{x \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2} dx$

Optimal result 620
Rubi [A] (verified) 621
Mathematica [A] (verified) 624
Maple [A] (verified) 624
Fricas [F] 625
Sympy [F(-1)] 625
Maxima [F] 626
Giac [F] 626
Mupad [F(-1)] 626

Optimal result

Integrand size = 30, antiderivative size = 403

$$\begin{aligned}\int \frac{x \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2} dx = & -\frac{n \log(a+bx) \log\left(\frac{b(\sqrt{f}-\sqrt{g}x)}{b\sqrt{f}+a\sqrt{g}}\right)}{2g} + \frac{n \log(c+dx) \log\left(\frac{d(\sqrt{f}-\sqrt{g}x)}{d\sqrt{f}+c\sqrt{g}}\right)}{2g} \\ & - \frac{n \log(a+bx) \log\left(\frac{b(\sqrt{f}+\sqrt{g}x)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g} + \frac{n \log(c+dx) \log\left(\frac{d(\sqrt{f}+\sqrt{g}x)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g} \\ & + \frac{\left(n \log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right) \log(f-gx^2)}{2g} \\ & - \frac{n \operatorname{PolyLog}\left(2, -\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g} - \frac{n \operatorname{PolyLog}\left(2, \frac{\sqrt{g}(a+bx)}{b\sqrt{f}+a\sqrt{g}}\right)}{2g} \\ & + \frac{n \operatorname{PolyLog}\left(2, -\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g} + \frac{n \operatorname{PolyLog}\left(2, \frac{\sqrt{g}(c+dx)}{d\sqrt{f}+c\sqrt{g}}\right)}{2g}\end{aligned}$$

```
[Out] 1/2*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))*ln(-g*x^2+f)/g-1/2*
n*ln(b*x+a)*ln(b*(f^(1/2)-x*g^(1/2))/(b*f^(1/2)+a*g^(1/2)))/g+1/2*n*ln(d*x+
c)*ln(d*(f^(1/2)-x*g^(1/2))/(d*f^(1/2)+c*g^(1/2)))/g-1/2*n*ln(b*x+a)*ln(b*(
f^(1/2)+x*g^(1/2))/(b*f^(1/2)-a*g^(1/2)))/g+1/2*n*ln(d*x+c)*ln(d*(f^(1/2)+x
*g^(1/2))/(d*f^(1/2)-c*g^(1/2)))/g-1/2*n*polylog(2,-(b*x+a)*g^(1/2)/(b*f^(1
/2)-a*g^(1/2)))/g-1/2*n*polylog(2,(b*x+a)*g^(1/2)/(b*f^(1/2)+a*g^(1/2)))/g+
1/2*n*polylog(2,-(d*x+c)*g^(1/2)/(d*f^(1/2)-c*g^(1/2)))/g+1/2*n*polylog(2,(
d*x+c)*g^(1/2)/(d*f^(1/2)+c*g^(1/2)))/g
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2593, 266, 2463, 2441, 2440, 2438}

$$\int \frac{x \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2} \, dx = \frac{\log(f-gx^2)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)}{2g} - \frac{n \operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g} - \frac{n \operatorname{PolyLog}\left(2,\frac{\sqrt{g}(a+bx)}{\sqrt{g}a+b\sqrt{f}}\right)}{2g} - \frac{n\log(a+bx)\log\left(\frac{b(\sqrt{f}-\sqrt{g}x)}{a\sqrt{g}+b\sqrt{f}}\right)}{2g} - \frac{n\log(a+bx)\log\left(\frac{b(\sqrt{f}+\sqrt{g}x)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g} + \frac{n \operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g} + \frac{n \operatorname{PolyLog}\left(2,\frac{\sqrt{g}(c+dx)}{\sqrt{g}c+d\sqrt{f}}\right)}{2g} + \frac{n\log(c+dx)\log\left(\frac{d(\sqrt{f}-\sqrt{g}x)}{c\sqrt{g}+d\sqrt{f}}\right)}{2g} + \frac{n\log(c+dx)\log\left(\frac{d(\sqrt{f}+\sqrt{g}x)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g}$$

```
[In] Int[(x*Log[e*((a + b*x)/(c + d*x))^n])/(f - g*x^2),x]
```

```
[Out] -1/2*(n*Log[a + b*x]*Log[(b*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])]
)/g + (n*Log[c + d*x]*Log[(d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g])
])/(2*g) - (n*Log[a + b*x]*Log[(b*(Sqrt[f] + Sqrt[g]*x))/(b*Sqrt[f] - a*Sqr
t[g])])/(2*g) + (n*Log[c + d*x]*Log[(d*(Sqrt[f] + Sqrt[g]*x))/(d*Sqrt[f] -
c*Sqrt[g])])/(2*g) + ((n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*
Log[c + d*x])*Log[f - g*x^2])/(2*g) - (n*PolyLog[2, -((Sqrt[g]*(a + b*x))/(
b*Sqrt[f] - a*Sqrt[g]))])/(2*g) - (n*PolyLog[2, (Sqrt[g]*(a + b*x))/(b*Sqrt
[f] + a*Sqrt[g])])/(2*g) + (n*PolyLog[2, -((Sqrt[g]*(c + d*x))/(d*Sqrt[f] -
 c*Sqrt[g]))])/(2*g) + (n*PolyLog[2, (Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqr
t[g])])/(2*g)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
```

```
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2593

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= n\int \frac{x\log(a+bx)}{f-gx^2}\,dx - n\int \frac{x\log(c+dx)}{f-gx^2}\,dx \\
&\quad - \left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\int \frac{x}{f-gx^2}\,dx \\
&= \frac{\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\log\left(f-gx^2\right)}{2g} \\
&\quad + n\int \left(\frac{\log(a+bx)}{2\sqrt{g}\left(\sqrt{f}-\sqrt{g}x\right)}-\frac{\log(a+bx)}{2\sqrt{g}\left(\sqrt{f}+\sqrt{g}x\right)}\right)dx \\
&\quad - n\int \left(\frac{\log(c+dx)}{2\sqrt{g}\left(\sqrt{f}-\sqrt{g}x\right)}-\frac{\log(c+dx)}{2\sqrt{g}\left(\sqrt{f}+\sqrt{g}x\right)}\right)dx \\
&= \frac{\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\log\left(f-gx^2\right)}{2g} \\
&\quad + \frac{n\int \frac{\log(a+bx)}{\sqrt{f}-\sqrt{g}x}\,dx}{2\sqrt{g}} - \frac{n\int \frac{\log(a+bx)}{\sqrt{f}+\sqrt{g}x}\,dx}{2\sqrt{g}} - \frac{n\int \frac{\log(c+dx)}{\sqrt{f}-\sqrt{g}x}\,dx}{2\sqrt{g}} + \frac{n\int \frac{\log(c+dx)}{\sqrt{f}+\sqrt{g}x}\,dx}{2\sqrt{g}}
\end{aligned}$$

$$= -\frac{n\log(a+bx)\log\left(\frac{b(\sqrt{f}-\sqrt{g}x)}{b\sqrt{f}+a\sqrt{g}}\right)}{2g} + \frac{n\log(c+dx)\log\left(\frac{d(\sqrt{f}-\sqrt{g}x)}{d\sqrt{f}+c\sqrt{g}}\right)}{2g} - \frac{n\log(a+bx)\log\left(\frac{b(\sqrt{f}+\sqrt{g}x)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g} + \frac{n\log(c+dx)\log\left(\frac{d(\sqrt{f}+\sqrt{g}x)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g} + \frac{\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\log\left(f-gx^2\right)}{2g} + \frac{(bn)\int\frac{\log\left(\frac{b(\sqrt{f}-\sqrt{g}x)}{b\sqrt{f}+a\sqrt{g}}\right)}{a+bx}dx}{2g} + \frac{(bn)\int\frac{\log\left(\frac{b(\sqrt{f}+\sqrt{g}x)}{b\sqrt{f}-a\sqrt{g}}\right)}{a+bx}dx}{2g} - \frac{(dn)\int\frac{\log\left(\frac{d(\sqrt{f}-\sqrt{g}x)}{d\sqrt{f}+c\sqrt{g}}\right)}{c+dx}dx}{2g} - \frac{(dn)\int\frac{\log\left(\frac{d(\sqrt{f}+\sqrt{g}x)}{d\sqrt{f}-c\sqrt{g}}\right)}{c+dx}dx}{2g}$$

$$= -\frac{n\log(a+bx)\log\left(\frac{b(\sqrt{f}-\sqrt{g}x)}{b\sqrt{f}+a\sqrt{g}}\right)}{2g} + \frac{n\log(c+dx)\log\left(\frac{d(\sqrt{f}-\sqrt{g}x)}{d\sqrt{f}+c\sqrt{g}}\right)}{2g} - \frac{n\log(a+bx)\log\left(\frac{b(\sqrt{f}+\sqrt{g}x)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g} + \frac{n\log(c+dx)\log\left(\frac{d(\sqrt{f}+\sqrt{g}x)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g} + \frac{\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\log\left(f-gx^2\right)}{2g} + \frac{n\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{g}x}{b\sqrt{f}-a\sqrt{g}}\right)}{x}dx,x,a+bx\right)}{2g} + \frac{n\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{g}x}{b\sqrt{f}+a\sqrt{g}}\right)}{x}dx,x,a+bx\right)}{2g} - \frac{n\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{g}x}{d\sqrt{f}-c\sqrt{g}}\right)}{x}dx,x,c+dx\right)}{2g} - \frac{n\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{g}x}{d\sqrt{f}+c\sqrt{g}}\right)}{x}dx,x,c+dx\right)}{2g}$$

$$= -\frac{n\log(a+bx)\log\left(\frac{b(\sqrt{f}-\sqrt{g}x)}{b\sqrt{f}+a\sqrt{g}}\right)}{2g} + \frac{n\log(c+dx)\log\left(\frac{d(\sqrt{f}-\sqrt{g}x)}{d\sqrt{f}+c\sqrt{g}}\right)}{2g} - \frac{n\log(a+bx)\log\left(\frac{b(\sqrt{f}+\sqrt{g}x)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g} + \frac{n\log(c+dx)\log\left(\frac{d(\sqrt{f}+\sqrt{g}x)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g} + \frac{\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\log\left(f-gx^2\right)}{2g} - \frac{n\text{Li}_2\left(-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g} - \frac{n\text{Li}_2\left(\frac{\sqrt{g}(a+bx)}{b\sqrt{f}+a\sqrt{g}}\right)}{2g} + \frac{n\text{Li}_2\left(-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g} + \frac{n\text{Li}_2\left(\frac{\sqrt{g}(c+dx)}{d\sqrt{f}+c\sqrt{g}}\right)}{2g}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.02

$$\int \frac{x\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx = -\frac{-n\log\left(\frac{\sqrt{g}(a+bx)}{b\sqrt{f}+a\sqrt{g}}\right)\log\left(\sqrt{f}-\sqrt{g}x\right)+\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\sqrt{f}-\sqrt{g}x\right)+n\log\left(\frac{\sqrt{g}(c+dx)}{d\sqrt{f}+c\sqrt{g}}\right)\log\left(\sqrt{f}-\sqrt{g}x\right)}{}$$

```
[In] Integrate[(x*Log[e*((a + b*x)/(c + d*x))^n])/(f - g*x^2),x]
```

```
[Out] -1/2*(-(n*Log[(Sqrt[g]*(a + b*x))/(b*Sqrt[f] + a*Sqrt[g])]*Log[Sqrt[f] - Sq
rt[g]*x]) + Log[e*((a + b*x)/(c + d*x))^n]*Log[Sqrt[f] - Sqrt[g]*x] + n*Log
[(Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])]*Log[Sqrt[f] - Sqrt[g]*x] - n*
Log[-((Sqrt[g]*(a + b*x))/(b*Sqrt[f] - a*Sqrt[g]))]*Log[Sqrt[f] + Sqrt[g]*x
] + Log[e*((a + b*x)/(c + d*x))^n]*Log[Sqrt[f] + Sqrt[g]*x] + n*Log[-((Sqrt
[g]*(c + d*x))/(d*Sqrt[f] - c*Sqrt[g]))]*Log[Sqrt[f] + Sqrt[g]*x] - n*PolyL
og[2, (b*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])] + n*PolyLog[2, (d*
(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g])] - n*PolyLog[2, (b*(Sqrt[f]
+ Sqrt[g]*x))/(b*Sqrt[f] - a*Sqrt[g])] + n*PolyLog[2, (d*(Sqrt[f] + Sqrt[g]
*x))/(d*Sqrt[f] - c*Sqrt[g])])/g
```

Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.09

method	result
parts	$-\frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\ln\left(-g\,x^{2}+f\right)}{2g}+\frac{n(-ad+cb)\left(\frac{\frac{\ln(dx+c)\ln\left(-g\,x^{2}+f\right)}{d}+\frac{2g\left(-\frac{\ln(dx+c)\left(\ln\left(\frac{d\sqrt{fg}-(dx+c)g+cg}{d\sqrt{fg}+cg}\right)+\ln\left(\frac{d\sqrt{fg}+(dx+c)g}{d\sqrt{fg}-cg}\right)\right)}{2g}\right)}{}}{ad-cb}\right)}{}$

[In] int(x*ln(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x,method=_RETURNVERBOSE)

[Out] -1/2*ln(e*((b*x+a)/(d*x+c))^n)/g*ln(-g*x^2+f)+1/2/g*n*(-a*d+b*c)*((ln(d*x+c)/d*ln(-g*x^2+f)+2/d*g*(-1/2*ln(d*x+c)*(ln((d*(f*g)^(1/2)-(d*x+c)*g+c*g)/(d*(f*g)^(1/2)+c*g))+ln((d*(f*g)^(1/2)+(d*x+c)*g-c*g)/(d*(f*g)^(1/2)-c*g)))/g-1/2*(dilog((d*(f*g)^(1/2)-(d*x+c)*g+c*g)/(d*(f*g)^(1/2)+c*g))+dilog((d*(f*g)^(1/2)+(d*x+c)*g-c*g)/(d*(f*g)^(1/2)-c*g)))/g))*d/(a*d-b*c)-(ln(b*x+a)/b*ln(-g*x^2+f)+2/b*g*(-1/2*ln(b*x+a)*(ln((b*(f*g)^(1/2)-g*(b*x+a)+a*g)/(b*(f*g)^(1/2)+a*g))+ln((b*(f*g)^(1/2)+g*(b*x+a)-a*g)/(b*(f*g)^(1/2)-a*g)))/g-1/2*(dilog((b*(f*g)^(1/2)-g*(b*x+a)+a*g)/(b*(f*g)^(1/2)+a*g))+dilog((b*(f*g)^(1/2)+g*(b*x+a)-a*g)/(b*(f*g)^(1/2)-a*g)))/g))*b/(a*d-b*c))

Fricas [F]

$$\int \frac{x\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}{f-gx^{2}}\,dx=\int -\frac{x\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{gx^{2}-f}\,dx$$

[In] integrate(x*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="fricas")

[Out] integral(-x*log(e*((b*x + a)/(d*x + c))^n)/(g*x^2 - f), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{x\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}{f-gx^{2}}\,dx=\text{Timed out}$$

[In] integrate(x*ln(e*((b*x+a)/(d*x+c))**n)/(-g*x**2+f),x)

[Out] Timed out

Maxima [F]

$$\int \frac{x\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx=\int -\frac{x\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{gx^2-f}\,dx$$

```
[In] integrate(x*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="maxima")
[Out] -integrate(x*log(e*((b*x + a)/(d*x + c))^n)/(g*x^2 - f), x)
```

Giac [F]

$$\int \frac{x\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx=\int -\frac{x\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{gx^2-f}\,dx$$

```
[In] integrate(x*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="giac")
[Out] integrate(-x*log(e*((b*x + a)/(d*x + c))^n)/(g*x^2 - f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx=\int \frac{x\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{f-g\,x^2}\,dx$$

```
[In] int((x*log(e*((a + b*x)/(c + d*x))^n))/(f - g*x^2),x)
[Out] int((x*log(e*((a + b*x)/(c + d*x))^n))/(f - g*x^2), x)
```

3.79 $\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx$

Optimal result 627
Rubi [A] (verified) 627
Mathematica [A] (verified) 630
Maple [F] 630
Fricas [F] 630
Sympy [F(-1)] 631
Maxima [A] (verification not implemented) 631
Giac [F] 632
Mupad [F(-1)] 632

Optimal result

Integrand size = 29, antiderivative size = 291

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx = \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{(d\sqrt{f}-c\sqrt{g})(a+bx)}{(b\sqrt{f}-a\sqrt{g})(c+dx)}\right)}{2\sqrt{f}\sqrt{g}} - \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{(d\sqrt{f}+c\sqrt{g})(a+bx)}{(b\sqrt{f}+a\sqrt{g})(c+dx)}\right)}{2\sqrt{f}\sqrt{g}} + \frac{n\,\text{PolyLog}\left(2,\frac{(d\sqrt{f}-c\sqrt{g})(a+bx)}{(b\sqrt{f}-a\sqrt{g})(c+dx)}\right)}{2\sqrt{f}\sqrt{g}} - \frac{n\,\text{PolyLog}\left(2,\frac{(d\sqrt{f}+c\sqrt{g})(a+bx)}{(b\sqrt{f}+a\sqrt{g})(c+dx)}\right)}{2\sqrt{f}\sqrt{g}}$$

```
[Out] 1/2*ln(e*((b*x+a)/(d*x+c))^n)*ln(1-(b*x+a)*(d*f^(1/2)-c*g^(1/2))/(d*x+c)/(b
*f^(1/2)-a*g^(1/2)))/f^(1/2)/g^(1/2)-1/2*ln(e*((b*x+a)/(d*x+c))^n)*ln(1-(b*
x+a)*(d*f^(1/2)+c*g^(1/2))/(d*x+c)/(b*f^(1/2)+a*g^(1/2)))/f^(1/2)/g^(1/2)+1
/2*n*polylog(2,(b*x+a)*(d*f^(1/2)-c*g^(1/2))/(d*x+c)/(b*f^(1/2)-a*g^(1/2)))
/f^(1/2)/g^(1/2)-1/2*n*polylog(2,(b*x+a)*(d*f^(1/2)+c*g^(1/2))/(d*x+c)/(b*f
^(1/2)+a*g^(1/2)))/f^(1/2)/g^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used

= {2576, 2404, 2354, 2438}

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx = \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{(a+bx)\left(d\sqrt{f}-c\sqrt{g}\right)}{(c+dx)\left(b\sqrt{f}-a\sqrt{g}\right)}\right)}{2\sqrt{f}\sqrt{g}} - \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{(a+bx)\left(c\sqrt{g}+d\sqrt{f}\right)}{(c+dx)\left(a\sqrt{g}+b\sqrt{f}\right)}\right)}{2\sqrt{f}\sqrt{g}} + \frac{n\,\text{PolyLog}\left(2,\frac{\left(d\sqrt{f}-c\sqrt{g}\right)(a+bx)}{\left(b\sqrt{f}-a\sqrt{g}\right)(c+dx)}\right)}{2\sqrt{f}\sqrt{g}} - \frac{n\,\text{PolyLog}\left(2,\frac{\left(\sqrt{g}c+d\sqrt{f}\right)(a+bx)}{\left(\sqrt{g}a+b\sqrt{f}\right)(c+dx)}\right)}{2\sqrt{f}\sqrt{g}}$$

```
[In] Int[Log[e*((a + b*x)/(c + d*x))^n]/(f - g*x^2),x]
```

```
[Out] (Log[e*((a + b*x)/(c + d*x))^n]*Log[1 - ((d*Sqrt[f] - c*Sqrt[g])*(a + b*x))
/((b*Sqrt[f] - a*Sqrt[g])*(c + d*x))])/(2*Sqrt[f]*Sqrt[g]) - (Log[e*((a + b
*x)/(c + d*x))^n]*Log[1 - ((d*Sqrt[f] + c*Sqrt[g])*(a + b*x))/((b*Sqrt[f] +
 a*Sqrt[g])*(c + d*x))])/(2*Sqrt[f]*Sqrt[g]) + (n*PolyLog[2, ((d*Sqrt[f] -
c*Sqrt[g])*(a + b*x))/((b*Sqrt[f] - a*Sqrt[g])*(c + d*x))])/(2*Sqrt[f]*Sqrt
[g]) - (n*PolyLog[2, ((d*Sqrt[f] + c*Sqrt[g])*(a + b*x))/((b*Sqrt[f] + a*Sq
rt[g])*(c + d*x))])/(2*Sqrt[f]*Sqrt[g])
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2576

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*(P2x_)^(m_.), x_Symbol] :> With[{f = Coeff[P2x, x, 0], g = Coef
f[P2x, x, 1], h = Coeff[P2x, x, 2]}, Dist[b*c - a*d, Subst[Int[(b^2*f - a*b
*g + a^2*h - (2*b*d*f - b*c*g - a*d*g + 2*a*c*h)*x + (d^2*f - c*d*g + c^2*h
)*x^2)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(2*(m + 1))), x], x, (a + b*x)/(c
+ d*x)], x]] /; FreeQ[{a, b, c, d, e, A, B, n}, x] && PolyQ[P2x, x, 2] && N
```

```
eQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = (bc-ad)\text{Subst}\left(\int \frac{\log(ex^n)}{b^2f-a^2g-(2bdf-2acg)x+(d^2f-c^2g)x^2}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= (bc-ad)\text{Subst}\left(\int \left(\frac{(d^2f-c^2g)\log(ex^n)}{(bc-ad)\sqrt{f}\sqrt{g}\left(2bdf-2(bc-ad)\sqrt{f}\sqrt{g}-2acg-2(d^2f-c^2g)x\right)} + \frac{(d^2f-c^2g)\log(ex^n)}{(bc-ad)\sqrt{f}\sqrt{g}\left(-2bdf-2(bc-ad)\sqrt{f}\sqrt{g}+2acg+2(d^2f-c^2g)x\right)}\right)dx, x, \frac{a+bx}{c+dx}\right)$$

$$= \frac{(d^2f-c^2g)\text{Subst}\left(\int \frac{\log(ex^n)}{2bdf-2(bc-ad)\sqrt{f}\sqrt{g}-2acg-2(d^2f-c^2g)x}\,dx, x, \frac{a+bx}{c+dx}\right)}{\sqrt{f}\sqrt{g}} + \frac{(d^2f-c^2g)\text{Subst}\left(\int \frac{\log(ex^n)}{-2bdf-2(bc-ad)\sqrt{f}\sqrt{g}+2acg+2(d^2f-c^2g)x}\,dx, x, \frac{a+bx}{c+dx}\right)}{\sqrt{f}\sqrt{g}}$$

$$= \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{(d\sqrt{f}-c\sqrt{g})(a+bx)}{(b\sqrt{f}-a\sqrt{g})(c+dx)}\right)}{2\sqrt{f}\sqrt{g}} - \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{(d\sqrt{f}+c\sqrt{g})(a+bx)}{(b\sqrt{f}+a\sqrt{g})(c+dx)}\right)}{2\sqrt{f}\sqrt{g}} + \frac{n\text{Subst}\left(\int \frac{\log\left(1-\frac{2(d^2f-c^2g)x}{2bdf-2(bc-ad)\sqrt{f}\sqrt{g}-2acg}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{2\sqrt{f}\sqrt{g}} - \frac{n\text{Subst}\left(\int \frac{\log\left(1+\frac{2(d^2f-c^2g)x}{-2bdf-2(bc-ad)\sqrt{f}\sqrt{g}+2acg}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{2\sqrt{f}\sqrt{g}}$$

$$= \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{(d\sqrt{f}-c\sqrt{g})(a+bx)}{(b\sqrt{f}-a\sqrt{g})(c+dx)}\right)}{2\sqrt{f}\sqrt{g}} - \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{(d\sqrt{f}+c\sqrt{g})(a+bx)}{(b\sqrt{f}+a\sqrt{g})(c+dx)}\right)}{2\sqrt{f}\sqrt{g}} + \frac{n\text{Li}_2\left(\frac{(d\sqrt{f}-c\sqrt{g})(a+bx)}{(b\sqrt{f}-a\sqrt{g})(c+dx)}\right)}{2\sqrt{f}\sqrt{g}} - \frac{n\text{Li}_2\left(\frac{(d\sqrt{f}+c\sqrt{g})(a+bx)}{(b\sqrt{f}+a\sqrt{g})(c+dx)}\right)}{2\sqrt{f}\sqrt{g}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.45

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx$$
$$= \frac{n\log\left(\frac{\sqrt{g}(a+bx)}{b\sqrt{f}+a\sqrt{g}}\right)\log\left(\sqrt{f}-\sqrt{g}x\right)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\sqrt{f}-\sqrt{g}x\right)-n\log\left(\frac{\sqrt{g}(c+dx)}{d\sqrt{f}+c\sqrt{g}}\right)\log\left(\sqrt{f}-\sqrt{g}x\right)-}{}$$

```
[In] Integrate[Log[e*((a + b*x)/(c + d*x))^n]/(f - g*x^2),x]
```

```
[Out] (n*Log[(Sqrt[g]*(a + b*x))/(b*Sqrt[f] + a*Sqrt[g])]*Log[Sqrt[f] - Sqrt[g]*x
] - Log[e*((a + b*x)/(c + d*x))^n]*Log[Sqrt[f] - Sqrt[g]*x] - n*Log[(Sqrt[g
]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])]*Log[Sqrt[f] - Sqrt[g]*x] - n*Log[-((S
qrt[g]*(a + b*x))/(b*Sqrt[f] - a*Sqrt[g]))]*Log[Sqrt[f] + Sqrt[g]*x] + Log[
e*((a + b*x)/(c + d*x))^n]*Log[Sqrt[f] + Sqrt[g]*x] + n*Log[-((Sqrt[g]*(c +
 d*x))/(d*Sqrt[f] - c*Sqrt[g]))]*Log[Sqrt[f] + Sqrt[g]*x] + n*PolyLog[2, (b
*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])] - n*PolyLog[2, (d*(Sqrt[f]
 - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g])] - n*PolyLog[2, (b*(Sqrt[f] + Sqrt[g
]*x))/(b*Sqrt[f] - a*Sqrt[g])] + n*PolyLog[2, (d*(Sqrt[f] + Sqrt[g]*x))/(d*
Sqrt[f] - c*Sqrt[g])])/(2*Sqrt[f]*Sqrt[g])
```

Maple [F]

$$\int \frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{-g\,x^2+f}dx$$

```
[In] int(ln(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x)
```

```
[Out] int(ln(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x)
```

Fricas [F]

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx = \int -\frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{gx^2-f}\,dx$$

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(-log(e*((b*x + a)/(d*x + c))^n)/(g*x^2 - f), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx = \text{Timed out}$$

```
[In] integrate(ln(e*((b*x+a)/(d*x+c))**n)/(-g*x**2+f),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.20

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx$$
$$= \frac{\left(\log\left(\sqrt{g}x-\sqrt{f}\right)\log\left(\frac{b\sqrt{g}x-b\sqrt{f}}{b\sqrt{f}+a\sqrt{g}}+1\right)-\log\left(\sqrt{g}x+\sqrt{f}\right)\log\left(-\frac{b\sqrt{g}x+b\sqrt{f}}{b\sqrt{f}-a\sqrt{g}}+1\right)-\log\left(\sqrt{g}x-\sqrt{f}\right)\log\left(\frac{d}{c}\right.\right.}{}$$
$$-\frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\log\left(\frac{gx-\sqrt{fg}}{gx+\sqrt{fg}}\right)}{2\sqrt{fg}}$$

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="maxima")
```

```
[Out] 1/2*(log(sqrt(g)*x - sqrt(f))*log((b*sqrt(g)*x - b*sqrt(f))/(b*sqrt(f) + a*
sqrt(g)) + 1) - log(sqrt(g)*x + sqrt(f))*log(-(b*sqrt(g)*x + b*sqrt(f))/(b*
sqrt(f) - a*sqrt(g)) + 1) - log(sqrt(g)*x - sqrt(f))*log((d*sqrt(g)*x - d*s
qrt(f))/(d*sqrt(f) + c*sqrt(g)) + 1) + log(sqrt(g)*x + sqrt(f))*log(-(d*sqr
t(g)*x + d*sqrt(f))/(d*sqrt(f) - c*sqrt(g)) + 1) + dilog(-(b*sqrt(g)*x - b*
sqrt(f))/(b*sqrt(f) + a*sqrt(g))) - dilog((b*sqrt(g)*x + b*sqrt(f))/(b*sqrt
(f) - a*sqrt(g))) - dilog(-(d*sqrt(g)*x - d*sqrt(f))/(d*sqrt(f) + c*sqrt(g)
)) + dilog((d*sqrt(g)*x + d*sqrt(f))/(d*sqrt(f) - c*sqrt(g))))*n/sqrt(f*g)
- 1/2*log(e*((b*x + a)/(d*x + c))^n)*log((g*x - sqrt(f*g))/(g*x + sqrt(f*g)
))/sqrt(f*g)
```

Giac [F]

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx = \int -\frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{gx^2-f}\,dx$$

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="giac")
[Out] integrate(-log(e*((b*x + a)/(d*x + c))^n)/(g*x^2 - f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx = \int \frac{\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{f-g\,x^2}\,dx$$

```
[In] int(log(e*((a + b*x)/(c + d*x))^n)/(f - g*x^2),x)
[Out] int(log(e*((a + b*x)/(c + d*x))^n)/(f - g*x^2), x)
```

3.80 $\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x(f-gx^2)}\,dx$

Optimal result . . . 633
Rubi [A] (verified) . . . 634
Mathematica [A] (verified) . . . 639
Maple [A] (verified) . . . 640
Fricas [F] . . . 641
Sympy [F(-1)] . . . 641
Maxima [F] . . . 641
Giac [F] . . . 641
Mupad [F(-1)] . . . 642

Optimal result

Integrand size = 32, antiderivative size = 518

$$\begin{aligned}\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x(f-gx^2)}\,dx &= \frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f}-\frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f}\\&-\frac{\log(x)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{f}\\&-\frac{n\log(a+bx)\log\left(\frac{b(\sqrt{f}-\sqrt{g}x)}{b\sqrt{f}+a\sqrt{g}}\right)}{2f}+\frac{n\log(c+dx)\log\left(\frac{d(\sqrt{f}-\sqrt{g}x)}{d\sqrt{f}+c\sqrt{g}}\right)}{2f}\\&-\frac{n\log(a+bx)\log\left(\frac{b(\sqrt{f}+\sqrt{g}x)}{b\sqrt{f}-a\sqrt{g}}\right)}{2f}+\frac{n\log(c+dx)\log\left(\frac{d(\sqrt{f}+\sqrt{g}x)}{d\sqrt{f}-c\sqrt{g}}\right)}{2f}\\&+\frac{\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\log(f-gx^2)}{2f}\\&-\frac{n\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2f}-\frac{n\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(a+bx)}{b\sqrt{f}+a\sqrt{g}}\right)}{2f}\\&+\frac{n\operatorname{PolyLog}\left(2,1+\frac{bx}{a}\right)}{f}+\frac{n\operatorname{PolyLog}\left(2,-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2f}\\&+\frac{n\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(c+dx)}{d\sqrt{f}+c\sqrt{g}}\right)}{2f}-\frac{n\operatorname{PolyLog}\left(2,1+\frac{dx}{c}\right)}{f}\end{aligned}$$

```
[Out] n*ln(-b*x/a)*ln(b*x+a)/f-n*ln(-d*x/c)*ln(d*x+c)/f-ln(x)*(n*ln(b*x+a)-ln(e*(
(b*x+a)/(d*x+c))^n)-n*ln(d*x+c))/f+1/2*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^
n)-n*ln(d*x+c))*ln(-g*x^2+f)/f-1/2*n*ln(b*x+a)*ln(b*(f^(1/2)-x*g^(1/2))/(b*
f^(1/2)+a*g^(1/2)))/f+1/2*n*ln(d*x+c)*ln(d*(f^(1/2)-x*g^(1/2))/(d*f^(1/2)+c
```

```
*g^(1/2)))/f-1/2*n*ln(b*x+a)*ln(b*(f^(1/2)+x*g^(1/2))/(b*f^(1/2)-a*g^(1/2))
)/f+1/2*n*ln(d*x+c)*ln(d*(f^(1/2)+x*g^(1/2))/(d*f^(1/2)-c*g^(1/2)))/f+n*pol
ylog(2,1+b*x/a)/f-n*polylog(2,1+d*x/c)/f-1/2*n*polylog(2,-(b*x+a)*g^(1/2)/(
b*f^(1/2)-a*g^(1/2)))/f-1/2*n*polylog(2,(b*x+a)*g^(1/2)/(b*f^(1/2)+a*g^(1/2
)))/f+1/2*n*polylog(2,-(d*x+c)*g^(1/2)/(d*f^(1/2)-c*g^(1/2)))/f+1/2*n*polyl
og(2,(d*x+c)*g^(1/2)/(d*f^(1/2)+c*g^(1/2)))/f
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.344, Rules used = {2593, 272, 36, 29, 31, 2463, 2441, 2352, 266, 2440, 2438}

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x(f-gx^2)}\,dx = \frac{\log(f-gx^2)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)}{2f} - \frac{\log(x)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)}{f} - \frac{n\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2f} - \frac{n\,\text{PolyLog}\left(2,\frac{\sqrt{g}(a+bx)}{\sqrt{g}a+b\sqrt{f}}\right)}{2f} - \frac{n\log(a+bx)\log\left(\frac{b(\sqrt{f}-\sqrt{g}x)}{a\sqrt{g}+b\sqrt{f}}\right)}{2f} - \frac{n\log(a+bx)\log\left(\frac{b(\sqrt{f}+\sqrt{g}x)}{b\sqrt{f}-a\sqrt{g}}\right)}{2f} + \frac{n\,\text{PolyLog}\left(2,\frac{bx}{a}+1\right)}{f} + \frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f} + \frac{n\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2f} + \frac{n\,\text{PolyLog}\left(2,\frac{\sqrt{g}(c+dx)}{\sqrt{g}c+d\sqrt{f}}\right)}{2f} + \frac{n\log(c+dx)\log\left(\frac{d(\sqrt{f}-\sqrt{g}x)}{c\sqrt{g}+d\sqrt{f}}\right)}{2f} + \frac{n\log(c+dx)\log\left(\frac{d(\sqrt{f}+\sqrt{g}x)}{d\sqrt{f}-c\sqrt{g}}\right)}{2f} - \frac{n\,\text{PolyLog}\left(2,\frac{dx}{c}+1\right)}{f} - \frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f}$$

```
[In] Int[Log[e*((a + b*x)/(c + d*x))^n]/(x*(f - g*x^2)),x]
```

```
[Out] (n*Log[-((b*x)/a)]*Log[a + b*x])/f - (n*Log[-((d*x)/c)]*Log[c + d*x])/f - (
Log[x]*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/
f - (n*Log[a + b*x]*Log[(b*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])])
/(2*f) + (n*Log[c + d*x]*Log[(d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[
g])])/(2*f) - (n*Log[a + b*x]*Log[(b*(Sqrt[f] + Sqrt[g]*x))/(b*Sqrt[f] - a*
Sqrt[g])])/(2*f) + (n*Log[c + d*x]*Log[(d*(Sqrt[f] + Sqrt[g]*x))/(d*Sqrt[f]
 - c*Sqrt[g])])/(2*f) + ((n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] -
 n*Log[c + d*x])*Log[f - g*x^2])/(2*f) - (n*PolyLog[2, -((Sqrt[g]*(a + b*x)
)/(b*Sqrt[f] - a*Sqrt[g]))])/(2*f) - (n*PolyLog[2, (Sqrt[g]*(a + b*x))/(b*S
```

```
qrt[f] + a*Sqrt[g])])/(2*f) + (n*PolyLog[2, 1 + (b*x)/a])/f + (n*PolyLog[2,
 -((Sqrt[g]*(c + d*x))/(d*Sqrt[f] - c*Sqrt[g]))])/(2*f) + (n*PolyLog[2, (Sq
rt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])])/(2*f) - (n*PolyLog[2, 1 + (d*x)/
c])/f
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2593

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= n\int \frac{\log(a+bx)}{x\left(f-gx^2\right)}\,dx - n\int \frac{\log(c+dx)}{x\left(f-gx^2\right)}\,dx \\
&\quad - \left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\int \frac{1}{x\left(f-gx^2\right)}\,dx \\
&= n\int \left(\frac{\log(a+bx)}{fx}-\frac{gx\log(a+bx)}{f\left(-f+gx^2\right)}\right)dx - n\int \left(\frac{\log(c+dx)}{fx}-\frac{gx\log(c+dx)}{f\left(-f+gx^2\right)}\right)dx \\
&\quad - \frac{1}{2}\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\text{Subst}\left(\int \frac{1}{x(f-gx)}\,dx, x, x^2\right) \\
&= \frac{n\int \frac{\log(a+bx)}{x}\,dx}{f} - \frac{n\int \frac{\log(c+dx)}{x}\,dx}{f} - \frac{(gn)\int \frac{x\log(a+bx)}{-f+gx^2}\,dx}{f} + \frac{(gn)\int \frac{x\log(c+dx)}{-f+gx^2}\,dx}{f} \\
&\quad - \frac{\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right)}{2f} \\
&\quad - \frac{\left(g\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\right)\text{Subst}\left(\int \frac{1}{f-gx}\,dx, x, x^2\right)}{2f}
\end{aligned}
$$

$$= \frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f} - \frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f} - \frac{\log(x)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{f} + \frac{\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\log\left(f-gx^2\right)}{2f} - \frac{(bn)\int\frac{\log\left(-\frac{bx}{a}\right)}{a+bx}\,dx}{f} + \frac{(dn)\int\frac{\log\left(-\frac{dx}{c}\right)}{c+dx}\,dx}{f} - \frac{(gn)\int\left(-\frac{\log(a+bx)}{2\sqrt{g}\left(\sqrt{f}-\sqrt{g}x\right)}+\frac{\log(a+bx)}{2\sqrt{g}\left(\sqrt{f}+\sqrt{g}x\right)}\right)dx}{f} + \frac{(gn)\int\left(-\frac{\log(c+dx)}{2\sqrt{g}\left(\sqrt{f}-\sqrt{g}x\right)}+\frac{\log(c+dx)}{2\sqrt{g}\left(\sqrt{f}+\sqrt{g}x\right)}\right)dx}{f}$$

$$= \frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f} - \frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f} - \frac{\log(x)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{f} + \frac{\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\log\left(f-gx^2\right)}{2f} + \frac{n\text{Li}_2\left(1+\frac{bx}{a}\right)}{f} - \frac{n\text{Li}_2\left(1+\frac{dx}{c}\right)}{f} + \frac{\left(\sqrt{g}n\right)\int\frac{\log(a+bx)}{\sqrt{f}-\sqrt{g}x}\,dx}{2f} - \frac{\left(\sqrt{g}n\right)\int\frac{\log(a+bx)}{\sqrt{f}+\sqrt{g}x}\,dx}{2f} - \frac{\left(\sqrt{g}n\right)\int\frac{\log(c+dx)}{\sqrt{f}-\sqrt{g}x}\,dx}{2f} + \frac{\left(\sqrt{g}n\right)\int\frac{\log(c+dx)}{\sqrt{f}+\sqrt{g}x}\,dx}{2f}$$

$$= \frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f} - \frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f} - \frac{\log(x)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{f} - \frac{n\log(a+bx)\log\left(\frac{b(\sqrt{f}-\sqrt{g}x)}{b\sqrt{f}+a\sqrt{g}}\right)}{2f} + \frac{n\log(c+dx)\log\left(\frac{d(\sqrt{f}-\sqrt{g}x)}{d\sqrt{f}+c\sqrt{g}}\right)}{2f} - \frac{n\log(a+bx)\log\left(\frac{b(\sqrt{f}+\sqrt{g}x)}{b\sqrt{f}-a\sqrt{g}}\right)}{2f} + \frac{n\log(c+dx)\log\left(\frac{d(\sqrt{f}+\sqrt{g}x)}{d\sqrt{f}-c\sqrt{g}}\right)}{2f} + \frac{\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\log\left(f-gx^2\right)}{2f} + \frac{n\text{Li}_2\left(1+\frac{bx}{a}\right)}{f} - \frac{n\text{Li}_2\left(1+\frac{dx}{c}\right)}{f} + \frac{(bn)\int\frac{\log\left(\frac{b(\sqrt{f}-\sqrt{g}x)}{b\sqrt{f}+a\sqrt{g}}\right)}{a+bx}dx}{2f} + \frac{(bn)\int\frac{\log\left(\frac{b(\sqrt{f}+\sqrt{g}x)}{b\sqrt{f}-a\sqrt{g}}\right)}{a+bx}dx}{2f} - \frac{(dn)\int\frac{\log\left(\frac{d(\sqrt{f}-\sqrt{g}x)}{d\sqrt{f}+c\sqrt{g}}\right)}{c+dx}dx}{2f} - \frac{(dn)\int\frac{\log\left(\frac{d(\sqrt{f}+\sqrt{g}x)}{d\sqrt{f}-c\sqrt{g}}\right)}{c+dx}dx}{2f}$$

$$= \frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f} - \frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f} - \frac{\log(x)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{f} - \frac{n\log(a+bx)\log\left(\frac{b(\sqrt{f}-\sqrt{g}x)}{b\sqrt{f}+a\sqrt{g}}\right)}{2f} + \frac{n\log(c+dx)\log\left(\frac{d(\sqrt{f}-\sqrt{g}x)}{d\sqrt{f}+c\sqrt{g}}\right)}{2f} - \frac{n\log(a+bx)\log\left(\frac{b(\sqrt{f}+\sqrt{g}x)}{b\sqrt{f}-a\sqrt{g}}\right)}{2f} + \frac{n\log(c+dx)\log\left(\frac{d(\sqrt{f}+\sqrt{g}x)}{d\sqrt{f}-c\sqrt{g}}\right)}{2f} + \frac{\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\log\left(f-gx^2\right)}{2f} + \frac{n\text{Li}_2\left(1+\frac{bx}{a}\right)}{f} - \frac{n\text{Li}_2\left(1+\frac{dx}{c}\right)}{f} + \frac{n\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{g}x}{b\sqrt{f}-a\sqrt{g}}\right)}{x}dx,x,a+bx\right)}{2f} + \frac{n\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{g}x}{b\sqrt{f}+a\sqrt{g}}\right)}{x}dx,x,a+bx\right)}{2f} - \frac{n\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{g}x}{d\sqrt{f}-c\sqrt{g}}\right)}{x}dx,x,c+dx\right)}{2f} - \frac{n\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{g}x}{d\sqrt{f}+c\sqrt{g}}\right)}{x}dx,x,c+dx\right)}{2f}$$

$$= \frac{n \log \left(-\frac{bx}{a}\right) \log(a+bx)}{f} - \frac{n \log \left(-\frac{dx}{c}\right) \log(c+dx)}{f} - \frac{\log(x) \left(n \log(a+bx) - \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{f} - \frac{n \log(a+bx) \log \left(\frac{b(\sqrt{f}-\sqrt{g}x)}{b\sqrt{f}+a\sqrt{g}}\right)}{2f} + \frac{n \log(c+dx) \log \left(\frac{d(\sqrt{f}-\sqrt{g}x)}{d\sqrt{f}+c\sqrt{g}}\right)}{2f} - \frac{n \log(a+bx) \log \left(\frac{b(\sqrt{f}+\sqrt{g}x)}{b\sqrt{f}-a\sqrt{g}}\right)}{2f} + \frac{n \log(c+dx) \log \left(\frac{d(\sqrt{f}+\sqrt{g}x)}{d\sqrt{f}-c\sqrt{g}}\right)}{2f} + \frac{\left(n \log(a+bx) - \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right) \log (f-gx^2)}{2f} - \frac{n \text{Li}_2\left(-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2f} - \frac{n \text{Li}_2\left(\frac{\sqrt{g}(a+bx)}{b\sqrt{f}+a\sqrt{g}}\right)}{2f} + \frac{n \text{Li}_2\left(1+\frac{bx}{a}\right)}{f} + \frac{n \text{Li}_2\left(-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2f} + \frac{n \text{Li}_2\left(\frac{\sqrt{g}(c+dx)}{d\sqrt{f}+c\sqrt{g}}\right)}{2f} - \frac{n \text{Li}_2\left(1+\frac{dx}{c}\right)}{f}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 487, normalized size of antiderivative = 0.94

$$\int \frac{\log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{x (f-gx^2)} \, dx = -\frac{2n \log(x) \log \left(1+\frac{bx}{a}\right) - 2 \log(x) \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right) - 2n \log(x) \log \left(1+\frac{dx}{c}\right) - n \log \left(\frac{\sqrt{g}(a+bx)}{b\sqrt{f}+a\sqrt{g}}\right) \log \left(\sqrt{f} - \right.}{}$$

```
[In] Integrate[Log[e*((a + b*x)/(c + d*x))^n]/(x*(f - g*x^2)),x]
```

```
[Out] -1/2*(2*n*Log[x]*Log[1 + (b*x)/a] - 2*Log[x]*Log[e*((a + b*x)/(c + d*x))^n]
 - 2*n*Log[x]*Log[1 + (d*x)/c] - n*Log[(Sqrt[g]*(a + b*x))/(b*Sqrt[f] + a*S
qrt[g])]*Log[Sqrt[f] - Sqrt[g]*x] + Log[e*((a + b*x)/(c + d*x))^n]*Log[Sqrt
[f] - Sqrt[g]*x] + n*Log[(Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])]*Log[S
qrt[f] - Sqrt[g]*x] - n*Log[-((Sqrt[g]*(a + b*x))/(b*Sqrt[f] - a*Sqrt[g]))]
*Log[Sqrt[f] + Sqrt[g]*x] + Log[e*((a + b*x)/(c + d*x))^n]*Log[Sqrt[f] + Sq
rt[g]*x] + n*Log[-((Sqrt[g]*(c + d*x))/(d*Sqrt[f] - c*Sqrt[g]))]*Log[Sqrt[f
] + Sqrt[g]*x] + 2*n*PolyLog[2, -((b*x)/a)] - 2*n*PolyLog[2, -((d*x)/c)] -
n*PolyLog[2, (b*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])] + n*PolyLog
[2, (d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g])] - n*PolyLog[2, (b*(S
qrt[f] + Sqrt[g]*x))/(b*Sqrt[f] - a*Sqrt[g])] + n*PolyLog[2, (d*(Sqrt[f] +
Sqrt[g]*x))/(d*Sqrt[f] - c*Sqrt[g])])/f
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 567, normalized size of antiderivative = 1.09

method	result
parts	$\frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\ln(x)}{f}-\frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\ln\left(-g\,x^{2}+f\right)}{2f}-\frac{n\left(\frac{(ad-cb)\left(\left(\frac{\ln(dx+c)\ln\left(-g\,x^{2}+f\right)}{d}+\frac{2g\left(-\frac{\ln(dx+c)\left(\ln\left(\frac{d\sqrt{fg}-(dx+c)g+cg}{d\sqrt{fg}+cg}\right.\right.}{2g}\right.}{\cdots}\right.\right.}{\cdots}\right.}{\cdots}$

[In] int(ln(e*((b*x+a)/(d*x+c))^n)/x/(-g*x^2+f),x,method=_RETURNVERBOSE)

```
[Out] ln(e*((b*x+a)/(d*x+c))^n)/f*ln(x)-1/2*ln(e*((b*x+a)/(d*x+c))^n)/f*ln(-g*x^2
+f)-1/2*n*((a*d-b*c)/f*((ln(d*x+c)/d*ln(-g*x^2+f)+2/d*g*(-1/2*ln(d*x+c)*(ln
((d*(f*g)^(1/2)-(d*x+c)*g+c*g)/(d*(f*g)^(1/2)+c*g))+ln((d*(f*g)^(1/2)+(d*x+
c)*g-c*g)/(d*(f*g)^(1/2)-c*g)))/g-1/2*(dilog((d*(f*g)^(1/2)-(d*x+c)*g+c*g)/
(d*(f*g)^(1/2)+c*g))+dilog((d*(f*g)^(1/2)+(d*x+c)*g-c*g)/(d*(f*g)^(1/2)-c*g
)))/g))*d/(a*d-b*c)-(ln(b*x+a)/b*ln(-g*x^2+f)+2/b*g*(-1/2*ln(b*x+a)*(ln((b*
(f*g)^(1/2)-g*(b*x+a)+a*g)/(b*(f*g)^(1/2)+a*g))+ln((b*(f*g)^(1/2)+g*(b*x+a)
-a*g)/(b*(f*g)^(1/2)-a*g)))/g-1/2*(dilog((b*(f*g)^(1/2)-g*(b*x+a)+a*g)/(b*(
f*g)^(1/2)+a*g))+dilog((b*(f*g)^(1/2)+g*(b*x+a)-a*g)/(b*(f*g)^(1/2)-a*g)))/
g))*b/(a*d-b*c))-2*(a*d-b*c)/f*(d/(a*d-b*c)*(dilog((d*x+c)/c)/d+ln(x)*ln((d
*x+c)/c)/d)-b/(a*d-b*c)*(dilog((b*x+a)/a)/b+ln(x)*ln((b*x+a)/a)/b)))
```

Fricas [F]

$$\int \frac{\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x\left(f-gx^2\right)}\,dx=\int -\frac{\log \left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(gx^2-f)x}\,dx$$

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x/(-g*x^2+f),x, algorithm="fricas")
[Out] integral(-log(e*((b*x + a)/(d*x + c))^n)/(g*x^3 - f*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x\left(f-gx^2\right)}\,dx=\text{Timed out}$$

```
[In] integrate(ln(e*((b*x+a)/(d*x+c))**n)/x/(-g*x**2+f),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x\left(f-gx^2\right)}\,dx=\int -\frac{\log \left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(gx^2-f)x}\,dx$$

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x/(-g*x^2+f),x, algorithm="maxima")
[Out] -integrate(log(e*((b*x + a)/(d*x + c))^n)/((g*x^2 - f)*x), x)
```

Giac [F]

$$\int \frac{\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x\left(f-gx^2\right)}\,dx=\int -\frac{\log \left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(gx^2-f)x}\,dx$$

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x/(-g*x^2+f),x, algorithm="giac")
[Out] integrate(-log(e*((b*x + a)/(d*x + c))^n)/((g*x^2 - f)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x\,(f-gx^2)}\,dx=\int \frac{\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{x\,\left(f-g\,x^2\right)}\,dx$$

```
[In] int(log(e*((a + b*x)/(c + d*x))^n)/(x*(f - g*x^2)),x)
[Out] int(log(e*((a + b*x)/(c + d*x))^n)/(x*(f - g*x^2)), x)
```

3.81 $\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2(f-gx^2)}\,dx$

Optimal result 643
Rubi [A] (verified) 644
Mathematica [A] (verified) 649
Maple [F] 650
Fricas [F] 650
Sympy [F(-1)] 650
Maxima [B] (verification not implemented) 650
Giac [F] 651
Mupad [F(-1)] 651

Optimal result

Integrand size = 32, antiderivative size = 596

$$
\begin{aligned}
\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2(f-gx^2)}\,dx &= \frac{bn\log(x)}{af}-\frac{dn\log(x)}{cf}-\frac{bn\log(a+bx)}{af} \\
&-\frac{n\log(a+bx)}{fx}+\frac{dn\log(c+dx)}{cf}+\frac{n\log(c+dx)}{fx} \\
&+\frac{n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)}{fx} \\
&-\frac{\sqrt{g}\,\mathrm{arctanh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{f^{3/2}} \\
&-\frac{\sqrt{g}n\log(a+bx)\log\left(\frac{b(\sqrt{f}-\sqrt{g}x)}{b\sqrt{f}+a\sqrt{g}}\right)}{2f^{3/2}} \\
&+\frac{\sqrt{g}n\log(c+dx)\log\left(\frac{d(\sqrt{f}-\sqrt{g}x)}{d\sqrt{f}+c\sqrt{g}}\right)}{2f^{3/2}} \\
&+\frac{\sqrt{g}n\log(a+bx)\log\left(\frac{b(\sqrt{f}+\sqrt{g}x)}{b\sqrt{f}-a\sqrt{g}}\right)}{2f^{3/2}} \\
&-\frac{\sqrt{g}n\log(c+dx)\log\left(\frac{d(\sqrt{f}+\sqrt{g}x)}{d\sqrt{f}-c\sqrt{g}}\right)}{2f^{3/2}} \\
&+\frac{\sqrt{g}n\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2f^{3/2}}-\frac{\sqrt{g}n\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(a+bx)}{b\sqrt{f}+a\sqrt{g}}\right)}{2f^{3/2}} \\
&-\frac{\sqrt{g}n\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2f^{3/2}}+\frac{\sqrt{g}n\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(c+dx)}{d\sqrt{f}+c\sqrt{g}}\right)}{2f^{3/2}}
\end{aligned}
$$

```
[Out] b*n*ln(x)/a/f-d*n*ln(x)/c/f-b*n*ln(b*x+a)/a/f-n*ln(b*x+a)/f/x+d*n*ln(d*x+c)
/c/f+n*ln(d*x+c)/f/x+(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))/f/
x-arctanh(x*g^(1/2)/f^(1/2))*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*
x+c))*g^(1/2)/f^(3/2)-1/2*n*ln(b*x+a)*ln(b*(f^(1/2)-x*g^(1/2))/(b*f^(1/2)+a
*g^(1/2)))*g^(1/2)/f^(3/2)+1/2*n*ln(d*x+c)*ln(d*(f^(1/2)-x*g^(1/2))/(d*f^(1
/2)+c*g^(1/2)))*g^(1/2)/f^(3/2)+1/2*n*ln(b*x+a)*ln(b*(f^(1/2)+x*g^(1/2))/(b
*f^(1/2)-a*g^(1/2)))*g^(1/2)/f^(3/2)-1/2*n*ln(d*x+c)*ln(d*(f^(1/2)+x*g^(1/2
))/(d*f^(1/2)-c*g^(1/2)))*g^(1/2)/f^(3/2)+1/2*n*polylog(2,-(b*x+a)*g^(1/2)/
(b*f^(1/2)-a*g^(1/2)))*g^(1/2)/f^(3/2)-1/2*n*polylog(2,(b*x+a)*g^(1/2)/(b*f
^(1/2)+a*g^(1/2)))*g^(1/2)/f^(3/2)-1/2*n*polylog(2,-(d*x+c)*g^(1/2)/(d*f^(1
/2)-c*g^(1/2)))*g^(1/2)/f^(3/2)+1/2*n*polylog(2,(d*x+c)*g^(1/2)/(d*f^(1/2)+
c*g^(1/2)))*g^(1/2)/f^(3/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 596, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2593, 331, 214, 2463, 2442, 36, 29, 31, 2456, 2441, 2440, 2438}

$$\begin{aligned}
&\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2(f-gx^2)}dx \\
&= -\frac{\sqrt{g}\text{arctanh}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)}{f^{3/2}} \\
&+\frac{-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)}{fx} \\
&+\frac{\sqrt{g}n\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2f^{3/2}}-\frac{\sqrt{g}n\,\text{PolyLog}\left(2,\frac{\sqrt{g}(a+bx)}{\sqrt{g}a+b\sqrt{f}}\right)}{2f^{3/2}} \\
&-\frac{\sqrt{g}n\log(a+bx)\log\left(\frac{b(\sqrt{f}-\sqrt{g}x)}{a\sqrt{g}+b\sqrt{f}}\right)}{2f^{3/2}}+\frac{\sqrt{g}n\log(a+bx)\log\left(\frac{b(\sqrt{f}+\sqrt{g}x)}{b\sqrt{f}-a\sqrt{g}}\right)}{2f^{3/2}} \\
&+\frac{bn\log(x)}{af}-\frac{bn\log(a+bx)}{af}-\frac{n\log(a+bx)}{fx}-\frac{\sqrt{g}n\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2f^{3/2}} \\
&+\frac{\sqrt{g}n\,\text{PolyLog}\left(2,\frac{\sqrt{g}(c+dx)}{\sqrt{g}c+d\sqrt{f}}\right)}{2f^{3/2}}+\frac{\sqrt{g}n\log(c+dx)\log\left(\frac{d(\sqrt{f}-\sqrt{g}x)}{c\sqrt{g}+d\sqrt{f}}\right)}{2f^{3/2}} \\
&-\frac{\sqrt{g}n\log(c+dx)\log\left(\frac{d(\sqrt{f}+\sqrt{g}x)}{d\sqrt{f}-c\sqrt{g}}\right)}{2f^{3/2}}-\frac{dn\log(x)}{cf}+\frac{dn\log(c+dx)}{cf}+\frac{n\log(c+dx)}{fx}
\end{aligned}$$

```
[In] Int[Log[e*((a + b*x)/(c + d*x))^n]/(x^2*(f - g*x^2)),x]
```

```
[Out] (b*n*Log[x])/(a*f) - (d*n*Log[x])/(c*f) - (b*n*Log[a + b*x])/(a*f) - (n*Log
[a + b*x])/(f*x) + (d*n*Log[c + d*x])/(c*f) + (n*Log[c + d*x])/(f*x) + (n*L
```

```
og[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])/(f*x) - (Sqr
t[g]*ArcTanh[(Sqrt[g]*x)/Sqrt[f]]*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d
*x))^n] - n*Log[c + d*x]))/f^(3/2) - (Sqrt[g]*n*Log[a + b*x]*Log[(b*(Sqrt[f
] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])])/(2*f^(3/2)) + (Sqrt[g]*n*Log[c +
d*x]*Log[(d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g])])/(2*f^(3/2)) +
(Sqrt[g]*n*Log[a + b*x]*Log[(b*(Sqrt[f] + Sqrt[g]*x))/(b*Sqrt[f] - a*Sqrt[g
])])/(2*f^(3/2)) - (Sqrt[g]*n*Log[c + d*x]*Log[(d*(Sqrt[f] + Sqrt[g]*x))/(d
*Sqrt[f] - c*Sqrt[g])])/(2*f^(3/2)) + (Sqrt[g]*n*PolyLog[2, -((Sqrt[g]*(a +
 b*x))/(b*Sqrt[f] - a*Sqrt[g]))])/(2*f^(3/2)) - (Sqrt[g]*n*PolyLog[2, (Sqrt
[g]*(a + b*x))/(b*Sqrt[f] + a*Sqrt[g])])/(2*f^(3/2)) - (Sqrt[g]*n*PolyLog[2
, -((Sqrt[g]*(c + d*x))/(d*Sqrt[f] - c*Sqrt[g]))])/(2*f^(3/2)) + (Sqrt[g]*n
*PolyLog[2, (Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])])/(2*f^(3/2))
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2593

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$\text{integral} = n\int \frac{\log(a+bx)}{x^2(f-gx^2)}\,dx - n\int \frac{\log(c+dx)}{x^2(f-gx^2)}\,dx - \left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\int \frac{1}{x^2(f-gx^2)}\,dx$$

$$= \frac{n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)}{fx} + n\int\left(\frac{\log(a+bx)}{fx^2}+\frac{g\log(a+bx)}{f(f-gx^2)}\right)dx - n\int\left(\frac{\log(c+dx)}{fx^2}+\frac{g\log(c+dx)}{f(f-gx^2)}\right)dx - \frac{\left(g\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\right)\int\frac{1}{f-gx^2}\,dx}{f}$$

$$= \frac{n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)}{fx} - \frac{\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{f^{3/2}} + \frac{n\int\frac{\log(a+bx)}{x^2}\,dx}{f} - \frac{n\int\frac{\log(c+dx)}{x^2}\,dx}{f} + \frac{(gn)\int\frac{\log(a+bx)}{f-gx^2}\,dx}{f} - \frac{(gn)\int\frac{\log(c+dx)}{f-gx^2}\,dx}{f}$$

$$= -\frac{n\log(a+bx)}{fx} + \frac{n\log(c+dx)}{fx} + \frac{n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)}{fx} - \frac{\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{f^{3/2}} + \frac{(bn)\int\frac{1}{x(a+bx)}\,dx}{f} - \frac{(dn)\int\frac{1}{x(c+dx)}\,dx}{f} + \frac{(gn)\int\left(\frac{\log(a+bx)}{2\sqrt{f}(\sqrt{f}-\sqrt{g}x)}+\frac{\log(a+bx)}{2\sqrt{f}(\sqrt{f}+\sqrt{g}x)}\right)dx}{f} - \frac{(gn)\int\left(\frac{\log(c+dx)}{2\sqrt{f}(\sqrt{f}-\sqrt{g}x)}+\frac{\log(c+dx)}{2\sqrt{f}(\sqrt{f}+\sqrt{g}x)}\right)dx}{f}$$

$$= -\frac{n\log(a+bx)}{fx} + \frac{n\log(c+dx)}{fx} + \frac{n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)}{fx} - \frac{\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{f^{3/2}} + \frac{(bn)\int\frac{1}{x}\,dx}{af} - \frac{(b^2n)\int\frac{1}{a+bx}\,dx}{af} - \frac{(dn)\int\frac{1}{x}\,dx}{cf} + \frac{(d^2n)\int\frac{1}{c+dx}\,dx}{cf} + \frac{(gn)\int\frac{\log(a+bx)}{\sqrt{f}-\sqrt{g}x}\,dx}{2f^{3/2}} + \frac{(gn)\int\frac{\log(a+bx)}{\sqrt{f}+\sqrt{g}x}\,dx}{2f^{3/2}} - \frac{(gn)\int\frac{\log(c+dx)}{\sqrt{f}-\sqrt{g}x}\,dx}{2f^{3/2}} - \frac{(gn)\int\frac{\log(c+dx)}{\sqrt{f}+\sqrt{g}x}\,dx}{2f^{3/2}}$$

$$=\frac{bn\log(x)}{af}-\frac{dn\log(x)}{cf}-\frac{bn\log(a+bx)}{af}-\frac{n\log(a+bx)}{fx}+\frac{dn\log(c+dx)}{cf}+\frac{n\log(c+dx)}{fx}+\frac{n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)}{fx}-\frac{\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{f^{3/2}}-\frac{\sqrt{g}n\log(a+bx)\log\left(\frac{b(\sqrt{f}-\sqrt{g}x)}{b\sqrt{f}+a\sqrt{g}}\right)}{2f^{3/2}}+\frac{\sqrt{g}n\log(c+dx)\log\left(\frac{d(\sqrt{f}-\sqrt{g}x)}{d\sqrt{f}+c\sqrt{g}}\right)}{2f^{3/2}}+\frac{\sqrt{g}n\log(a+bx)\log\left(\frac{b(\sqrt{f}+\sqrt{g}x)}{b\sqrt{f}-a\sqrt{g}}\right)}{2f^{3/2}}-\frac{\sqrt{g}n\log(c+dx)\log\left(\frac{d(\sqrt{f}+\sqrt{g}x)}{d\sqrt{f}-c\sqrt{g}}\right)}{2f^{3/2}}+\frac{\left(b\sqrt{g}n\right)\int\frac{\log\left(\frac{b(\sqrt{f}-\sqrt{g}x)}{b\sqrt{f}+a\sqrt{g}}\right)}{a+bx}dx}{2f^{3/2}}-\frac{\left(b\sqrt{g}n\right)\int\frac{\log\left(\frac{b(\sqrt{f}+\sqrt{g}x)}{b\sqrt{f}-a\sqrt{g}}\right)}{a+bx}dx}{2f^{3/2}}-\frac{\left(d\sqrt{g}n\right)\int\frac{\log\left(\frac{d(\sqrt{f}-\sqrt{g}x)}{d\sqrt{f}+c\sqrt{g}}\right)}{c+dx}dx}{2f^{3/2}}+\frac{\left(d\sqrt{g}n\right)\int\frac{\log\left(\frac{d(\sqrt{f}+\sqrt{g}x)}{d\sqrt{f}-c\sqrt{g}}\right)}{c+dx}dx}{2f^{3/2}}$$

$$=\frac{bn\log(x)}{af}-\frac{dn\log(x)}{cf}-\frac{bn\log(a+bx)}{af}-\frac{n\log(a+bx)}{fx}+\frac{dn\log(c+dx)}{cf}+\frac{n\log(c+dx)}{fx}+\frac{n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)}{fx}-\frac{\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{f^{3/2}}-\frac{\sqrt{g}n\log(a+bx)\log\left(\frac{b(\sqrt{f}-\sqrt{g}x)}{b\sqrt{f}+a\sqrt{g}}\right)}{2f^{3/2}}+\frac{\sqrt{g}n\log(c+dx)\log\left(\frac{d(\sqrt{f}-\sqrt{g}x)}{d\sqrt{f}+c\sqrt{g}}\right)}{2f^{3/2}}+\frac{\sqrt{g}n\log(a+bx)\log\left(\frac{b(\sqrt{f}+\sqrt{g}x)}{b\sqrt{f}-a\sqrt{g}}\right)}{2f^{3/2}}-\frac{\sqrt{g}n\log(c+dx)\log\left(\frac{d(\sqrt{f}+\sqrt{g}x)}{d\sqrt{f}-c\sqrt{g}}\right)}{2f^{3/2}}-\frac{\left(\sqrt{g}n\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{g}x}{b\sqrt{f}-a\sqrt{g}}\right)}{x}dx,x,a+bx\right)}{2f^{3/2}}+\frac{\left(\sqrt{g}n\right)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{g}x}{b\sqrt{f}+a\sqrt{g}}\right)}{x}dx,x,a+bx\right)}{2f^{3/2}}+\frac{\left(\sqrt{g}n\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{g}x}{d\sqrt{f}-c\sqrt{g}}\right)}{x}dx,x,c+dx\right)}{2f^{3/2}}-\frac{\left(\sqrt{g}n\right)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{g}x}{d\sqrt{f}+c\sqrt{g}}\right)}{x}dx,x,c+dx\right)}{2f^{3/2}}$$

$$
\begin{aligned}
&= \frac{bn\log(x)}{af} - \frac{dn\log(x)}{cf} - \frac{bn\log(a+bx)}{af} - \frac{n\log(a+bx)}{fx} + \frac{dn\log(c+dx)}{cf} \\
&+ \frac{n\log(c+dx)}{fx} + \frac{n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)}{fx} \\
&- \frac{\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{f^{3/2}} \\
&- \frac{\sqrt{g}n\log(a+bx)\log\left(\frac{b(\sqrt{f}-\sqrt{g}x)}{b\sqrt{f}+a\sqrt{g}}\right)}{2f^{3/2}} + \frac{\sqrt{g}n\log(c+dx)\log\left(\frac{d(\sqrt{f}-\sqrt{g}x)}{d\sqrt{f}+c\sqrt{g}}\right)}{2f^{3/2}} \\
&+ \frac{\sqrt{g}n\log(a+bx)\log\left(\frac{b(\sqrt{f}+\sqrt{g}x)}{b\sqrt{f}-a\sqrt{g}}\right)}{2f^{3/2}} - \frac{\sqrt{g}n\log(c+dx)\log\left(\frac{d(\sqrt{f}+\sqrt{g}x)}{d\sqrt{f}-c\sqrt{g}}\right)}{2f^{3/2}} \\
&+ \frac{\sqrt{g}n\operatorname{Li}_2\left(-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2f^{3/2}} - \frac{\sqrt{g}n\operatorname{Li}_2\left(\frac{\sqrt{g}(a+bx)}{b\sqrt{f}+a\sqrt{g}}\right)}{2f^{3/2}} \\
&- \frac{\sqrt{g}n\operatorname{Li}_2\left(-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2f^{3/2}} + \frac{\sqrt{g}n\operatorname{Li}_2\left(\frac{\sqrt{g}(c+dx)}{d\sqrt{f}+c\sqrt{g}}\right)}{2f^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 479, normalized size of antiderivative = 0.80

$$
\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2(f-gx^2)}\,dx
$$

$$
= \frac{-\frac{2\sqrt{f}\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x} + \frac{2\sqrt{f}n((bc-ad)\log(x)-bc\log(a+bx)+ad\log(c+dx))}{ac} - \sqrt{g}\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\sqrt{f}-\sqrt{g}x\right) + \sqrt{g}\,\mathrm{lo}}{}
$$

```
[In] Integrate[Log[e*((a + b*x)/(c + d*x))^n]/(x^2*(f - g*x^2)),x]
```

```
[Out] ((-2*Sqrt[f]*Log[e*((a + b*x)/(c + d*x))^n])/x + (2*Sqrt[f]*n*((b*c - a*d)*
Log[x] - b*c*Log[a + b*x] + a*d*Log[c + d*x]))/(a*c) - Sqrt[g]*Log[e*((a +
b*x)/(c + d*x))^n]*Log[Sqrt[f] - Sqrt[g]*x] + Sqrt[g]*Log[e*((a + b*x)/(c +
 d*x))^n]*Log[Sqrt[f] + Sqrt[g]*x] + Sqrt[g]*n*((Log[(Sqrt[g]*(a + b*x))/(b
*Sqrt[f] + a*Sqrt[g])] - Log[(Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])])*
Log[Sqrt[f] - Sqrt[g]*x] + PolyLog[2, (b*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f]
+ a*Sqrt[g])] - PolyLog[2, (d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g]
)]) - Sqrt[g]*n*((Log[-((Sqrt[g]*(a + b*x))/(b*Sqrt[f] - a*Sqrt[g]))] - Log
[-((Sqrt[g]*(c + d*x))/(d*Sqrt[f] - c*Sqrt[g]))])*Log[Sqrt[f] + Sqrt[g]*x]
+ PolyLog[2, (b*(Sqrt[f] + Sqrt[g]*x))/(b*Sqrt[f] - a*Sqrt[g])] - PolyLog[2
, (d*(Sqrt[f] + Sqrt[g]*x))/(d*Sqrt[f] - c*Sqrt[g])]))/(2*f^(3/2))
```

Maple [F]

$$\int \frac{\ln \left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{x^{2} \left(-g\,x^{2}+f\right)}dx$$

```
[In] int(ln(e*((b*x+a)/(d*x+c))^n)/x^2/(-g*x^2+f),x)
[Out] int(ln(e*((b*x+a)/(d*x+c))^n)/x^2/(-g*x^2+f),x)
```

Fricas [F]

$$\int \frac{\log \left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}{x^{2}\left(f-gx^{2}\right)}\,dx = \int -\frac{\log \left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{(gx^{2}-f)x^{2}}\,dx$$

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x^2/(-g*x^2+f),x, algorithm="fricas")
[Out] integral(-log(e*((b*x + a)/(d*x + c))^n)/(g*x^4 - f*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}{x^{2}\left(f-gx^{2}\right)}\,dx = \text{Timed out}$$

```
[In] integrate(ln(e*((b*x+a)/(d*x+c))**n)/x**2/(-g*x**2+f),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 969 vs. $2(484) = 968$.

Time = 0.40 (sec) , antiderivative size = 969, normalized size of antiderivative = 1.63

$$\begin{aligned}&\int \frac{\log \left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}{x^{2}\left(f-gx^{2}\right)}\,dx\\&=\frac{1}{2}\left(2\,acd\left(\frac{b^{2}\log \left(bx+a\right)}{(ab^{2}c^{2}-2\,a^{2}bcd+a^{3}d^{2})f}+\frac{d}{(bc^{2}d-acd^{2})fx+(bc^{3}-ac^{2}d)f}-\frac{(2\,bcd-ad^{2})\log \left(dx+c\right)}{(b^{2}c^{4}-2\,abc^{3}d+a^{2}c^{2}d^{2})f}-\right.\right.\\&\quad-\frac{1}{2}\left(\frac{g\log \left(\frac{gx-\sqrt{fg}}{gx+\sqrt{fg}}\right)}{\sqrt{fg}f}+\frac{2}{fx}\right)\log \left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\end{aligned}$$

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x^2/(-g*x^2+f),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a*c*d*(b^2*log(b*x + a)/((a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*f) + d/
((b*c^2*d - a*c*d^2)*f*x + (b*c^3 - a*c^2*d)*f) - (2*b*c*d - a*d^2)*log(d*x
 + c)/((b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*f) - log(x)/(a*c^2*f)) + 2*b*d
^2*(c/((b*c*d^2 - a*d^3)*f*x + (b*c^2*d - a*c*d^2)*f) + a*log(b*x + a)/((b^
2*c^2 - 2*a*b*c*d + a^2*d^2)*f) - a*log(d*x + c)/((b^2*c^2 - 2*a*b*c*d + a^
2*d^2)*f)) - 2*b*c*d*(b*log(b*x + a)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*f) -
b*log(d*x + c)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*f) + 1/((b*c*d - a*d^2)*f*x
 + (b*c^2 - a*c*d)*f)) - 2*a*d^2*(b*log(b*x + a)/((b^2*c^2 - 2*a*b*c*d + a^
2*d^2)*f) - b*log(d*x + c)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*f) + 1/((b*c*d
- a*d^2)*f*x + (b*c^2 - a*c*d)*f)) - 2*b*c*(b*log(b*x + a)/((a*b*c - a^2*d)
*f) - d*log(d*x + c)/((b*c^2 - a*c*d)*f) - log(x)/(a*c*f)) + 2*b*d*(log(b*x
 + a)/((b*c - a*d)*f) - log(d*x + c)/((b*c - a*d)*f)) + (log(sqrt(g)*x - sq
rt(f))*log((b*sqrt(g)*x - b*sqrt(f))/(b*sqrt(f) + a*sqrt(g)) + 1) + dilog(-
(b*sqrt(g)*x - b*sqrt(f))/(b*sqrt(f) + a*sqrt(g))))*sqrt(g)/f^(3/2) - (log(
sqrt(g)*x + sqrt(f))*log(-(b*sqrt(g)*x + b*sqrt(f))/(b*sqrt(f) - a*sqrt(g))
 + 1) + dilog((b*sqrt(g)*x + b*sqrt(f))/(b*sqrt(f) - a*sqrt(g))))*sqrt(g)/f
^(3/2) - (log(sqrt(g)*x - sqrt(f))*log((d*sqrt(g)*x - d*sqrt(f))/(d*sqrt(f)
 + c*sqrt(g)) + 1) + dilog(-(d*sqrt(g)*x - d*sqrt(f))/(d*sqrt(f) + c*sqrt(g
))))*sqrt(g)/f^(3/2) + (log(sqrt(g)*x + sqrt(f))*log(-(d*sqrt(g)*x + d*sqrt
(f))/(d*sqrt(f) - c*sqrt(g)) + 1) + dilog((d*sqrt(g)*x + d*sqrt(f))/(d*sqrt
(f) - c*sqrt(g))))*sqrt(g)/f^(3/2))*n - 1/2*(g*log((g*x - sqrt(f*g))/(g*x +
 sqrt(f*g)))/(sqrt(f*g)*f) + 2/(f*x))*log(e*((b*x + a)/(d*x + c))^n)
```

Giac [F]

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2\left(f-gx^2\right)}\,dx = \int -\frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(gx^2-f)x^2}\,dx$$

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x^2/(-g*x^2+f),x, algorithm="giac")
[Out] integrate(-log(e*((b*x + a)/(d*x + c))^n)/((g*x^2 - f)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2\left(f-gx^2\right)}\,dx = \int \frac{\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{x^2\,\left(f-g\,x^2\right)}\,dx$$

```
[In] int(log(e*((a + b*x)/(c + d*x))^n)/(x^2*(f - g*x^2)),x)
[Out] int(log(e*((a + b*x)/(c + d*x))^n)/(x^2*(f - g*x^2)), x)
```

3.82 $\int \frac{x^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx$

Optimal result 653
Rubi [A] (verified) 654
Mathematica [A] (verified) 664
Maple [F] 665
Fricas [F] 665
Sympy [F(-1)] 665
Maxima [F(-2)] 665
Giac [F] 666
Mupad [F(-1)] 666

Optimal result

Integrand size = 34, antiderivative size = 1046

$$\int \frac{x^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx$$
$$= \frac{anx}{2bh} - \frac{cnx}{2dh} - \frac{a^2 n\log(a+bx)}{2b^2h} + \frac{nx^2\log(a+bx)}{2h} - \frac{gn(a+bx)\log(a+bx)}{bh^2}$$
$$+ \frac{c^2 n\log(c+dx)}{2d^2h} - \frac{nx^2\log(c+dx)}{2h} + \frac{gn(c+dx)\log(c+dx)}{dh^2}$$
$$+ \frac{gx\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{h^2}$$
$$- \frac{x^2\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{2h}$$
$$- \frac{g(g^2-3fh)\operatorname{arctanh}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{h^3\sqrt{g^2-4fh}}$$
$$+ \frac{\left(g^2 - fh - \frac{g(g^2-3fh)}{\sqrt{g^2-4fh}}\right) n\log(a+bx)\log\left(-\frac{b\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h^3}$$
$$- \frac{\left(g^2 - fh - \frac{g(g^2-3fh)}{\sqrt{g^2-4fh}}\right) n\log(c+dx)\log\left(-\frac{d\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h^3}$$
$$+ \frac{\left(g^2 - fh + \frac{g(g^2-3fh)}{\sqrt{g^2-4fh}}\right) n\log(a+bx)\log\left(-\frac{b\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h^3}$$
$$- \frac{\left(g^2 - fh + \frac{g(g^2-3fh)}{\sqrt{g^2-4fh}}\right) n\log(c+dx)\log\left(-\frac{d\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h^3}$$
$$- \frac{(g^2-fh)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)\log\left(f+gx+hx^2\right)}{2h^3}$$
$$+ \frac{\left(g^2 - fh - \frac{g(g^2-3fh)}{\sqrt{g^2-4fh}}\right) n\operatorname{PolyLog}\left(2, \frac{2h(a+bx)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h^3}$$
$$+ \frac{\left(g^2 - fh + \frac{g(g^2-3fh)}{\sqrt{g^2-4fh}}\right) n\operatorname{PolyLog}\left(2, \frac{2h(a+bx)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h^3}$$
$$- \frac{\left(g^2 - fh - \frac{g(g^2-3fh)}{\sqrt{g^2-4fh}}\right) n\operatorname{PolyLog}\left(2, \frac{2h(c+dx)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h^3}$$
$$- \frac{\left(g^2 - fh + \frac{g(g^2-3fh)}{\sqrt{g^2-4fh}}\right) n\operatorname{PolyLog}\left(2, \frac{2h(c+dx)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h^3}$$

```
[Out] 1/2*a*n*x/b/h-1/2*c*n*x/d/h-1/2*a^2*n*ln(b*x+a)/b^2/h+1/2*n*x^2*ln(b*x+a)/h
-g*n*(b*x+a)*ln(b*x+a)/b/h^2+1/2*c^2*n*ln(d*x+c)/d^2/h-1/2*n*x^2*ln(d*x+c)/
h+g*n*(d*x+c)*ln(d*x+c)/d/h^2+g*x*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*
ln(d*x+c))/h^2-1/2*x^2*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))/
h-1/2*(-f*h+g^2)*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))*ln(h*x
^2+g*x+f)/h^3+1/2*n*ln(b*x+a)*ln(-b*(g+2*h*x-(-4*f*h+g^2)^(1/2))/(2*a*h-b*(
g-(-4*f*h+g^2)^(1/2))))*(g^2-f*h-g*(-3*f*h+g^2)/(-4*f*h+g^2)^(1/2))/h^3-1/2
*n*ln(d*x+c)*ln(-d*(g+2*h*x-(-4*f*h+g^2)^(1/2))/(2*c*h-d*(g-(-4*f*h+g^2)^(1
/2))))*(g^2-f*h-g*(-3*f*h+g^2)/(-4*f*h+g^2)^(1/2))/h^3+1/2*n*polylog(2,2*h*
(b*x+a)/(2*a*h-b*(g-(-4*f*h+g^2)^(1/2))))*(g^2-f*h-g*(-3*f*h+g^2)/(-4*f*h+g
^2)^(1/2))/h^3-1/2*n*polylog(2,2*h*(d*x+c)/(2*c*h-d*(g-(-4*f*h+g^2)^(1/2)))
)*(g^2-f*h-g*(-3*f*h+g^2)/(-4*f*h+g^2)^(1/2))/h^3+1/2*n*ln(b*x+a)*ln(-b*(g+
2*h*x+(-4*f*h+g^2)^(1/2))/(2*a*h-b*(g+(-4*f*h+g^2)^(1/2))))*(g^2-f*h+g*(-3*
f*h+g^2)/(-4*f*h+g^2)^(1/2))/h^3-1/2*n*ln(d*x+c)*ln(-d*(g+2*h*x+(-4*f*h+g^2
)^(1/2))/(2*c*h-d*(g+(-4*f*h+g^2)^(1/2))))*(g^2-f*h+g*(-3*f*h+g^2)/(-4*f*h+
g^2)^(1/2))/h^3+1/2*n*polylog(2,2*h*(b*x+a)/(2*a*h-b*(g+(-4*f*h+g^2)^(1/2))
))*(g^2-f*h+g*(-3*f*h+g^2)/(-4*f*h+g^2)^(1/2))/h^3-1/2*n*polylog(2,2*h*(d*x
+c)/(2*c*h-d*(g+(-4*f*h+g^2)^(1/2))))*(g^2-f*h+g*(-3*f*h+g^2)/(-4*f*h+g^2)^
(1/2))/h^3-g*(-3*f*h+g^2)*arctanh((2*h*x+g)/(-4*f*h+g^2)^(1/2))*(n*ln(b*x+a
)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))/h^3/(-4*f*h+g^2)^(1/2)
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 1046, normalized size of antiderivative = 1.00, number of steps used = 37, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules

used = {2593, 2465, 2436, 2332, 2442, 45, 2441, 2440, 2438, 715, 648, 632, 212, 642}

$$\int \frac{x^3 \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2} dx$$
$$= -\frac{n \log(a+bx) a^2}{2b^2h} + \frac{nxa}{2bh} - \frac{cnx}{2dh} + \frac{nx^2 \log(a+bx)}{2h} - \frac{gn(a+bx) \log(a+bx)}{bh^2}$$
$$- \frac{nx^2 \log(c+dx)}{2h} + \frac{c^2 n \log(c+dx)}{2d^2h} + \frac{gn(c+dx) \log(c+dx)}{dh^2}$$
$$- \frac{x^2 \left(n \log(a+bx) - \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{2h}$$
$$+ \frac{gx \left(n \log(a+bx) - \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{h^2}$$
$$- \frac{g(g^2-3fh) \operatorname{arctanh}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right) \left(n \log(a+bx) - \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right)}{h^3 \sqrt{g^2-4fh}}$$
$$+ \frac{\left(g^2 - \frac{(g^2-3fh)g}{\sqrt{g^2-4fh}} - fh\right) n \log(a+bx) \log \left(-\frac{b\left(g+2hx-\sqrt{g^2-4fh}\right)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h^3}$$
$$- \frac{\left(g^2 - \frac{(g^2-3fh)g}{\sqrt{g^2-4fh}} - fh\right) n \log(c+dx) \log \left(-\frac{d\left(g+2hx-\sqrt{g^2-4fh}\right)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h^3}$$
$$+ \frac{\left(g^2 + \frac{(g^2-3fh)g}{\sqrt{g^2-4fh}} - fh\right) n \log(a+bx) \log \left(-\frac{b\left(g+2hx+\sqrt{g^2-4fh}\right)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h^3}$$
$$- \frac{\left(g^2 + \frac{(g^2-3fh)g}{\sqrt{g^2-4fh}} - fh\right) n \log(c+dx) \log \left(-\frac{d\left(g+2hx+\sqrt{g^2-4fh}\right)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h^3}$$
$$- \frac{(g^2-fh) \left(n \log(a+bx) - \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right) - n \log(c+dx)\right) \log (hx^2+gx+f)}{2h^3}$$
$$+ \frac{\left(g^2 - \frac{(g^2-3fh)g}{\sqrt{g^2-4fh}} - fh\right) n \operatorname{PolyLog} \left(2, \frac{2h(a+bx)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h^3}$$
$$+ \frac{\left(g^2 + \frac{(g^2-3fh)g}{\sqrt{g^2-4fh}} - fh\right) n \operatorname{PolyLog} \left(2, \frac{2h(a+bx)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h^3}$$
$$- \frac{\left(g^2 - \frac{(g^2-3fh)g}{\sqrt{g^2-4fh}} - fh\right) n \operatorname{PolyLog} \left(2, \frac{2h(c+dx)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h^3}$$
$$- \frac{\left(g^2 + \frac{(g^2-3fh)g}{\sqrt{g^2-4fh}} - fh\right) n \operatorname{PolyLog} \left(2, \frac{2h(c+dx)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h^3}$$

```
[In] Int[(x^3*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x + h*x^2),x]
```

```
[Out] (a*n*x)/(2*b*h) - (c*n*x)/(2*d*h) - (a^2*n*Log[a + b*x])/(2*b^2*h) + (n*x^2
*Log[a + b*x])/(2*h) - (g*n*(a + b*x)*Log[a + b*x])/(b*h^2) + (c^2*n*Log[c
+ d*x])/(2*d^2*h) - (n*x^2*Log[c + d*x])/(2*h) + (g*n*(c + d*x)*Log[c + d*x
])/(d*h^2) + (g*x*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[
c + d*x]))/h^2 - (x^2*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*
Log[c + d*x]))/(2*h) - (g*(g^2 - 3*f*h)*ArcTanh[(g + 2*h*x)/Sqrt[g^2 - 4*f*
h]]*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/(h^
3*Sqrt[g^2 - 4*f*h]) + ((g^2 - f*h - (g*(g^2 - 3*f*h))/Sqrt[g^2 - 4*f*h])*n
*Log[a + b*x]*Log[-((b*(g - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a*h - b*(g - Sqr
t[g^2 - 4*f*h])))])/(2*h^3) - ((g^2 - f*h - (g*(g^2 - 3*f*h))/Sqrt[g^2 - 4*
f*h])*n*Log[c + d*x]*Log[-((d*(g - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*c*h - d*(
g - Sqrt[g^2 - 4*f*h])))])/(2*h^3) + ((g^2 - f*h + (g*(g^2 - 3*f*h))/Sqrt[g
^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a*
h - b*(g + Sqrt[g^2 - 4*f*h])))])/(2*h^3) - ((g^2 - f*h + (g*(g^2 - 3*f*h))
/Sqrt[g^2 - 4*f*h])*n*Log[c + d*x]*Log[-((d*(g + Sqrt[g^2 - 4*f*h] + 2*h*x)
)/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h])))])/(2*h^3) - ((g^2 - f*h)*(n*Log[a +
b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])*Log[f + g*x + h*x^2
])/(2*h^3) + ((g^2 - f*h - (g*(g^2 - 3*f*h))/Sqrt[g^2 - 4*f*h])*n*PolyLog[2
, (2*h*(a + b*x))/(2*a*h - b*(g - Sqrt[g^2 - 4*f*h]))])/(2*h^3) + ((g^2 - f
*h + (g*(g^2 - 3*f*h))/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(a + b*x))/(2*a
*h - b*(g + Sqrt[g^2 - 4*f*h]))])/(2*h^3) - ((g^2 - f*h - (g*(g^2 - 3*f*h))
/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g - Sqrt[g^2 -
 4*f*h]))])/(2*h^3) - ((g^2 - f*h + (g*(g^2 - 3*f*h))/Sqrt[g^2 - 4*f*h])*n*
PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))])/(2*h^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
```

```
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2593

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= n\int \frac{x^3\log(a+bx)}{f+gx+hx^2}\,dx - n\int \frac{x^3\log(c+dx)}{f+gx+hx^2}\,dx \\
&\quad - \left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\int \frac{x^3}{f+gx+hx^2}\,dx \\
&= n\int \left(-\frac{g\log(a+bx)}{h^2}+\frac{x\log(a+bx)}{h}+\frac{(fg+(g^2-fh)\,x)\log(a+bx)}{h^2\,(f+gx+hx^2)}\right)dx \\
&\quad - n\int \left(-\frac{g\log(c+dx)}{h^2}+\frac{x\log(c+dx)}{h}+\frac{(fg+(g^2-fh)\,x)\log(c+dx)}{h^2\,(f+gx+hx^2)}\right)dx \\
&\quad - \left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\int \left(-\frac{g}{h^2}+\frac{x}{h}\right. \\
&\qquad \left.+\frac{fg+(g^2-fh)\,x}{h^2\,(f+gx+hx^2)}\right)dx
\end{aligned}$$

$$= \frac{gx\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{h^2} - \frac{x^2\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{2h} + \frac{n\int\frac{(fg+(g^2-fh)x)\log(a+bx)}{f+gx+hx^2}\,dx}{h^2} - \frac{n\int\frac{(fg+(g^2-fh)x)\log(c+dx)}{f+gx+hx^2}\,dx}{h^2} - \frac{(gn)\int\log(a+bx)\,dx}{h^2} + \frac{(gn)\int\log(c+dx)\,dx}{h^2} + \frac{n\int x\log(a+bx)\,dx}{h} - \frac{n\int x\log(c+dx)\,dx}{h} - \frac{\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\int\frac{fg+(g^2-fh)x}{f+gx+hx^2}\,dx}{h^2}$$

$$= \frac{nx^2\log(a+bx)}{2h} - \frac{nx^2\log(c+dx)}{2h} + \frac{gx\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{h^2} - \frac{x^2\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{2h} + \frac{n\int\left(\frac{\left(g^2-fh+\frac{g(-g^2+3fh)}{\sqrt{g^2-4fh}}\right)\log(a+bx)}{g-\sqrt{g^2-4fh}+2hx}+\frac{\left(g^2-fh-\frac{g(-g^2+3fh)}{\sqrt{g^2-4fh}}\right)\log(a+bx)}{g+\sqrt{g^2-4fh}+2hx}\right)dx}{h^2} - \frac{n\int\left(\frac{\left(g^2-fh+\frac{g(-g^2+3fh)}{\sqrt{g^2-4fh}}\right)\log(c+dx)}{g-\sqrt{g^2-4fh}+2hx}+\frac{\left(g^2-fh-\frac{g(-g^2+3fh)}{\sqrt{g^2-4fh}}\right)\log(c+dx)}{g+\sqrt{g^2-4fh}+2hx}\right)dx}{h^2} - \frac{(gn)\mathrm{Subst}\left(\int\log(x)\,dx,x,a+bx\right)}{bh^2} + \frac{(gn)\mathrm{Subst}\left(\int\log(x)\,dx,x,c+dx\right)}{dh^2} - \frac{(bn)\int\frac{x^2}{a+bx}\,dx}{2h} + \frac{(dn)\int\frac{x^2}{c+dx}\,dx}{2h} + \frac{\left(g(g^2-3fh)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\right)\int\frac{1}{f+gx+hx^2}\,dx}{2h^3} - \frac{\left((g^2-fh)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\right)\int\frac{g+2hx}{f+gx+hx^2}\,dx}{2h^3}$$

$$
\begin{aligned}
&= \frac{nx^2\log(a+bx)}{2h} - \frac{gn(a+bx)\log(a+bx)}{bh^2} - \frac{nx^2\log(c+dx)}{2h} \\
&\quad + \frac{gn(c+dx)\log(c+dx)}{dh^2} + \frac{gx\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{h^2} \\
&\quad - \frac{x^2\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{2h} \\
&\quad - \frac{(g^2-fh)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)\log\left(f+gx+hx^2\right)}{2h^3} \\
&\quad - \frac{(bn)\int\left(-\frac{a}{b^2} + \frac{x}{b} + \frac{a^2}{b^2(a+bx)}\right)dx}{2h} + \frac{(dn)\int\left(-\frac{c}{d^2} + \frac{x}{d} + \frac{c^2}{d^2(c+dx)}\right)dx}{2h} \\
&\quad + \frac{\left(\left(g^2 - fh - \frac{g(g^2-3fh)}{\sqrt{g^2-4fh}}\right)n\right)\int\frac{\log(a+bx)}{g-\sqrt{g^2-4fh}+2hx}\,dx}{h^2} \\
&\quad - \frac{\left(\left(g^2 - fh - \frac{g(g^2-3fh)}{\sqrt{g^2-4fh}}\right)n\right)\int\frac{\log(c+dx)}{g-\sqrt{g^2-4fh}+2hx}\,dx}{h^2} \\
&\quad + \frac{\left(\left(g^2 - fh + \frac{g(g^2-3fh)}{\sqrt{g^2-4fh}}\right)n\right)\int\frac{\log(a+bx)}{g+\sqrt{g^2-4fh}+2hx}\,dx}{h^2} \\
&\quad - \frac{\left(\left(g^2 - fh + \frac{g(g^2-3fh)}{\sqrt{g^2-4fh}}\right)n\right)\int\frac{\log(c+dx)}{g+\sqrt{g^2-4fh}+2hx}\,dx}{h^2} \\
&\quad - \frac{\left(g(g^2-3fh)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)\right)\mathrm{Subst}\left(\int\frac{1}{g^2-4fh-x^2}\,dx, x, g+2h\right.}{h^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{anx}{2bh} - \frac{cnx}{2dh} - \frac{a^2 n \log(a+bx)}{2b^2h} + \frac{nx^2\log(a+bx)}{2h} - \frac{gn(a+bx)\log(a+bx)}{bh^2} \\
&+ \frac{c^2 n \log(c+dx)}{2d^2h} - \frac{nx^2\log(c+dx)}{2h} + \frac{gn(c+dx)\log(c+dx)}{dh^2} \\
&+ \frac{gx\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{h^2} \\
&- \frac{x^2\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{2h} \\
&- \frac{g(g^2-3fh)\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{h^3\sqrt{g^2-4fh}} \\
&+ \frac{\left(g^2 - fh - \frac{g(g^2-3fh)}{\sqrt{g^2-4fh}}\right) n\log(a+bx)\log\left(-\frac{b\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h^3} \\
&- \frac{\left(g^2 - fh - \frac{g(g^2-3fh)}{\sqrt{g^2-4fh}}\right) n\log(c+dx)\log\left(-\frac{d\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h^3} \\
&+ \frac{\left(g^2 - fh + \frac{g(g^2-3fh)}{\sqrt{g^2-4fh}}\right) n\log(a+bx)\log\left(-\frac{b\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h^3} \\
&- \frac{\left(g^2 - fh + \frac{g(g^2-3fh)}{\sqrt{g^2-4fh}}\right) n\log(c+dx)\log\left(-\frac{d\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h^3} \\
&- \frac{(g^2-fh)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)\log\left(f+gx+hx^2\right)}{2h^3} \\
&- \frac{\left(b\left(g^2 - fh - \frac{g(g^2-3fh)}{\sqrt{g^2-4fh}}\right)n\right)\int \frac{\log\left(\frac{b\left(g-\sqrt{g^2-4fh}+2hx\right)}{-2ah+b\left(g-\sqrt{g^2-4fh}\right)}\right)}{a+bx}\,dx}{2h^3} \\
&+ \frac{\left(d\left(g^2 - fh - \frac{g(g^2-3fh)}{\sqrt{g^2-4fh}}\right)n\right)\int \frac{\log\left(\frac{d\left(g-\sqrt{g^2-4fh}+2hx\right)}{-2ch+d\left(g-\sqrt{g^2-4fh}\right)}\right)}{c+dx}\,dx}{2h^3} \\
&- \frac{\left(b\left(g^2 - fh + \frac{g(g^2-3fh)}{\sqrt{g^2-4fh}}\right)n\right)\int \frac{\log\left(\frac{b\left(g+\sqrt{g^2-4fh}+2hx\right)}{-2ah+b\left(g+\sqrt{g^2-4fh}\right)}\right)}{a+bx}\,dx}{2h^3} \\
&+ \frac{\left(d\left(g^2 - fh + \frac{g(g^2-3fh)}{\sqrt{g^2-4fh}}\right)n\right)\int \frac{\log\left(\frac{d\left(g+\sqrt{g^2-4fh}+2hx\right)}{-2ch+d\left(g+\sqrt{g^2-4fh}\right)}\right)}{c+dx}\,dx}{2h^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{anx}{2bh} - \frac{cnx}{2dh} - \frac{a^2 n \log(a+bx)}{2b^2 h} + \frac{nx^2 \log(a+bx)}{2h} - \frac{gn(a+bx)\log(a+bx)}{bh^2} \\
&+ \frac{c^2 n \log(c+dx)}{2d^2 h} - \frac{nx^2 \log(c+dx)}{2h} + \frac{gn(c+dx)\log(c+dx)}{dh^2} \\
&+ \frac{gx\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{h^2} \\
&- \frac{x^2\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{2h} \\
&- \frac{g(g^2-3fh)\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{h^3\sqrt{g^2-4fh}} \\
&+ \frac{\left(g^2 - fh - \frac{g(g^2-3fh)}{\sqrt{g^2-4fh}}\right) n\log(a+bx)\log\left(-\frac{b\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h^3} \\
&- \frac{\left(g^2 - fh - \frac{g(g^2-3fh)}{\sqrt{g^2-4fh}}\right) n\log(c+dx)\log\left(-\frac{d\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h^3} \\
&+ \frac{\left(g^2 - fh + \frac{g(g^2-3fh)}{\sqrt{g^2-4fh}}\right) n\log(a+bx)\log\left(-\frac{b\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h^3} \\
&- \frac{\left(g^2 - fh + \frac{g(g^2-3fh)}{\sqrt{g^2-4fh}}\right) n\log(c+dx)\log\left(-\frac{d\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h^3} \\
&- \frac{(g^2-fh)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)\log\left(f+gx+hx^2\right)}{2h^3} \\
&- \frac{\left(\left(g^2 - fh - \frac{g(g^2-3fh)}{\sqrt{g^2-4fh}}\right) n\right) \text{Subst}\left(\int \frac{\log\left(1+\frac{2hx}{-2ah+b\left(g-\sqrt{g^2-4fh}\right)}\right)}{x}dx, x, a+bx\right)}{2h^3} \\
&+ \frac{\left(\left(g^2 - fh - \frac{g(g^2-3fh)}{\sqrt{g^2-4fh}}\right) n\right) \text{Subst}\left(\int \frac{\log\left(1+\frac{2hx}{-2ch+d\left(g-\sqrt{g^2-4fh}\right)}\right)}{x}dx, x, c+dx\right)}{2h^3} \\
&- \frac{\left(\left(g^2 - fh + \frac{g(g^2-3fh)}{\sqrt{g^2-4fh}}\right) n\right) \text{Subst}\left(\int \frac{\log\left(1+\frac{2hx}{-2ah+b\left(g+\sqrt{g^2-4fh}\right)}\right)}{x}dx, x, a+bx\right)}{2h^3} \\
&+ \frac{\left(\left(g^2 - fh + \frac{g(g^2-3fh)}{\sqrt{g^2-4fh}}\right) n\right) \text{Subst}\left(\int \frac{\log\left(1+\frac{2hx}{-2ch+d\left(g+\sqrt{g^2-4fh}\right)}\right)}{x}dx, x, c+dx\right)}{2h^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{anx}{2bh} - \frac{cnx}{2dh} - \frac{a^2 n \log(a+bx)}{2b^2 h} + \frac{nx^2 \log(a+bx)}{2h} - \frac{gn(a+bx)\log(a+bx)}{bh^2} \\
&+ \frac{c^2 n \log(c+dx)}{2d^2 h} - \frac{nx^2 \log(c+dx)}{2h} + \frac{gn(c+dx)\log(c+dx)}{dh^2} \\
&+ \frac{gx\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{h^2} \\
&- \frac{x^2\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{2h} \\
&- \frac{g(g^2-3fh)\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{h^3\sqrt{g^2-4fh}} \\
&+ \frac{\left(g^2 - fh - \frac{g(g^2-3fh)}{\sqrt{g^2-4fh}}\right) n\log(a+bx)\log\left(-\frac{b\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h^3} \\
&- \frac{\left(g^2 - fh - \frac{g(g^2-3fh)}{\sqrt{g^2-4fh}}\right) n\log(c+dx)\log\left(-\frac{d\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h^3} \\
&+ \frac{\left(g^2 - fh + \frac{g(g^2-3fh)}{\sqrt{g^2-4fh}}\right) n\log(a+bx)\log\left(-\frac{b\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h^3} \\
&- \frac{\left(g^2 - fh + \frac{g(g^2-3fh)}{\sqrt{g^2-4fh}}\right) n\log(c+dx)\log\left(-\frac{d\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h^3} \\
&- \frac{(g^2-fh)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)\log\left(f+gx+hx^2\right)}{2h^3} \\
&+ \frac{\left(g^2 - fh - \frac{g(g^2-3fh)}{\sqrt{g^2-4fh}}\right) n\operatorname{Li}_2\left(\frac{2h(a+bx)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h^3} \\
&+ \frac{\left(g^2 - fh + \frac{g(g^2-3fh)}{\sqrt{g^2-4fh}}\right) n\operatorname{Li}_2\left(\frac{2h(a+bx)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h^3} \\
&- \frac{\left(g^2 - fh - \frac{g(g^2-3fh)}{\sqrt{g^2-4fh}}\right) n\operatorname{Li}_2\left(\frac{2h(c+dx)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h^3} \\
&- \frac{\left(g^2 - fh + \frac{g(g^2-3fh)}{\sqrt{g^2-4fh}}\right) n\operatorname{Li}_2\left(\frac{2h(c+dx)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 1240, normalized size of antiderivative = 1.19

$$\int \frac{x^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx$$

$$= \frac{h^2x^2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - \frac{2gh(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b} + \frac{2(bc-ad)ghn\log(c+dx)}{bd} + \frac{h^2n(-a^2d^2\log(a+bx)+b(d(-bc+ad)x+bc^2\log(c+dx)}{b^2d^2}}{}$$

```
[In] Integrate[(x^3*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x + h*x^2),x]
```

```
[Out] (h^2*x^2*Log[e*((a + b*x)/(c + d*x))^n] - (2*g*h*(a + b*x)*Log[e*((a + b*x)
/(c + d*x))^n])/b + (2*(b*c - a*d)*g*h*n*Log[c + d*x])/(b*d) + (h^2*n*(-(a^
2*d^2*Log[a + b*x]) + b*(d*(-(b*c) + a*d)*x + b*c^2*Log[c + d*x])))/(b^2*d^
2) + (2*f*g*h*Log[e*((a + b*x)/(c + d*x))^n]*Log[g - Sqrt[g^2 - 4*f*h] + 2*
h*x])/Sqrt[g^2 - 4*f*h] + (g^2 - f*h)*(1 - g/Sqrt[g^2 - 4*f*h])*Log[e*((a +
 b*x)/(c + d*x))^n]*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] - (2*f*g*h*Log[e*((a
 + b*x)/(c + d*x))^n]*Log[g + Sqrt[g^2 - 4*f*h] + 2*h*x])/Sqrt[g^2 - 4*f*h]
 + (g^2 - f*h)*(1 + g/Sqrt[g^2 - 4*f*h])*Log[e*((a + b*x)/(c + d*x))^n]*Log
[g + Sqrt[g^2 - 4*f*h] + 2*h*x] - (2*f*g*h*n*((Log[(2*h*(a + b*x))/(-(b*g)
+ 2*a*h + b*Sqrt[g^2 - 4*f*h])] - Log[(2*h*(c + d*x))/(-(d*g) + 2*c*h + d*S
qrt[g^2 - 4*f*h])])*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] + PolyLog[2, (b*(-g
+ Sqrt[g^2 - 4*f*h] - 2*h*x))/(-(b*g) + 2*a*h + b*Sqrt[g^2 - 4*f*h])] - Pol
yLog[2, (d*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(2*c*h + d*(-g + Sqrt[g^2 - 4*
f*h]))]))/Sqrt[g^2 - 4*f*h] - ((g^2 - f*h)*(-g + Sqrt[g^2 - 4*f*h])*n*((Log
[(2*h*(a + b*x))/(-(b*g) + 2*a*h + b*Sqrt[g^2 - 4*f*h])] - Log[(2*h*(c + d*
x))/(-(d*g) + 2*c*h + d*Sqrt[g^2 - 4*f*h])])*Log[g - Sqrt[g^2 - 4*f*h] + 2*
h*x] + PolyLog[2, (b*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(-(b*g) + 2*a*h + b*
Sqrt[g^2 - 4*f*h])] - PolyLog[2, (d*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(2*c*
h + d*(-g + Sqrt[g^2 - 4*f*h]))]))/Sqrt[g^2 - 4*f*h] + (2*f*g*h*n*((Log[(2*
h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))] - Log[(2*h*(c + d*x))/(2*
c*h - d*(g + Sqrt[g^2 - 4*f*h]))])*Log[g + Sqrt[g^2 - 4*f*h] + 2*h*x] + Pol
yLog[2, (b*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*a*h + b*(g + Sqrt[g^2 - 4*f
*h]))] - PolyLog[2, (d*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*c*h + d*(g + Sq
rt[g^2 - 4*f*h]))]))/Sqrt[g^2 - 4*f*h] - ((g^2 - f*h)*(g + Sqrt[g^2 - 4*f*h
])*n*((Log[(2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))] - Log[(2*h*
(c + d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))])*Log[g + Sqrt[g^2 - 4*f*h]
+ 2*h*x] + PolyLog[2, (b*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*a*h + b*(g +
Sqrt[g^2 - 4*f*h]))] - PolyLog[2, (d*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*c
*h + d*(g + Sqrt[g^2 - 4*f*h]))]))/Sqrt[g^2 - 4*f*h])/(2*h^3)
```

Maple [F]

$$\int \frac{x^3 \ln \left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{h\,x^2+gx+f}dx$$

```
[In] int(x^3*ln(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x)
```

```
[Out] int(x^3*ln(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x)
```

Fricas [F]

$$\int \frac{x^3 \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx = \int \frac{x^3 \log \left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{hx^2+gx+f}\,dx$$

```
[In] integrate(x^3*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="fricas
")
```

```
[Out] integral(x^3*log(e*((b*x + a)/(d*x + c))^n)/(h*x^2 + g*x + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3 \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**3*ln(e*((b*x+a)/(d*x+c))**n)/(h*x**2+g*x+f),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3 \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*f*h-g^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int \frac{x^3 \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2} dx = \int \frac{x^3 \log \left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{hx^2+gx+f} dx$$

```
[In] integrate(x^3*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="giac")
[Out] integrate(x^3*log(e*((b*x + a)/(d*x + c))^n)/(h*x^2 + g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2} dx = \int \frac{x^3 \ln \left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{h\,x^2+g\,x+f} dx$$

```
[In] int((x^3*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x + h*x^2),x)
[Out] int((x^3*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x + h*x^2), x)
```

3.83 $\int \frac{x^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx$

Optimal result 668
Rubi [A] (verified) 669
Mathematica [A] (verified) 678
Maple [F] 679
Fricas [F] 679
Sympy [F(-1)] 679
Maxima [F(-2)] 679
Giac [F] 680
Mupad [F(-1)] 680

Optimal result

Integrand size = 34, antiderivative size = 831

$$\int \frac{x^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx = \frac{n(a+bx)\log(a+bx)}{bh} - \frac{n(c+dx)\log(c+dx)}{dh} - \frac{x\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{h} + \frac{(g^2-2fh)\operatorname{arctanh}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{h^2\sqrt{g^2-4fh}} - \frac{\left(g-\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\log(a+bx)\log\left(-\frac{b\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h^2} + \frac{\left(g-\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\log(c+dx)\log\left(-\frac{d\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h^2} - \frac{\left(g+\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\log(a+bx)\log\left(-\frac{b\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h^2} + \frac{\left(g+\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\log(c+dx)\log\left(-\frac{d\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h^2} + \frac{g\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\log\left(f+gx+hx^2\right)}{2h^2} - \frac{\left(g-\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\operatorname{PolyLog}\left(2,\frac{2h(a+bx)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h^2} - \frac{\left(g+\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\operatorname{PolyLog}\left(2,\frac{2h(a+bx)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h^2} + \frac{\left(g-\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\operatorname{PolyLog}\left(2,\frac{2h(c+dx)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h^2} + \frac{\left(g+\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\operatorname{PolyLog}\left(2,\frac{2h(c+dx)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h^2}$$

```
[Out] n*(b*x+a)*ln(b*x+a)/b/h-n*(d*x+c)*ln(d*x+c)/d/h-x*(n*ln(b*x+a)-ln(e*((b*x+a
)/(d*x+c))^n)-n*ln(d*x+c))/h+1/2*g*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n
*ln(d*x+c))*ln(h*x^2+g*x+f)/h^2-1/2*n*ln(b*x+a)*ln(-b*(g+2*h*x-(-4*f*h+g^2)
^(1/2))/(2*a*h-b*(g-(-4*f*h+g^2)^(1/2))))*(g+(2*f*h-g^2)/(-4*f*h+g^2)^(1/2)
```

```
)/h^2+1/2*n*ln(d*x+c)*ln(-d*(g+2*h*x-(-4*f*h+g^2)^(1/2))/(2*c*h-d*(g-(-4*f*
h+g^2)^(1/2))))*(g+(2*f*h-g^2)/(-4*f*h+g^2)^(1/2))/h^2-1/2*n*polylog(2,2*h*
(b*x+a)/(2*a*h-b*(g-(-4*f*h+g^2)^(1/2))))*(g+(2*f*h-g^2)/(-4*f*h+g^2)^(1/2)
)/h^2+1/2*n*polylog(2,2*h*(d*x+c)/(2*c*h-d*(g-(-4*f*h+g^2)^(1/2))))*(g+(2*f
*h-g^2)/(-4*f*h+g^2)^(1/2))/h^2-1/2*n*ln(b*x+a)*ln(-b*(g+2*h*x+(-4*f*h+g^2)
^(1/2))/(2*a*h-b*(g+(-4*f*h+g^2)^(1/2))))*(g+(-2*f*h+g^2)/(-4*f*h+g^2)^(1/2
))/h^2+1/2*n*ln(d*x+c)*ln(-d*(g+2*h*x+(-4*f*h+g^2)^(1/2))/(2*c*h-d*(g+(-4*f
*h+g^2)^(1/2))))*(g+(-2*f*h+g^2)/(-4*f*h+g^2)^(1/2))/h^2-1/2*n*polylog(2,2*
h*(b*x+a)/(2*a*h-b*(g+(-4*f*h+g^2)^(1/2))))*(g+(-2*f*h+g^2)/(-4*f*h+g^2)^(1
/2))/h^2+1/2*n*polylog(2,2*h*(d*x+c)/(2*c*h-d*(g+(-4*f*h+g^2)^(1/2))))*(g+(
-2*f*h+g^2)/(-4*f*h+g^2)^(1/2))/h^2+(-2*f*h+g^2)*arctanh((2*h*x+g)/(-4*f*h+
g^2)^(1/2))*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))/h^2/(-4*f*h
+g^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 831, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules

used = {2593, 2465, 2436, 2332, 2441, 2440, 2438, 717, 648, 632, 212, 642}

$$\int \frac{x^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx$$
$$= \frac{n(a+bx)\log(a+bx)}{bh} - \frac{\left(g-\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\log\left(-\frac{b\left(g+2hx-\sqrt{g^2-4fh}\right)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)\log(a+bx)}{2h^2}$$
$$- \frac{\left(g+\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\log\left(-\frac{b\left(g+2hx+\sqrt{g^2-4fh}\right)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)\log(a+bx)}{2h^2}$$
$$- \frac{n(c+dx)\log(c+dx)}{dh} - \frac{x\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{h}$$
$$+ \frac{(g^2-2fh)\,\mathrm{arctanh}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{h^2\sqrt{g^2-4fh}}$$
$$+ \frac{\left(g-\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\log(c+dx)\log\left(-\frac{d\left(g+2hx-\sqrt{g^2-4fh}\right)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h^2}$$
$$+ \frac{\left(g+\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\log(c+dx)\log\left(-\frac{d\left(g+2hx+\sqrt{g^2-4fh}\right)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h^2}$$
$$+ \frac{g\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\log\left(hx^2+gx+f\right)}{2h^2}$$
$$- \frac{\left(g-\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\,\mathrm{PolyLog}\left(2,\frac{2h(a+bx)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h^2}$$
$$- \frac{\left(g+\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\,\mathrm{PolyLog}\left(2,\frac{2h(a+bx)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h^2}$$
$$+ \frac{\left(g-\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\,\mathrm{PolyLog}\left(2,\frac{2h(c+dx)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h^2}$$
$$+ \frac{\left(g+\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\,\mathrm{PolyLog}\left(2,\frac{2h(c+dx)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h^2}$$

```
[In] Int[(x^2*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x + h*x^2),x]
```

```
[Out] (n*(a + b*x)*Log[a + b*x])/(b*h) - (n*(c + d*x)*Log[c + d*x])/(d*h) - (x*(n
*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/h + ((g^2
 - 2*f*h)*ArcTanh[(g + 2*h*x)/Sqrt[g^2 - 4*f*h]]*(n*Log[a + b*x] - Log[e*((
a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/(h^2*Sqrt[g^2 - 4*f*h]) - ((g - (
g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*(g - Sqrt[g^2 - 4*f
*h] + 2*h*x))/(2*a*h - b*(g - Sqrt[g^2 - 4*f*h])))])/(2*h^2) + ((g - (g^2 -
```

```
 2*f*h)/Sqrt[g^2 - 4*f*h])*n*Log[c + d*x]*Log[-((d*(g - Sqrt[g^2 - 4*f*h] +
 2*h*x))/(2*c*h - d*(g - Sqrt[g^2 - 4*f*h]))))])/(2*h^2) - ((g + (g^2 - 2*f*
h)/Sqrt[g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*(g + Sqrt[g^2 - 4*f*h] + 2*h*
x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))))])/(2*h^2) + ((g + (g^2 - 2*f*h)/Sq
rt[g^2 - 4*f*h])*n*Log[c + d*x]*Log[-((d*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(
2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))))])/(2*h^2) + (g*(n*Log[a + b*x] - Log[e*
((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])*Log[f + g*x + h*x^2])/(2*h^2) -
((g - (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(a + b*x))/(2*a*h
- b*(g - Sqrt[g^2 - 4*f*h]))])/(2*h^2) - ((g + (g^2 - 2*f*h)/Sqrt[g^2 - 4*f
*h])*n*PolyLog[2, (2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))])/(2*
h^2) + ((g - (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(c + d*x))/
(2*c*h - d*(g - Sqrt[g^2 - 4*f*h]))])/(2*h^2) + ((g + (g^2 - 2*f*h)/Sqrt[g^
2 - 4*f*h])*n*PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h])
)])/(2*h^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 717

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(
m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
```

```
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2593

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
```

```
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= n\int \frac{x^2\log(a+bx)}{f+gx+hx^2}\,dx - n\int \frac{x^2\log(c+dx)}{f+gx+hx^2}\,dx \\
&\quad -\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\int \frac{x^2}{f+gx+hx^2}\,dx \\
&= -\frac{x\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{h} \\
&\quad +n\int\left(\frac{\log(a+bx)}{h}-\frac{(f+gx)\log(a+bx)}{h\left(f+gx+hx^2\right)}\right)dx \\
&\quad -n\int\left(\frac{\log(c+dx)}{h}-\frac{(f+gx)\log(c+dx)}{h\left(f+gx+hx^2\right)}\right)dx \\
&\quad -\frac{\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\int \frac{-f-gx}{f+gx+hx^2}\,dx}{h} \\
&= -\frac{x\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{h}+\frac{n\int \log(a+bx)\,dx}{h} \\
&\quad -\frac{n\int \frac{(f+gx)\log(a+bx)}{f+gx+hx^2}\,dx}{h}-\frac{n\int \log(c+dx)\,dx}{h}+\frac{n\int \frac{(f+gx)\log(c+dx)}{f+gx+hx^2}\,dx}{h} \\
&\quad +\frac{\left(g\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\right)\int \frac{g+2hx}{f+gx+hx^2}\,dx}{2h^2} \\
&\quad -\frac{\left(\left(g^2-2fh\right)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\right)\int \frac{1}{f+gx+hx^2}\,dx}{2h^2}
\end{aligned}$$

$$= -\frac{x\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{h}$$
$$+\frac{g\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\log\left(f+gx+hx^2\right)}{2h^2}$$
$$-\frac{n\int\left(\frac{\left(g+\frac{-g^2+2fh}{\sqrt{g^2-4fh}}\right)\log(a+bx)}{g-\sqrt{g^2-4fh}+2hx}+\frac{\left(g-\frac{-g^2+2fh}{\sqrt{g^2-4fh}}\right)\log(a+bx)}{g+\sqrt{g^2-4fh}+2hx}\right)dx}{h}$$
$$+\frac{n\int\left(\frac{\left(g+\frac{-g^2+2fh}{\sqrt{g^2-4fh}}\right)\log(c+dx)}{g-\sqrt{g^2-4fh}+2hx}+\frac{\left(g-\frac{-g^2+2fh}{\sqrt{g^2-4fh}}\right)\log(c+dx)}{g+\sqrt{g^2-4fh}+2hx}\right)dx}{h}$$
$$+\frac{n\text{Subst}\left(\int\log(x)\,dx,x,a+bx\right)}{bh}-\frac{n\text{Subst}\left(\int\log(x)\,dx,x,c+dx\right)}{dh}$$
$$+\frac{\left(\left(g^2-2fh\right)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\right)\text{Subst}\left(\int\frac{1}{g^2-4fh-x^2}\,dx,x,g+2hx\right)}{h^2}$$

$$= \frac{n(a+bx)\log(a+bx)}{bh}-\frac{n(c+dx)\log(c+dx)}{dh}$$
$$-\frac{x\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{h}$$
$$+\frac{\left(g^2-2fh\right)\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{h^2\sqrt{g^2-4fh}}$$
$$+\frac{g\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\log\left(f+gx+hx^2\right)}{2h^2}$$
$$-\frac{\left(\left(g-\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\right)\int\frac{\log(a+bx)}{g-\sqrt{g^2-4fh}+2hx}\,dx}{h}$$
$$+\frac{\left(\left(g-\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\right)\int\frac{\log(c+dx)}{g-\sqrt{g^2-4fh}+2hx}\,dx}{h}$$
$$-\frac{\left(\left(g+\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\right)\int\frac{\log(a+bx)}{g+\sqrt{g^2-4fh}+2hx}\,dx}{h}$$
$$+\frac{\left(\left(g+\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\right)\int\frac{\log(c+dx)}{g+\sqrt{g^2-4fh}+2hx}\,dx}{h}$$

$$
\begin{aligned}
&= \frac{n(a+bx)\log(a+bx)}{bh} - \frac{n(c+dx)\log(c+dx)}{dh} \\
&- \frac{x\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{h} \\
&+ \frac{(g^2-2fh)\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{h^2\sqrt{g^2-4fh}} \\
&- \frac{\left(g - \frac{g^2-2fh}{\sqrt{g^2-4fh}}\right) n\log(a+bx)\log\left(-\frac{b\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h^2} \\
&+ \frac{\left(g - \frac{g^2-2fh}{\sqrt{g^2-4fh}}\right) n\log(c+dx)\log\left(-\frac{d\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h^2} \\
&- \frac{\left(g + \frac{g^2-2fh}{\sqrt{g^2-4fh}}\right) n\log(a+bx)\log\left(-\frac{b\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h^2} \\
&+ \frac{\left(g + \frac{g^2-2fh}{\sqrt{g^2-4fh}}\right) n\log(c+dx)\log\left(-\frac{d\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h^2} \\
&+ \frac{g\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)\log\left(f+gx+hx^2\right)}{2h^2} \\
&+ \frac{\left(b\left(g - \frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\right)\int \frac{\log\left(\frac{b\left(g-\sqrt{g^2-4fh}+2hx\right)}{-2ah+b\left(g-\sqrt{g^2-4fh}\right)}\right)}{a+bx}\,dx}{2h^2} \\
&- \frac{\left(d\left(g - \frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\right)\int \frac{\log\left(\frac{d\left(g-\sqrt{g^2-4fh}+2hx\right)}{-2ch+d\left(g-\sqrt{g^2-4fh}\right)}\right)}{c+dx}\,dx}{2h^2} \\
&+ \frac{\left(b\left(g + \frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\right)\int \frac{\log\left(\frac{b\left(g+\sqrt{g^2-4fh}+2hx\right)}{-2ah+b\left(g+\sqrt{g^2-4fh}\right)}\right)}{a+bx}\,dx}{2h^2} \\
&- \frac{\left(d\left(g + \frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\right)\int \frac{\log\left(\frac{d\left(g+\sqrt{g^2-4fh}+2hx\right)}{-2ch+d\left(g+\sqrt{g^2-4fh}\right)}\right)}{c+dx}\,dx}{2h^2}
\end{aligned}
$$

$$= \frac{n(a+bx)\log(a+bx)}{bh} - \frac{n(c+dx)\log(c+dx)}{dh} - \frac{x\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{h} + \frac{(g^2-2fh)\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{h^2\sqrt{g^2-4fh}} - \frac{\left(g - \frac{g^2-2fh}{\sqrt{g^2-4fh}}\right) n\log(a+bx)\log\left(-\frac{b\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h^2} + \frac{\left(g - \frac{g^2-2fh}{\sqrt{g^2-4fh}}\right) n\log(c+dx)\log\left(-\frac{d\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h^2} - \frac{\left(g + \frac{g^2-2fh}{\sqrt{g^2-4fh}}\right) n\log(a+bx)\log\left(-\frac{b\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h^2} + \frac{\left(g + \frac{g^2-2fh}{\sqrt{g^2-4fh}}\right) n\log(c+dx)\log\left(-\frac{d\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h^2} + \frac{g\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)\log\left(f+gx+hx^2\right)}{2h^2} + \frac{\left(\left(g - \frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\right)\text{Subst}\left(\int \frac{\log\left(1+\frac{2hx}{-2ah+b\left(g-\sqrt{g^2-4fh}\right)}\right)}{x}dx, x, a+bx\right)}{2h^2} - \frac{\left(\left(g - \frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\right)\text{Subst}\left(\int \frac{\log\left(1+\frac{2hx}{-2ch+d\left(g-\sqrt{g^2-4fh}\right)}\right)}{x}dx, x, c+dx\right)}{2h^2} + \frac{\left(\left(g + \frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\right)\text{Subst}\left(\int \frac{\log\left(1+\frac{2hx}{-2ah+b\left(g+\sqrt{g^2-4fh}\right)}\right)}{x}dx, x, a+bx\right)}{2h^2} - \frac{\left(\left(g + \frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\right)\text{Subst}\left(\int \frac{\log\left(1+\frac{2hx}{-2ch+d\left(g+\sqrt{g^2-4fh}\right)}\right)}{x}dx, x, c+dx\right)}{2h^2}$$

$$
\begin{aligned}
&= \frac{n(a+bx)\log(a+bx)}{bh} - \frac{n(c+dx)\log(c+dx)}{dh} \\
&\quad - \frac{x\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{h} \\
&\quad + \frac{(g^2-2fh)\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{h^2\sqrt{g^2-4fh}} \\
&\quad - \frac{\left(g-\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\log(a+bx)\log\left(-\frac{b\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h^2} \\
&\quad + \frac{\left(g-\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\log(c+dx)\log\left(-\frac{d\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h^2} \\
&\quad - \frac{\left(g+\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\log(a+bx)\log\left(-\frac{b\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h^2} \\
&\quad + \frac{\left(g+\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\log(c+dx)\log\left(-\frac{d\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h^2} \\
&\quad + \frac{g\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)\log\left(f+gx+hx^2\right)}{2h^2} \\
&\quad - \frac{\left(g-\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\,\mathrm{Li}_2\left(\frac{2h(a+bx)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h^2} \\
&\quad - \frac{\left(g+\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\,\mathrm{Li}_2\left(\frac{2h(a+bx)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h^2} \\
&\quad + \frac{\left(g-\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\,\mathrm{Li}_2\left(\frac{2h(c+dx)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h^2} \\
&\quad + \frac{\left(g+\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\,\mathrm{Li}_2\left(\frac{2h(c+dx)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 1105, normalized size of antiderivative = 1.33

$$\int \frac{x^2 \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx$$

$$= \frac{2dh\sqrt{g^2-4fh}(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-2(bc-ad)h\sqrt{g^2-4fh}n\log(c+dx)-2bdfh\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(g\right.}{}$$

```
[In] Integrate[(x^2*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x + h*x^2),x]
```

```
[Out] (2*d*h*Sqrt[g^2 - 4*f*h]*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] - 2*(b*c
- a*d)*h*Sqrt[g^2 - 4*f*h]*n*Log[c + d*x] - 2*b*d*f*h*Log[e*((a + b*x)/(c +
 d*x))^n]*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] + b*d*g*(g - Sqrt[g^2 - 4*f*h]
)*Log[e*((a + b*x)/(c + d*x))^n]*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] + 2*b*d
*f*h*Log[e*((a + b*x)/(c + d*x))^n]*Log[g + Sqrt[g^2 - 4*f*h] + 2*h*x] - b*
d*g*(g + Sqrt[g^2 - 4*f*h])*Log[e*((a + b*x)/(c + d*x))^n]*Log[g + Sqrt[g^2
 - 4*f*h] + 2*h*x] + 2*b*d*f*h*n*((Log[(2*h*(a + b*x))/(-(b*g) + 2*a*h + b*
Sqrt[g^2 - 4*f*h])] - Log[(2*h*(c + d*x))/(-(d*g) + 2*c*h + d*Sqrt[g^2 - 4*
f*h])])*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] + PolyLog[2, (b*(-g + Sqrt[g^2 -
 4*f*h] - 2*h*x))/(-(b*g) + 2*a*h + b*Sqrt[g^2 - 4*f*h])] - PolyLog[2, (d*(
-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(2*c*h + d*(-g + Sqrt[g^2 - 4*f*h]))]) - b
*d*g*(g - Sqrt[g^2 - 4*f*h])*n*((Log[(2*h*(a + b*x))/(-(b*g) + 2*a*h + b*Sq
rt[g^2 - 4*f*h])] - Log[(2*h*(c + d*x))/(-(d*g) + 2*c*h + d*Sqrt[g^2 - 4*f*
h])])*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] + PolyLog[2, (b*(-g + Sqrt[g^2 - 4
*f*h] - 2*h*x))/(-(b*g) + 2*a*h + b*Sqrt[g^2 - 4*f*h])] - PolyLog[2, (d*(-g
 + Sqrt[g^2 - 4*f*h] - 2*h*x))/(2*c*h + d*(-g + Sqrt[g^2 - 4*f*h]))]) - 2*b
*d*f*h*n*((Log[(2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))] - Log[(
2*h*(c + d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))])*Log[g + Sqrt[g^2 - 4*f
*h] + 2*h*x] + PolyLog[2, (b*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*a*h + b*(
g + Sqrt[g^2 - 4*f*h]))] - PolyLog[2, (d*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(
-2*c*h + d*(g + Sqrt[g^2 - 4*f*h]))]) + b*d*g*(g + Sqrt[g^2 - 4*f*h])*n*((L
og[(2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))] - Log[(2*h*(c + d*x
))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))])*Log[g + Sqrt[g^2 - 4*f*h] + 2*h*x]
 + PolyLog[2, (b*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*a*h + b*(g + Sqrt[g^2
 - 4*f*h]))] - PolyLog[2, (d*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*c*h + d*(
g + Sqrt[g^2 - 4*f*h]))]))/(2*b*d*h^2*Sqrt[g^2 - 4*f*h])
```

Maple [F]

$$\int \frac{x^2 \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{h\,x^2+gx+f}dx$$

```
[In] int(x^2*ln(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x)
```

```
[Out] int(x^2*ln(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x)
```

Fricas [F]

$$\int \frac{x^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx = \int \frac{x^2 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{hx^2+gx+f}\,dx$$

```
[In] integrate(x^2*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="fricas
")
```

```
[Out] integral(x^2*log(e*((b*x + a)/(d*x + c))^n)/(h*x^2 + g*x + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**2*ln(e*((b*x+a)/(d*x+c))**n)/(h*x**2+g*x+f),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*f*h-g^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int \frac{x^2 \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2} \, dx = \int \frac{x^2 \log \left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{hx^2+gx+f} \, dx$$

```
[In] integrate(x^2*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="giac")
[Out] integrate(x^2*log(e*((b*x + a)/(d*x + c))^n)/(h*x^2 + g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2} \, dx = \int \frac{x^2 \ln \left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{h\,x^2+g\,x+f} \, dx$$

```
[In] int((x^2*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x + h*x^2),x)
[Out] int((x^2*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x + h*x^2), x)
```

3.84 $\int \frac{x \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2} \, dx$

Optimal result 682
Rubi [A] (verified) 683
Mathematica [A] (verified) 690
Maple [F] 691
Fricas [F] 691
Sympy [F(-1)] 691
Maxima [F(-2)] 692
Giac [F] 692
Mupad [F(-1)] 692

Optimal result

Integrand size = 32, antiderivative size = 685

$$\int \frac{x\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx$$
$$= -\frac{g\operatorname{arctanh}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{h\sqrt{g^2-4fh}}$$
$$+\frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\log(a+bx)\log\left(-\frac{b\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h}$$
$$-\frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\log(c+dx)\log\left(-\frac{d\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h}$$
$$+\frac{\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)n\log(a+bx)\log\left(-\frac{b\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h}$$
$$-\frac{\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)n\log(c+dx)\log\left(-\frac{d\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h}$$
$$-\frac{\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\log\left(f+gx+hx^2\right)}{2h}$$
$$+\frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\operatorname{PolyLog}\left(2,\frac{2h(a+bx)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h}$$
$$+\frac{\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)n\operatorname{PolyLog}\left(2,\frac{2h(a+bx)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h}$$
$$-\frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\operatorname{PolyLog}\left(2,\frac{2h(c+dx)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h}$$
$$-\frac{\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)n\operatorname{PolyLog}\left(2,\frac{2h(c+dx)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h}$$

```
[Out] -1/2*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))*ln(h*x^2+g*x+f)/h+
1/2*n*ln(b*x+a)*ln(-b*(g+2*h*x-(-4*f*h+g^2)^(1/2))/(2*a*h-b*(g-(-4*f*h+g^2)
^(1/2))))*(1-g/(-4*f*h+g^2)^(1/2))/h-1/2*n*ln(d*x+c)*ln(-d*(g+2*h*x-(-4*f*h
+g^2)^(1/2))/(2*c*h-d*(g-(-4*f*h+g^2)^(1/2))))*(1-g/(-4*f*h+g^2)^(1/2))/h+1
/2*n*polylog(2,2*h*(b*x+a)/(2*a*h-b*(g-(-4*f*h+g^2)^(1/2))))*(1-g/(-4*f*h+g
^2)^(1/2))/h-1/2*n*polylog(2,2*h*(d*x+c)/(2*c*h-d*(g-(-4*f*h+g^2)^(1/2))))*
(1-g/(-4*f*h+g^2)^(1/2))/h+1/2*n*ln(b*x+a)*ln(-b*(g+2*h*x+(-4*f*h+g^2)^(1/2
))/(2*a*h-b*(g+(-4*f*h+g^2)^(1/2))))*(1+g/(-4*f*h+g^2)^(1/2))/h-1/2*n*ln(d*
```

```
x+c)*ln(-d*(g+2*h*x+(-4*f*h+g^2)^(1/2))/(2*c*h-d*(g+(-4*f*h+g^2)^(1/2))))*(
1+g/(-4*f*h+g^2)^(1/2))/h+1/2*n*polylog(2,2*h*(b*x+a)/(2*a*h-b*(g+(-4*f*h+g
^2)^(1/2))))*(1+g/(-4*f*h+g^2)^(1/2))/h-1/2*n*polylog(2,2*h*(d*x+c)/(2*c*h-
d*(g+(-4*f*h+g^2)^(1/2))))*(1+g/(-4*f*h+g^2)^(1/2))/h-g*arctanh((2*h*x+g)/(
-4*f*h+g^2)^(1/2))*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))/h/(-
4*f*h+g^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 685, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used

= {2593, 2465, 2441, 2440, 2438, 648, 632, 212, 642}

$$\int \frac{x\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx$$
$$= -\frac{g\,\mathrm{arctanh}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)}{h\sqrt{g^2-4fh}}$$
$$-\frac{\log\left(f+gx+hx^2\right)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)}{2h}$$
$$+\frac{n\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)\mathrm{PolyLog}\left(2,\frac{2h(a+bx)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h}$$
$$+\frac{n\left(\frac{g}{\sqrt{g^2-4fh}}+1\right)\mathrm{PolyLog}\left(2,\frac{2h(a+bx)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h}$$
$$+\frac{n\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)\log(a+bx)\log\left(-\frac{b\left(-\sqrt{g^2-4fh}+g+2hx\right)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h}$$
$$+\frac{n\left(\frac{g}{\sqrt{g^2-4fh}}+1\right)\log(a+bx)\log\left(-\frac{b\left(\sqrt{g^2-4fh}+g+2hx\right)}{2ah-b\left(\sqrt{g^2-4fh}+g\right)}\right)}{2h}$$
$$-\frac{n\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)\mathrm{PolyLog}\left(2,\frac{2h(c+dx)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h}$$
$$-\frac{n\left(\frac{g}{\sqrt{g^2-4fh}}+1\right)\mathrm{PolyLog}\left(2,\frac{2h(c+dx)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h}$$
$$-\frac{n\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)\log(c+dx)\log\left(-\frac{d\left(-\sqrt{g^2-4fh}+g+2hx\right)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h}$$
$$-\frac{n\left(\frac{g}{\sqrt{g^2-4fh}}+1\right)\log(c+dx)\log\left(-\frac{d\left(\sqrt{g^2-4fh}+g+2hx\right)}{2ch-d\left(\sqrt{g^2-4fh}+g\right)}\right)}{2h}$$

```
[In] Int[(x*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x + h*x^2),x]
```

```
[Out] -((g*ArcTanh[(g + 2*h*x)/Sqrt[g^2 - 4*f*h]]*(n*Log[a + b*x] - Log[e*((a + b
*x)/(c + d*x))^n] - n*Log[c + d*x]))/(h*Sqrt[g^2 - 4*f*h])) + ((1 - g/Sqrt[
g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*(g - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a
*h - b*(g - Sqrt[g^2 - 4*f*h])))])/(2*h) - ((1 - g/Sqrt[g^2 - 4*f*h])*n*Log
[c + d*x]*Log[-((d*(g - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*c*h - d*(g - Sqrt[g^
2 - 4*f*h])))])/(2*h) + ((1 + g/Sqrt[g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*
(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h])))])/(2*
h) - ((1 + g/Sqrt[g^2 - 4*f*h])*n*Log[c + d*x]*Log[-((d*(g + Sqrt[g^2 - 4*f
```

```
*h] + 2*h*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))))])/(2*h) - ((n*Log[a + b*
x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])*Log[f + g*x + h*x^2])
/(2*h) + ((1 - g/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(a + b*x))/(2*a*h - b
*(g - Sqrt[g^2 - 4*f*h]))])/(2*h) + ((1 + g/Sqrt[g^2 - 4*f*h])*n*PolyLog[2,
 (2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))])/(2*h) - ((1 - g/Sqrt
[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g - Sqrt[g^2 - 4*f*
h]))])/(2*h) - ((1 + g/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(c + d*x))/(2*c
*h - d*(g + Sqrt[g^2 - 4*f*h]))])/(2*h)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2593

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= n\int \frac{x\log(a+bx)}{f+gx+hx^2}\,dx - n\int \frac{x\log(c+dx)}{f+gx+hx^2}\,dx \\
&\quad - \left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)\int \frac{x}{f+gx+hx^2}\,dx \\
&= n\int \left(\frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)\log(a+bx)}{g-\sqrt{g^2-4fh}+2hx} + \frac{\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)\log(a+bx)}{g+\sqrt{g^2-4fh}+2hx}\right)dx \\
&\quad - n\int \left(\frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)\log(c+dx)}{g-\sqrt{g^2-4fh}+2hx} + \frac{\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)\log(c+dx)}{g+\sqrt{g^2-4fh}+2hx}\right)dx \\
&\quad - \frac{\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)\int \frac{g+2hx}{f+gx+hx^2}\,dx}{2h} \\
&\quad - \frac{\left(g\left(-n\log(a+bx) + \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + n\log(c+dx)\right)\right)\int \frac{1}{f+gx+hx^2}\,dx}{2h}
\end{aligned}
$$

$$= -\frac{\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\log\left(f+gx+hx^2\right)}{2h}$$
$$+\left(\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\right)\int\frac{\log(a+bx)}{g-\sqrt{g^2-4fh}+2hx}\,dx$$
$$-\left(\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\right)\int\frac{\log(c+dx)}{g-\sqrt{g^2-4fh}+2hx}\,dx$$
$$+\left(\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)n\right)\int\frac{\log(a+bx)}{g+\sqrt{g^2-4fh}+2hx}\,dx$$
$$-\left(\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)n\right)\int\frac{\log(c+dx)}{g+\sqrt{g^2-4fh}+2hx}\,dx$$
$$+\frac{\left(g\left(-n\log(a+bx)+\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(c+dx)\right)\right)\operatorname{Subst}\left(\int\frac{1}{g^2-4fh-x^2}\,dx,x,g+2hx\right)}{h}$$

$$= -\frac{g\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{h\sqrt{g^2-4fh}}$$
$$+\frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\log(a+bx)\log\left(-\frac{b\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h}$$
$$-\frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\log(c+dx)\log\left(-\frac{d\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h}$$
$$+\frac{\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)n\log(a+bx)\log\left(-\frac{b\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h}$$
$$-\frac{\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)n\log(c+dx)\log\left(-\frac{d\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h}$$
$$-\frac{\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\log\left(f+gx+hx^2\right)}{2h}$$
$$-\frac{\left(b\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\right)\int\frac{\log\left(\frac{b\left(g-\sqrt{g^2-4fh}+2hx\right)}{-2ah+b\left(g-\sqrt{g^2-4fh}\right)}\right)}{a+bx}\,dx}{2h}$$
$$+\frac{\left(d\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\right)\int\frac{\log\left(\frac{d\left(g-\sqrt{g^2-4fh}+2hx\right)}{-2ch+d\left(g-\sqrt{g^2-4fh}\right)}\right)}{c+dx}\,dx}{2h}$$
$$-\frac{\left(b\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)n\right)\int\frac{\log\left(\frac{b\left(g+\sqrt{g^2-4fh}+2hx\right)}{-2ah+b\left(g+\sqrt{g^2-4fh}\right)}\right)}{a+bx}\,dx}{2h}$$
$$+\frac{\left(d\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)n\right)\int\frac{\log\left(\frac{d\left(g+\sqrt{g^2-4fh}+2hx\right)}{-2ch+d\left(g+\sqrt{g^2-4fh}\right)}\right)}{c+dx}\,dx}{2h}$$

$$= -\frac{g\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{h\sqrt{g^2-4fh}}$$
$$+\frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\log(a+bx)\log\left(-\frac{b\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h}$$
$$-\frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\log(c+dx)\log\left(-\frac{d\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h}$$
$$+\frac{\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)n\log(a+bx)\log\left(-\frac{b\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h}$$
$$-\frac{\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)n\log(c+dx)\log\left(-\frac{d\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h}$$
$$-\frac{\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\log\left(f+gx+hx^2\right)}{2h}$$
$$-\frac{\left(\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{2hx}{-2ah+b\left(g-\sqrt{g^2-4fh}\right)}\right)}{x}dx,x,a+bx\right)}{2h}$$
$$+\frac{\left(\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{2hx}{-2ch+d\left(g-\sqrt{g^2-4fh}\right)}\right)}{x}dx,x,c+dx\right)}{2h}$$
$$-\frac{\left(\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)n\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{2hx}{-2ah+b\left(g+\sqrt{g^2-4fh}\right)}\right)}{x}dx,x,a+bx\right)}{2h}$$
$$+\frac{\left(\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)n\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{2hx}{-2ch+d\left(g+\sqrt{g^2-4fh}\right)}\right)}{x}dx,x,c+dx\right)}{2h}$$

$$= -\frac{g\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{h\sqrt{g^2-4fh}}$$
$$+\frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\log(a+bx)\log\left(-\frac{b\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h}$$
$$-\frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\log(c+dx)\log\left(-\frac{d\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h}$$
$$+\frac{\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)n\log(a+bx)\log\left(-\frac{b\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h}$$
$$-\frac{\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)n\log(c+dx)\log\left(-\frac{d\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h}$$
$$-\frac{\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\log\left(f+gx+hx^2\right)}{2h}$$
$$+\frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\text{Li}_2\left(\frac{2h(a+bx)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h}$$
$$+\frac{\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)n\text{Li}_2\left(\frac{2h(a+bx)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h}$$
$$-\frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\text{Li}_2\left(\frac{2h(c+dx)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h}$$
$$-\frac{\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)n\text{Li}_2\left(\frac{2h(c+dx)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2h}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 539, normalized size of antiderivative = 0.79

$$\int \frac{x\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx$$
$$= \frac{\left(-g+\sqrt{g^2-4fh}\right)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(g-\sqrt{g^2-4fh}+2hx\right)+\left(g+\sqrt{g^2-4fh}\right)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(g+\right.}{}$$

```
[In] Integrate[(x*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x + h*x^2),x]
```

```
[Out] ((-g + Sqrt[g^2 - 4*f*h])*Log[e*((a + b*x)/(c + d*x))^n]*Log[g - Sqrt[g^2 -
 4*f*h] + 2*h*x] + (g + Sqrt[g^2 - 4*f*h])*Log[e*((a + b*x)/(c + d*x))^n]*L
```

```
og[g + Sqrt[g^2 - 4*f*h] + 2*h*x] + (g - Sqrt[g^2 - 4*f*h])*n*((Log[(2*h*(a
 + b*x))/(-(b*g) + 2*a*h + b*Sqrt[g^2 - 4*f*h])] - Log[(2*h*(c + d*x))/(-(d
*g) + 2*c*h + d*Sqrt[g^2 - 4*f*h])])*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] + P
olyLog[2, (b*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(-(b*g) + 2*a*h + b*Sqrt[g^2
 - 4*f*h])] - PolyLog[2, (d*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(2*c*h + d*(-
g + Sqrt[g^2 - 4*f*h]))]) - (g + Sqrt[g^2 - 4*f*h])*n*((Log[(2*h*(a + b*x))
/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))] - Log[(2*h*(c + d*x))/(2*c*h - d*(g +
 Sqrt[g^2 - 4*f*h]))])*Log[g + Sqrt[g^2 - 4*f*h] + 2*h*x] + PolyLog[2, (b*(
g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*a*h + b*(g + Sqrt[g^2 - 4*f*h]))] - Pol
yLog[2, (d*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*c*h + d*(g + Sqrt[g^2 - 4*f
*h]))]))/(2*h*Sqrt[g^2 - 4*f*h])
```

Maple [F]

$$\int \frac{x \ln \left(e \left(\frac{bx+a}{dx+c}\right)^{n}\right)}{h\,x^{2}+gx+f}dx$$

```
[In] int(x*ln(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x)
[Out] int(x*ln(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x)
```

Fricas [F]

$$\int \frac{x \log \left(e \left(\frac{a+bx}{c+dx}\right)^{n}\right)}{f+gx+hx^{2}}\,dx = \int \frac{x \log \left(e \left(\frac{bx+a}{dx+c}\right)^{n}\right)}{hx^{2}+gx+f}\,dx$$

```
[In] integrate(x*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="fricas")
[Out] integral(x*log(e*((b*x + a)/(d*x + c))^n)/(h*x^2 + g*x + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x \log \left(e \left(\frac{a+bx}{c+dx}\right)^{n}\right)}{f+gx+hx^{2}}\,dx = \text{Timed out}$$

```
[In] integrate(x*ln(e*((b*x+a)/(d*x+c))**n)/(h*x**2+g*x+f),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2} \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*f*h-g^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int \frac{x \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2} \, dx = \int \frac{x \log \left(e \left(\frac{bx+a}{dx+c}\right)^n\right)}{hx^2+gx+f} \, dx$$

```
[In] integrate(x*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="giac")
```

```
[Out] integrate(x*log(e*((b*x + a)/(d*x + c))^n)/(h*x^2 + g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2} \, dx = \int \frac{x \ln \left(e \left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{h\,x^2+g\,x+f} \, dx$$

```
[In] int((x*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x + h*x^2),x)
```

```
[Out] int((x*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x + h*x^2), x)
```

3.85 $\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx$

Optimal result 693
Rubi [A] (verified) 694
Mathematica [A] (verified) 696
Maple [F] 697
Fricas [F] 697
Sympy [F(-1)] 697
Maxima [F(-2)] 698
Giac [F] 698
Mupad [F(-1)] 698

Optimal result

Integrand size = 31, antiderivative size = 401

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx = -\frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{2(d^2f-cdg+c^2h)(a+bx)}{\left(2bdf-bcg-adg+2ach-(bc-ad)\sqrt{g^2-4fh}\right)(c+dx)}\right)}{\sqrt{g^2-4fh}} + \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{2(d^2f-cdg+c^2h)(a+bx)}{\left(2bdf-bcg-adg+2ach+(bc-ad)\sqrt{g^2-4fh}\right)(c+dx)}\right)}{\sqrt{g^2-4fh}} - \frac{n\,\mathrm{PolyLog}\left(2,\frac{2(d^2f-cdg+c^2h)(a+bx)}{\left(2bdf-bcg-adg+2ach-(bc-ad)\sqrt{g^2-4fh}\right)(c+dx)}\right)}{\sqrt{g^2-4fh}} + \frac{n\,\mathrm{PolyLog}\left(2,\frac{2(d^2f-cdg+c^2h)(a+bx)}{\left(2bdf-bcg-adg+2ach+(bc-ad)\sqrt{g^2-4fh}\right)(c+dx)}\right)}{\sqrt{g^2-4fh}}$$

```
[Out] -ln(e*((b*x+a)/(d*x+c))^n)*ln(1-2*(c^2*h-c*d*g+d^2*f)*(b*x+a)/(d*x+c)/(2*b*
d*f-b*c*g-a*d*g+2*a*c*h-(-a*d+b*c)*(-4*f*h+g^2)^(1/2)))/(-4*f*h+g^2)^(1/2)+
ln(e*((b*x+a)/(d*x+c))^n)*ln(1-2*(c^2*h-c*d*g+d^2*f)*(b*x+a)/(d*x+c)/(2*b*d
*f-b*c*g-a*d*g+2*a*c*h+(-a*d+b*c)*(-4*f*h+g^2)^(1/2)))/(-4*f*h+g^2)^(1/2)-n
*polylog(2,2*(c^2*h-c*d*g+d^2*f)*(b*x+a)/(d*x+c)/(2*b*d*f-b*c*g-a*d*g+2*a*c
*h-(-a*d+b*c)*(-4*f*h+g^2)^(1/2)))/(-4*f*h+g^2)^(1/2)+n*polylog(2,2*(c^2*h-
c*d*g+d^2*f)*(b*x+a)/(d*x+c)/(2*b*d*f-b*c*g-a*d*g+2*a*c*h+(-a*d+b*c)*(-4*f*
h+g^2)^(1/2)))/(-4*f*h+g^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2576, 2404, 2354, 2438}

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx = -\frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{2(a+bx)(c^2h-cdg+d^2f)}{(c+dx)\left(-\sqrt{g^2-4fh}(bc-ad)+2ach-adg-bcg+2bdf\right)}\right)}{\sqrt{g^2-4fh}} + \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{2(a+bx)(c^2h-cdg+d^2f)}{(c+dx)\left(\sqrt{g^2-4fh}(bc-ad)+2ach-adg-bcg+2bdf\right)}\right)}{\sqrt{g^2-4fh}} - \frac{n\,\text{PolyLog}\left(2,\frac{2(hc^2-dgc+d^2f)(a+bx)}{\left(-\sqrt{g^2-4fh}(bc-ad)+2bdf-bcg-adg+2ach\right)(c+dx)}\right)}{\sqrt{g^2-4fh}} + \frac{n\,\text{PolyLog}\left(2,\frac{2(hc^2-dgc+d^2f)(a+bx)}{\left(\sqrt{g^2-4fh}(bc-ad)+2bdf-bcg-adg+2ach\right)(c+dx)}\right)}{\sqrt{g^2-4fh}}$$

```
[In] Int[Log[e*((a + b*x)/(c + d*x))^n]/(f + g*x + h*x^2),x]
```

```
[Out] -((Log[e*((a + b*x)/(c + d*x))^n]*Log[1 - (2*(d^2*f - c*d*g + c^2*h)*(a + b
*x))/((2*b*d*f - b*c*g - a*d*g + 2*a*c*h - (b*c - a*d)*Sqrt[g^2 - 4*f*h])*(
c + d*x))]])/Sqrt[g^2 - 4*f*h]) + (Log[e*((a + b*x)/(c + d*x))^n]*Log[1 - (2
*(d^2*f - c*d*g + c^2*h)*(a + b*x))/((2*b*d*f - b*c*g - a*d*g + 2*a*c*h + (
b*c - a*d)*Sqrt[g^2 - 4*f*h])*(c + d*x))]])/Sqrt[g^2 - 4*f*h] - (n*PolyLog[2
, (2*(d^2*f - c*d*g + c^2*h)*(a + b*x))/((2*b*d*f - b*c*g - a*d*g + 2*a*c*h
 - (b*c - a*d)*Sqrt[g^2 - 4*f*h])*(c + d*x))]])/Sqrt[g^2 - 4*f*h] + (n*PolyL
og[2, (2*(d^2*f - c*d*g + c^2*h)*(a + b*x))/((2*b*d*f - b*c*g - a*d*g + 2*a
*c*h + (b*c - a*d)*Sqrt[g^2 - 4*f*h])*(c + d*x))]])/Sqrt[g^2 - 4*f*h]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2576

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*(P2x_)^(m_.), x_Symbol] :> With[{f = Coeff[P2x, x, 0], g = Coef
f[P2x, x, 1], h = Coeff[P2x, x, 2]}, Dist[b*c - a*d, Subst[Int[(b^2*f - a*b
*g + a^2*h - (2*b*d*f - b*c*g - a*d*g + 2*a*c*h)*x + (d^2*f - c*d*g + c^2*h
)*x^2)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(2*(m + 1))), x], x, (a + b*x)/(c
+ d*x)], x]] /; FreeQ[{a, b, c, d, e, A, B, n}, x] && PolyQ[P2x, x, 2] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = (bc-ad)\text{Subst}\left(\int \frac{\log(ex^n)}{b^2f-abg+a^2h-(2bdf-bcg-adg+2ach)x+(d^2f-cdg+c^2h)x^2}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= (bc-ad)\text{Subst}\left(\int \left(\frac{2(d^2f-cdg+c^2h)\log(ex^n)}{(bc-ad)\sqrt{g^2-4fh}\left(2bdf-bcg-adg+2ach-(bc-ad)\sqrt{g^2-4fh}-2\left(d^2f\right.\right.} + \frac{2(d^2f-cdg+c^2h)\log(ex^n)}{(bc-ad)\sqrt{g^2-4fh}\left(-2bdf+bcg+adg-2ach-(bc-ad)\sqrt{g^2-4fh}+2\left(d^2f-cdg+c^2h\right)\right.}\right)\right)$$

$$= \frac{(2(d^2f-cdg+c^2h))\,\text{Subst}\left(\int \frac{\log(ex^n)}{2bdf-bcg-adg+2ach-(bc-ad)\sqrt{g^2-4fh}-2(d^2f-cdg+c^2h)x}\,dx, x, \frac{a+bx}{c+dx}\right)}{\sqrt{g^2-4fh}} + \frac{(2(d^2f-cdg+c^2h))\,\text{Subst}\left(\int \frac{\log(ex^n)}{-2bdf+bcg+adg-2ach-(bc-ad)\sqrt{g^2-4fh}+2(d^2f-cdg+c^2h)x}\,dx, x, \frac{a+bx}{c+dx}\right)}{\sqrt{g^2-4fh}}$$

$$= -\frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{2(d^2f-cdg+c^2h)(a+bx)}{\left(2bdf-bcg-adg+2ach-(bc-ad)\sqrt{g^2-4fh}\right)(c+dx)}\right)}{\sqrt{g^2-4fh}} + \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{2(d^2f-cdg+c^2h)(a+bx)}{\left(2bdf-bcg-adg+2ach+(bc-ad)\sqrt{g^2-4fh}\right)(c+dx)}\right)}{\sqrt{g^2-4fh}} - \frac{n\text{Subst}\left(\int\frac{\log\left(1+\frac{2\left(d^2f-cdg+c^2h\right)x}{-2bdf+bcg+adg-2ach-(bc-ad)\sqrt{g^2-4fh}}\right)}{x}dx,x,\frac{a+bx}{c+dx}\right)}{\sqrt{g^2-4fh}} + \frac{n\text{Subst}\left(\int\frac{\log\left(1-\frac{2\left(d^2f-cdg+c^2h\right)x}{2bdf-bcg-adg+2ach-(bc-ad)\sqrt{g^2-4fh}}\right)}{x}dx,x,\frac{a+bx}{c+dx}\right)}{\sqrt{g^2-4fh}}$$

$$= -\frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{2(d^2f-cdg+c^2h)(a+bx)}{\left(2bdf-bcg-adg+2ach-(bc-ad)\sqrt{g^2-4fh}\right)(c+dx)}\right)}{\sqrt{g^2-4fh}} + \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{2(d^2f-cdg+c^2h)(a+bx)}{\left(2bdf-bcg-adg+2ach+(bc-ad)\sqrt{g^2-4fh}\right)(c+dx)}\right)}{\sqrt{g^2-4fh}} - \frac{n\text{Li}_2\left(\frac{2(d^2f-cdg+c^2h)(a+bx)}{\left(2bdf-bcg-adg+2ach-(bc-ad)\sqrt{g^2-4fh}\right)(c+dx)}\right)}{\sqrt{g^2-4fh}} + \frac{n\text{Li}_2\left(\frac{2(d^2f-cdg+c^2h)(a+bx)}{\left(2bdf-bcg-adg+2ach+(bc-ad)\sqrt{g^2-4fh}\right)(c+dx)}\right)}{\sqrt{g^2-4fh}}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.28

$$\int\frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx = \frac{-n\log\left(\frac{2h(a+bx)}{-bg+2ah+b\sqrt{g^2-4fh}}\right)\log\left(g-\sqrt{g^2-4fh}+2hx\right)+\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(g-\sqrt{g^2-4fh}+2hx\right)+n\log}{}$$

```
[In] Integrate[Log[e*((a + b*x)/(c + d*x))^n]/(f + g*x + h*x^2),x]
```

```
[Out] (-(n*Log[(2*h*(a + b*x))/(-(b*g) + 2*a*h + b*Sqrt[g^2 - 4*f*h])]*Log[g - Sq
rt[g^2 - 4*f*h] + 2*h*x]) + Log[e*((a + b*x)/(c + d*x))^n]*Log[g - Sqrt[g^2
```

```
 - 4*f*h] + 2*h*x] + n*Log[(2*h*(c + d*x))/(-(d*g) + 2*c*h + d*Sqrt[g^2 - 4
*f*h])]*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] + n*Log[(2*h*(a + b*x))/(2*a*h -
 b*(g + Sqrt[g^2 - 4*f*h]))]*Log[g + Sqrt[g^2 - 4*f*h] + 2*h*x] - Log[e*((a
 + b*x)/(c + d*x))^n]*Log[g + Sqrt[g^2 - 4*f*h] + 2*h*x] - n*Log[(2*h*(c +
d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))]*Log[g + Sqrt[g^2 - 4*f*h] + 2*h*
x] + n*PolyLog[2, (d*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(-(d*g) + 2*c*h + d*
Sqrt[g^2 - 4*f*h])] - n*PolyLog[2, (b*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(2*
a*h + b*(-g + Sqrt[g^2 - 4*f*h]))] + n*PolyLog[2, (b*(g + Sqrt[g^2 - 4*f*h]
 + 2*h*x))/(-2*a*h + b*(g + Sqrt[g^2 - 4*f*h]))] - n*PolyLog[2, (d*(g + Sqr
t[g^2 - 4*f*h] + 2*h*x))/(-2*c*h + d*(g + Sqrt[g^2 - 4*f*h]))])/Sqrt[g^2 -
4*f*h]
```

Maple [F]

$$\int \frac{\ln \left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{h\,x^{2}+gx+f}dx$$

```
[In] int(ln(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x)
[Out] int(ln(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x)
```

Fricas [F]

$$\int \frac{\log \left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}{f+gx+hx^{2}}\,dx=\int \frac{\log \left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{hx^{2}+gx+f}\,dx$$

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="fricas")
[Out] integral(log(e*((b*x + a)/(d*x + c))^n)/(h*x^2 + g*x + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}{f+gx+hx^{2}}\,dx=\text{Timed out}$$

```
[In] integrate(ln(e*((b*x+a)/(d*x+c))**n)/(h*x**2+g*x+f),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*f*h-g^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx = \int \frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{hx^2+gx+f}\,dx$$

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="giac")
```

```
[Out] integrate(log(e*((b*x + a)/(d*x + c))^n)/(h*x^2 + g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx = \int \frac{\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{h\,x^2+g\,x+f}\,dx$$

```
[In] int(log(e*((a + b*x)/(c + d*x))^n)/(f + g*x + h*x^2),x)
```

```
[Out] int(log(e*((a + b*x)/(c + d*x))^n)/(f + g*x + h*x^2), x)
```

3.86 $\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x(f+gx+hx^2)}\,dx$

Optimal result 700
Rubi [A] (verified) 701
Mathematica [A] (verified) 710
Maple [F] 710
Fricas [F] 711
Sympy [F(-1)] 711
Maxima [F(-2)] 711
Giac [F] 711
Mupad [F(-1)] 712

Optimal result

Integrand size = 34, antiderivative size = 800

$$\int \frac{\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x(f+gx+hx^2)}\,dx = \frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f}-\frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f} - \frac{g\,\mathrm{arctanh}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{f\sqrt{g^2-4fh}} - \frac{\log(x)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{f} - \frac{\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)n\log(a+bx)\log\left(-\frac{b\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2f} + \frac{\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)n\log(c+dx)\log\left(-\frac{d\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2f} - \frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\log(a+bx)\log\left(-\frac{b\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2f} + \frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\log(c+dx)\log\left(-\frac{d\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2f} + \frac{\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\log\left(f+gx+hx^2\right)}{2f} - \frac{\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)n\,\mathrm{PolyLog}\left(2,\frac{2h(a+bx)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2f} - \frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\,\mathrm{PolyLog}\left(2,\frac{2h(a+bx)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2f} + \frac{n\,\mathrm{PolyLog}\left(2,1+\frac{bx}{a}\right)}{f} + \frac{\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)n\,\mathrm{PolyLog}\left(2,\frac{2h(c+dx)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2f} + \frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\,\mathrm{PolyLog}\left(2,\frac{2h(c+dx)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2f} - \frac{n\,\mathrm{PolyLog}\left(2,1+\frac{dx}{c}\right)}{f}$$

```
[Out] n*ln(-b*x/a)*ln(b*x+a)/f-n*ln(-d*x/c)*ln(d*x+c)/f-ln(x)*(n*ln(b*x+a)-ln(e*(
(b*x+a)/(d*x+c))^n)-n*ln(d*x+c))/f+1/2*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^
```

```
n)-n*ln(d*x+c))*ln(h*x^2+g*x+f)/f+n*polylog(2,1+b*x/a)/f-n*polylog(2,1+d*x/
c)/f-1/2*n*ln(b*x+a)*ln(-b*(g+2*h*x+(-4*f*h+g^2)^(1/2))/(2*a*h-b*(g+(-4*f*h
+g^2)^(1/2))))*(1-g/(-4*f*h+g^2)^(1/2))/f+1/2*n*ln(d*x+c)*ln(-d*(g+2*h*x+(-
4*f*h+g^2)^(1/2))/(2*c*h-d*(g+(-4*f*h+g^2)^(1/2))))*(1-g/(-4*f*h+g^2)^(1/2)
)/f-1/2*n*polylog(2,2*h*(b*x+a)/(2*a*h-b*(g+(-4*f*h+g^2)^(1/2))))*(1-g/(-4*
f*h+g^2)^(1/2))/f+1/2*n*polylog(2,2*h*(d*x+c)/(2*c*h-d*(g+(-4*f*h+g^2)^(1/2
))))*(1-g/(-4*f*h+g^2)^(1/2))/f-1/2*n*ln(b*x+a)*ln(-b*(g+2*h*x-(-4*f*h+g^2)
^(1/2))/(2*a*h-b*(g-(-4*f*h+g^2)^(1/2))))*(1+g/(-4*f*h+g^2)^(1/2))/f+1/2*n*
ln(d*x+c)*ln(-d*(g+2*h*x-(-4*f*h+g^2)^(1/2))/(2*c*h-d*(g-(-4*f*h+g^2)^(1/2)
)))*(1+g/(-4*f*h+g^2)^(1/2))/f-1/2*n*polylog(2,2*h*(b*x+a)/(2*a*h-b*(g-(-4*
f*h+g^2)^(1/2))))*(1+g/(-4*f*h+g^2)^(1/2))/f+1/2*n*polylog(2,2*h*(d*x+c)/(2
*c*h-d*(g-(-4*f*h+g^2)^(1/2))))*(1+g/(-4*f*h+g^2)^(1/2))/f-g*arctanh((2*h*x
+g)/(-4*f*h+g^2)^(1/2))*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))
/f/(-4*f*h+g^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 800, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules

used = {2593, 2465, 2441, 2352, 2440, 2438, 719, 29, 648, 632, 212, 642}

$$\int \frac{\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x\left(f+gx+hx^2\right)}\,dx$$
$$= \frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f} - \frac{\left(\frac{g}{\sqrt{g^2-4fh}}+1\right)n\log\left(-\frac{b\left(g+2hx-\sqrt{g^2-4fh}\right)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)\log(a+bx)}{2f}$$
$$- \frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\log\left(-\frac{b\left(g+2hx+\sqrt{g^2-4fh}\right)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)\log(a+bx)}{2f} - \frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f}$$
$$- \frac{g\operatorname{arctanh}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{f\sqrt{g^2-4fh}}$$
$$- \frac{\log(x)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{f}$$
$$+ \frac{\left(\frac{g}{\sqrt{g^2-4fh}}+1\right)n\log(c+dx)\log\left(-\frac{d\left(g+2hx-\sqrt{g^2-4fh}\right)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2f}$$
$$+ \frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\log(c+dx)\log\left(-\frac{d\left(g+2hx+\sqrt{g^2-4fh}\right)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2f}$$
$$+ \frac{\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\log\left(hx^2+gx+f\right)}{2f}$$
$$- \frac{\left(\frac{g}{\sqrt{g^2-4fh}}+1\right)n\operatorname{PolyLog}\left(2,\frac{2h(a+bx)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2f}$$
$$- \frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\operatorname{PolyLog}\left(2,\frac{2h(a+bx)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2f}$$
$$+ \frac{n\operatorname{PolyLog}\left(2,\frac{bx}{a}+1\right)}{f} + \frac{\left(\frac{g}{\sqrt{g^2-4fh}}+1\right)n\operatorname{PolyLog}\left(2,\frac{2h(c+dx)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2f}$$
$$+ \frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\operatorname{PolyLog}\left(2,\frac{2h(c+dx)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2f} - \frac{n\operatorname{PolyLog}\left(2,\frac{dx}{c}+1\right)}{f}$$

```
[In] Int[Log[e*((a + b*x)/(c + d*x))^n]/(x*(f + g*x + h*x^2)),x]
```

```
[Out] (n*Log[-((b*x)/a)]*Log[a + b*x])/f - (n*Log[-((d*x)/c)]*Log[c + d*x])/f - (
g*ArcTanh[(g + 2*h*x)/Sqrt[g^2 - 4*f*h]]*(n*Log[a + b*x] - Log[e*((a + b*x)
/(c + d*x))^n] - n*Log[c + d*x]))/(f*Sqrt[g^2 - 4*f*h]) - (Log[x]*(n*Log[a
+ b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/f - ((1 + g/Sqrt
```

```
[g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*(g - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*
a*h - b*(g - Sqrt[g^2 - 4*f*h]))))])/(2*f) + ((1 + g/Sqrt[g^2 - 4*f*h])*n*Lo
g[c + d*x]*Log[-((d*(g - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*c*h - d*(g - Sqrt[g
^2 - 4*f*h]))))])/(2*f) - ((1 - g/Sqrt[g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b
*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))))])/(2
*f) + ((1 - g/Sqrt[g^2 - 4*f*h])*n*Log[c + d*x]*Log[-((d*(g + Sqrt[g^2 - 4*
f*h] + 2*h*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))))])/(2*f) + ((n*Log[a + b
*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])*Log[f + g*x + h*x^2]
)/(2*f) - ((1 + g/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(a + b*x))/(2*a*h -
b*(g - Sqrt[g^2 - 4*f*h]))])/(2*f) - ((1 - g/Sqrt[g^2 - 4*f*h])*n*PolyLog[2
, (2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))])/(2*f) + (n*PolyLog[
2, 1 + (b*x)/a])/f + ((1 + g/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(c + d*x)
)/(2*c*h - d*(g - Sqrt[g^2 - 4*f*h]))])/(2*f) + ((1 - g/Sqrt[g^2 - 4*f*h])*
n*PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))])/(2*f) -
(n*PolyLog[2, 1 + (d*x)/c])/f
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2593

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= n\int \frac{\log(a+bx)}{x\,(f+gx+hx^2)}\,dx - n\int \frac{\log(c+dx)}{x\,(f+gx+hx^2)}\,dx \\
&\quad - \left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)\int \frac{1}{x\,(f+gx+hx^2)}\,dx \\
&= n\int \left(\frac{\log(a+bx)}{fx} + \frac{(-g-hx)\log(a+bx)}{f\,(f+gx+hx^2)}\right)dx \\
&\quad - n\int \left(\frac{\log(c+dx)}{fx} + \frac{(-g-hx)\log(c+dx)}{f\,(f+gx+hx^2)}\right)dx \\
&\quad - \frac{\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)\int \frac{1}{x}\,dx}{f} \\
&\quad - \frac{\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)\int \frac{-g-hx}{f+gx+hx^2}\,dx}{f} \\
&= -\frac{\log(x)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{f} + \frac{n\int \frac{\log(a+bx)}{x}\,dx}{f} \\
&\quad + \frac{n\int \frac{(-g-hx)\log(a+bx)}{f+gx+hx^2}\,dx}{f} - \frac{n\int \frac{\log(c+dx)}{x}\,dx}{f} - \frac{n\int \frac{(-g-hx)\log(c+dx)}{f+gx+hx^2}\,dx}{f} \\
&\quad + \frac{\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)\int \frac{g+2hx}{f+gx+hx^2}\,dx}{2f} \\
&\quad + \frac{\left(g\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)\right)\int \frac{1}{f+gx+hx^2}\,dx}{2f}
\end{aligned}
$$

$$= \frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f} - \frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f} - \frac{\log(x)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{f} + \frac{\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\log\left(f+gx+hx^2\right)}{2f} + \frac{n\int\left(\frac{\left(-h-\frac{gh}{\sqrt{g^2-4fh}}\right)\log(a+bx)}{g-\sqrt{g^2-4fh}+2hx}+\frac{\left(-h+\frac{gh}{\sqrt{g^2-4fh}}\right)\log(a+bx)}{g+\sqrt{g^2-4fh}+2hx}\right)dx}{f} - \frac{n\int\left(\frac{\left(-h-\frac{gh}{\sqrt{g^2-4fh}}\right)\log(c+dx)}{g-\sqrt{g^2-4fh}+2hx}+\frac{\left(-h+\frac{gh}{\sqrt{g^2-4fh}}\right)\log(c+dx)}{g+\sqrt{g^2-4fh}+2hx}\right)dx}{f} - \frac{(bn)\int\frac{\log\left(-\frac{bx}{a}\right)}{a+bx}dx}{f} + \frac{(dn)\int\frac{\log\left(-\frac{dx}{c}\right)}{c+dx}dx}{f} - \frac{\left(g\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\right)\text{Subst}\left(\int\frac{1}{g^2-4fh-x^2}dx,x,g+2hx\right)}{f}$$

$$= \frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f} - \frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f} - \frac{g\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{f\sqrt{g^2-4fh}} - \frac{\log(x)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{f} + \frac{\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\log\left(f+gx+hx^2\right)}{2f} + \frac{n\text{Li}_2\left(1+\frac{bx}{a}\right)}{f} - \frac{n\text{Li}_2\left(1+\frac{dx}{c}\right)}{f} - \frac{\left(h\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\right)\int\frac{\log(a+bx)}{g+\sqrt{g^2-4fh}+2hx}dx}{f} + \frac{\left(h\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\right)\int\frac{\log(c+dx)}{g+\sqrt{g^2-4fh}+2hx}dx}{f} - \frac{\left(h\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)n\right)\int\frac{\log(a+bx)}{g-\sqrt{g^2-4fh}+2hx}dx}{f} + \frac{\left(h\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)n\right)\int\frac{\log(c+dx)}{g-\sqrt{g^2-4fh}+2hx}dx}{f}$$

$$= \frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f} - \frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f}$$

$$- \frac{g\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{f\sqrt{g^2-4fh}}$$

$$- \frac{\log(x)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{f}$$

$$- \frac{\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)n\log(a+bx)\log\left(-\frac{b\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2f}$$

$$+ \frac{\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)n\log(c+dx)\log\left(-\frac{d\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2f}$$

$$- \frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\log(a+bx)\log\left(-\frac{b\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2f}$$

$$+ \frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\log(c+dx)\log\left(-\frac{d\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2f}$$

$$+ \frac{\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\log\left(f+gx+hx^2\right)}{2f}$$

$$+ \frac{n\text{Li}_2\left(1+\frac{bx}{a}\right)}{f} - \frac{n\text{Li}_2\left(1+\frac{dx}{c}\right)}{f} + \frac{\left(b\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\right)\int\frac{\log\left(\frac{b\left(g+\sqrt{g^2-4fh}+2hx\right)}{-2ah+b\left(g+\sqrt{g^2-4fh}\right)}\right)}{a+bx}dx}{2f}$$

$$- \frac{\left(d\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\right)\int\frac{\log\left(\frac{d\left(g+\sqrt{g^2-4fh}+2hx\right)}{-2ch+d\left(g+\sqrt{g^2-4fh}\right)}\right)}{c+dx}dx}{2f}$$

$$+ \frac{\left(b\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)n\right)\int\frac{\log\left(\frac{b\left(g-\sqrt{g^2-4fh}+2hx\right)}{-2ah+b\left(g-\sqrt{g^2-4fh}\right)}\right)}{a+bx}dx}{2f}$$

$$- \frac{\left(d\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)n\right)\int\frac{\log\left(\frac{d\left(g-\sqrt{g^2-4fh}+2hx\right)}{-2ch+d\left(g-\sqrt{g^2-4fh}\right)}\right)}{c+dx}dx}{2f}$$

$$
\begin{aligned}
&= \frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f} - \frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f} \\
&- \frac{g\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{f\sqrt{g^2-4fh}} \\
&- \frac{\log(x)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{f} \\
&- \frac{\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)n\log(a+bx)\log\left(-\frac{b\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2f} \\
&+ \frac{\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)n\log(c+dx)\log\left(-\frac{d\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2f} \\
&- \frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\log(a+bx)\log\left(-\frac{b\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2f} \\
&+ \frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\log(c+dx)\log\left(-\frac{d\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2f} \\
&+ \frac{\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\log\left(f+gx+hx^2\right)}{2f} \\
&+ \frac{n\text{Li}_2\left(1+\frac{bx}{a}\right)}{f} - \frac{n\text{Li}_2\left(1+\frac{dx}{c}\right)}{f} \\
&+ \frac{\left(\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{2hx}{-2ah+b\left(g+\sqrt{g^2-4fh}\right)}\right)}{x}dx,x,a+bx\right)}{2f} \\
&- \frac{\left(\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{2hx}{-2ch+d\left(g+\sqrt{g^2-4fh}\right)}\right)}{x}dx,x,c+dx\right)}{2f} \\
&+ \frac{\left(\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)n\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{2hx}{-2ah+b\left(g-\sqrt{g^2-4fh}\right)}\right)}{x}dx,x,a+bx\right)}{2f} \\
&- \frac{\left(\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)n\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{2hx}{-2ch+d\left(g-\sqrt{g^2-4fh}\right)}\right)}{x}dx,x,c+dx\right)}{2f}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f} - \frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f} \\
&- \frac{g\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{f\sqrt{g^2-4fh}} \\
&- \frac{\log(x)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{f} \\
&- \frac{\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)n\log(a+bx)\log\left(-\frac{b\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2f} \\
&+ \frac{\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)n\log(c+dx)\log\left(-\frac{d\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2f} \\
&- \frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\log(a+bx)\log\left(-\frac{b\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2f} \\
&+ \frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\log(c+dx)\log\left(-\frac{d\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2f} \\
&+ \frac{\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\log\left(f+gx+hx^2\right)}{2f} \\
&- \frac{\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)n\operatorname{Li}_2\left(\frac{2h(a+bx)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2f} \\
&- \frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\operatorname{Li}_2\left(\frac{2h(a+bx)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2f} + \frac{n\operatorname{Li}_2\left(1+\frac{bx}{a}\right)}{f} \\
&+ \frac{\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)n\operatorname{Li}_2\left(\frac{2h(c+dx)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2f} \\
&+ \frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\operatorname{Li}_2\left(\frac{2h(c+dx)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2f} - \frac{n\operatorname{Li}_2\left(1+\frac{dx}{c}\right)}{f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 625, normalized size of antiderivative = 0.78

$$\int \frac{\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x\left(f+gx+hx^2\right)}\,dx$$

$$= \frac{2\log(x)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(g-\sqrt{g^2-4fh}+2hx\right)-\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)\log\left(\right.}{}$$

```
[In] Integrate[Log[e*((a + b*x)/(c + d*x))^n]/(x*(f + g*x + h*x^2)),x]
```

```
[Out] (2*Log[x]*Log[e*((a + b*x)/(c + d*x))^n] - (1 + g/Sqrt[g^2 - 4*f*h])*Log[e*
((a + b*x)/(c + d*x))^n]*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] - (1 - g/Sqrt[g
^2 - 4*f*h])*Log[e*((a + b*x)/(c + d*x))^n]*Log[g + Sqrt[g^2 - 4*f*h] + 2*h
*x] - 2*n*(Log[x]*(Log[1 + (b*x)/a] - Log[1 + (d*x)/c]) + PolyLog[2, -((b*x
)/a)] - PolyLog[2, -((d*x)/c)]) + ((g + Sqrt[g^2 - 4*f*h])*n*((Log[(2*h*(a
+ b*x))/(-(b*g) + 2*a*h + b*Sqrt[g^2 - 4*f*h])] - Log[(2*h*(c + d*x))/(-(d*
g) + 2*c*h + d*Sqrt[g^2 - 4*f*h])])*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] + Po
lyLog[2, (b*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(-(b*g) + 2*a*h + b*Sqrt[g^2
- 4*f*h])] - PolyLog[2, (d*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(2*c*h + d*(-g
 + Sqrt[g^2 - 4*f*h]))]))/Sqrt[g^2 - 4*f*h] + ((-g + Sqrt[g^2 - 4*f*h])*n*(
(Log[(2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))] - Log[(2*h*(c + d
*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))])*Log[g + Sqrt[g^2 - 4*f*h] + 2*h*
x] + PolyLog[2, (b*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*a*h + b*(g + Sqrt[g
^2 - 4*f*h]))] - PolyLog[2, (d*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*c*h + d
*(g + Sqrt[g^2 - 4*f*h]))]))/Sqrt[g^2 - 4*f*h])/(2*f)
```

Maple [F]

$$\int \frac{\ln \left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{x\left(h\,x^2+gx+f\right)}dx$$

```
[In] int(ln(e*((b*x+a)/(d*x+c))^n)/x/(h*x^2+g*x+f),x)
```

```
[Out] int(ln(e*((b*x+a)/(d*x+c))^n)/x/(h*x^2+g*x+f),x)
```

Fricas [F]

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x\left(f+gx+hx^2\right)}\,dx=\int \frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(hx^2+gx+f)x}\,dx$$

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x/(h*x^2+g*x+f),x, algorithm="fricas")
[Out] integral(log(e*((b*x + a)/(d*x + c))^n)/(h*x^3 + g*x^2 + f*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x\left(f+gx+hx^2\right)}\,dx=\text{Timed out}$$

```
[In] integrate(ln(e*((b*x+a)/(d*x+c))**n)/x/(h*x**2+g*x+f),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x\left(f+gx+hx^2\right)}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x/(h*x^2+g*x+f),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*f*h-g^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x\left(f+gx+hx^2\right)}\,dx=\int \frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(hx^2+gx+f)x}\,dx$$

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x/(h*x^2+g*x+f),x, algorithm="giac")
[Out] integrate(log(e*((b*x + a)/(d*x + c))^n)/((h*x^2 + g*x + f)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x(f+gx+hx^2)}\,dx=\int \frac{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{x\,\left(h\,x^2+g\,x+f\right)}\,dx$$

```
[In] int(log(e*((a + b*x)/(c + d*x))^n)/(x*(f + g*x + h*x^2)),x)
[Out] int(log(e*((a + b*x)/(c + d*x))^n)/(x*(f + g*x + h*x^2)), x)
```

3.87 $\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2(f+gx+hx^2)}\,dx$

Optimal result 714
Rubi [A] (verified) 715
Mathematica [A] (verified) 725
Maple [F] 725
Fricas [F] 726
Sympy [F(-1)] 726
Maxima [F(-2)] 726
Giac [F] 727
Mupad [F(-1)] 727

Optimal result

Integrand size = 34, antiderivative size = 995

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2(f+gx+hx^2)}\,dx = \frac{bn\log(x)}{af}-\frac{dn\log(x)}{cf}-\frac{bn\log(a+bx)}{af}-\frac{n\log(a+bx)}{fx}-\frac{gn\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f^2}+\frac{dn\log(c+dx)}{cf}+\frac{n\log(c+dx)}{fx}+\frac{gn\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f^2}+\frac{n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)}{fx}+\frac{(g^2-2fh)\,\text{arctanh}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{f^2\sqrt{g^2-4fh}}+\frac{g\log(x)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{f^2}+\frac{\left(g+\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\log(a+bx)\log\left(-\frac{b\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2f^2}-\frac{\left(g+\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\log(c+dx)\log\left(-\frac{d\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2f^2}+\frac{\left(g-\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\log(a+bx)\log\left(-\frac{b\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2f^2}-\frac{\left(g-\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\log(c+dx)\log\left(-\frac{d\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2f^2}-\frac{g\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\log\left(f+gx+hx^2\right)}{2f^2}+\frac{\left(g+\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\,\text{PolyLog}\left(2,\frac{2h(a+bx)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2f^2}+\frac{\left(g-\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\,\text{PolyLog}\left(2,\frac{2h(a+bx)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2f^2}-\frac{gn\,\text{PolyLog}\left(2,1+\frac{bx}{a}\right)}{f^2}-\frac{\left(g+\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\,\text{PolyLog}\left(2,\frac{2h(c+dx)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2f^2}-\frac{\left(g-\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\,\text{PolyLog}\left(2,\frac{2h(c+dx)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2f^2}+\frac{gn\,\text{PolyLog}\left(2,1+\frac{dx}{c}\right)}{f^2}$$

```
[Out] b*n*ln(x)/a/f-d*n*ln(x)/c/f-b*n*ln(b*x+a)/a/f-n*ln(b*x+a)/f/x-g*n*ln(-b*x/a
)*ln(b*x+a)/f^2+d*n*ln(d*x+c)/c/f+n*ln(d*x+c)/f/x+g*n*ln(-d*x/c)*ln(d*x+c)/
f^2+(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))/f/x+g*ln(x)*(n*ln(b
*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))/f^2-1/2*g*(n*ln(b*x+a)-ln(e*((
b*x+a)/(d*x+c))^n)-n*ln(d*x+c))*ln(h*x^2+g*x+f)/f^2-g*n*polylog(2,1+b*x/a)/
f^2+g*n*polylog(2,1+d*x/c)/f^2+1/2*n*ln(b*x+a)*ln(-b*(g+2*h*x+(-4*f*h+g^2)^
(1/2))/(2*a*h-b*(g+(-4*f*h+g^2)^(1/2))))*(g+(2*f*h-g^2)/(-4*f*h+g^2)^(1/2))
/f^2-1/2*n*ln(d*x+c)*ln(-d*(g+2*h*x+(-4*f*h+g^2)^(1/2))/(2*c*h-d*(g+(-4*f*h
+g^2)^(1/2))))*(g+(2*f*h-g^2)/(-4*f*h+g^2)^(1/2))/f^2+1/2*n*polylog(2,2*h*(
b*x+a)/(2*a*h-b*(g+(-4*f*h+g^2)^(1/2))))*(g+(2*f*h-g^2)/(-4*f*h+g^2)^(1/2))
/f^2-1/2*n*polylog(2,2*h*(d*x+c)/(2*c*h-d*(g+(-4*f*h+g^2)^(1/2))))*(g+(2*f*
h-g^2)/(-4*f*h+g^2)^(1/2))/f^2+1/2*n*ln(b*x+a)*ln(-b*(g+2*h*x-(-4*f*h+g^2)^
(1/2))/(2*a*h-b*(g-(-4*f*h+g^2)^(1/2))))*(g+(-2*f*h+g^2)/(-4*f*h+g^2)^(1/2)
)/f^2-1/2*n*ln(d*x+c)*ln(-d*(g+2*h*x-(-4*f*h+g^2)^(1/2))/(2*c*h-d*(g-(-4*f*
h+g^2)^(1/2))))*(g+(-2*f*h+g^2)/(-4*f*h+g^2)^(1/2))/f^2+1/2*n*polylog(2,2*h
*(b*x+a)/(2*a*h-b*(g-(-4*f*h+g^2)^(1/2))))*(g+(-2*f*h+g^2)/(-4*f*h+g^2)^(1/
2))/f^2-1/2*n*polylog(2,2*h*(d*x+c)/(2*c*h-d*(g-(-4*f*h+g^2)^(1/2))))*(g+(-
2*f*h+g^2)/(-4*f*h+g^2)^(1/2))/f^2+(-2*f*h+g^2)*arctanh((2*h*x+g)/(-4*f*h+g
^2)^(1/2))*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))/f^2/(-4*f*h+
g^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 995, normalized size of antiderivative = 1.00, number of steps used = 40, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules

used = {2593, 2465, 2442, 36, 29, 31, 2441, 2352, 2440, 2438, 723, 814, 648, 632, 212, 642}

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2(f+gx+hx^2)}\,dx$$
$$= \frac{bn\log(x)}{af} - \frac{dn\log(x)}{cf} + \frac{g\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)\log(x)}{f^2}$$
$$- \frac{bn\log(a+bx)}{af} - \frac{gn\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f^2} - \frac{n\log(a+bx)}{fx}$$
$$+ \frac{dn\log(c+dx)}{cf} + \frac{gn\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f^2} + \frac{n\log(c+dx)}{fx}$$
$$+ \frac{(g^2-2fh)\,\mathrm{arctanh}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{f^2\sqrt{g^2-4fh}}$$
$$+ \frac{n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)}{fx}$$
$$+ \frac{\left(g+\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\log(a+bx)\log\left(-\frac{b\left(g+2hx-\sqrt{g^2-4fh}\right)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2f^2}$$
$$- \frac{\left(g+\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\log(c+dx)\log\left(-\frac{d\left(g+2hx-\sqrt{g^2-4fh}\right)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2f^2}$$
$$+ \frac{\left(g-\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\log(a+bx)\log\left(-\frac{b\left(g+2hx+\sqrt{g^2-4fh}\right)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2f^2}$$
$$- \frac{\left(g-\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\log(c+dx)\log\left(-\frac{d\left(g+2hx+\sqrt{g^2-4fh}\right)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2f^2}$$
$$- \frac{g\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)\log\left(hx^2+gx+f\right)}{2f^2}$$
$$+ \frac{\left(g+\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\,\mathrm{PolyLog}\left(2, \frac{2h(a+bx)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2f^2}$$
$$+ \frac{\left(g-\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\,\mathrm{PolyLog}\left(2, \frac{2h(a+bx)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2f^2}$$
$$- \frac{gn\,\mathrm{PolyLog}\left(2, \frac{bx}{a}+1\right)}{f^2} - \frac{\left(g+\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\,\mathrm{PolyLog}\left(2, \frac{2h(c+dx)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2f^2}$$
$$- \frac{\left(g-\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\,\mathrm{PolyLog}\left(2, \frac{2h(c+dx)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2f^2} + \frac{gn\,\mathrm{PolyLog}\left(2, \frac{dx}{c}+1\right)}{f^2}$$

```
[In] Int[Log[e*((a + b*x)/(c + d*x))^n]/(x^2*(f + g*x + h*x^2)),x]
```

```
[Out] (b*n*Log[x])/(a*f) - (d*n*Log[x])/(c*f) - (b*n*Log[a + b*x])/(a*f) - (n*Log
[a + b*x])/(f*x) - (g*n*Log[-((b*x)/a)]*Log[a + b*x])/f^2 + (d*n*Log[c + d*
x])/(c*f) + (n*Log[c + d*x])/(f*x) + (g*n*Log[-((d*x)/c)]*Log[c + d*x])/f^2
 + (n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])/(f*x)
 + ((g^2 - 2*f*h)*ArcTanh[(g + 2*h*x)/Sqrt[g^2 - 4*f*h]]*(n*Log[a + b*x] -
Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/(f^2*Sqrt[g^2 - 4*f*h]) +
 (g*Log[x]*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x
]))/f^2 + ((g + (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*(g
 - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a*h - b*(g - Sqrt[g^2 - 4*f*h])))])/(2*f^
2) - ((g + (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*Log[c + d*x]*Log[-((d*(g - Sq
rt[g^2 - 4*f*h] + 2*h*x))/(2*c*h - d*(g - Sqrt[g^2 - 4*f*h])))])/(2*f^2) +
((g - (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*(g + Sqrt[g^
2 - 4*f*h] + 2*h*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h])))])/(2*f^2) - ((g -
 (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*Log[c + d*x]*Log[-((d*(g + Sqrt[g^2 - 4
*f*h] + 2*h*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h])))])/(2*f^2) - (g*(n*Log[
a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])*Log[f + g*x + h
*x^2])/(2*f^2) + ((g + (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(
a + b*x))/(2*a*h - b*(g - Sqrt[g^2 - 4*f*h]))])/(2*f^2) + ((g - (g^2 - 2*f*
h)/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2
 - 4*f*h]))])/(2*f^2) - (g*n*PolyLog[2, 1 + (b*x)/a])/f^2 - ((g + (g^2 - 2*
f*h)/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g - Sqrt[g
^2 - 4*f*h]))])/(2*f^2) - ((g - (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*PolyLog[
2, (2*h*(c + d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))])/(2*f^2) + (g*n*Pol
yLog[2, 1 + (d*x)/c])/f^2
```

<u>Rule 29</u>

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

<u>Rule 31</u>

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

<u>Rule 36</u>

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

<u>Rule 212</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2593

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= n\int \frac{\log(a+bx)}{x^2\left(f+gx+hx^2\right)}\,dx - n\int \frac{\log(c+dx)}{x^2\left(f+gx+hx^2\right)}\,dx\\ &\quad -\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\int \frac{1}{x^2\left(f+gx+hx^2\right)}\,dx\end{aligned}$$

$$= \frac{n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)}{fx} + n\int\left(\frac{\log(a+bx)}{fx^2}-\frac{g\log(a+bx)}{f^2x}+\frac{(g^2-fh+ghx)\log(a+bx)}{f^2(f+gx+hx^2)}\right)dx - n\int\left(\frac{\log(c+dx)}{fx^2}-\frac{g\log(c+dx)}{f^2x}+\frac{(g^2-fh+ghx)\log(c+dx)}{f^2(f+gx+hx^2)}\right)dx - \frac{\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\int\frac{-g-hx}{x(f+gx+hx^2)}dx}{f}$$

$$= \frac{n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)}{fx} + \frac{n\int\frac{(g^2-fh+ghx)\log(a+bx)}{f+gx+hx^2}dx}{f^2} - \frac{n\int\frac{(g^2-fh+ghx)\log(c+dx)}{f+gx+hx^2}dx}{f^2} + \frac{n\int\frac{\log(a+bx)}{x^2}dx}{f} - \frac{n\int\frac{\log(c+dx)}{x^2}dx}{f} - \frac{(gn)\int\frac{\log(a+bx)}{x}dx}{f^2} + \frac{(gn)\int\frac{\log(c+dx)}{x}dx}{f^2} - \frac{\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\int\left(-\frac{g}{fx}+\frac{g^2-fh+ghx}{f(f+gx+hx^2)}\right)dx}{f}$$

$$= -\frac{n\log(a+bx)}{fx} - \frac{gn\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f^2} + \frac{n\log(c+dx)}{fx} + \frac{gn\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f^2} + \frac{n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)}{fx} + \frac{g\log(x)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{f^2} + \frac{n\int\left(\frac{\left(gh+\frac{h(g^2-2fh)}{\sqrt{g^2-4fh}}\right)\log(a+bx)}{g-\sqrt{g^2-4fh}+2hx}+\frac{\left(gh-\frac{h(g^2-2fh)}{\sqrt{g^2-4fh}}\right)\log(a+bx)}{g+\sqrt{g^2-4fh}+2hx}\right)dx}{f^2} - \frac{n\int\left(\frac{\left(gh+\frac{h(g^2-2fh)}{\sqrt{g^2-4fh}}\right)\log(c+dx)}{g-\sqrt{g^2-4fh}+2hx}+\frac{\left(gh-\frac{h(g^2-2fh)}{\sqrt{g^2-4fh}}\right)\log(c+dx)}{g+\sqrt{g^2-4fh}+2hx}\right)dx}{f^2} + \frac{(bn)\int\frac{1}{x(a+bx)}dx}{f} - \frac{(dn)\int\frac{1}{x(c+dx)}dx}{f} + \frac{(bgn)\int\frac{\log\left(-\frac{bx}{a}\right)}{a+bx}dx}{f^2} - \frac{(dgn)\int\frac{\log\left(-\frac{dx}{c}\right)}{c+dx}dx}{f^2} - \frac{\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\int\frac{g^2-fh+ghx}{f+gx+hx^2}dx}{f^2}$$

$$
\begin{aligned}
&= -\frac{n\log(a+bx)}{fx} - \frac{gn\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f^2} + \frac{n\log(c+dx)}{fx} \\
&+ \frac{gn\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f^2} + \frac{n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)}{fx} \\
&+ \frac{g\log(x)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{f^2} - \frac{gn\text{Li}_2\left(1+\frac{bx}{a}\right)}{f^2} \\
&+ \frac{gn\text{Li}_2\left(1+\frac{dx}{c}\right)}{f^2} + \frac{(bn)\int\frac{1}{x}\,dx}{af} - \frac{(b^2n)\int\frac{1}{a+bx}\,dx}{af} - \frac{(dn)\int\frac{1}{x}\,dx}{cf} \\
&+ \frac{(d^2n)\int\frac{1}{c+dx}\,dx}{cf} + \frac{\left(h\left(g-\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\right)\int\frac{\log(a+bx)}{g+\sqrt{g^2-4fh}+2hx}\,dx}{f^2} \\
&- \frac{\left(h\left(g-\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\right)\int\frac{\log(c+dx)}{g+\sqrt{g^2-4fh}+2hx}\,dx}{f^2} \\
&+ \frac{\left(h\left(g+\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\right)\int\frac{\log(a+bx)}{g-\sqrt{g^2-4fh}+2hx}\,dx}{f^2} \\
&- \frac{\left(h\left(g+\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\right)\int\frac{\log(c+dx)}{g-\sqrt{g^2-4fh}+2hx}\,dx}{f^2} \\
&- \frac{\left(g\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)\right)\int\frac{g+2hx}{f+gx+hx^2}\,dx}{2f^2} \\
&- \frac{\left((g^2-2fh)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)\right)\int\frac{1}{f+gx+hx^2}\,dx}{2f^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{bn\log(x)}{af} - \frac{dn\log(x)}{cf} - \frac{bn\log(a+bx)}{af} - \frac{n\log(a+bx)}{fx} \\
&- \frac{gn\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f^2} + \frac{dn\log(c+dx)}{cf} + \frac{n\log(c+dx)}{fx} \\
&+ \frac{gn\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f^2} + \frac{n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)}{fx} \\
&+ \frac{g\log(x)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{f^2} \\
&+ \frac{\left(g+\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\log(a+bx)\log\left(-\frac{b\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2f^2} \\
&- \frac{\left(g+\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\log(c+dx)\log\left(-\frac{d\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2f^2} \\
&+ \frac{\left(g-\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\log(a+bx)\log\left(-\frac{b\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2f^2} \\
&- \frac{\left(g-\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\log(c+dx)\log\left(-\frac{d\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2f^2} \\
&- \frac{g\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)\log\left(f+gx+hx^2\right)}{2f^2} \\
&- \frac{gn\text{Li}_2\left(1+\frac{bx}{a}\right)}{f^2} + \frac{gn\text{Li}_2\left(1+\frac{dx}{c}\right)}{f^2} \\
&- \frac{\left(b\left(g-\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\right)\int\frac{\log\left(\frac{b\left(g+\sqrt{g^2-4fh}+2hx\right)}{-2ah+b\left(g+\sqrt{g^2-4fh}\right)}\right)}{a+bx}\,dx}{2f^2} \\
&+ \frac{\left(d\left(g-\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\right)\int\frac{\log\left(\frac{d\left(g+\sqrt{g^2-4fh}+2hx\right)}{-2ch+d\left(g+\sqrt{g^2-4fh}\right)}\right)}{c+dx}\,dx}{2f^2} \\
&- \frac{\left(b\left(g+\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\right)\int\frac{\log\left(\frac{b\left(g-\sqrt{g^2-4fh}+2hx\right)}{-2ah+b\left(g-\sqrt{g^2-4fh}\right)}\right)}{a+bx}\,dx}{2f^2} \\
&+ \frac{\left(d\left(g+\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\right)\int\frac{\log\left(\frac{d\left(g-\sqrt{g^2-4fh}+2hx\right)}{-2ch+d\left(g-\sqrt{g^2-4fh}\right)}\right)}{c+dx}\,dx}{2f^2} \\
&+ \frac{\left(\left(g^2-2fh\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)\right)\text{Subst}\left(\int\frac{1}{g^2-4fh-x^2}\,dx, x, g+2hx\right)}{f^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{bn\log(x)}{af} - \frac{dn\log(x)}{cf} - \frac{bn\log(a+bx)}{af} - \frac{n\log(a+bx)}{fx} \\
&- \frac{gn\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f^2} + \frac{dn\log(c+dx)}{cf} + \frac{n\log(c+dx)}{fx} \\
&+ \frac{gn\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f^2} + \frac{n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)}{fx} \\
&+ \frac{(g^2-2fh)\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{f^2\sqrt{g^2-4fh}} \\
&+ \frac{g\log(x)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{f^2} \\
&+ \frac{\left(g + \frac{g^2-2fh}{\sqrt{g^2-4fh}}\right) n\log(a+bx)\log\left(-\frac{b\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2f^2} \\
&- \frac{\left(g + \frac{g^2-2fh}{\sqrt{g^2-4fh}}\right) n\log(c+dx)\log\left(-\frac{d\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2f^2} \\
&+ \frac{\left(g - \frac{g^2-2fh}{\sqrt{g^2-4fh}}\right) n\log(a+bx)\log\left(-\frac{b\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2f^2} \\
&- \frac{\left(g - \frac{g^2-2fh}{\sqrt{g^2-4fh}}\right) n\log(c+dx)\log\left(-\frac{d\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2f^2} \\
&- \frac{g\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)\log\left(f+gx+hx^2\right)}{2f^2} \\
&- \frac{gn\text{Li}_2\left(1+\frac{bx}{a}\right)}{f^2} + \frac{gn\text{Li}_2\left(1+\frac{dx}{c}\right)}{f^2} \\
&- \frac{\left(\left(g - \frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\right)\text{Subst}\left(\int \frac{\log\left(1+\frac{2hx}{-2ah+b\left(g+\sqrt{g^2-4fh}\right)}\right)}{x}dx, x, a+bx\right)}{2f^2} \\
&+ \frac{\left(\left(g - \frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\right)\text{Subst}\left(\int \frac{\log\left(1+\frac{2hx}{-2ch+d\left(g+\sqrt{g^2-4fh}\right)}\right)}{x}dx, x, c+dx\right)}{2f^2} \\
&- \frac{\left(\left(g + \frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\right)\text{Subst}\left(\int \frac{\log\left(1+\frac{2hx}{-2ah+b\left(g-\sqrt{g^2-4fh}\right)}\right)}{x}dx, x, a+bx\right)}{2f^2} \\
&\quad \left(\left(g + \frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\right)\text{Subst}\left(\int \frac{\log\left(1+\frac{2hx}{-2ch+d\left(g-\sqrt{g^2-4fh}\right)}\right)}{x}dx, x, c+dx\right)
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{bn\log(x)}{af} - \frac{dn\log(x)}{cf} - \frac{bn\log(a+bx)}{af} - \frac{n\log(a+bx)}{fx} \\
&- \frac{gn\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f^2} + \frac{dn\log(c+dx)}{cf} + \frac{n\log(c+dx)}{fx} \\
&+ \frac{gn\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f^2} + \frac{n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)}{fx} \\
&+ \frac{(g^2-2fh)\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{f^2\sqrt{g^2-4fh}} \\
&+ \frac{g\log(x)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{f^2} \\
&+ \frac{\left(g+\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\log(a+bx)\log\left(-\frac{b\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2f^2} \\
&- \frac{\left(g+\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\log(c+dx)\log\left(-\frac{d\left(g-\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2f^2} \\
&+ \frac{\left(g-\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\log(a+bx)\log\left(-\frac{b\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2f^2} \\
&- \frac{\left(g-\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\log(c+dx)\log\left(-\frac{d\left(g+\sqrt{g^2-4fh}+2hx\right)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2f^2} \\
&- \frac{g\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)\log\left(f+gx+hx^2\right)}{2f^2} \\
&+ \frac{\left(g+\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\operatorname{Li}_2\left(\frac{2h(a+bx)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2f^2} \\
&+ \frac{\left(g-\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\operatorname{Li}_2\left(\frac{2h(a+bx)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2f^2} \\
&- \frac{gn\operatorname{Li}_2\left(1+\frac{bx}{a}\right)}{f^2} - \frac{\left(g+\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\operatorname{Li}_2\left(\frac{2h(c+dx)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{2f^2} \\
&- \frac{\left(g-\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)n\operatorname{Li}_2\left(\frac{2h(c+dx)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)}{2f^2} + \frac{gn\operatorname{Li}_2\left(1+\frac{dx}{c}\right)}{f^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 721, normalized size of antiderivative = 0.72

$$\int \frac{\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2(f+gx+hx^2)}\,dx$$

$$= \frac{-\frac{2f\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x} - 2g\log(x)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + \frac{2fn((bc-ad)\log(x)-bc\log(a+bx)+ad\log(c+dx))}{ac} + \left(g+\frac{g^2-2fh}{\sqrt{g^2-4fh}}\right)\log}{}$$

```
[In] Integrate[Log[e*((a + b*x)/(c + d*x))^n]/(x^2*(f + g*x + h*x^2)),x]
```

```
[Out] ((-2*f*Log[e*((a + b*x)/(c + d*x))^n])/x - 2*g*Log[x]*Log[e*((a + b*x)/(c +
 d*x))^n] + (2*f*n*((b*c - a*d)*Log[x] - b*c*Log[a + b*x] + a*d*Log[c + d*x
]))/(a*c) + (g + (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*Log[e*((a + b*x)/(c + d*x
))^n]*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] + (g + (-g^2 + 2*f*h)/Sqrt[g^2 - 4
*f*h])*Log[e*((a + b*x)/(c + d*x))^n]*Log[g + Sqrt[g^2 - 4*f*h] + 2*h*x] +
2*g*n*(Log[x]*(Log[1 + (b*x)/a] - Log[1 + (d*x)/c]) + PolyLog[2, -((b*x)/a)
] - PolyLog[2, -((d*x)/c)]) - ((g^2 - 2*f*h + g*Sqrt[g^2 - 4*f*h])*n*((Log[
(2*h*(a + b*x))/(-(b*g) + 2*a*h + b*Sqrt[g^2 - 4*f*h])] - Log[(2*h*(c + d*x
))/(-(d*g) + 2*c*h + d*Sqrt[g^2 - 4*f*h])])*Log[g - Sqrt[g^2 - 4*f*h] + 2*h
*x] + PolyLog[2, (b*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(-(b*g) + 2*a*h + b*S
qrt[g^2 - 4*f*h])] - PolyLog[2, (d*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(2*c*h
 + d*(-g + Sqrt[g^2 - 4*f*h]))]))/Sqrt[g^2 - 4*f*h] + ((g^2 - 2*f*h - g*Sqr
t[g^2 - 4*f*h])*n*((Log[(2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))
] - Log[(2*h*(c + d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))])*Log[g + Sqrt[
g^2 - 4*f*h] + 2*h*x] + PolyLog[2, (b*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*
a*h + b*(g + Sqrt[g^2 - 4*f*h]))] - PolyLog[2, (d*(g + Sqrt[g^2 - 4*f*h] +
2*h*x))/(-2*c*h + d*(g + Sqrt[g^2 - 4*f*h]))]))/Sqrt[g^2 - 4*f*h])/(2*f^2)
```

Maple [F]

$$\int \frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{x^2(h\,x^2+gx+f)}dx$$

```
[In] int(ln(e*((b*x+a)/(d*x+c))^n)/x^2/(h*x^2+g*x+f),x)
```

```
[Out] int(ln(e*((b*x+a)/(d*x+c))^n)/x^2/(h*x^2+g*x+f),x)
```

Fricas [F]

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2\left(f+gx+hx^2\right)}\,dx = \int \frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(hx^2+gx+f)x^2}\,dx$$

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x^2/(h*x^2+g*x+f),x, algorithm="fricas
")
```

```
[Out] integral(log(e*((b*x + a)/(d*x + c))^n)/(h*x^4 + g*x^3 + f*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2\left(f+gx+hx^2\right)}\,dx = \text{Timed out}$$

```
[In] integrate(ln(e*((b*x+a)/(d*x+c))**n)/x**2/(h*x**2+g*x+f),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2\left(f+gx+hx^2\right)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x^2/(h*x^2+g*x+f),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*f*h-g^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2(f+gx+hx^2)}\,dx = \int \frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(hx^2+gx+f)x^2}\,dx$$

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x^2/(h*x^2+g*x+f),x, algorithm="giac")
[Out] integrate(log(e*((b*x + a)/(d*x + c))^n)/((h*x^2 + g*x + f)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2(f+gx+hx^2)}\,dx = \int \frac{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{x^2\,\left(h\,x^2+g\,x+f\right)}\,dx$$

```
[In] int(log(e*((a + b*x)/(c + d*x))^n)/(x^2*(f + g*x + h*x^2)),x)
[Out] int(log(e*((a + b*x)/(c + d*x))^n)/(x^2*(f + g*x + h*x^2)), x)
```

3.88 $\int \frac{\log\left(\frac{cx}{a+bx}\right)}{a+bx}\,dx$

Optimal result 728
Rubi [A] (verified) 728
Mathematica [A] (verified) 730
Maple [A] (verified) 730
Fricas [F] 731
Sympy [F] 731
Maxima [B] (verification not implemented) 731
Giac [F] 732
Mupad [F(-1)] 732

Optimal result

Integrand size = 19, antiderivative size = 46

$$\int \frac{\log\left(\frac{cx}{a+bx}\right)}{a+bx}\,dx=-\frac{\log\left(\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{b}-\frac{\operatorname{PolyLog}\left(2,1-\frac{a}{a+bx}\right)}{b}$$

```
[Out] -ln(a/(b*x+a))*ln(c*x/(b*x+a))/b-polylog(2,1-a/(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2544, 2458, 2378, 2370, 2352}

$$\int \frac{\log\left(\frac{cx}{a+bx}\right)}{a+bx}\,dx=-\frac{\log\left(\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{b}-\frac{\operatorname{PolyLog}\left(2,1-\frac{a}{a+bx}\right)}{b}$$

```
[In] Int[Log[(c*x)/(a + b*x)]/(a + b*x),x]
```

```
[Out] -((Log[a/(a + b*x)]*Log[(c*x)/(a + b*x)])/b) - PolyLog[2, 1 - a/(a + b*x)]/
b
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2370

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
```

```
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2378

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2544

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[(b*c - a*d)/(b*(c +
 d*x))])*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/g), x] + Dist[B*n*((b*c
- a*d)/g), Int[Log[(b*c - a*d)/(b*(c + d*x))]/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && EqQ[d*f - c*g, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\log\left(\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{b}+\frac{a\int\frac{\log\left(\frac{a}{a+bx}\right)}{x(a+bx)}\,dx}{b}\\
&= -\frac{\log\left(\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{b}+\frac{a\text{Subst}\left(\int\frac{\log\left(\frac{a}{x}\right)}{x\left(-\frac{a}{b}+\frac{x}{b}\right)}\,dx,x,a+bx\right)}{b^2}\\
&= -\frac{\log\left(\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{b}-\frac{a\text{Subst}\left(\int\frac{\log(ax)}{\left(-\frac{a}{b}+\frac{1}{bx}\right)x}\,dx,x,\frac{1}{a+bx}\right)}{b^2}\\
&= -\frac{\log\left(\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{b}-\frac{a\text{Subst}\left(\int\frac{\log(ax)}{\frac{1}{b}-\frac{ax}{b}}\,dx,x,\frac{1}{a+bx}\right)}{b^2}\\
&= -\frac{\log\left(\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{b}-\frac{\text{Li}_2\left(\frac{bx}{a+bx}\right)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.83

$$\int \frac{\log\left(\frac{cx}{a+bx}\right)}{a+bx}\,dx = \frac{\log\left(-\frac{bx}{a}\right)\log\left(\frac{a}{a+bx}\right)}{b} + \frac{\log^2\left(\frac{a}{a+bx}\right)}{2b} - \frac{\log\left(\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{b} - \frac{\text{PolyLog}\left(2,\frac{a+bx}{a}\right)}{b}$$

```
[In] Integrate[Log[(c*x)/(a + b*x)]/(a + b*x),x]
```

```
[Out] (Log[-((b*x)/a)]*Log[a/(a + b*x)])/b + Log[a/(a + b*x)]^2/(2*b) - (Log[a/(a
 + b*x)]*Log[(c*x)/(a + b*x)])/b - PolyLog[2, (a + b*x)/a]/b
```

Maple [A] (verified)

Time = 1.90 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.50

method	result	size
parts	$\frac{\ln\left(\frac{cx}{bx+a}\right)\ln(bx+a)}{b}-\frac{-\frac{c\ln(bx+a)^2}{2}+c\left(\operatorname{dilog}\left(-\frac{xb}{a}\right)+\ln(bx+a)\ln\left(-\frac{xb}{a}\right)\right)}{bc}$	69
derivativedivides	$-\frac{\operatorname{dilog}\left(-\frac{\left(\frac{c}{b}-\frac{ac}{b(bx+a)}\right)b-c}{c}\right)}{b}-\frac{\ln\left(\frac{c}{b}-\frac{ac}{b(bx+a)}\right)\ln\left(-\frac{\left(\frac{c}{b}-\frac{ac}{b(bx+a)}\right)b-c}{c}\right)}{b}$	97
default	$-\frac{\operatorname{dilog}\left(-\frac{\left(\frac{c}{b}-\frac{ac}{b(bx+a)}\right)b-c}{c}\right)}{b}-\frac{\ln\left(\frac{c}{b}-\frac{ac}{b(bx+a)}\right)\ln\left(-\frac{\left(\frac{c}{b}-\frac{ac}{b(bx+a)}\right)b-c}{c}\right)}{b}$	97
risch	$-\frac{\operatorname{dilog}\left(-\frac{\left(\frac{c}{b}-\frac{ac}{b(bx+a)}\right)b-c}{c}\right)}{b}-\frac{\ln\left(\frac{c}{b}-\frac{ac}{b(bx+a)}\right)\ln\left(-\frac{\left(\frac{c}{b}-\frac{ac}{b(bx+a)}\right)b-c}{c}\right)}{b}$	97

```
[In] int(ln(c*x/(b*x+a))/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] ln(c*x/(b*x+a))*ln(b*x+a)/b-1/b/c*(-1/2*c*ln(b*x+a)^2+c*(dilog(-x/a*b)+ln(b
*x+a)*ln(-x/a*b)))
```

Fricas [F]

$$\int \frac{\log\left(\frac{cx}{a+bx}\right)}{a+bx}\,dx = \int \frac{\log\left(\frac{cx}{bx+a}\right)}{bx+a}\,dx$$

```
[In] integrate(log(c*x/(b*x+a))/(b*x+a),x, algorithm="fricas")
[Out] integral(log(c*x/(b*x + a))/(b*x + a), x)
```

Sympy [F]

$$\int \frac{\log\left(\frac{cx}{a+bx}\right)}{a+bx}\,dx = \int \frac{\log\left(\frac{cx}{a+bx}\right)}{a+bx}\,dx$$

```
[In] integrate(ln(c*x/(b*x+a))/(b*x+a),x)
[Out] Integral(log(c*x/(a + b*x))/(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(45) = 90$.

Time = 0.19 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.07

$$\int \frac{\log\left(\frac{cx}{a+bx}\right)}{a+bx}\,dx = \frac{\log(bx+a)\log\left(\frac{cx}{bx+a}\right)}{b} - \frac{\frac{c\log(bx+a)^2}{b} - \frac{2\left(\log\left(\frac{bx}{a}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{bx}{a}\right)\right)c}{b}}{2\,c} + \frac{(c\log(bx+a) - c\log(x))\log(bx+a)}{bc}$$

```
[In] integrate(log(c*x/(b*x+a))/(b*x+a),x, algorithm="maxima")
[Out] log(b*x + a)*log(c*x/(b*x + a))/b - 1/2*(c*log(b*x + a)^2/b - 2*(log(b*x/a
+ 1)*log(x) + dilog(-b*x/a))*c/b)/c + (c*log(b*x + a) - c*log(x))*log(b*x +
 a)/(b*c)
```

Giac [F]

$$\int \frac{\log\left(\frac{cx}{a+bx}\right)}{a+bx}\,dx = \int \frac{\log\left(\frac{cx}{bx+a}\right)}{bx+a}\,dx$$

```
[In] integrate(log(c*x/(b*x+a))/(b*x+a),x, algorithm="giac")
[Out] integrate(log(c*x/(b*x + a))/(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{cx}{a+bx}\right)}{a+bx}\,dx = \int \frac{\ln\left(\frac{c\,x}{a+b\,x}\right)}{a+b\,x}\,dx$$

```
[In] int(log((c*x)/(a + b*x))/(a + b*x),x)
[Out] int(log((c*x)/(a + b*x))/(a + b*x), x)
```

3.89 $\int \frac{\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)}\,dx$

Optimal result 733
Rubi [A] (verified) 733
Mathematica [A] (verified) 734
Maple [A] (verified) 734
Fricas [A] (verification not implemented) 735
Sympy [A] (verification not implemented) 735
Maxima [B] (verification not implemented) 736
Giac [F] 736
Mupad [B] (verification not implemented) 736

Optimal result

Integrand size = 24, antiderivative size = 20

$$\int \frac{\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)}\,dx = \frac{\log^3\left(\frac{cx}{a+bx}\right)}{3a}$$

```
[Out] 1/3*ln(c*x/(b*x+a))^3/a
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2562, 2339, 30}

$$\int \frac{\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)}\,dx = \frac{\log^3\left(\frac{cx}{a+bx}\right)}{3a}$$

```
[In] Int[Log[(c*x)/(a + b*x)]^2/(x*(a + b*x)),x]
```

```
[Out] Log[(c*x)/(a + b*x)]^3/(3*a)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
```

```
x]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\log^2(cx)}{x}\,dx, x, \frac{x}{a+bx}\right)}{a}\\
&= \frac{\text{Subst}\left(\int x^2\,dx, x, \log\left(\frac{cx}{a+bx}\right)\right)}{a}\\
&= \frac{\log^3\left(\frac{cx}{a+bx}\right)}{3a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)}\,dx = \frac{\log^3\left(\frac{cx}{a+bx}\right)}{3a}$$

```
[In] Integrate[Log[(c*x)/(a + b*x)]^2/(x*(a + b*x)),x]
[Out] Log[(c*x)/(a + b*x)]^3/(3*a)
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$\frac{\ln\left(\frac{cx}{bx+a}\right)^{3}}{3a}$	19
default	$\frac{\ln\left(\frac{cx}{bx+a}\right)^{3}}{3a}$	19
norman	$\frac{\ln\left(\frac{cx}{bx+a}\right)^{3}}{3a}$	19
risch	$\frac{\ln\left(\frac{cx}{bx+a}\right)^{3}}{3a}$	19
parallelrisch	$\frac{\ln\left(\frac{cx}{bx+a}\right)^{3}}{3a}$	19

```
[In] int(ln(c*x/(b*x+a))^2/x/(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/3*ln(c*x/(b*x+a))^3/a
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)}\,dx = \frac{\log\left(\frac{cx}{bx+a}\right)^3}{3\,a}$$

```
[In] integrate(log(c*x/(b*x+a))^2/x/(b*x+a),x, algorithm="fricas")
[Out] 1/3*log(c*x/(b*x + a))^3/a
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)}\,dx = \frac{\log\left(\frac{cx}{a+bx}\right)^3}{3a}$$

```
[In] integrate(ln(c*x/(b*x+a))**2/x/(b*x+a),x)
[Out] log(c*x/(a + b*x))**3/(3*a)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(18) = 36$.

Time = 0.21 (sec) , antiderivative size = 141, normalized size of antiderivative = 7.05

$$\int \frac{\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)}\,dx = -\left(\frac{\log(bx+a)}{a} - \frac{\log(x)}{a}\right)\log\left(\frac{cx}{bx+a}\right)^2 - \frac{\left(c\log(bx+a)^2 - 2\,c\log(bx+a)\log(x) + c\log(x)^2\right)\log\left(\frac{cx}{bx+a}\right)}{ac} - \frac{c^2\log(bx+a)^3 - 3\,c^2\log(bx+a)^2\log(x) + 3\,c^2\log(bx+a)\log(x)^2 - c^2\log(x)^3}{3\,ac^2}$$

```
[In] integrate(log(c*x/(b*x+a))^2/x/(b*x+a),x, algorithm="maxima")
```

```
[Out] -(log(b*x + a)/a - log(x)/a)*log(c*x/(b*x + a))^2 - (c*log(b*x + a)^2 - 2*c
*log(b*x + a)*log(x) + c*log(x)^2)*log(c*x/(b*x + a))/(a*c) - 1/3*(c^2*log(
b*x + a)^3 - 3*c^2*log(b*x + a)^2*log(x) + 3*c^2*log(b*x + a)*log(x)^2 - c^
2*log(x)^3)/(a*c^2)
```

Giac [F]

$$\int \frac{\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)}\,dx = \int \frac{\log\left(\frac{cx}{bx+a}\right)^2}{(bx+a)x}\,dx$$

```
[In] integrate(log(c*x/(b*x+a))^2/x/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(log(c*x/(b*x + a))^2/((b*x + a)*x), x)
```

Mupad [B] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)}\,dx = \frac{\ln\left(\frac{c\,x}{a+b\,x}\right)^3}{3\,a}$$

```
[In] int(log((c*x)/(a + b*x))^2/(x*(a + b*x)),x)
```

```
[Out] log((c*x)/(a + b*x))^3/(3*a)
```

3.90 $\int \frac{\log\left(\frac{a}{a+bx}\right)\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)}\,dx$

Optimal result . . . 737
Rubi [A] (verified) . . . 737
Mathematica [A] (verified) . . . 739
Maple [F] . . . 739
Fricas [F] . . . 739
Sympy [F] . . . 739
Maxima [F] . . . 740
Giac [F] . . . 740
Mupad [F(-1)] . . . 740

Optimal result

Integrand size = 34, antiderivative size = 82

$$\int \frac{\log\left(\frac{a}{a+bx}\right)\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)}\,dx = -\frac{\log^2\left(\frac{cx}{a+bx}\right)\operatorname{PolyLog}\left(2,1-\frac{a}{a+bx}\right)}{a} + \frac{2\log\left(\frac{cx}{a+bx}\right)\operatorname{PolyLog}\left(3,1-\frac{a}{a+bx}\right)}{a} - \frac{2\operatorname{PolyLog}\left(4,1-\frac{a}{a+bx}\right)}{a}$$

```
[Out] -ln(c*x/(b*x+a))^2*polylog(2,1-a/(b*x+a))/a+2*ln(c*x/(b*x+a))*polylog(3,1-a
/(b*x+a))/a-2*polylog(4,1-a/(b*x+a))/a
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {2588, 2590, 6745}

$$\int \frac{\log\left(\frac{a}{a+bx}\right)\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)}\,dx = -\frac{\operatorname{PolyLog}\left(2,1-\frac{a}{a+bx}\right)\log^2\left(\frac{cx}{a+bx}\right)}{a} + \frac{2\operatorname{PolyLog}\left(3,1-\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{a} - \frac{2\operatorname{PolyLog}\left(4,1-\frac{a}{a+bx}\right)}{a}$$

```
[In] Int[(Log[a/(a + b*x)]*Log[(c*x)/(a + b*x)]^2)/(x*(a + b*x)),x]
```

```
[Out] -((Log[(c*x)/(a + b*x)]^2*PolyLog[2, 1 - a/(a + b*x)])/a) + (2*Log[(c*x)/(a
 + b*x)]*PolyLog[3, 1 - a/(a + b*x)])/a - (2*PolyLog[4, 1 - a/(a + b*x)])/a
```

Rule 2588

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[(v - 1)*((c + d
*x)/(a + b*x))], h = Simplify[u*(a + b*x)*(c + d*x)]}, Simp[(-h)*PolyLog[2,
 1 - v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(b*c - a*d)), x] + Dist[h*p
*r*s, Int[PolyLog[2, 1 - v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(
(a + b*x)*(c + d*x))), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e,
 f, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 2590

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_)*PolyLog[n_, v_], x_Symbol] :> With[{g = Simplify[v*((c +
 d*x)/(a + b*x))], h = Simplify[u*(a + b*x)*(c + d*x)]}, Simp[h*PolyLog[n +
 1, v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(b*c - a*d)), x] - Dist[h*p*
r*s, Int[PolyLog[n + 1, v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/((
a + b*x)*(c + d*x))), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e,
f, n, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\log^2\left(\frac{cx}{a+bx}\right)\text{Li}_2\left(1-\frac{a}{a+bx}\right)}{a}+2\int\frac{\log\left(\frac{cx}{a+bx}\right)\text{Li}_2\left(1-\frac{a}{a+bx}\right)}{x(a+bx)}\,dx\\
&= -\frac{\log^2\left(\frac{cx}{a+bx}\right)\text{Li}_2\left(1-\frac{a}{a+bx}\right)}{a}+\frac{2\log\left(\frac{cx}{a+bx}\right)\text{Li}_3\left(1-\frac{a}{a+bx}\right)}{a}-2\int\frac{\text{Li}_3\left(1-\frac{a}{a+bx}\right)}{x(a+bx)}\,dx\\
&= -\frac{\log^2\left(\frac{cx}{a+bx}\right)\text{Li}_2\left(1-\frac{a}{a+bx}\right)}{a}+\frac{2\log\left(\frac{cx}{a+bx}\right)\text{Li}_3\left(1-\frac{a}{a+bx}\right)}{a}-\frac{2\text{Li}_4\left(1-\frac{a}{a+bx}\right)}{a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.93

$$\int \frac{\log\left(\frac{a}{a+bx}\right)\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)}\,dx = -\frac{\log^2\left(\frac{cx}{a+bx}\right)\text{PolyLog}\left(2,\frac{bx}{a+bx}\right)}{a} + \frac{2\log\left(\frac{cx}{a+bx}\right)\text{PolyLog}\left(3,\frac{bx}{a+bx}\right)}{a} - \frac{2\,\text{PolyLog}\left(4,\frac{bx}{a+bx}\right)}{a}$$

```
[In] Integrate[(Log[a/(a + b*x)]*Log[(c*x)/(a + b*x)]^2)/(x*(a + b*x)),x]
[Out] -((Log[(c*x)/(a + b*x)]^2*PolyLog[2, (b*x)/(a + b*x)])/a) + (2*Log[(c*x)/(a
 + b*x)]*PolyLog[3, (b*x)/(a + b*x)])/a - (2*PolyLog[4, (b*x)/(a + b*x)])/a
```

Maple [F]

$$\int \frac{\ln\left(\frac{a}{bx+a}\right)\ln\left(\frac{cx}{bx+a}\right)^2}{x\,(bx+a)}dx$$

```
[In] int(ln(a/(b*x+a))*ln(c*x/(b*x+a))^2/x/(b*x+a),x)
[Out] int(ln(a/(b*x+a))*ln(c*x/(b*x+a))^2/x/(b*x+a),x)
```

Fricas [F]

$$\int \frac{\log\left(\frac{a}{a+bx}\right)\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)}\,dx = \int \frac{\log\left(\frac{cx}{bx+a}\right)^2\log\left(\frac{a}{bx+a}\right)}{(bx+a)x}\,dx$$

```
[In] integrate(log(a/(b*x+a))*log(c*x/(b*x+a))^2/x/(b*x+a),x, algorithm="fricas"
)
[Out] integral(log(c*x/(b*x + a))^2*log(a/(b*x + a))/(b*x^2 + a*x), x)
```

Sympy [F]

$$\int \frac{\log\left(\frac{a}{a+bx}\right)\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)}\,dx = \int \frac{\log\left(\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)^2}{x\,(a+bx)}\,dx$$

```
[In] integrate(ln(a/(b*x+a))*ln(c*x/(b*x+a))**2/x/(b*x+a),x)
[Out] Integral(log(a/(a + b*x))*log(c*x/(a + b*x))**2/(x*(a + b*x)), x)
```

Maxima [F]

$$\int \frac{\log\left(\frac{a}{a+bx}\right)\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)}\,dx = \int \frac{\log\left(\frac{cx}{bx+a}\right)^2\log\left(\frac{a}{bx+a}\right)}{(bx+a)x}\,dx$$

```
[In] integrate(log(a/(b*x+a))*log(c*x/(b*x+a))^2/x/(b*x+a),x, algorithm="maxima"
)
```

```
[Out] 1/4*(log(b*x + a)^4 - 4*log(b*x + a)^3*log(x))/a + integrate((a*log(a)*log(
c)^2 + 2*a*log(a)*log(c)*log(x) + a*log(a)*log(x)^2 + (a*(log(a) + 2*log(c)
) + (3*b*x + 2*a)*log(x))*log(b*x + a)^2 - (2*a*(log(a) + log(c))*log(x) +
a*log(x)^2 + (2*log(a)*log(c) + log(c)^2)*a)*log(b*x + a))/(a*b*x^2 + a^2*x
), x)
```

Giac [F]

$$\int \frac{\log\left(\frac{a}{a+bx}\right)\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)}\,dx = \int \frac{\log\left(\frac{cx}{bx+a}\right)^2\log\left(\frac{a}{bx+a}\right)}{(bx+a)x}\,dx$$

```
[In] integrate(log(a/(b*x+a))*log(c*x/(b*x+a))^2/x/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(log(c*x/(b*x + a))^2*log(a/(b*x + a))/((b*x + a)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{a}{a+bx}\right)\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)}\,dx = \int \frac{\ln\left(\frac{c\,x}{a+b\,x}\right)^2\ln\left(\frac{a}{a+b\,x}\right)}{x\,\left(a+b\,x\right)}\,dx$$

```
[In] int((log((c*x)/(a + b*x))^2*log(a/(a + b*x)))/(x*(a + b*x)),x)
```

```
[Out] int((log((c*x)/(a + b*x))^2*log(a/(a + b*x)))/(x*(a + b*x)), x)
```

3.91 $\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{e(a+bx)}{c+dx}\right)}{(c+dx)(ag+bgx)}\,dx$

Optimal result . . . 741
Rubi [A] (verified) . . . 741
Mathematica [A] (verified) . . . 743
Maple [A] (verified) . . . 743
Fricas [F] . . . 743
Sympy [F] . . . 744
Maxima [F] . . . 744
Giac [F] . . . 745
Mupad [F(-1)] . . . 745

Optimal result

Integrand size = 55, antiderivative size = 150

$$\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{e(a+bx)}{c+dx}\right)}{(c+dx)(ag+bgx)}\,dx = -\frac{\log^2\left(\frac{e(a+bx)}{c+dx}\right)\operatorname{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g} + \frac{2\log\left(\frac{e(a+bx)}{c+dx}\right)\operatorname{PolyLog}\left(3,1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g} - \frac{2\operatorname{PolyLog}\left(4,1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g}$$

```
[Out] -ln(e*(b*x+a)/(d*x+c))^2*polylog(2,1+(a*d-b*c)/b/(d*x+c))/(-a*d+b*c)/g+2*ln
(e*(b*x+a)/(d*x+c))*polylog(3,1+(a*d-b*c)/b/(d*x+c))/(-a*d+b*c)/g-2*polylog
(4,1+(a*d-b*c)/b/(d*x+c))/(-a*d+b*c)/g
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.055, Rules used = {2588, 2590, 6745}

$$\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{e(a+bx)}{c+dx}\right)}{(c+dx)(ag+bgx)}\,dx = -\frac{\operatorname{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{e(a+bx)}{c+dx}\right)}{g(bc-ad)} + \frac{2\operatorname{PolyLog}\left(3,1-\frac{bc-ad}{b(c+dx)}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)}{g(bc-ad)} - \frac{2\operatorname{PolyLog}\left(4,1-\frac{bc-ad}{b(c+dx)}\right)}{g(bc-ad)}$$

```
[In] Int[(Log[(b*c - a*d)/(b*(c + d*x))]*Log[(e*(a + b*x))/(c + d*x)]^2)/((c + d
*x)*(a*g + b*g*x)),x]
```

```
[Out] -((Log[(e*(a + b*x))/(c + d*x)]^2*PolyLog[2, 1 - (b*c - a*d)/(b*(c + d*x))]
)/((b*c - a*d)*g)) + (2*Log[(e*(a + b*x))/(c + d*x)]*PolyLog[3, 1 - (b*c -
a*d)/(b*(c + d*x))])/((b*c - a*d)*g) - (2*PolyLog[4, 1 - (b*c - a*d)/(b*(c
+ d*x))])/((b*c - a*d)*g)
```

Rule 2588

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[(v - 1)*((c + d
*x)/(a + b*x))], h = Simplify[u*(a + b*x)*(c + d*x)]}, Simp[(-h)*PolyLog[2,
 1 - v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(b*c - a*d)), x] + Dist[h*p
*r*s, Int[PolyLog[2, 1 - v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(
(a + b*x)*(c + d*x))), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e,
 f, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 2590

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_)*PolyLog[n_, v_], x_Symbol] :> With[{g = Simplify[v*((c +
 d*x)/(a + b*x))], h = Simplify[u*(a + b*x)*(c + d*x)]}, Simp[h*PolyLog[n +
 1, v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(b*c - a*d)), x] - Dist[h*p*
r*s, Int[PolyLog[n + 1, v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/((
a + b*x)*(c + d*x))), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e,
f, n, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\log^2\left(\frac{e(a+bx)}{c+dx}\right)\text{Li}_2\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g}+\frac{2\int\frac{\log\left(\frac{e(a+bx)}{c+dx}\right)\text{Li}_2\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(a+bx)(c+dx)}\,dx}{g}\\
&= -\frac{\log^2\left(\frac{e(a+bx)}{c+dx}\right)\text{Li}_2\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g}+\frac{2\log\left(\frac{e(a+bx)}{c+dx}\right)\text{Li}_3\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g}-\frac{2\int\frac{\text{Li}_3\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(a+bx)(c+dx)}\,dx}{g}\\
&= -\frac{\log^2\left(\frac{e(a+bx)}{c+dx}\right)\text{Li}_2\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g}+\frac{2\log\left(\frac{e(a+bx)}{c+dx}\right)\text{Li}_3\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g}-\frac{2\text{Li}_4\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.73

$$\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{e(a+bx)}{c+dx}\right)}{(c+dx)(ag+bgx)}\,dx = \frac{-\log^2\left(\frac{e(a+bx)}{c+dx}\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)+2\log\left(\frac{e(a+bx)}{c+dx}\right)\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)-2\operatorname{PolyLog}\left(4,\frac{d(a+bx)}{b(c+dx)}\right)}{(bc-ad)g}$$

```
[In] Integrate[(Log[(b*c - a*d)/(b*(c + d*x))]*Log[(e*(a + b*x))/(c + d*x)]^2)/(
(c + d*x)*(a*g + b*g*x)),x]
[Out] (-(Log[(e*(a + b*x))/(c + d*x)]^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])
+ 2*Log[(e*(a + b*x))/(c + d*x)]*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))] -
2*PolyLog[4, (d*(a + b*x))/(b*(c + d*x))])/((b*c - a*d)*g)
```

Maple [A] (verified)

Time = 4.11 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.33

method	result
default	$-\frac{\frac{\ln\left(-\frac{\frac{e(bx+a)d}{dx+c}-be}{be}\right)\ln\left(\frac{e(bx+a)}{dx+c}\right)^3}{3}-\frac{\ln\left(\frac{e(bx+a)}{dx+c}\right)^3\ln\left(1-\frac{d(bx+a)}{b(dx+c)}\right)}{3}-\ln\left(\frac{e(bx+a)}{dx+c}\right)^2\operatorname{Li}_2\left(\frac{d(bx+a)}{b(dx+c)}\right)+2\ln\left(\frac{e(bx+a)}{dx+c}\right)\operatorname{Li}_3\left(\frac{d(bx+a)}{b(dx+c)}\right)-}{g(ad-cb)}$

```
[In] int(ln((-a*d+b*c)/b/(d*x+c))*ln(e*(b*x+a)/(d*x+c))^2/(d*x+c)/(b*g*x+a*g),x,
method=_RETURNVERBOSE)
[Out] -1/g/(a*d-b*c)*(1/3*ln(-(e*(b*x+a)/(d*x+c)*d-b*e)/b/e)*ln(e*(b*x+a)/(d*x+c)
)^3-1/3*ln(e*(b*x+a)/(d*x+c))^3*ln(1-d*(b*x+a)/b/(d*x+c))-ln(e*(b*x+a)/(d*x
+c))^2*polylog(2,d*(b*x+a)/b/(d*x+c))+2*ln(e*(b*x+a)/(d*x+c))*polylog(3,d*(
b*x+a)/b/(d*x+c))-2*polylog(4,d*(b*x+a)/b/(d*x+c)))
```

Fricas [F]

$$\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{e(a+bx)}{c+dx}\right)}{(c+dx)(ag+bgx)}\,dx = \int \frac{\log\left(\frac{(bx+a)e}{dx+c}\right)^2\log\left(\frac{bc-ad}{(dx+c)b}\right)}{(bgx+ag)(dx+c)}\,dx$$

```
[In] integrate(log((-a*d+b*c)/b/(d*x+c))*log(e*(b*x+a)/(d*x+c))^2/(d*x+c)/(b*g*x
+a*g),x, algorithm="fricas")
[Out] integral(log((b*c - a*d)/(b*d*x + b*c))*log((b*e*x + a*e)/(d*x + c))^2/(b*d
*g*x^2 + a*c*g + (b*c + a*d)*g*x), x)
```

Sympy [F]

$$\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{e(a+bx)}{c+dx}\right)}{(c+dx)(ag+bgx)}\,dx = -\frac{d\int \frac{\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^3}{c+dx}\,dx}{3g\,(ad-bc)} - \frac{\log\left(\frac{-ad+bc}{b(c+dx)}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)^3}{3adg-3bcg}$$

```
[In] integrate(ln((-a*d+b*c)/b/(d*x+c))*ln(e*(b*x+a)/(d*x+c))**2/(d*x+c)/(b*g*x+
a*g),x)
```

```
[Out] -d*Integral(log(a*e/(c + d*x) + b*e*x/(c + d*x))**3/(c + d*x), x)/(3*g*(a*d
 - b*c)) - log((-a*d + b*c)/(b*(c + d*x)))*log(e*(a + b*x)/(c + d*x))**3/(3
*a*d*g - 3*b*c*g)
```

Maxima [F]

$$\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{e(a+bx)}{c+dx}\right)}{(c+dx)(ag+bgx)}\,dx = \int \frac{\log\left(\frac{(bx+a)e}{dx+c}\right)^2\log\left(\frac{bc-ad}{(dx+c)b}\right)}{(bgx+ag)(dx+c)}\,dx$$

```
[In] integrate(log((-a*d+b*c)/b/(d*x+c))*log(e*(b*x+a)/(d*x+c))^2/(d*x+c)/(b*g*x
+a*g),x, algorithm="maxima")
```

```
[Out] -1/4*(4*log(b*x + a)*log(d*x + c)^3 - log(d*x + c)^4)/(b*c*g - a*d*g) - int
egrate((((d*log(b*c - a*d) - d*log(b))*a - (c*log(b*c - a*d) - c*log(b))*b)
*log(b*x + a)^2 + ((d*log(b*c - a*d) - d*log(b) + 2*d*log(e))*a - (c*(log(b
*c - a*d) + 2*log(e)) - c*log(b))*b - (3*b*d*x + 2*b*c + a*d)*log(b*x + a))
*log(d*x + c)^2 + (d*log(b*c - a*d)*log(e)^2 - d*log(b)*log(e)^2)*a - (c*lo
g(b*c - a*d)*log(e)^2 - c*log(b)*log(e)^2)*b + 2*((d*log(b*c - a*d)*log(e)
- d*log(b)*log(e))*a - (c*log(b*c - a*d)*log(e) - c*log(b)*log(e))*b)*log(b
*x + a) + ((b*c - a*d)*log(b*x + a)^2 - (2*d*log(b*c - a*d)*log(e) - 2*d*lo
g(b)*log(e) + d*log(e)^2)*a - (2*c*log(b)*log(e) - (2*log(b*c - a*d)*log(e)
 + log(e)^2)*c)*b - 2*((d*log(b*c - a*d) - d*log(b) + d*log(e))*a - (c*(log
(b*c - a*d) + log(e)) - c*log(b))*b)*log(b*x + a))*log(d*x + c))/(a*b*c^2*g
 - a^2*c*d*g + (b^2*c*d*g - a*b*d^2*g)*x^2 + (b^2*c^2*g - a^2*d^2*g)*x), x)
```

Giac [F]

$$\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{e(a+bx)}{c+dx}\right)}{(c+dx)(ag+bgx)}\,dx = \int \frac{\log\left(\frac{(bx+a)e}{dx+c}\right)^2\log\left(\frac{bc-ad}{(dx+c)b}\right)}{(bgx+ag)(dx+c)}\,dx$$

[In] integrate(log((-a*d+b*c)/b/(d*x+c))*log(e*(b*x+a)/(d*x+c))^2/(d*x+c)/(b*g*x+a*g),x, algorithm="giac")

[Out] integrate(log((b*x + a)*e/(d*x + c))^2*log((b*c - a*d)/((d*x + c)*b))/((b*g*x + a*g)*(d*x + c)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{e(a+bx)}{c+dx}\right)}{(c+dx)(ag+bgx)}\,dx = \int \frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2\ln\left(-\frac{a\,d-b\,c}{b\,(c+d\,x)}\right)}{(a\,g+b\,g\,x)\,(c+d\,x)}\,dx$$

[In] int((log((e*(a + b*x))/(c + d*x))^2*log(-(a*d - b*c)/(b*(c + d*x))))/((a*g + b*g*x)*(c + d*x)),x)

[Out] int((log((e*(a + b*x))/(c + d*x))^2*log(-(a*d - b*c)/(b*(c + d*x))))/((a*g + b*g*x)*(c + d*x)), x)

3.92 $\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(ag+bgx)}\,dx$

Optimal result 746
Rubi [A] (verified) 746
Mathematica [B] (verified) 748
Maple [F] 749
Fricas [F] 749
Sympy [F] 749
Maxima [F] 750
Giac [F] 750
Mupad [F(-1)] 750

Optimal result

Integrand size = 58, antiderivative size = 160

$$\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(ag+bgx)}\,dx = -\frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g} + \frac{2n\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{PolyLog}\left(3,1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g} - \frac{2n^2\,\text{PolyLog}\left(4,1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g}$$

```
[Out] -ln(e*((b*x+a)/(d*x+c))^n)^2*polylog(2,1+(a*d-b*c)/b/(d*x+c))/(-a*d+b*c)/g+
2*n*ln(e*((b*x+a)/(d*x+c))^n)*polylog(3,1+(a*d-b*c)/b/(d*x+c))/(-a*d+b*c)/g
-2*n^2*polylog(4,1+(a*d-b*c)/b/(d*x+c))/(-a*d+b*c)/g
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.052, Rules used = {2588, 2590, 6745}

$$\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(ag+bgx)}\,dx = -\frac{\text{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g(bc-ad)} + \frac{2n\,\text{PolyLog}\left(3,1-\frac{bc-ad}{b(c+dx)}\right)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g(bc-ad)} - \frac{2n^2\,\text{PolyLog}\left(4,1-\frac{bc-ad}{b(c+dx)}\right)}{g(bc-ad)}$$

```
[In] Int[(Log[e*((a + b*x)/(c + d*x))^n]^2*Log[(b*c - a*d)/(b*(c + d*x))])/((c +
 d*x)*(a*g + b*g*x)),x]
```

```
[Out] -((Log[e*((a + b*x)/(c + d*x))^n]^2*PolyLog[2, 1 - (b*c - a*d)/(b*(c + d*x)
)])/((b*c - a*d)*g)) + (2*n*Log[e*((a + b*x)/(c + d*x))^n]*PolyLog[3, 1 - (
b*c - a*d)/(b*(c + d*x))])/((b*c - a*d)*g) - (2*n^2*PolyLog[4, 1 - (b*c - a
*d)/(b*(c + d*x))])/((b*c - a*d)*g)
```

Rule 2588

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[(v - 1)*((c + d
*x)/(a + b*x))], h = Simplify[u*(a + b*x)*(c + d*x)]}, Simp[(-h)*PolyLog[2,
 1 - v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(b*c - a*d)), x] + Dist[h*p
*r*s, Int[PolyLog[2, 1 - v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(
(a + b*x)*(c + d*x))), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e,
 f, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 2590

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_)*PolyLog[n_, v_], x_Symbol] :> With[{g = Simplify[v*((c +
 d*x)/(a + b*x))], h = Simplify[u*(a + b*x)*(c + d*x)]}, Simp[h*PolyLog[n +
 1, v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(b*c - a*d)), x] - Dist[h*p*
r*s, Int[PolyLog[n + 1, v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/((
a + b*x)*(c + d*x))), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e,
f, n, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{Li}_2\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g}+\frac{(2n)\int\frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{Li}_2\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(a+bx)(c+dx)}\,dx}{g}\\
&= -\frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{Li}_2\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g}\\
&\quad+\frac{2n\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{Li}_3\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g}-\frac{(2n^2)\int\frac{\text{Li}_3\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(a+bx)(c+dx)}\,dx}{g}
\end{aligned}$$

$$= -\frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{Li}_2\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g} + \frac{2n\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{Li}_3\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g} - \frac{2n^2\operatorname{Li}_4\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 785 vs. $2(160) = 320$.

Time = 0.31 (sec) , antiderivative size = 785, normalized size of antiderivative = 4.91

$$\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(ag+bgx)}\,dx$$

$$= \frac{\log\left(\frac{a+bx}{c+dx}\right)\left(3\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - 3n\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{a+bx}{c+dx}\right) + n^2\log^2\left(\frac{a+bx}{c+dx}\right)\right)\log\left(\frac{bc-ad}{bc+bdx}\right) + \frac{3}{2}\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)\right.\right.}{}$$

```
[In] Integrate[(Log[e*((a + b*x)/(c + d*x))^n]^2*Log[(b*c - a*d)/(b*(c + d*x))])
/((c + d*x)*(a*g + b*g*x)),x]
```

```
[Out] (Log[(a + b*x)/(c + d*x)]*(3*Log[e*((a + b*x)/(c + d*x))^n]^2 - 3*n*Log[e*(
(a + b*x)/(c + d*x))^n]*Log[(a + b*x)/(c + d*x)] + n^2*Log[(a + b*x)/(c + d
*x)]^2)*Log[(b*c - a*d)/(b*c + b*d*x)] + (3*(Log[e*((a + b*x)/(c + d*x))^n]
 - n*Log[(a + b*x)/(c + d*x)])^2*(-Log[c/d + x]^2 - 2*Log[a/b + x]*Log[c +
d*x] + 2*Log[c/d + x]*Log[c + d*x] + 2*Log[(a + b*x)/(c + d*x)]*Log[c + d*x
] + 2*Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)] + 2*PolyLog[2, (d*(a + b*
x))/(-(b*c) + a*d)]))/2 + n*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*
x)/(c + d*x)])*(Log[c/d + x]^3 + 3*Log[c/d + x]^2*(-Log[a/b + x] + Log[(d*(
a + b*x))/(-(b*c) + a*d)]) + 3*(-Log[a/b + x] + Log[c/d + x] + Log[(a + b*x
)/(c + d*x)])^2*Log[c + d*x] + 3*Log[a/b + x]^2*Log[(b*(c + d*x))/(b*c - a*
d)] + 6*Log[a/b + x]*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 3*(Log[a/b
+ x] - Log[c/d + x] - Log[(a + b*x)/(c + d*x)])*(Log[c/d + x]^2 - 2*(Log[a/
b + x]*Log[(b*(c + d*x))/(b*c - a*d)] + PolyLog[2, (d*(a + b*x))/(-(b*c) +
a*d)])) + 6*Log[c/d + x]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - 6*PolyLog[
3, (d*(a + b*x))/(-(b*c) + a*d)] - 6*PolyLog[3, (b*(c + d*x))/(b*c - a*d)])
 - n^2*(Log[(a + b*x)/(c + d*x)]^3*Log[(b*c - a*d)/(b*c + b*d*x)] + 3*Log[(
a + b*x)/(c + d*x)]^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))] - 6*Log[(a +
b*x)/(c + d*x)]*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))] + 6*PolyLog[4, (d*(
a + b*x))/(b*(c + d*x))]))/(3*(b*c - a*d)*g)
```

Maple [F]

$$\int \frac{\ln \left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)^{2} \ln \left(\frac{-ad+cb}{b(dx+c)}\right)}{(dx+c)\,(bgx+ag)}dx$$

```
[In] int(ln(e*((b*x+a)/(d*x+c))^n)^2*ln((-a*d+b*c)/b/(d*x+c))/(d*x+c)/(b*g*x+a*g
),x)
```

```
[Out] int(ln(e*((b*x+a)/(d*x+c))^n)^2*ln((-a*d+b*c)/b/(d*x+c))/(d*x+c)/(b*g*x+a*g
),x)
```

Fricas [F]

$$\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(ag+bgx)}\,dx = \int \frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2\log\left(\frac{bc-ad}{(dx+c)b}\right)}{(bgx+ag)(dx+c)}\,dx$$

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^2*log((-a*d+b*c)/b/(d*x+c))/(d*x+c)/(b
*g*x+a*g),x, algorithm="fricas")
```

```
[Out] integral(log(e*((b*x + a)/(d*x + c))^n)^2*log((b*c - a*d)/(b*d*x + b*c))/(b
*d*g*x^2 + a*c*g + (b*c + a*d)*g*x), x)
```

Sympy [F]

$$\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(ag+bgx)}\,dx = \frac{\int \frac{\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2\log\left(-\frac{ad}{bc+bdx}+\frac{bc}{bc+bdx}\right)}{ac+adx+bcx+bdx^2}\,dx}{g}$$

```
[In] integrate(ln(e*((b*x+a)/(d*x+c))**n)**2*ln((-a*d+b*c)/b/(d*x+c))/(d*x+c)/(b
*g*x+a*g),x)
```

```
[Out] Integral(log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2*log(-a*d/(b*c + b*d*x)
+ b*c/(b*c + b*d*x))/(a*c + a*d*x + b*c*x + b*d*x**2), x)/g
```

Maxima [F]

$$\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(ag+bgx)}\,dx=\int \frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2\log\left(\frac{bc-ad}{(dx+c)b}\right)}{(bgx+ag)(dx+c)}\,dx$$

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^2*log((-a*d+b*c)/b/(d*x+c))/(d*x+c)/(b
*g*x+a*g),x, algorithm="maxima")
```

```
[Out] integrate(log(e*((b*x + a)/(d*x + c))^n)^2*log((b*c - a*d)/((d*x + c)*b))/(
(b*g*x + a*g)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(ag+bgx)}\,dx=\int \frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2\log\left(\frac{bc-ad}{(dx+c)b}\right)}{(bgx+ag)(dx+c)}\,dx$$

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^2*log((-a*d+b*c)/b/(d*x+c))/(d*x+c)/(b
*g*x+a*g),x, algorithm="giac")
```

```
[Out] integrate(log(e*((b*x + a)/(d*x + c))^n)^2*log((b*c - a*d)/((d*x + c)*b))/(
(b*g*x + a*g)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(ag+bgx)}\,dx=\int \frac{\ln\left(-\frac{a\,d-b\,c}{b\,(c+d\,x)}\right)\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2}{(a\,g+b\,g\,x)\,(c+d\,x)}\,dx$$

```
[In] int((log(-(a*d - b*c)/(b*(c + d*x)))*log(e*((a + b*x)/(c + d*x))^n)^2)/((a*
g + b*g*x)*(c + d*x)),x)
```

```
[Out] int((log(-(a*d - b*c)/(b*(c + d*x)))*log(e*((a + b*x)/(c + d*x))^n)^2)/((a*
g + b*g*x)*(c + d*x)), x)
```

3.93 $\int \log\left(\frac{c(b+ax)}{x}\right)\,dx$

Optimal result 751
Rubi [A] (verified) 751
Mathematica [A] (verified) 752
Maple [A] (verified) 752
Fricas [A] (verification not implemented) 753
Sympy [A] (verification not implemented) 753
Maxima [A] (verification not implemented) 754
Giac [B] (verification not implemented) 754
Mupad [B] (verification not implemented) 754

Optimal result

Integrand size = 11, antiderivative size = 25

$$\int \log\left(\frac{c(b+ax)}{x}\right)\,dx = x\log\left(ac+\frac{bc}{x}\right)+\frac{b\log(b+ax)}{a}$$

```
[Out] x*ln(a*c+b*c/x)+b*ln(a*x+b)/a
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2503, 2498, 269, 31}

$$\int \log\left(\frac{c(b+ax)}{x}\right)\,dx = x\log\left(ac+\frac{bc}{x}\right)+\frac{b\log(ax+b)}{a}$$

```
[In] Int[Log[(c*(b + a*x))/x],x]
```

```
[Out] x*Log[a*c + (b*c)/x] + (b*Log[b + a*x])/a
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2503

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.), x_Symbol] :> Int[(a + b*Lo
g[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b, c, p, q}, x] && BinomialQ[v
, x] &&  !BinomialMatchQ[v, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \log\left(ac+\frac{bc}{x}\right)\,dx\\
&= x\log\left(ac+\frac{bc}{x}\right)+(bc)\int\frac{1}{\left(ac+\frac{bc}{x}\right)x}\,dx\\
&= x\log\left(ac+\frac{bc}{x}\right)+(bc)\int\frac{1}{bc+acx}\,dx\\
&= x\log\left(ac+\frac{bc}{x}\right)+\frac{b\log(b+ax)}{a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \log\left(\frac{c(b+ax)}{x}\right)\,dx=\frac{b\log(x)}{a}+\frac{(b+ax)\log\left(\frac{c(b+ax)}{x}\right)}{a}$$

```
[In] Integrate[Log[(c*(b + a*x))/x],x]
[Out] (b*Log[x])/a + ((b + a*x)*Log[(c*(b + a*x))/x])/a
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

method	result	size
risch	$x\ln\left(\frac{c(ax+b)}{x}\right)+\frac{b\ln(ax+b)}{a}$	26
parts	$x\ln\left(\frac{c(ax+b)}{x}\right)+\frac{b\ln(ax+b)}{a}$	26
parallelrisch	$-\frac{-\ln\left(\frac{c(ax+b)}{x}\right)xab-\ln(x)b^2-b^2\ln\left(\frac{c(ax+b)}{x}\right)}{ab}$	49
derivativedivides	$-cb\left(\frac{\ln\left(-\frac{bc}{x}\right)}{ac}-\frac{\ln\left(ca+\frac{bc}{x}\right)\left(ca+\frac{bc}{x}\right)x}{a\,c^2b}\right)$	54
default	$-cb\left(\frac{\ln\left(-\frac{bc}{x}\right)}{ac}-\frac{\ln\left(ca+\frac{bc}{x}\right)\left(ca+\frac{bc}{x}\right)x}{a\,c^2b}\right)$	54

```
[In] int(ln(c*(a*x+b)/x),x,method=_RETURNVERBOSE)
[Out] x*ln(c*(a*x+b)/x)+b*ln(a*x+b)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int\log\left(\frac{c(b+ax)}{x}\right)\,dx=\frac{ax\log\left(\frac{acx+bc}{x}\right)+b\log(ax+b)}{a}$$

```
[In] integrate(log(c*(a*x+b)/x),x, algorithm="fricas")
[Out] (a*x*log((a*c*x + b*c)/x) + b*log(a*x + b))/a
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int\log\left(\frac{c(b+ax)}{x}\right)\,dx=x\log\left(\frac{c(ax+b)}{x}\right)+\frac{b\log(ax+b)}{a}$$

```
[In] integrate(ln(c*(a*x+b)/x),x)
[Out] x*log(c*(a*x + b)/x) + b*log(a*x + b)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \log \left(\frac{c(b+ax)}{x}\right) \, dx = x \log \left(\frac{(ax+b)c}{x}\right) + \frac{b \log (ax+b)}{a}$$

```
[In] integrate(log(c*(a*x+b)/x),x, algorithm="maxima")
```

```
[Out] x*log((a*x + b)*c/x) + b*log(a*x + b)/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(25) = 50.

Time = 0.31 (sec) , antiderivative size = 153, normalized size of antiderivative = 6.12

$$\int \log \left(\frac{c(b+ax)}{x}\right) \, dx = \frac{b^2c^2\left(\frac{\log\left(\frac{|acx+bc|}{|x|}\right)}{ac} - \frac{\log\left(\left|-ac+\frac{acx+bc}{x}\right|\right)}{ac}\right) - \frac{b^2c^2 \log\left(-\left(b-\frac{a}{\frac{a}{b}-\frac{acx+bc}{bcx}}\right)c\left(\frac{a}{b}-\frac{acx+bc}{bcx}\right)\right)}{ac-\frac{acx+bc}{x}}}{bc}$$

```
[In] integrate(log(c*(a*x+b)/x),x, algorithm="giac")
```

```
[Out] (b^2*c^2*(log(abs(a*c*x + b*c)/abs(x))/(a*c) - log(abs(-a*c + (a*c*x + b*c)
/x))/(a*c)) - b^2*c^2*log(-(b - a/(a/b - (a*c*x + b*c)/(b*c*x)))*c*(a/b - (
a*c*x + b*c)/(b*c*x)))/(a*c - (a*c*x + b*c)/x))/(b*c)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \log \left(\frac{c(b+ax)}{x}\right) \, dx = x \, \ln\left(\frac{c\,(b+a\,x)}{x}\right) + \frac{b\,\ln\left(b+a\,x\right)}{a}$$

```
[In] int(log((c*(b + a*x))/x),x)
```

```
[Out] x*log((c*(b + a*x))/x) + (b*log(b + a*x))/a
```

3.94 $\int \log^2\left(\frac{c(b+ax)}{x}\right)\,dx$

Optimal result 755
Rubi [A] (verified) 755
Mathematica [A] (verified) 757
Maple [F] 757
Fricas [F] 757
Sympy [F] 757
Maxima [A] (verification not implemented) 758
Giac [F] 758
Mupad [F(-1)] 758

Optimal result

Integrand size = 13, antiderivative size = 67

$$\int \log^2\left(\frac{c(b+ax)}{x}\right)\,dx=\frac{(b+ax)\log^2\left(ac+\frac{bc}{x}\right)}{a}-\frac{2b\log\left(c\left(a+\frac{b}{x}\right)\right)\log\left(-\frac{b}{ax}\right)}{a}-\frac{2b\,\text{PolyLog}\left(2,1+\frac{b}{ax}\right)}{a}$$

```
[Out] (a*x+b)*ln(a*c+b*c/x)^2/a-2*b*ln(c*(a+b/x))*ln(-b/a/x)/a-2*b*polylog(2,1+b/
a/x)/a
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2503, 2499, 2504, 2441, 2352}

$$\int \log^2\left(\frac{c(b+ax)}{x}\right)\,dx=\frac{(ax+b)\log^2\left(ac+\frac{bc}{x}\right)}{a}-\frac{2b\log\left(-\frac{b}{ax}\right)\log\left(c\left(a+\frac{b}{x}\right)\right)}{a}-\frac{2b\,\text{PolyLog}\left(2,\frac{b}{ax}+1\right)}{a}$$

```
[In] Int[Log[(c*(b + a*x))/x]^2,x]
```

```
[Out] ((b + a*x)*Log[a*c + (b*c)/x]^2)/a - (2*b*Log[c*(a + b/x)]*Log[-(b/(a*x))])
/a - (2*b*PolyLog[2, 1 + b/(a*x)])/a
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2499

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)/(x_))^(p_.)]*(b_.))^(q_), x_Symbol] :>
 Simp[(e + d*x)*((a + b*Log[c*(d + e/x)^p])^q/d), x] + Dist[b*e*p*(q/d), In
t[(a + b*Log[c*(d + e/x)^p])^(q - 1)/x, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && IGtQ[q, 0]
```

Rule 2503

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.), x_Symbol] :> Int[(a + b*Lo
g[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b, c, p, q}, x] && BinomialQ[v
, x] &&  !BinomialMatchQ[v, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\text{integral} = \int \log^2\left(ac+\frac{bc}{x}\right)\,dx$$

$$= \frac{(b+ax)\log^2\left(ac+\frac{bc}{x}\right)}{a} + \frac{(2b)\int \frac{\log\left(ac+\frac{bc}{x}\right)}{x}\,dx}{a}$$

$$= \frac{(b+ax)\log^2\left(ac+\frac{bc}{x}\right)}{a} - \frac{(2b)\text{Subst}\left(\int \frac{\log(ac+bcx)}{x}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{(b+ax)\log^2\left(ac+\frac{bc}{x}\right)}{a} - \frac{2b\log\left(c\left(a+\frac{b}{x}\right)\right)\log\left(-\frac{b}{ax}\right)}{a} + \frac{(2b^2c)\,\text{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)}{ac+bcx}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{(b+ax)\log^2\left(ac+\frac{bc}{x}\right)}{a} - \frac{2b\log\left(c\left(a+\frac{b}{x}\right)\right)\log\left(-\frac{b}{ax}\right)}{a} - \frac{2b\text{Li}_2\left(1+\frac{b}{ax}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94

$$\int \log^2\left(\frac{c(b+ax)}{x}\right)\,dx$$
$$=\frac{\log\left(\frac{c(b+ax)}{x}\right)\left(-2b\log\left(-\frac{b}{ax}\right)+(b+ax)\log\left(\frac{c(b+ax)}{x}\right)\right)-2b\,\text{PolyLog}\left(2,1+\frac{b}{ax}\right)}{a}$$

```
[In] Integrate[Log[(c*(b + a*x))/x]^2,x]
[Out] (Log[(c*(b + a*x))/x]*(-2*b*Log[-(b/(a*x))] + (b + a*x)*Log[(c*(b + a*x))/x
]) - 2*b*PolyLog[2, 1 + b/(a*x)])/a
```

Maple [F]

$$\int \ln\left(\frac{c(ax+b)}{x}\right)^2 dx$$

```
[In] int(ln(c*(a*x+b)/x)^2,x)
[Out] int(ln(c*(a*x+b)/x)^2,x)
```

Fricas [F]

$$\int \log^2\left(\frac{c(b+ax)}{x}\right)\,dx=\int \log\left(\frac{(ax+b)c}{x}\right)^2 dx$$

```
[In] integrate(log(c*(a*x+b)/x)^2,x, algorithm="fricas")
[Out] integral(log((a*c*x + b*c)/x)^2, x)
```

Sympy [F]

$$\int \log^2\left(\frac{c(b+ax)}{x}\right)\,dx=2b\int \frac{\log\left(ac+\frac{bc}{x}\right)}{ax+b}\,dx+x\log\left(\frac{c(ax+b)}{x}\right)^2$$

```
[In] integrate(ln(c*(a*x+b)/x)**2,x)
[Out] 2*b*Integral(log(a*c + b*c/x)/(a*x + b), x) + x*log(c*(a*x + b)/x)**2
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.69

$$\int \log^2\left(\frac{c(b+ax)}{x}\right)\,dx$$
$$= x\log\left(\frac{(ax+b)c}{x}\right)^2 + \frac{2\,b\log(ax+b)\log\left(\frac{(ax+b)c}{x}\right)}{a}$$
$$+ \frac{\left(\frac{c\log(ax+b)^2}{a} - \frac{2\left(\log\left(\frac{ax}{b}+1\right)\log(x)+\text{Li}_2\left(-\frac{ax}{b}\right)\right)c}{a}\right)b - \frac{2\left(c\log(ax+b)-c\log(x)\right)b\log(ax+b)}{a}}{c}$$

```
[In] integrate(log(c*(a*x+b)/x)^2,x, algorithm="maxima")
[Out] x*log((a*x + b)*c/x)^2 + 2*b*log(a*x + b)*log((a*x + b)*c/x)/a + ((c*log(a*
x + b)^2/a - 2*(log(a*x/b + 1)*log(x) + dilog(-a*x/b))*c/a)*b - 2*(c*log(a*
x + b) - c*log(x))*b*log(a*x + b)/a)/c
```

Giac [F]

$$\int \log^2\left(\frac{c(b+ax)}{x}\right)\,dx = \int \log\left(\frac{(ax+b)c}{x}\right)^2\,dx$$

```
[In] integrate(log(c*(a*x+b)/x)^2,x, algorithm="giac")
[Out] integrate(log((a*x + b)*c/x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \log^2\left(\frac{c(b+ax)}{x}\right)\,dx = \int \ln\left(\frac{c\,(b+a\,x)}{x}\right)^2\,dx$$

```
[In] int(log((c*(b + a*x))/x)^2,x)
[Out] int(log((c*(b + a*x))/x)^2, x)
```

3.95 $\int \log^3\left(\frac{c(b+ax)}{x}\right)\,dx$

Optimal result 759
Rubi [A] (verified) 759
Mathematica [A] (verified) 762
Maple [F] 762
Fricas [F] 762
Sympy [F] 762
Maxima [F] 763
Giac [F] 763
Mupad [F(-1)] 763

Optimal result

Integrand size = 13, antiderivative size = 97

$$\int \log^3\left(\frac{c(b+ax)}{x}\right)\,dx = \frac{(b+ax)\log^3\left(ac+\frac{bc}{x}\right)}{a}-\frac{3b\log^2\left(c\left(a+\frac{b}{x}\right)\right)\log\left(-\frac{b}{ax}\right)}{a} - \frac{6b\log\left(c\left(a+\frac{b}{x}\right)\right)\text{PolyLog}\left(2,1+\frac{b}{ax}\right)}{a}+\frac{6b\,\text{PolyLog}\left(3,1+\frac{b}{ax}\right)}{a}$$

```
[Out] (a*x+b)*ln(a*c+b*c/x)^3/a-3*b*ln(c*(a+b/x))^2*ln(-b/a/x)/a-6*b*ln(c*(a+b/x)
)*polylog(2,1+b/a/x)/a+6*b*polylog(3,1+b/a/x)/a
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2503, 2499, 2504, 2443, 2481, 2421, 6724}

$$\int \log^3\left(\frac{c(b+ax)}{x}\right)\,dx = -\frac{6b\,\text{PolyLog}\left(2,\frac{b}{ax}+1\right)\log\left(c\left(a+\frac{b}{x}\right)\right)}{a}+\frac{(ax+b)\log^3\left(ac+\frac{bc}{x}\right)}{a} - \frac{3b\log\left(-\frac{b}{ax}\right)\log^2\left(c\left(a+\frac{b}{x}\right)\right)}{a}+\frac{6b\,\text{PolyLog}\left(3,\frac{b}{ax}+1\right)}{a}$$

```
[In] Int[Log[(c*(b + a*x))/x]^3,x]
```

```
[Out] ((b + a*x)*Log[a*c + (b*c)/x]^3)/a - (3*b*Log[c*(a + b/x)]^2*Log[-(b/(a*x))
])/a - (6*b*Log[c*(a + b/x)]*PolyLog[2, 1 + b/(a*x)])/a + (6*b*PolyLog[3, 1
 + b/(a*x)])/a
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2499

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)/(x_))^(p_.)]*(b_.))^(q_), x_Symbol] :>
 Simp[(e + d*x)*((a + b*Log[c*(d + e/x)^p])^q/d), x] + Dist[b*e*p*(q/d), In
t[(a + b*Log[c*(d + e/x)^p])^(q - 1)/x, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && IGtQ[q, 0]
```

Rule 2503

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.), x_Symbol] :> Int[(a + b*Lo
g[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b, c, p, q}, x] && BinomialQ[v
, x] &&  !BinomialMatchQ[v, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \log^3\left(ac+\frac{bc}{x}\right)dx \\
&= \frac{(b+ax)\log^3\left(ac+\frac{bc}{x}\right)}{a}+\frac{(3b)\int\frac{\log^2\left(ac+\frac{bc}{x}\right)}{x}\,dx}{a} \\
&= \frac{(b+ax)\log^3\left(ac+\frac{bc}{x}\right)}{a}-\frac{(3b)\text{Subst}\left(\int\frac{\log^2(ac+bcx)}{x}\,dx,x,\frac{1}{x}\right)}{a} \\
&= \frac{(b+ax)\log^3\left(ac+\frac{bc}{x}\right)}{a}-\frac{3b\log^2\left(c\left(a+\frac{b}{x}\right)\right)\log\left(-\frac{b}{ax}\right)}{a} \\
&\quad+\frac{(6b^2c)\,\text{Subst}\left(\int\frac{\log\left(-\frac{bx}{a}\right)\log(ac+bcx)}{ac+bcx}\,dx,x,\frac{1}{x}\right)}{a} \\
&= \frac{(b+ax)\log^3\left(ac+\frac{bc}{x}\right)}{a}-\frac{3b\log^2\left(c\left(a+\frac{b}{x}\right)\right)\log\left(-\frac{b}{ax}\right)}{a} \\
&\quad+\frac{(6b)\text{Subst}\left(\int\frac{\log(x)\log\left(-\frac{b\left(-\frac{a}{b}+\frac{x}{bc}\right)}{a}\right)}{x}\,dx,x,ac+\frac{bc}{x}\right)}{a} \\
&= \frac{(b+ax)\log^3\left(ac+\frac{bc}{x}\right)}{a}-\frac{3b\log^2\left(c\left(a+\frac{b}{x}\right)\right)\log\left(-\frac{b}{ax}\right)}{a} \\
&\quad-\frac{6b\log\left(c\left(a+\frac{b}{x}\right)\right)\text{Li}_2\left(1+\frac{b}{ax}\right)}{a}+\frac{(6b)\text{Subst}\left(\int\frac{\text{Li}_2\left(\frac{x}{ac}\right)}{x}\,dx,x,ac+\frac{bc}{x}\right)}{a} \\
&= \frac{(b+ax)\log^3\left(ac+\frac{bc}{x}\right)}{a}-\frac{3b\log^2\left(c\left(a+\frac{b}{x}\right)\right)\log\left(-\frac{b}{ax}\right)}{a} \\
&\quad-\frac{6b\log\left(c\left(a+\frac{b}{x}\right)\right)\text{Li}_2\left(1+\frac{b}{ax}\right)}{a}+\frac{6b\text{Li}_3\left(1+\frac{b}{ax}\right)}{a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.94

$$\int \log^3\left(\frac{c(b+ax)}{x}\right)\,dx$$
$$= \frac{\log^2\left(\frac{c(b+ax)}{x}\right)\left(-3b\log\left(-\frac{b}{ax}\right)+(b+ax)\log\left(\frac{c(b+ax)}{x}\right)\right)-6b\log\left(\frac{c(b+ax)}{x}\right)\mathrm{PolyLog}\left(2,1+\frac{b}{ax}\right)+6b\,\mathrm{PolyLo}}{a}$$

```
[In] Integrate[Log[(c*(b + a*x))/x]^3,x]
[Out] (Log[(c*(b + a*x))/x]^2*(-3*b*Log[-(b/(a*x))] + (b + a*x)*Log[(c*(b + a*x))
/x]) - 6*b*Log[(c*(b + a*x))/x]*PolyLog[2, 1 + b/(a*x)] + 6*b*PolyLog[3, 1
+ b/(a*x)])/a
```

Maple [F]

$$\int \ln\left(\frac{c(ax+b)}{x}\right)^3 dx$$

```
[In] int(ln(c*(a*x+b)/x)^3,x)
[Out] int(ln(c*(a*x+b)/x)^3,x)
```

Fricas [F]

$$\int \log^3\left(\frac{c(b+ax)}{x}\right)\,dx = \int \log\left(\frac{(ax+b)c}{x}\right)^3\,dx$$

```
[In] integrate(log(c*(a*x+b)/x)^3,x, algorithm="fricas")
[Out] integral(log((a*c*x + b*c)/x)^3, x)
```

Sympy [F]

$$\int \log^3\left(\frac{c(b+ax)}{x}\right)\,dx = 3b\int \frac{\log\left(ac+\frac{bc}{x}\right)^2}{ax+b}\,dx + x\log\left(\frac{c(ax+b)}{x}\right)^3$$

```
[In] integrate(ln(c*(a*x+b)/x)**3,x)
[Out] 3*b*Integral(log(a*c + b*c/x)**2/(a*x + b), x) + x*log(c*(a*x + b)/x)**3
```

Maxima [F]

$$\int \log^3\left(\frac{c(b+ax)}{x}\right)\,dx = \int \log\left(\frac{(ax+b)c}{x}\right)^3\,dx$$

```
[In] integrate(log(c*(a*x+b)/x)^3,x, algorithm="maxima")
```

```
[Out] ((a*x + b)*log(a*x + b)^3 + 3*(a*x*log(c) - a*x*log(x))*log(a*x + b)^2)/a +
 integrate((a*x*log(c)^3 + b*log(c)^3 - (a*x + b)*log(x)^3 + 3*(a*x*log(c)
+ b*log(c))*log(x)^2 + 3*((log(c)^2 - 2*log(c))*a*x + b*log(c)^2 + (a*x + b
)*log(x)^2 - 2*(a*x*(log(c) - 1) + b*log(c))*log(x))*log(a*x + b) - 3*(a*x*
log(c)^2 + b*log(c)^2)*log(x))/(a*x + b), x)
```

Giac [F]

$$\int \log^3\left(\frac{c(b+ax)}{x}\right)\,dx = \int \log\left(\frac{(ax+b)c}{x}\right)^3\,dx$$

```
[In] integrate(log(c*(a*x+b)/x)^3,x, algorithm="giac")
```

```
[Out] integrate(log((a*x + b)*c/x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \log^3\left(\frac{c(b+ax)}{x}\right)\,dx = \int \ln\left(\frac{c\,(b+a\,x)}{x}\right)^3\,dx$$

```
[In] int(log((c*(b + a*x))/x)^3,x)
```

```
[Out] int(log((c*(b + a*x))/x)^3, x)
```

3.96 $\int \log \left(\frac{c(b+ax)^2}{x^2}\right) dx$

Optimal result . . . 764
Rubi [A] (verified) . . . 764
Mathematica [A] (verified) . . . 765
Maple [A] (verified) . . . 765
Fricas [A] (verification not implemented) . . . 766
Sympy [A] (verification not implemented) . . . 766
Maxima [A] (verification not implemented) . . . 766
Giac [A] (verification not implemented) . . . 767
Mupad [B] (verification not implemented) . . . 767

Optimal result

Integrand size = 13, antiderivative size = 28

$$\int \log \left(\frac{c(b+ax)^2}{x^2}\right) dx = \frac{2b\log(b+ax)}{a} + x\log\left(\frac{c(b+ax)^2}{x^2}\right)$$

```
[Out] 2*b*ln(a*x+b)/a+x*ln(c*(a*x+b)^2/x^2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2536, 31}

$$\int \log \left(\frac{c(b+ax)^2}{x^2}\right) dx = x\log\left(\frac{c(ax+b)^2}{x^2}\right) + \frac{2b\log(ax+b)}{a}$$

```
[In] Int[Log[(c*(b + a*x)^2)/x^2],x]
```

```
[Out] (2*b*Log[b + a*x])/a + x*Log[(c*(b + a*x)^2)/x^2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2536

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)^n/(c
+ d*x)^n)])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b
*x)^n/(c + d*x)^n)])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A,
```

```
B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x\log\left(\frac{c(b+ax)^2}{x^2}\right)+(2b)\int\frac{1}{b+ax}\,dx\\ &= \frac{2b\log(b+ax)}{a}+x\log\left(\frac{c(b+ax)^2}{x^2}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int\log\left(\frac{c(b+ax)^2}{x^2}\right)\,dx=\frac{2b\log(b+ax)}{a}+x\log\left(\frac{c(b+ax)^2}{x^2}\right)$$

```
[In] Integrate[Log[(c*(b + a*x)^2)/x^2],x]
[Out] (2*b*Log[b + a*x])/a + x*Log[(c*(b + a*x)^2)/x^2]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

method	result	size
risch	$\frac{2b\ln(ax+b)}{a}+x\ln\left(\frac{c(ax+b)^2}{x^2}\right)$	29
parts	$\frac{2b\ln(ax+b)}{a}+x\ln\left(\frac{c(ax+b)^2}{x^2}\right)$	29
derivativedivides	$x\ln\left(c\left(a+\frac{b}{x}\right)^2\right)-2b\left(\frac{\ln\left(\frac{1}{x}\right)}{a}-\frac{\ln\left(a+\frac{b}{x}\right)}{a}\right)$	41
default	$x\ln\left(c\left(a+\frac{b}{x}\right)^2\right)-2b\left(\frac{\ln\left(\frac{1}{x}\right)}{a}-\frac{\ln\left(a+\frac{b}{x}\right)}{a}\right)$	41
parallelrisch	$-\frac{-2\ln\left(\frac{c(ax+b)^2}{x^2}\right)xa-4\ln(x)b-2b\ln\left(\frac{c(ax+b)^2}{x^2}\right)}{2a}$	45

```
[In] int(ln(c*(a*x+b)^2/x^2),x,method=_RETURNVERBOSE)
[Out] 2*b*ln(a*x+b)/a+x*ln(c*(a*x+b)^2/x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.50

$$\int \log \left(\frac{c(b+ax)^2}{x^2}\right) dx = \frac{ax \log \left(\frac{a^2cx^2+2\,abcx+b^2c}{x^2}\right) + 2\,b \log (ax+b)}{a}$$

```
[In] integrate(log(c*(a*x+b)^2/x^2),x, algorithm="fricas")
[Out] (a*x*log((a^2*c*x^2 + 2*a*b*c*x + b^2*c)/x^2) + 2*b*log(a*x + b))/a
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \log \left(\frac{c(b+ax)^2}{x^2}\right) dx = x \log \left(\frac{c(ax+b)^2}{x^2}\right) + \frac{2b \log (ax+b)}{a}$$

```
[In] integrate(ln(c*(a*x+b)**2/x**2),x)
[Out] x*log(c*(a*x + b)**2/x**2) + 2*b*log(a*x + b)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \log \left(\frac{c(b+ax)^2}{x^2}\right) dx = x \log \left(\frac{(ax+b)^2c}{x^2}\right) + \frac{2\,b \log (ax+b)}{a}$$

```
[In] integrate(log(c*(a*x+b)^2/x^2),x, algorithm="maxima")
[Out] x*log((a*x + b)^2*c/x^2) + 2*b*log(a*x + b)/a
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \log\left(\frac{c(b+ax)^2}{x^2}\right)\,dx = x\log\left(\frac{(ax+b)^2c}{x^2}\right) + \frac{2\,b\log\left(|ax+b|\right)}{a}$$

```
[In] integrate(log(c*(a*x+b)^2/x^2),x, algorithm="giac")
[Out] x*log((a*x + b)^2*c/x^2) + 2*b*log(abs(a*x + b))/a
```

Mupad [B] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \log\left(\frac{c(b+ax)^2}{x^2}\right)\,dx = x\,\ln\left(\frac{c\,(b+a\,x)^2}{x^2}\right) + \frac{2\,b\,\ln\left(b+a\,x\right)}{a}$$

```
[In] int(log((c*(b + a*x)^2)/x^2),x)
[Out] x*log((c*(b + a*x)^2)/x^2) + (2*b*log(b + a*x))/a
```

3.97 $\int \log^2\left(\frac{c(b+ax)^2}{x^2}\right)\,dx$

Optimal result 768
Rubi [A] (verified) 768
Mathematica [A] (verified) 770
Maple [F] 771
Fricas [F] 771
Sympy [F] 771
Maxima [A] (verification not implemented) 771
Giac [F] 772
Mupad [F(-1)] 772

Optimal result

Integrand size = 15, antiderivative size = 67

$$\int \log^2\left(\frac{c(b+ax)^2}{x^2}\right)\,dx = -\frac{4b\log\left(\frac{b}{b+ax}\right)\log\left(\frac{c(b+ax)^2}{x^2}\right)}{a} + x\log^2\left(\frac{c(b+ax)^2}{x^2}\right) + \frac{8b\,\mathrm{PolyLog}\left(2,1-\frac{b}{b+ax}\right)}{a}$$

```
[Out] -4*b*ln(b/(a*x+b))*ln(c*(a*x+b)^2/x^2)/a+x*ln(c*(a*x+b)^2/x^2)^2+8*b*polylo
g(2,1-b/(a*x+b))/a
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2536, 2542, 2458, 2378, 2370, 2352}

$$\int \log^2\left(\frac{c(b+ax)^2}{x^2}\right)\,dx = x\log^2\left(\frac{c(ax+b)^2}{x^2}\right) - \frac{4b\log\left(\frac{b}{ax+b}\right)\log\left(\frac{c(ax+b)^2}{x^2}\right)}{a} + \frac{8b\,\mathrm{PolyLog}\left(2,1-\frac{b}{b+ax}\right)}{a}$$

```
[In] Int[Log[(c*(b + a*x)^2)/x^2]^2,x]
```

```
[Out] (-4*b*Log[b/(b + a*x)]*Log[(c*(b + a*x)^2)/x^2])/a + x*Log[(c*(b + a*x)^2)/
x^2]^2 + (8*b*PolyLog[2, 1 - b/(b + a*x)])/a
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2370

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2378

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2536

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)^n/(c
+ d*x)^n)])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b
*x)^n/(c + d*x)^n)])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A,
B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rule 2542

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[-(b*c - a*d)/(d*(a
+ b*x))])*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/g), x] + Dist[B*n*((b*c
 - a*d)/g), Int[Log[-(b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)), x],
x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
 - a*d, 0] && EqQ[b*f - a*g, 0]
```

Rubi steps

$$\text{integral} = x\log^2\left(\frac{c(b+ax)^2}{x^2}\right) + (4b)\int \frac{\log\left(\frac{c(b+ax)^2}{x^2}\right)}{b+ax}\,dx$$

$$= -\frac{4b\log\left(\frac{b}{b+ax}\right)\log\left(\frac{c(b+ax)^2}{x^2}\right)}{a} + x\log^2\left(\frac{c(b+ax)^2}{x^2}\right) - \frac{(8b^2)\int\frac{\log\left(\frac{b}{b+ax}\right)}{x(b+ax)}\,dx}{a}$$

$$= -\frac{4b\log\left(\frac{b}{b+ax}\right)\log\left(\frac{c(b+ax)^2}{x^2}\right)}{a} + x\log^2\left(\frac{c(b+ax)^2}{x^2}\right) - \frac{(8b^2)\,\text{Subst}\left(\int\frac{\log\left(\frac{b}{x}\right)}{x\left(-\frac{b}{a}+\frac{x}{a}\right)}\,dx, x, b+ax\right)}{a^2}$$

$$= -\frac{4b\log\left(\frac{b}{b+ax}\right)\log\left(\frac{c(b+ax)^2}{x^2}\right)}{a} + x\log^2\left(\frac{c(b+ax)^2}{x^2}\right) + \frac{(8b^2)\,\text{Subst}\left(\int\frac{\log(bx)}{\left(-\frac{b}{a}+\frac{1}{ax}\right)x}\,dx, x, \frac{1}{b+ax}\right)}{a^2}$$

$$= -\frac{4b\log\left(\frac{b}{b+ax}\right)\log\left(\frac{c(b+ax)^2}{x^2}\right)}{a} + x\log^2\left(\frac{c(b+ax)^2}{x^2}\right) + \frac{(8b^2)\,\text{Subst}\left(\int\frac{\log(bx)}{\frac{1}{a}-\frac{bx}{a}}\,dx, x, \frac{1}{b+ax}\right)}{a^2}$$

$$= -\frac{4b\log\left(\frac{b}{b+ax}\right)\log\left(\frac{c(b+ax)^2}{x^2}\right)}{a} + x\log^2\left(\frac{c(b+ax)^2}{x^2}\right) + \frac{8b\text{Li}_2\left(\frac{ax}{b+ax}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.58

$$\int\log^2\left(\frac{c(b+ax)^2}{x^2}\right)\,dx = -\frac{8b\log\left(-\frac{ax}{b}\right)\log\left(\frac{b}{b+ax}\right)}{a} - \frac{4b\log^2\left(\frac{b}{b+ax}\right)}{a} - \frac{4b\log\left(\frac{b}{b+ax}\right)\log\left(\frac{c(b+ax)^2}{x^2}\right)}{a} + x\log^2\left(\frac{c(b+ax)^2}{x^2}\right) + \frac{8b\,\text{PolyLog}\left(2,\frac{b+ax}{b}\right)}{a}$$

```
[In] Integrate[Log[(c*(b + a*x)^2)/x^2]^2,x]
```

```
[Out] (-8*b*Log[-((a*x)/b)]*Log[b/(b + a*x)])/a - (4*b*Log[b/(b + a*x)]^2)/a - (4
*b*Log[b/(b + a*x)]*Log[(c*(b + a*x)^2)/x^2])/a + x*Log[(c*(b + a*x)^2)/x^2
]^2 + (8*b*PolyLog[2, (b + a*x)/b])/a
```

Maple [F]

$$\int \ln \left(\frac{c(ax+b)^2}{x^2}\right)^2 dx$$

```
[In] int(ln(c*(a*x+b)^2/x^2)^2,x)
[Out] int(ln(c*(a*x+b)^2/x^2)^2,x)
```

Fricas [F]

$$\int \log^2\left(\frac{c(b+ax)^2}{x^2}\right) dx = \int \log \left(\frac{(ax+b)^2c}{x^2}\right)^2 dx$$

```
[In] integrate(log(c*(a*x+b)^2/x^2)^2,x, algorithm="fricas")
[Out] integral(log((a^2*c*x^2 + 2*a*b*c*x + b^2*c)/x^2)^2, x)
```

Sympy [F]

$$\int \log^2\left(\frac{c(b+ax)^2}{x^2}\right) dx = 4b\int \frac{\log \left(a^2c+\frac{2abc}{x}+\frac{b^2c}{x^2}\right)}{ax+b}\, dx + x\log \left(\frac{c(ax+b)^2}{x^2}\right)^2$$

```
[In] integrate(ln(c*(a*x+b)**2/x**2)**2,x)
[Out] 4*b*Integral(log(a**2*c + 2*a*b*c/x + b**2*c/x**2)/(a*x + b), x) + x*log(c*
(a*x + b)**2/x**2)**2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.76

$$\begin{aligned}
&\int \log^2\left(\frac{c(b+ax)^2}{x^2}\right) dx \\
&= x\log \left(\frac{(ax+b)^2c}{x^2}\right)^2 + \frac{4\,b\log\left(ax+b\right)\log\left(\frac{(ax+b)^2c}{x^2}\right)}{a} \\
&\quad + \frac{4\left(\left(\frac{c\log(ax+b)^2}{a} - \frac{2\left(\log\left(\frac{ax}{b}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{ax}{b}\right)\right)c}{a}\right)b - \frac{2\left(c\log(ax+b)-c\log(x)\right)b\log(ax+b)}{a}\right)}{c}
\end{aligned}$$

```
[In] integrate(log(c*(a*x+b)^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] x*log((a*x + b)^2*c/x^2)^2 + 4*b*log(a*x + b)*log((a*x + b)^2*c/x^2)/a + 4*
((c*log(a*x + b)^2/a - 2*(log(a*x/b + 1)*log(x) + dilog(-a*x/b))*c/a)*b - 2
*(c*log(a*x + b) - c*log(x))*b*log(a*x + b)/a)/c
```

Giac [F]

$$\int \log^2\left(\frac{c(b+ax)^2}{x^2}\right)\,dx = \int \log\left(\frac{(ax+b)^2c}{x^2}\right)^2\,dx$$

```
[In] integrate(log(c*(a*x+b)^2/x^2)^2,x, algorithm="giac")
```

```
[Out] integrate(log((a*x + b)^2*c/x^2)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \log^2\left(\frac{c(b+ax)^2}{x^2}\right)\,dx = \int \ln\left(\frac{c\,(b+a\,x)^2}{x^2}\right)^2\,dx$$

```
[In] int(log((c*(b + a*x)^2)/x^2)^2,x)
```

```
[Out] int(log((c*(b + a*x)^2)/x^2)^2, x)
```

3.98 $\int \log^3\left(\frac{c(b+ax)^2}{x^2}\right)\,dx$

Optimal result 773
Rubi [A] (verified) 773
Mathematica [A] (verified) 775
Maple [F] 775
Fricas [F] 776
Sympy [F] 776
Maxima [F] 776
Giac [F] 777
Mupad [F(-1)] 777

Optimal result

Integrand size = 15, antiderivative size = 102

$$\int \log^3\left(\frac{c(b+ax)^2}{x^2}\right)\,dx = x\log^3\left(\frac{c(b+ax)^2}{x^2}\right)-\frac{6b\log^2\left(\frac{c(b+ax)^2}{x^2}\right)\log\left(1-\frac{ax}{b+ax}\right)}{a}+\frac{24b\log\left(\frac{c(b+ax)^2}{x^2}\right)\operatorname{PolyLog}\left(2,\frac{ax}{b+ax}\right)}{a}+\frac{48b\operatorname{PolyLog}\left(3,\frac{ax}{b+ax}\right)}{a}$$

```
[Out] x*ln(c*(a*x+b)^2/x^2)^3-6*b*ln(c*(a*x+b)^2/x^2)^2*ln(1-a*x/(a*x+b))/a+24*b*
ln(c*(a*x+b)^2/x^2)*polylog(2,a*x/(a*x+b))/a+48*b*polylog(3,a*x/(a*x+b))/a
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2536, 2550, 2379, 2421, 6724}

$$\int \log^3\left(\frac{c(b+ax)^2}{x^2}\right)\,dx = \frac{24b\operatorname{PolyLog}\left(2,\frac{ax}{b+ax}\right)\log\left(\frac{c(ax+b)^2}{x^2}\right)}{a}+x\log^3\left(\frac{c(ax+b)^2}{x^2}\right)-\frac{6b\log\left(1-\frac{ax}{ax+b}\right)\log^2\left(\frac{c(ax+b)^2}{x^2}\right)}{a}+\frac{48b\operatorname{PolyLog}\left(3,\frac{ax}{b+ax}\right)}{a}$$

```
[In] Int[Log[(c*(b + a*x)^2)/x^2]^3,x]
```

```
[Out] x*Log[(c*(b + a*x)^2)/x^2]^3 - (6*b*Log[(c*(b + a*x)^2)/x^2]^2*Log[1 - (a*x
)/(b + a*x)])/a + (24*b*Log[(c*(b + a*x)^2)/x^2]*PolyLog[2, (a*x)/(b + a*x)
])/a + (48*b*PolyLog[3, (a*x)/(b + a*x)])/a
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2536

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)^n/(c
+ d*x)^n)])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b
*x)^n/(c + d*x)^n)])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A,
B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x\log^3\left(\frac{c(b+ax)^2}{x^2}\right)+(6b)\int\frac{\log^2\left(\frac{c(b+ax)^2}{x^2}\right)}{b+ax}\,dx\\ &= x\log^3\left(\frac{c(b+ax)^2}{x^2}\right)+(6b)\text{Subst}\left(\int\frac{\log^2\left(cx^2\right)}{(a-x)x}\,dx,x,\frac{b+ax}{x}\right)\end{aligned}$$

$$= x\log^3\left(\frac{c(b+ax)^2}{x^2}\right) - \frac{6b\log^2\left(\frac{c(b+ax)^2}{x^2}\right)\log\left(1-\frac{ax}{b+ax}\right)}{a} + \frac{(24b)\text{Subst}\left(\int \frac{\log\left(1-\frac{a}{x}\right)\log(cx^2)}{x}\,dx, x, \frac{b+ax}{x}\right)}{a}$$

$$= x\log^3\left(\frac{c(b+ax)^2}{x^2}\right) - \frac{6b\log^2\left(\frac{c(b+ax)^2}{x^2}\right)\log\left(1-\frac{ax}{b+ax}\right)}{a} + \frac{24b\log\left(\frac{c(b+ax)^2}{x^2}\right)\text{Li}_2\left(\frac{ax}{b+ax}\right)}{a} - \frac{(48b)\text{Subst}\left(\int \frac{\text{Li}_2\left(\frac{a}{x}\right)}{x}\,dx, x, \frac{b+ax}{x}\right)}{a}$$

$$= x\log^3\left(\frac{c(b+ax)^2}{x^2}\right) - \frac{6b\log^2\left(\frac{c(b+ax)^2}{x^2}\right)\log\left(1-\frac{ax}{b+ax}\right)}{a} + \frac{24b\log\left(\frac{c(b+ax)^2}{x^2}\right)\text{Li}_2\left(\frac{ax}{b+ax}\right)}{a} + \frac{48b\text{Li}_3\left(\frac{ax}{b+ax}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.96

$$\int \log^3\left(\frac{c(b+ax)^2}{x^2}\right)\,dx = -\frac{6b\log\left(\frac{b}{b+ax}\right)\log^2\left(\frac{c(b+ax)^2}{x^2}\right)}{a} + x\log^3\left(\frac{c(b+ax)^2}{x^2}\right) + \frac{24b\log\left(\frac{c(b+ax)^2}{x^2}\right)\text{PolyLog}\left(2,\frac{ax}{b+ax}\right)}{a} + \frac{48b\,\text{PolyLog}\left(3,\frac{ax}{b+ax}\right)}{a}$$

```
[In] Integrate[Log[(c*(b + a*x)^2)/x^2]^3,x]
```

```
[Out] (-6*b*Log[b/(b + a*x)]*Log[(c*(b + a*x)^2)/x^2]^2)/a + x*Log[(c*(b + a*x)^2
)/x^2]^3 + (24*b*Log[(c*(b + a*x)^2)/x^2]*PolyLog[2, (a*x)/(b + a*x)])/a +
(48*b*PolyLog[3, (a*x)/(b + a*x)])/a
```

Maple [F]

$$\int \ln\left(\frac{c(ax+b)^2}{x^2}\right)^3 dx$$

```
[In] int(ln(c*(a*x+b)^2/x^2)^3,x)
```

```
[Out] int(ln(c*(a*x+b)^2/x^2)^3,x)
```

Fricas [F]

$$\int \log^3\left(\frac{c(b+ax)^2}{x^2}\right)\,dx = \int \log\left(\frac{(ax+b)^2c}{x^2}\right)^3\,dx$$

```
[In] integrate(log(c*(a*x+b)^2/x^2)^3,x, algorithm="fricas")
[Out] integral(log((a^2*c*x^2 + 2*a*b*c*x + b^2*c)/x^2)^3, x)
```

Sympy [F]

$$\int \log^3\left(\frac{c(b+ax)^2}{x^2}\right)\,dx = 6b\int \frac{\log\left(a^2c+\frac{2abc}{x}+\frac{b^2c}{x^2}\right)^2}{ax+b}\,dx + x\log\left(\frac{c(ax+b)^2}{x^2}\right)^3$$

```
[In] integrate(ln(c*(a*x+b)**2/x**2)**3,x)
[Out] 6*b*Integral(log(a**2*c + 2*a*b*c/x + b**2*c/x**2)**2/(a*x + b), x) + x*log
(c*(a*x + b)**2/x**2)**3
```

Maxima [F]

$$\int \log^3\left(\frac{c(b+ax)^2}{x^2}\right)\,dx = \int \log\left(\frac{(ax+b)^2c}{x^2}\right)^3\,dx$$

```
[In] integrate(log(c*(a*x+b)^2/x^2)^3,x, algorithm="maxima")
[Out] 4*(2*(a*x + b)*log(a*x + b)^3 + 3*(a*x*log(c) - 2*a*x*log(x))*log(a*x + b)^
2)/a + integrate((a*x*log(c)^3 + b*log(c)^3 - 8*(a*x + b)*log(x)^3 + 12*(a*
x*log(c) + b*log(c))*log(x)^2 + 6*((log(c)^2 - 4*log(c))*a*x + b*log(c)^2 +
 4*(a*x + b)*log(x)^2 - 4*(a*x*(log(c) - 2) + b*log(c))*log(x))*log(a*x + b
) - 6*(a*x*log(c)^2 + b*log(c)^2)*log(x))/(a*x + b), x)
```

Giac [F]

$$\int \log^3\left(\frac{c(b+ax)^2}{x^2}\right)\,dx = \int \log\left(\frac{(ax+b)^2c}{x^2}\right)^3\,dx$$

```
[In] integrate(log(c*(a*x+b)^2/x^2)^3,x, algorithm="giac")
[Out] integrate(log((a*x + b)^2*c/x^2)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \log^3\left(\frac{c(b+ax)^2}{x^2}\right)\,dx = \int \ln\left(\frac{c\,(b+a\,x)^2}{x^2}\right)^3\,dx$$

```
[In] int(log((c*(b + a*x)^2)/x^2)^3,x)
[Out] int(log((c*(b + a*x)^2)/x^2)^3, x)
```

3.99 $\int \log\left(\frac{cx^2}{(b+ax)^2}\right) dx$

Optimal result . . . 778
Rubi [A] (verified) . . . 778
Mathematica [A] (verified) . . . 779
Maple [A] (verified) . . . 779
Fricas [A] (verification not implemented) . . . 780
Sympy [A] (verification not implemented) . . . 780
Maxima [A] (verification not implemented) . . . 780
Giac [A] (verification not implemented) . . . 780
Mupad [B] (verification not implemented) . . . 781

Optimal result

Integrand size = 13, antiderivative size = 28

$$\int \log\left(\frac{cx^2}{(b+ax)^2}\right) dx = x\log\left(\frac{cx^2}{(b+ax)^2}\right) - \frac{2b\log(b+ax)}{a}$$

```
[Out] x*ln(c*x^2/(a*x+b)^2)-2*b*ln(a*x+b)/a
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2536, 31}

$$\int \log\left(\frac{cx^2}{(b+ax)^2}\right) dx = x\log\left(\frac{cx^2}{(ax+b)^2}\right) - \frac{2b\log(ax+b)}{a}$$

```
[In] Int[Log[(c*x^2)/(b + a*x)^2],x]
```

```
[Out] x*Log[(c*x^2)/(b + a*x)^2] - (2*b*Log[b + a*x])/a
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2536

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)^n/(c
+ d*x)^n)])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b
*x)^n/(c + d*x)^n)])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A,
```

```
B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x\log\left(\frac{cx^2}{(b+ax)^2}\right)-(2b)\int\frac{1}{b+ax}\,dx\\ &= x\log\left(\frac{cx^2}{(b+ax)^2}\right)-\frac{2b\log(b+ax)}{a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int\log\left(\frac{cx^2}{(b+ax)^2}\right)\,dx = x\log\left(\frac{cx^2}{(b+ax)^2}\right)-\frac{2b\log(b+ax)}{a}$$

```
[In] Integrate[Log[(c*x^2)/(b + a*x)^2],x]
[Out] x*Log[(c*x^2)/(b + a*x)^2] - (2*b*Log[b + a*x])/a
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

method	result	size
risch	$x\ln\left(\frac{cx^2}{(ax+b)^2}\right)-\frac{2b\ln(ax+b)}{a}$	29
parts	$x\ln\left(\frac{cx^2}{(ax+b)^2}\right)-\frac{2b\ln(ax+b)}{a}$	29
parallelrisch	$-\frac{-2\ln\left(\frac{cx^2}{(ax+b)^2}\right)xab+4\ln(x)b^2-2b^2\ln\left(\frac{cx^2}{(ax+b)^2}\right)}{2ab}$	53
derivativedivides	$-\frac{-(ax+b)\ln\left(\frac{c\left(\frac{b}{ax+b}-1\right)^2}{a^2}\right)+2b\left(-\ln\left(\frac{1}{ax+b}\right)+\ln\left(\frac{b}{ax+b}-1\right)\right)}{a}$	59
default	$-\frac{-(ax+b)\ln\left(\frac{c\left(\frac{b}{ax+b}-1\right)^2}{a^2}\right)+2b\left(-\ln\left(\frac{1}{ax+b}\right)+\ln\left(\frac{b}{ax+b}-1\right)\right)}{a}$	59

```
[In] int(ln(c*x^2/(a*x+b)^2),x,method=_RETURNVERBOSE)
[Out] x*ln(c*x^2/(a*x+b)^2)-2*b*ln(a*x+b)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.46

$$\int \log \left(\frac{cx^2}{(b+ax)^2}\right) dx = \frac{ax \log \left(\frac{cx^2}{a^2x^2+2\,abx+b^2}\right) - 2\,b \log (ax+b)}{a}$$

```
[In] integrate(log(c*x^2/(a*x+b)^2),x, algorithm="fricas")
[Out] (a*x*log(c*x^2/(a^2*x^2 + 2*a*b*x + b^2)) - 2*b*log(a*x + b))/a
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \log \left(\frac{cx^2}{(b+ax)^2}\right) dx = x \log \left(\frac{cx^2}{(ax+b)^2}\right) - \frac{2b \log (ax+b)}{a}$$

```
[In] integrate(ln(c*x**2/(a*x+b)**2),x)
[Out] x*log(c*x**2/(a*x + b)**2) - 2*b*log(a*x + b)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \log \left(\frac{cx^2}{(b+ax)^2}\right) dx = x \log \left(\frac{cx^2}{(ax+b)^2}\right) - \frac{2\,b \log (ax+b)}{a}$$

```
[In] integrate(log(c*x^2/(a*x+b)^2),x, algorithm="maxima")
[Out] x*log(c*x^2/(a*x + b)^2) - 2*b*log(a*x + b)/a
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \log \left(\frac{cx^2}{(b+ax)^2}\right) dx = x \log \left(\frac{cx^2}{(ax+b)^2}\right) - \frac{2\,b \log (|ax+b|)}{a}$$

```
[In] integrate(log(c*x^2/(a*x+b)^2),x, algorithm="giac")
[Out] x*log(c*x^2/(a*x + b)^2) - 2*b*log(abs(a*x + b))/a
```

Mupad [B] (verification not implemented)

Time = 1.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \log \left(\frac{cx^2}{(b+ax)^2}\right) \, dx = x \ln \left(\frac{c\,x^2}{(b+a\,x)^2}\right) - \frac{2\,b\,\ln (b+a\,x)}{a}$$

```
[In] int(log((c*x^2)/(b + a*x)^2),x)
[Out] x*log((c*x^2)/(b + a*x)^2) - (2*b*log(b + a*x))/a
```

3.100 $\int \log^2\left(\frac{cx^2}{(b+ax)^2}\right) dx$

Optimal result . . . 782
Rubi [A] (verified) . . . 782
Mathematica [A] (verified) . . . 784
Maple [F] . . . 784
Fricas [F] . . . 785
Sympy [F] . . . 785
Maxima [A] (verification not implemented) . . . 785
Giac [F] . . . 786
Mupad [F(-1)] . . . 786

Optimal result

Integrand size = 15, antiderivative size = 67

$$\int \log^2\left(\frac{cx^2}{(b+ax)^2}\right) dx = x\log^2\left(\frac{cx^2}{(b+ax)^2}\right) + \frac{4b\log\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(\frac{b}{b+ax}\right)}{a} + \frac{8b\,\text{PolyLog}\left(2,1-\frac{b}{b+ax}\right)}{a}$$

```
[Out] x*ln(c*x^2/(a*x+b)^2)^2+4*b*ln(c*x^2/(a*x+b)^2)*ln(b/(a*x+b))/a+8*b*polylog
(2,1-b/(a*x+b))/a
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2536, 2542, 2458, 2378, 2370, 2352}

$$\int \log^2\left(\frac{cx^2}{(b+ax)^2}\right) dx = x\log^2\left(\frac{cx^2}{(ax+b)^2}\right) + \frac{4b\log\left(\frac{b}{ax+b}\right)\log\left(\frac{cx^2}{(ax+b)^2}\right)}{a} + \frac{8b\,\text{PolyLog}\left(2,1-\frac{b}{b+ax}\right)}{a}$$

```
[In] Int[Log[(c*x^2)/(b + a*x)^2]^2,x]
```

```
[Out] x*Log[(c*x^2)/(b + a*x)^2]^2 + (4*b*Log[(c*x^2)/(b + a*x)^2]*Log[b/(b + a*x
)])/a + (8*b*PolyLog[2, 1 - b/(b + a*x)])/a
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2370

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2378

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2536

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)^n/(c
+ d*x)^n)])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b
*x)^n/(c + d*x)^n)])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A,
B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rule 2542

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[-(b*c - a*d)/(d*(a
+ b*x))])*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/g), x] + Dist[B*n*((b*c
 - a*d)/g), Int[Log[-(b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)), x],
x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
 - a*d, 0] && EqQ[b*f - a*g, 0]
```

Rubi steps

$$\text{integral} = x\log^2\left(\frac{cx^2}{(b+ax)^2}\right) - (4b)\int \frac{\log\left(\frac{cx^2}{(b+ax)^2}\right)}{b+ax}\,dx$$

$$= x\log^2\left(\frac{cx^2}{(b+ax)^2}\right)+\frac{4b\log\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(\frac{b}{b+ax}\right)}{a}-\frac{(8b^2)\int\frac{\log\left(\frac{b}{b+ax}\right)}{x(b+ax)}dx}{a}$$

$$= x\log^2\left(\frac{cx^2}{(b+ax)^2}\right)+\frac{4b\log\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(\frac{b}{b+ax}\right)}{a}-\frac{(8b^2)\,\mathrm{Subst}\left(\int\frac{\log\left(\frac{b}{x}\right)}{x\left(-\frac{b}{a}+\frac{x}{a}\right)}dx,x,b+ax\right)}{a^2}$$

$$= x\log^2\left(\frac{cx^2}{(b+ax)^2}\right)+\frac{4b\log\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(\frac{b}{b+ax}\right)}{a}+\frac{(8b^2)\,\mathrm{Subst}\left(\int\frac{\log(bx)}{\left(-\frac{b}{a}+\frac{1}{ax}\right)x}dx,x,\frac{1}{b+ax}\right)}{a^2}$$

$$= x\log^2\left(\frac{cx^2}{(b+ax)^2}\right)+\frac{4b\log\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(\frac{b}{b+ax}\right)}{a}+\frac{(8b^2)\,\mathrm{Subst}\left(\int\frac{\log(bx)}{\frac{1}{a}-\frac{bx}{a}}dx,x,\frac{1}{b+ax}\right)}{a^2}$$

$$= x\log^2\left(\frac{cx^2}{(b+ax)^2}\right)+\frac{4b\log\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(\frac{b}{b+ax}\right)}{a}+\frac{8b\mathrm{Li}_2\left(\frac{ax}{b+ax}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.43

$$\int\log^2\left(\frac{cx^2}{(b+ax)^2}\right)dx$$
$$=\frac{ax\log^2\left(\frac{cx^2}{(b+ax)^2}\right)+4b\log\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(\frac{b}{b+ax}\right)-4b\log\left(\frac{b}{b+ax}\right)\left(2\log\left(-\frac{ax}{b}\right)+\log\left(\frac{b}{b+ax}\right)\right)+8b\,\mathrm{PolyLog}(2,1}{a}$$

```
[In] Integrate[Log[(c*x^2)/(b + a*x)^2]^2,x]
```

```
[Out] (a*x*Log[(c*x^2)/(b + a*x)^2]^2 + 4*b*Log[(c*x^2)/(b + a*x)^2]*Log[b/(b + a
*x)] - 4*b*Log[b/(b + a*x)]*(2*Log[-((a*x)/b)] + Log[b/(b + a*x)]) + 8*b*Po
lyLog[2, 1 + (a*x)/b])/a
```

Maple [F]

$$\int\ln\left(\frac{c\,x^2}{(ax+b)^2}\right)^2dx$$

```
[In] int(ln(c*x^2/(a*x+b)^2)^2,x)
```

```
[Out] int(ln(c*x^2/(a*x+b)^2)^2,x)
```

Fricas [F]

$$\int \log^2\left(\frac{cx^2}{(b+ax)^2}\right)\,dx=\int \log\left(\frac{cx^2}{(ax+b)^2}\right)^2\,dx$$

```
[In] integrate(log(c*x^2/(a*x+b)^2)^2,x, algorithm="fricas")
[Out] integral(log(c*x^2/(a^2*x^2 + 2*a*b*x + b^2))^2, x)
```

Sympy [F]

$$\int \log^2\left(\frac{cx^2}{(b+ax)^2}\right)\,dx=-4b\int \frac{\log\left(\frac{cx^2}{a^2x^2+2abx+b^2}\right)}{ax+b}\,dx+x\log\left(\frac{cx^2}{(ax+b)^2}\right)^2$$

```
[In] integrate(ln(c*x**2/(a*x+b)**2)**2,x)
[Out] -4*b*Integral(log(c*x**2/(a**2*x**2 + 2*a*b*x + b**2))/(a*x + b), x) + x*lo
g(c*x**2/(a*x + b)**2)**2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.76

$$\begin{aligned}&\int \log^2\left(\frac{cx^2}{(b+ax)^2}\right)\,dx\\&=x\log\left(\frac{cx^2}{(ax+b)^2}\right)^2-\frac{4\,b\log(ax+b)\log\left(\frac{cx^2}{(ax+b)^2}\right)}{a}\\&\quad+\frac{4\left(\left(\frac{c\log(ax+b)^2}{a}-\frac{2\left(\log\left(\frac{ax}{b}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{ax}{b}\right)\right)c}{a}\right)b-\frac{2\,(c\log(ax+b)-c\log(x))b\log(ax+b)}{a}\right)}{c}\end{aligned}$$

```
[In] integrate(log(c*x^2/(a*x+b)^2)^2,x, algorithm="maxima")
[Out] x*log(c*x^2/(a*x + b)^2)^2 - 4*b*log(a*x + b)*log(c*x^2/(a*x + b)^2)/a + 4*
((c*log(a*x + b)^2/a - 2*(log(a*x/b + 1)*log(x) + dilog(-a*x/b))*c/a)*b - 2
*(c*log(a*x + b) - c*log(x))*b*log(a*x + b)/a)/c
```

Giac [F]

$$\int \log^2\left(\frac{cx^2}{(b+ax)^2}\right)\,dx = \int \log\left(\frac{cx^2}{(ax+b)^2}\right)^2\,dx$$

```
[In] integrate(log(c*x^2/(a*x+b)^2)^2,x, algorithm="giac")
[Out] integrate(log(c*x^2/(a*x + b)^2)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \log^2\left(\frac{cx^2}{(b+ax)^2}\right)\,dx = \int \ln\left(\frac{c\,x^2}{(b+a\,x)^2}\right)^2\,dx$$

```
[In] int(log((c*x^2)/(b + a*x)^2)^2,x)
[Out] int(log((c*x^2)/(b + a*x)^2)^2, x)
```

3.101 $\int \log^3\left(\frac{cx^2}{(b+ax)^2}\right) dx$

Optimal result 787
Rubi [A] (verified) 787
Mathematica [A] (verified) 789
Maple [F] 789
Fricas [F] 790
Sympy [F] 790
Maxima [F] 790
Giac [F] 790
Mupad [F(-1)] 791

Optimal result

Integrand size = 15, antiderivative size = 98

$$\int \log^3\left(\frac{cx^2}{(b+ax)^2}\right) dx = x\log^3\left(\frac{cx^2}{(b+ax)^2}\right) + \frac{6b\log^2\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(\frac{b}{b+ax}\right)}{a} + \frac{24b\log\left(\frac{cx^2}{(b+ax)^2}\right)\text{PolyLog}\left(2,\frac{ax}{b+ax}\right)}{a} - \frac{48b\,\text{PolyLog}\left(3,\frac{ax}{b+ax}\right)}{a}$$

```
[Out] x*ln(c*x^2/(a*x+b)^2)^3+6*b*ln(c*x^2/(a*x+b)^2)^2*ln(b/(a*x+b))/a+24*b*ln(c
*x^2/(a*x+b)^2)*polylog(2,a*x/(a*x+b))/a-48*b*polylog(3,a*x/(a*x+b))/a
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2536, 2552, 2354, 2421, 6724}

$$\int \log^3\left(\frac{cx^2}{(b+ax)^2}\right) dx = \frac{24b\,\text{PolyLog}\left(2,\frac{ax}{b+ax}\right)\log\left(\frac{cx^2}{(ax+b)^2}\right)}{a} + x\log^3\left(\frac{cx^2}{(ax+b)^2}\right) + \frac{6b\log\left(\frac{b}{ax+b}\right)\log^2\left(\frac{cx^2}{(ax+b)^2}\right)}{a} - \frac{48b\,\text{PolyLog}\left(3,\frac{ax}{b+ax}\right)}{a}$$

```
[In] Int[Log[(c*x^2)/(b + a*x)^2]^3,x]
```

```
[Out] x*Log[(c*x^2)/(b + a*x)^2]^3 + (6*b*Log[(c*x^2)/(b + a*x)^2]^2*Log[b/(b + a
*x)])/a + (24*b*Log[(c*x^2)/(b + a*x)^2]*PolyLog[2, (a*x)/(b + a*x)])/a - (
48*b*PolyLog[3, (a*x)/(b + a*x)])/a
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2536

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)^n/(c
+ d*x)^n)])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b
*x)^n/(c + d*x)^n)])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A,
B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x\log^3\left(\frac{cx^2}{(b+ax)^2}\right)-(6b)\int\frac{\log^2\left(\frac{cx^2}{(b+ax)^2}\right)}{b+ax}\,dx\\ &= x\log^3\left(\frac{cx^2}{(b+ax)^2}\right)-(6b)\text{Subst}\left(\int\frac{\log^2\left(cx^2\right)}{1-ax}\,dx,x,\frac{x}{b+ax}\right)\end{aligned}$$

$$= x\log^3\left(\frac{cx^2}{(b+ax)^2}\right)+\frac{6b\log^2\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(\frac{b}{b+ax}\right)}{a}-\frac{(24b)\mathrm{Subst}\left(\int\frac{\log(cx^2)\log(1-ax)}{x}\,dx,x,\frac{x}{b+ax}\right)}{a}$$

$$= x\log^3\left(\frac{cx^2}{(b+ax)^2}\right)+\frac{6b\log^2\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(\frac{b}{b+ax}\right)}{a} + \frac{24b\log\left(\frac{cx^2}{(b+ax)^2}\right)\mathrm{Li}_2\left(\frac{ax}{b+ax}\right)}{a}-\frac{(48b)\mathrm{Subst}\left(\int\frac{\mathrm{Li}_2(ax)}{x}\,dx,x,\frac{x}{b+ax}\right)}{a}$$

$$= x\log^3\left(\frac{cx^2}{(b+ax)^2}\right)+\frac{6b\log^2\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(\frac{b}{b+ax}\right)}{a} + \frac{24b\log\left(\frac{cx^2}{(b+ax)^2}\right)\mathrm{Li}_2\left(\frac{ax}{b+ax}\right)}{a}-\frac{48b\mathrm{Li}_3\left(\frac{ax}{b+ax}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int\log^3\left(\frac{cx^2}{(b+ax)^2}\right)\,dx = x\log^3\left(\frac{cx^2}{(b+ax)^2}\right)+\frac{6b\log^2\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(\frac{b}{b+ax}\right)}{a} + \frac{24b\log\left(\frac{cx^2}{(b+ax)^2}\right)\mathrm{PolyLog}\left(2,\frac{ax}{b+ax}\right)}{a}-\frac{48b\,\mathrm{PolyLog}\left(3,\frac{ax}{b+ax}\right)}{a}$$

```
[In] Integrate[Log[(c*x^2)/(b + a*x)^2]^3,x]
```

```
[Out] x*Log[(c*x^2)/(b + a*x)^2]^3 + (6*b*Log[(c*x^2)/(b + a*x)^2]^2*Log[b/(b + a
*x)])/a + (24*b*Log[(c*x^2)/(b + a*x)^2]*PolyLog[2, (a*x)/(b + a*x)])/a - (
48*b*PolyLog[3, (a*x)/(b + a*x)])/a
```

Maple [F]

$$\int\ln\left(\frac{c\,x^2}{(ax+b)^2}\right)^3dx$$

```
[In] int(ln(c*x^2/(a*x+b)^2)^3,x)
```

```
[Out] int(ln(c*x^2/(a*x+b)^2)^3,x)
```

Fricas [F]

$$\int \log^3\left(\frac{cx^2}{(b+ax)^2}\right)\,dx = \int \log\left(\frac{cx^2}{(ax+b)^2}\right)^3\,dx$$

```
[In] integrate(log(c*x^2/(a*x+b)^2)^3,x, algorithm="fricas")
[Out] integral(log(c*x^2/(a^2*x^2 + 2*a*b*x + b^2))^3, x)
```

Sympy [F]

$$\int \log^3\left(\frac{cx^2}{(b+ax)^2}\right)\,dx = -6b\int \frac{\log\left(\frac{cx^2}{a^2x^2+2abx+b^2}\right)^2}{ax+b}\,dx + x\log\left(\frac{cx^2}{(ax+b)^2}\right)^3$$

```
[In] integrate(ln(c*x**2/(a*x+b)**2)**3,x)
[Out] -6*b*Integral(log(c*x**2/(a**2*x**2 + 2*a*b*x + b**2))**2/(a*x + b), x) + x
*log(c*x**2/(a*x + b)**2)**3
```

Maxima [F]

$$\int \log^3\left(\frac{cx^2}{(b+ax)^2}\right)\,dx = \int \log\left(\frac{cx^2}{(ax+b)^2}\right)^3\,dx$$

```
[In] integrate(log(c*x^2/(a*x+b)^2)^3,x, algorithm="maxima")
[Out] -4*(2*(a*x + b)*log(a*x + b)^3 - 3*(a*x*log(c) + 2*a*x*log(x))*log(a*x + b)
^2)/a - integrate(-(a*x*log(c)^3 + b*log(c)^3 + 8*(a*x + b)*log(x)^3 + 12*(
a*x*log(c) + b*log(c))*log(x)^2 - 6*((log(c)^2 + 4*log(c))*a*x + b*log(c)^2
 + 4*(a*x + b)*log(x)^2 + 4*(a*x*(log(c) + 2) + b*log(c))*log(x))*log(a*x +
 b) + 6*(a*x*log(c)^2 + b*log(c)^2)*log(x))/(a*x + b), x)
```

Giac [F]

$$\int \log^3\left(\frac{cx^2}{(b+ax)^2}\right)\,dx = \int \log\left(\frac{cx^2}{(ax+b)^2}\right)^3\,dx$$

```
[In] integrate(log(c*x^2/(a*x+b)^2)^3,x, algorithm="giac")
[Out] integrate(log(c*x^2/(a*x + b)^2)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \log^3\left(\frac{cx^2}{(b+ax)^2}\right)\,dx = \int \ln\left(\frac{c\,x^2}{{(b+a\,x)}^2}\right)^3\,dx$$

```
[In] int(log((c*x^2)/(b + a*x)^2)^3,x)
[Out] int(log((c*x^2)/(b + a*x)^2)^3, x)
```

3.102 $\int \frac{\text{PolyLog}\left(2,1+\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(c+dx)}\,dx$

Optimal result 792
Rubi [A] (verified) 792
Mathematica [A] (verified) 793
Maple [A] (verified) 793
Fricas [F] 793
Sympy [F(-1)] 794
Maxima [F] 794
Giac [F] 794
Mupad [F(-1)] 794

Optimal result

Integrand size = 38, antiderivative size = 35

$$\int \frac{\text{PolyLog}\left(2,1+\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(c+dx)}\,dx=-\frac{\text{PolyLog}\left(3,1+\frac{bc-ad}{d(a+bx)}\right)}{bc-ad}$$

```
[Out] -polylog(3,1+(-a*d+b*c)/d/(b*x+a))/(-a*d+b*c)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {6745}

$$\int \frac{\text{PolyLog}\left(2,1+\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(c+dx)}\,dx=-\frac{\text{PolyLog}\left(3,\frac{bc-ad}{d(a+bx)}+1\right)}{bc-ad}$$

```
[In] Int[PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)),x]
[Out] -(PolyLog[3, 1 + (b*c - a*d)/(d*(a + b*x))]/(b*c - a*d))
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\text{integral}=-\frac{\text{Li}_3\left(1+\frac{bc-ad}{d(a+bx)}\right)}{bc-ad}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \frac{\text{PolyLog}\left(2,1+\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(c+dx)}\,dx=\frac{\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{-bc+ad}$$

```
[In] Integrate[PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)),x
]
```

```
[Out] PolyLog[3, (b*(c + d*x))/(d*(a + b*x))]/(-(b*c) + a*d)
```

Maple [A] (verified)

Time = 8.38 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

method	result	size
derivativedivides	$\frac{\operatorname{Li}_3\left(1-\frac{ad-cb}{d(bx+a)}\right)}{ad-cb}$	36
default	$\frac{\operatorname{Li}_3\left(1-\frac{ad-cb}{d(bx+a)}\right)}{ad-cb}$	36

```
[In] int(polylog(2,1+(-a*d+b*c)/d/(b*x+a))/(b*x+a)/(d*x+c),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/(a*d-b*c)*polylog(3,1-(a*d-b*c)/d/(b*x+a))
```

Fricas [F]

$$\int \frac{\text{PolyLog}\left(2,1+\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(c+dx)}\,dx=\int \frac{\operatorname{Li}_2\left(\frac{bc-ad}{(bx+a)d}+1\right)}{(bx+a)(dx+c)}\,dx$$

```
[In] integrate(polylog(2,1+(-a*d+b*c)/d/(b*x+a))/(b*x+a)/(d*x+c),x, algorithm="f
ricas")
```

```
[Out] integral(dilog((b*c - a*d)/(b*d*x + a*d) + 1)/(b*d*x^2 + a*c + (b*c + a*d)*
x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}\left(2,1+\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(c+dx)}\,dx=\text{Timed out}$$

```
[In] integrate(polylog(2,1+(-a*d+b*c)/d/(b*x+a))/(b*x+a)/(d*x+c),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\text{PolyLog}\left(2,1+\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(c+dx)}\,dx=\int \frac{\text{Li}_2\left(\frac{bc-ad}{(bx+a)d}+1\right)}{(bx+a)(dx+c)}\,dx$$

```
[In] integrate(polylog(2,1+(-a*d+b*c)/d/(b*x+a))/(b*x+a)/(d*x+c),x, algorithm="m
axima")
[Out] integrate(dilog((b*c - a*d)/((b*x + a)*d) + 1)/((b*x + a)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{\text{PolyLog}\left(2,1+\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(c+dx)}\,dx=\int \frac{\text{Li}_2\left(\frac{bc-ad}{(bx+a)d}+1\right)}{(bx+a)(dx+c)}\,dx$$

```
[In] integrate(polylog(2,1+(-a*d+b*c)/d/(b*x+a))/(b*x+a)/(d*x+c),x, algorithm="g
iac")
[Out] integrate(dilog((b*c - a*d)/((b*x + a)*d) + 1)/((b*x + a)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}\left(2,1+\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(c+dx)}\,dx=\int \frac{\text{polylog}\left(2,1-\frac{a\,d-b\,c}{d\,(a+b\,x)}\right)}{(a+b\,x)\,(c+d\,x)}\,dx$$

```
[In] int(polylog(2, 1 - (a*d - b*c)/(d*(a + b*x)))/((a + b*x)*(c + d*x)),x)
[Out] int(polylog(2, 1 - (a*d - b*c)/(d*(a + b*x)))/((a + b*x)*(c + d*x)), x)
```

3.103 $\int \frac{\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx$

Optimal result 795
Rubi [A] (verified) 795
Mathematica [A] (verified) 796
Maple [A] (verified) 797
Fricas [F] 797
Sympy [F] 797
Maxima [F] 798
Giac [F] 798
Mupad [F(-1)] 798

Optimal result

Integrand size = 50, antiderivative size = 85

$$\int \frac{\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\operatorname{PolyLog}\left(2,1+\frac{bc-ad}{d(a+bx)}\right)}{bc-ad} - \frac{\operatorname{PolyLog}\left(3,1+\frac{bc-ad}{d(a+bx)}\right)}{bc-ad}$$

```
[Out] ln(e*(d*x+c)/(b*x+a))*polylog(2,1+(-a*d+b*c)/d/(b*x+a))/(-a*d+b*c)-polylog(
3,1+(-a*d+b*c)/d/(b*x+a))/(-a*d+b*c)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2588, 6745}

$$\int \frac{\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \frac{\operatorname{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)\log\left(\frac{e(c+dx)}{a+bx}\right)}{bc-ad} - \frac{\operatorname{PolyLog}\left(3,\frac{bc-ad}{d(a+bx)}+1\right)}{bc-ad}$$

```
[In] Int[(Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[(e*(c + d*x))/(a + b*x)])/((a +
b*x)*(c + d*x)),x]
```

```
[Out] (Log[(e*(c + d*x))/(a + b*x)]*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/(b
*c - a*d) - PolyLog[3, 1 + (b*c - a*d)/(d*(a + b*x))]/(b*c - a*d)
```

Rule 2588

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[(v - 1)*((c + d
*x)/(a + b*x))], h = Simplify[u*(a + b*x)*(c + d*x)]}, Simp[(-h)*PolyLog[2,
 1 - v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(b*c - a*d)), x] + Dist[h*p
*r*s, Int[PolyLog[2, 1 - v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(
(a + b*x)*(c + d*x))), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e,
 f, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\text{integral} = \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\text{Li}_2\left(1+\frac{bc-ad}{d(a+bx)}\right)}{bc-ad} + \int \frac{\text{Li}_2\left(1-\frac{-bc+ad}{d(a+bx)}\right)}{(a+bx)(c+dx)}\,dx$$

$$= \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\text{Li}_2\left(1+\frac{bc-ad}{d(a+bx)}\right)}{bc-ad} - \frac{\text{Li}_3\left(1+\frac{bc-ad}{d(a+bx)}\right)}{bc-ad}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.80

$$\int \frac{\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right) - \text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{bc-ad}$$

```
[In] Integrate[(Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[(e*(c + d*x))/(a + b*x)])/
((a + b*x)*(c + d*x)),x]
```

```
[Out] (Log[(e*(c + d*x))/(a + b*x)]*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] - Pol
yLog[3, (b*(c + d*x))/(d*(a + b*x))])/(b*c - a*d)
```

Maple [A] (verified)

Time = 3.80 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.84

method	result	size
default	$\frac{\frac{\ln\left(-\frac{\frac{e(dx+c)b}{bx+a}-de}{de}\right)\ln\left(\frac{e(dx+c)}{bx+a}\right)^{2}}{2}-\frac{\ln\left(\frac{e(dx+c)}{bx+a}\right)^{2}\ln\left(1-\frac{b(dx+c)}{d(bx+a)}\right)}{2}-\ln\left(\frac{e(dx+c)}{bx+a}\right)\operatorname{Li}_{2}\left(\frac{b(dx+c)}{d(bx+a)}\right)+\operatorname{Li}_{3}\left(\frac{b(dx+c)}{d(bx+a)}\right)}{ad-cb}$	156

```
[In] int(ln((a*d-b*c)/d/(b*x+a))*ln(e*(d*x+c)/(b*x+a))/(b*x+a)/(d*x+c),x,method=
_RETURNVERBOSE)
[Out] 1/(a*d-b*c)*(1/2*ln(-(e*(d*x+c)/(b*x+a)*b-d*e)/d/e)*ln(e*(d*x+c)/(b*x+a))^2
-1/2*ln(e*(d*x+c)/(b*x+a))^2*ln(1-b*(d*x+c)/d/(b*x+a))-ln(e*(d*x+c)/(b*x+a)
)*polylog(2,b*(d*x+c)/d/(b*x+a))+polylog(3,b*(d*x+c)/d/(b*x+a)))
```

Fricas [F]

$$\int \frac{\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx=\int \frac{\log\left(\frac{(dx+c)e}{bx+a}\right)\log\left(-\frac{bc-ad}{(bx+a)d}\right)}{(bx+a)(dx+c)}\,dx$$

```
[In] integrate(log((a*d-b*c)/d/(b*x+a))*log(e*(d*x+c)/(b*x+a))/(b*x+a)/(d*x+c),x
, algorithm="fricas")
[Out] integral(log(-(b*c - a*d)/(b*d*x + a*d))*log((d*e*x + c*e)/(b*x + a))/(b*d*
x^2 + a*c + (b*c + a*d)*x), x)
```

Sympy [F]

$$\int \frac{\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx=\frac{b\int \frac{\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)^{2}}{a+bx}\,dx}{2\,(ad-bc)}+\frac{\log\left(\frac{ad-bc}{d(a+bx)}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)^{2}}{2ad-2bc}$$

```
[In] integrate(ln((a*d-b*c)/d/(b*x+a))*ln(e*(d*x+c)/(b*x+a))/(b*x+a)/(d*x+c),x)
[Out] b*Integral(log(c*e/(a + b*x) + d*e*x/(a + b*x))**2/(a + b*x), x)/(2*(a*d -
b*c)) + log((a*d - b*c)/(d*(a + b*x)))*log(e*(c + d*x)/(a + b*x))**2/(2*a*d
 - 2*b*c)
```

Maxima [F]

$$\int \frac{\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\log\left(\frac{(dx+c)e}{bx+a}\right)\log\left(-\frac{bc-ad}{(bx+a)d}\right)}{(bx+a)(dx+c)}\,dx$$

```
[In] integrate(log((a*d-b*c)/d/(b*x+a))*log(e*(d*x+c)/(b*x+a))/(b*x+a)/(d*x+c),x
, algorithm="maxima")
```

```
[Out] integrate(log((d*x + c)*e/(b*x + a))*log(-(b*c - a*d)/((b*x + a)*d))/((b*x
+ a)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\log\left(\frac{(dx+c)e}{bx+a}\right)\log\left(-\frac{bc-ad}{(bx+a)d}\right)}{(bx+a)(dx+c)}\,dx$$

```
[In] integrate(log((a*d-b*c)/d/(b*x+a))*log(e*(d*x+c)/(b*x+a))/(b*x+a)/(d*x+c),x
, algorithm="giac")
```

```
[Out] integrate(log((d*x + c)*e/(b*x + a))*log(-(b*c - a*d)/((b*x + a)*d))/((b*x
+ a)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\,\ln\left(\frac{a\,d-b\,c}{d\,(a+b\,x)}\right)}{(a+b\,x)\,(c+d\,x)}\,dx$$

```
[In] int((log((e*(c + d*x))/(a + b*x))*log((a*d - b*c)/(d*(a + b*x))))/((a + b*x
)*(c + d*x)),x)
```

```
[Out] int((log((e*(c + d*x))/(a + b*x))*log((a*d - b*c)/(d*(a + b*x))))/((a + b*x
)*(c + d*x)), x)
```

3.104 $\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{a+bx}\,dx$

Optimal result 799
Rubi [A] (verified) 799
Mathematica [A] (verified) 801
Maple [B] (verified) 801
Fricas [F] 802
Sympy [F] 802
Maxima [F] 803
Giac [F] 803
Mupad [F(-1)] 803

Optimal result

Integrand size = 42, antiderivative size = 140

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{a+bx}\,dx = -\frac{\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{b} - \frac{2\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b} + \frac{2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b}$$

```
[Out] -ln((a*d-b*c)/d/(b*x+a))*ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/b-2*ln
((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*polylog(2,b*(d*x+c)/d/(b*x+a))/b+2*
polylog(3,b*(d*x+c)/d/(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2552, 2354, 2421, 6724}

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{a+bx}\,dx = -\frac{2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{b} - \frac{\log\left(\frac{ad-bc}{d(a+bx)}\right)\log^2\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{b} + \frac{2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b}$$

```
[In] Int[Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2/(a + b*x),x]
```

```
[Out] -((Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*
f)*(a + b*x))]^2)/b) - (2*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x
))]*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/b + (2*PolyLog[3, (b*(c + d*x)
)/(d*(a + b*x))])/b
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{\log^2\left(\frac{(be-af)x}{de-cf}\right)}{d-bx}\,dx, x, \frac{c+dx}{a+bx}\right)$$

$$= -\frac{\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{b} + \frac{2\text{Subst}\left(\int \frac{\log\left(\frac{(be-af)x}{de-cf}\right)\log\left(1-\frac{bx}{d}\right)}{x}\,dx, x, \frac{c+dx}{a+bx}\right)}{b}$$

$$= -\frac{\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{b} - \frac{2\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b} + \frac{2\text{Subst}\left(\int\frac{\text{Li}_2\left(\frac{bx}{d}\right)}{x}\,dx, x, \frac{c+dx}{a+bx}\right)}{b}$$

$$= -\frac{\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{b} - \frac{2\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b} + \frac{2\text{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.96

$$\int\frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{a+bx}\,dx = \frac{-\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right) - 2\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\text{PolyLog}\left(2, \frac{b(c+dx)}{d(a+bx)}\right) + 2\,\text{PolyLog}\left(3, \frac{b(c+dx)}{d(a+bx)}\right)}{b}$$

```
[In] Integrate[Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2/(a + b*x),
x]
```

```
[Out] (-(Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*
f)*(a + b*x))]^2) - 2*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*
PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] + 2*PolyLog[3, (b*(c + d*x))/(d*(a
+ b*x))])/b
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(140) = 280$.

Time = 3.77 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.99

method	result
derivativedivides	$\frac{(cf-de)\left(\ln\left(-\frac{(af-be)(ad-cb)}{b(cf-de)(bx+a)}+\frac{d(af-be)}{(cf-de)b}\right)^2\ln\left(1-\frac{(-bcf+bde)\left(-\frac{(af-be)(ad-cb)}{b(cf-de)(bx+a)}+\frac{d(af-be)}{(cf-de)b}\right)}{-adf+bde}\right)+2\ln\left(-\frac{(af-be)(ad-cb)}{b(cf-de)(bx+a)} \ldots\right.\right.}{-bcf \ldots}$
default	$\frac{(cf-de)\left(\ln\left(-\frac{(af-be)(ad-cb)}{b(cf-de)(bx+a)}+\frac{d(af-be)}{(cf-de)b}\right)^2\ln\left(1-\frac{(-bcf+bde)\left(-\frac{(af-be)(ad-cb)}{b(cf-de)(bx+a)}+\frac{d(af-be)}{(cf-de)b}\right)}{-adf+bde}\right)+2\ln\left(-\frac{(af-be)(ad-cb)}{b(cf-de)(bx+a)} \ldots\right.\right.}{-bcf \ldots}$
risch	$\frac{\ln\left(-\frac{(af-be)(ad-cb)}{b(cf-de)(bx+a)}+\frac{d(af-be)}{(cf-de)b}\right)^2\ln\left(1-\frac{(-bcf+bde)\left(-\frac{(af-be)(ad-cb)}{b(cf-de)(bx+a)}+\frac{d(af-be)}{(cf-de)b}\right)}{-adf+bde}\right)cf}{-bcf+bde} - \frac{\ln\left(-\frac{(af-be)(ad-cb)}{b(cf-de)(bx+a)}+\frac{d(af \ldots}{(cf- \ldots}\right.}{\ldots}$

```
[In] int(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(b*x+a),x,method=_RETURNVER
BOSE)
```

```
[Out] (c*f-d*e)/(-b*c*f+b*d*e)*(ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*
f-d*e)*(a*f-b*e)/b)^2*ln(1-(-b*c*f+b*d*e)/(-a*d*f+b*d*e)*(-(a*f-b*e)*(a*d-b
*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b))+2*ln(-(a*f-b*e)*(a*d-b*c)
/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)*polylog(2,(-b*c*f+b*d*e)/(-a*
d*f+b*d*e)*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/
b))-2*polylog(3,(-b*c*f+b*d*e)/(-a*d*f+b*d*e)*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-
d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)))
```

Fricas [F]

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{a+bx}\,dx=\int \frac{\log\left(\frac{(be-af)(dx+c)}{(de-cf)(bx+a)}\right)^2}{bx+a}\,dx$$

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(b*x+a),x, algorithm
="fricas")
```

```
[Out] integral(log((b*c*e - a*c*f + (b*d*e - a*d*f)*x)/(a*d*e - a*c*f + (b*d*e -
b*c*f)*x))^2/(b*x + a), x)
```

Sympy [F]

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{a+bx}\,dx=\int \frac{\log\left(-\frac{acf}{-acf+ade-bcfx+bdex}-\frac{adfx}{-acf+ade-bcfx+bdex}+\frac{bce}{-acf+ade-bcfx+bdex}+\frac{bdex}{-acf+ade-bcfx+bdex}\right)^2}{a+bx}\,dx$$

```
[In] integrate(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))**2/(b*x+a),x)
```

```
[Out] Integral(log(-a*c*f/(-a*c*f + a*d*e - b*c*f*x + b*d*e*x) - a*d*f*x/(-a*c*f
+ a*d*e - b*c*f*x + b*d*e*x) + b*c*e/(-a*c*f + a*d*e - b*c*f*x + b*d*e*x) +
 b*d*e*x/(-a*c*f + a*d*e - b*c*f*x + b*d*e*x))**2/(a + b*x), x)
```

Maxima [F]

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{a+bx}\,dx = \int \frac{\log\left(\frac{(be-af)(dx+c)}{(de-cf)(bx+a)}\right)^2}{bx+a}\,dx$$

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(b*x+a),x, algorithm
="maxima")
```

```
[Out] log(d*x + c)^3/a - integrate(-((log(-b*e + a*f)^2 - 2*log(-b*e + a*f)*log(-
d*e + c*f) + log(-d*e + c*f)^2)*b*d*x + (log(-b*e + a*f)^2 - 2*log(-b*e + a
*f)*log(-d*e + c*f) + log(-d*e + c*f)^2)*b*c + (b*d*x + b*c)*log(b*x + a)^2
 - 2*(b*d*x*(log(-b*e + a*f) - log(-d*e + c*f)) + b*c*(log(-b*e + a*f) - lo
g(-d*e + c*f)))*log(b*x + a) + 2*(b*d*x*(log(-b*e + a*f) - log(-d*e + c*f))
 + b*c*(log(-b*e + a*f) - log(-d*e + c*f)) - (2*b*d*x + b*c + a*d)*log(b*x
+ a))*log(d*x + c))/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x), x)
```

Giac [F]

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{a+bx}\,dx = \int \frac{\log\left(\frac{(be-af)(dx+c)}{(de-cf)(bx+a)}\right)^2}{bx+a}\,dx$$

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(b*x+a),x, algorithm
="giac")
```

```
[Out] integrate(log((b*e - a*f)*(d*x + c)/((d*e - c*f)*(b*x + a)))^2/(b*x + a), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{a+bx}\,dx = \int \frac{\ln\left(\frac{(a\,f-b\,e)\,(c+d\,x)}{(c\,f-d\,e)\,(a+b\,x)}\right)^2}{a+b\,x}\,dx$$

```
[In] int(log(((a*f - b*e)*(c + d*x))/((c*f - d*e)*(a + b*x)))^2/(a + b*x),x)
```

```
[Out] int(log(((a*f - b*e)*(c + d*x))/((c*f - d*e)*(a + b*x)))^2/(a + b*x), x)
```

3.105 $\int \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\log\left(\frac{(-bc+ad)(e+fx)}{(de-cf)(a+bx)}\right)}{(a+bx)(c+dx)}\,dx$

Optimal result 804
Rubi [A] (verified) 804
Mathematica [A] (verified) 805
Maple [B] (verified) 806
Fricas [F] 806
Sympy [F] 806
Maxima [F] 807
Giac [F] 807
Mupad [F(-1)] 808

Optimal result

Integrand size = 62, antiderivative size = 109

$$\int \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\log\left(\frac{(-bc+ad)(e+fx)}{(de-cf)(a+bx)}\right)}{(a+bx)(c+dx)}\,dx = \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\text{PolyLog}\left(2,1+\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}\right)}{bc-ad} - \frac{\text{PolyLog}\left(3,1+\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}\right)}{bc-ad}$$

```
[Out] ln(e*(d*x+c)/(b*x+a))*polylog(2,1+(-a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))/(-
a*d+b*c)-polylog(3,1+(-a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))/(-a*d+b*c)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {2588, 6745}

$$\int \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\log\left(\frac{(-bc+ad)(e+fx)}{(de-cf)(a+bx)}\right)}{(a+bx)(c+dx)}\,dx = \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\text{PolyLog}\left(2,\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}+1\right)}{bc-ad} - \frac{\text{PolyLog}\left(3,\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}+1\right)}{bc-ad}$$

```
[In] Int[(Log[(e*(c + d*x))/(a + b*x)]*Log[((-(b*c) + a*d)*(e + f*x))/((d*e - c*
f)*(a + b*x))])/((a + b*x)*(c + d*x)),x]
```

```
[Out] (Log[(e*(c + d*x))/(a + b*x)]*PolyLog[2, 1 + ((b*c - a*d)*(e + f*x))/((d*e
- c*f)*(a + b*x))])/(b*c - a*d) - PolyLog[3, 1 + ((b*c - a*d)*(e + f*x))/((
d*e - c*f)*(a + b*x))]/(b*c - a*d)
```

Rule 2588

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[(v - 1)*((c + d
*x)/(a + b*x))], h = Simplify[u*(a + b*x)*(c + d*x)]}, Simp[(-h)*PolyLog[2,
 1 - v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(b*c - a*d)), x] + Dist[h*p
*r*s, Int[PolyLog[2, 1 - v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(
(a + b*x)*(c + d*x))), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e,
 f, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\text{Li}_2\left(1+\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}\right)}{bc-ad}+\int\frac{\text{Li}_2\left(1-\frac{(-bc+ad)(e+fx)}{(de-cf)(a+bx)}\right)}{(a+bx)(c+dx)}\,dx\\ &= \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\text{Li}_2\left(1+\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}\right)}{bc-ad}-\frac{\text{Li}_3\left(1+\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}\right)}{bc-ad}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.88

$$\int\frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\log\left(\frac{(-bc+ad)(e+fx)}{(de-cf)(a+bx)}\right)}{(a+bx)(c+dx)}\,dx = \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\text{PolyLog}\left(2,\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)-\text{PolyLog}\left(3,\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{bc-ad}$$

```
[In] Integrate[(Log[(e*(c + d*x))/(a + b*x)]*Log[((-(b*c) + a*d)*(e + f*x))/((d*
e - c*f)*(a + b*x))])/((a + b*x)*(c + d*x)),x]
```

```
[Out] (Log[(e*(c + d*x))/(a + b*x)]*PolyLog[2, ((b*e - a*f)*(c + d*x))/((d*e - c*
f)*(a + b*x))] - PolyLog[3, ((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))
])/(b*c - a*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. 2(109) = 218.

Time = 25.00 (sec) , antiderivative size = 423, normalized size of antiderivative = 3.88

method	result
default	$\frac{\frac{\ln\left(-\frac{\frac{e(dx+c)af}{bx+a}-\frac{e^2(dx+c)b}{bx+a}-cef+d\,e^2}{e(cf-de)}\right)\ln\left(\frac{e(dx+c)}{bx+a}\right)^2}{2}-\frac{af\left(\ln\left(\frac{e(dx+c)}{bx+a}\right)^2\ln\left(1-\frac{(af-be)e(dx+c)}{(bx+a)\left(cef-d\,e^2\right)}\right)+2\ln\left(\frac{e(dx+c)}{bx+a}\right)\mathrm{Li}_2\left(\frac{(af-be)e(dx+}{(bx+a)\left(cef-d\,e\right.}\right.\right.}{2(af-be)}}{}$

```
[In] int(ln(e*(d*x+c)/(b*x+a))*ln((a*d-b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))/(b*x+a)/
(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(a*d-b*c)*(1/2*ln(-(e*(d*x+c)/(b*x+a)*a*f-e^2*(d*x+c)/(b*x+a)*b-c*e*f+d*e
^2)/e/(c*f-d*e))*ln(e*(d*x+c)/(b*x+a))^2-1/2*a*f/(a*f-b*e)*(ln(e*(d*x+c)/(b
*x+a))^2*ln(1-(a*f-b*e)*e*(d*x+c)/(b*x+a)/(c*e*f-d*e^2))+2*ln(e*(d*x+c)/(b*
x+a))*polylog(2,(a*f-b*e)*e*(d*x+c)/(b*x+a)/(c*e*f-d*e^2))-2*polylog(3,(a*f
-b*e)*e*(d*x+c)/(b*x+a)/(c*e*f-d*e^2)))+1/2*b*e/(a*f-b*e)*(ln(e*(d*x+c)/(b*
x+a))^2*ln(1-(a*f-b*e)*e*(d*x+c)/(b*x+a)/(c*e*f-d*e^2))+2*ln(e*(d*x+c)/(b*x
+a))*polylog(2,(a*f-b*e)*e*(d*x+c)/(b*x+a)/(c*e*f-d*e^2))-2*polylog(3,(a*f-
b*e)*e*(d*x+c)/(b*x+a)/(c*e*f-d*e^2))))
```

Fricas [F]

$$\int \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\log\left(\frac{(-bc+ad)(e+fx)}{(de-cf)(a+bx)}\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\log\left(\frac{(dx+c)e}{bx+a}\right)\log\left(-\frac{(bc-ad)(fx+e)}{(de-cf)(bx+a)}\right)}{(bx+a)(dx+c)}\,dx$$

```
[In] integrate(log(e*(d*x+c)/(b*x+a))*log((a*d-b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))/
(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(log(-((b*c - a*d)*f*x + (b*c - a*d)*e)/(a*d*e - a*c*f + (b*d*e - b
*c*f)*x))*log((d*e*x + c*e)/(b*x + a))/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

Sympy [F]

$$\int \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\log\left(\frac{(-bc+ad)(e+fx)}{(de-cf)(a+bx)}\right)}{(a+bx)(c+dx)}\,dx = \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)^2\log\left(\frac{(e+fx)(ad-bc)}{(a+bx)(-cf+de)}\right)}{2ad-2bc} - \frac{(af-be)\int \frac{\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)^2}{ae+afx+bex+bfx^2}\,dx}{2\,(ad-bc)}$$

```
[In] integrate(ln(e*(d*x+c)/(b*x+a))*ln((a*d-b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))/(b
*x+a)/(d*x+c),x)
```

```
[Out] log(e*(c + d*x)/(a + b*x))**2*log((e + f*x)*(a*d - b*c)/((a + b*x)*(-c*f +
d*e)))/(2*a*d - 2*b*c) - (a*f - b*e)*Integral(log(c*e/(a + b*x) + d*e*x/(a
+ b*x))**2/(a*e + a*f*x + b*e*x + b*f*x**2), x)/(2*(a*d - b*c))
```

Maxima [F]

$$\int \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\log\left(\frac{(-bc+ad)(e+fx)}{(de-cf)(a+bx)}\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\log\left(\frac{(dx+c)e}{bx+a}\right)\log\left(-\frac{(bc-ad)(fx+e)}{(de-cf)(bx+a)}\right)}{(bx+a)(dx+c)}\,dx$$

```
[In] integrate(log(e*(d*x+c)/(b*x+a))*log((a*d-b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))/
(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*(log(b*x + a)^2 - 2*(log(b*x + a) - log(e))*log(d*x + c) + log(d*x + c
)^2 - 2*log(b*x + a)*log(e))*log(f*x + e)/(b*c - a*d) + integrate(1/2*(2*(e
*log(-b*c + a*d)*log(e) - e*log(d*e - c*f)*log(e))*b*c + (b*d*f*x^2 + 2*b*c
*e - (2*d*e - c*f)*a + (3*b*c*f - a*d*f)*x)*log(b*x + a)^2 - 2*(d*e*log(-b*
c + a*d)*log(e) - d*e*log(d*e - c*f)*log(e))*a + 2*((f*log(-b*c + a*d)*log(
e) - f*log(d*e - c*f)*log(e))*b*c - (d*f*log(-b*c + a*d)*log(e) - d*f*log(d
*e - c*f)*log(e))*a)*x - 2*(b*d*f*x^2*log(e) - (e*(log(d*e - c*f) - log(e))
 - e*log(-b*c + a*d))*b*c + (d*e*(log(d*e - c*f) - log(e)) - d*e*log(-b*c +
 a*d) + c*f*log(e))*a + ((f*log(-b*c + a*d) - f*log(d*e - c*f) + 2*f*log(e)
)*b*c - (d*f*log(-b*c + a*d) - d*f*log(d*e - c*f))*a)*x)*log(b*x + a) + 2*(
b*d*f*x^2*log(e) + (e*log(-b*c + a*d) - e*log(d*e - c*f))*b*c - (d*e*log(-b
*c + a*d) - d*e*log(d*e - c*f) - c*f*log(e))*a + ((f*log(-b*c + a*d) - f*lo
g(d*e - c*f) + f*log(e))*b*c - (d*f*log(-b*c + a*d) - (f*log(d*e - c*f) + f
*log(e))*d)*a)*x - (b*d*f*x^2 + 2*b*c*f*x + b*c*e - (d*e - c*f)*a)*log(b*x
+ a))*log(d*x + c))/(a*b*c^2*e - a^2*c*d*e + (b^2*c*d*f - a*b*d^2*f)*x^3 -
(a*b*d^2*e + a^2*d^2*f - (c*d*e + c^2*f)*b^2)*x^2 + (b^2*c^2*e + a*b*c^2*f
- (d^2*e + c*d*f)*a^2)*x), x)
```

Giac [F]

$$\int \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\log\left(\frac{(-bc+ad)(e+fx)}{(de-cf)(a+bx)}\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\log\left(\frac{(dx+c)e}{bx+a}\right)\log\left(-\frac{(bc-ad)(fx+e)}{(de-cf)(bx+a)}\right)}{(bx+a)(dx+c)}\,dx$$

```
[In] integrate(log(e*(d*x+c)/(b*x+a))*log((a*d-b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))/
(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(log((d*x + c)*e/(b*x + a))*log(-(b*c - a*d)*(f*x + e)/((d*e - c*f
)*(b*x + a)))/((b*x + a)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\log\left(\frac{(-bc+ad)(e+fx)}{(de-cf)(a+bx)}\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\,\ln\left(-\frac{(e+f\,x)\,(a\,d-b\,c)}{(c\,f-d\,e)\,(a+b\,x)}\right)}{(a+b\,x)\,(c+d\,x)}\,dx$$

```
[In] int((log((e*(c + d*x))/(a + b*x))*log(-((e + f*x)*(a*d - b*c))/((c*f - d*e)
*(a + b*x))))/((a + b*x)*(c + d*x)),x)
```

```
[Out] int((log((e*(c + d*x))/(a + b*x))*log(-((e + f*x)*(a*d - b*c))/((c*f - d*e)
*(a + b*x))))/((a + b*x)*(c + d*x)), x)
```

3.106 $\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{(a+bx)(e+fx)} dx$

Optimal result 809
Rubi [A] (verified) 809
Mathematica [B] (verified) 811
Maple [A] (verified) 813
Fricas [A] (verification not implemented) 813
Sympy [F] 814
Maxima [F(-2)] 814
Giac [F] 814
Mupad [F(-1)] 815

Optimal result

Integrand size = 49, antiderivative size = 204

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{(a+bx)(e+fx)} dx = -\frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(1-\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{be-af} - \frac{2\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\text{PolyLog}\left(2,\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{be-af} + \frac{2\,\text{PolyLog}\left(3,\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{be-af}$$

```
[Out] -ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2*ln(1-(-a*f+b*e)*(d*x+c)/(-c*f+
d*e)/(b*x+a))/(-a*f+b*e)-2*ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*polylo
g(2,(-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))/(-a*f+b*e)+2*polylog(3,(-a*f+b*e
)*(d*x+c)/(-c*f+d*e)/(b*x+a))/(-a*f+b*e)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.082, Rules used

$= \{2566, 2354, 2421, 6724\}$

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{(a+bx)(e+fx)}\,dx = \frac{2\,\text{PolyLog}\left(3,\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{be-af} - \frac{2\,\text{PolyLog}\left(2,\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{be-af} - \frac{\log\left(1-\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)\log^2\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{be-af}$$

```
[In] Int[Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2/((a + b*x)*(e +
f*x)),x]
```

```
[Out] -((Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2*Log[1 - ((b*e - a
*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/(b*e - a*f)) - (2*Log[((b*e - a*f)
*(c + d*x))/((d*e - c*f)*(a + b*x))]*PolyLog[2, ((b*e - a*f)*(c + d*x))/((d
*e - c*f)*(a + b*x))])/(b*e - a*f) + (2*PolyLog[3, ((b*e - a*f)*(c + d*x))/
((d*e - c*f)*(a + b*x))])/(b*e - a*f)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2566

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(q + 1)*(i/d)^q, Subst[Int[(b*f - a*g - (d*f - c*
g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c +
 d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n}, x] && EqQ[n + mn,
 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, q] && IGtQ[p, 0] &&
EqQ[d*h - c*i, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{\log^2\left(\frac{(be-af)x}{de-cf}\right)}{de-cf+(-be+af)x}\,dx, x, \frac{c+dx}{a+bx}\right)$$

$$= -\frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(1-\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{be-af} + \frac{2\text{Subst}\left(\int \frac{\log\left(\frac{(be-af)x}{de-cf}\right)\log\left(1+\frac{(-be+af)x}{de-cf}\right)}{x}\,dx, x, \frac{c+dx}{a+bx}\right)}{be-af}$$

$$= -\frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(1-\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{be-af} - \frac{2\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\text{Li}_2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{be-af} + \frac{2\text{Subst}\left(\int \frac{\text{Li}_2\left(-\frac{(-be+af)x}{de-cf}\right)}{x}\,dx, x, \frac{c+dx}{a+bx}\right)}{be-af}$$

$$= -\frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(1-\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{be-af} - \frac{2\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\text{Li}_2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{be-af} + \frac{2\text{Li}_3\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{be-af}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1636 vs. 2(204) = 408.

Time = 0.63 (sec) , antiderivative size = 1636, normalized size of antiderivative = 8.02

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{(a+bx)(e+fx)}\,dx$$

$$= \frac{-2\log^3\left(\frac{a}{b}+x\right)+3\log^2\left(\frac{a}{b}+x\right)\log(a+bx)-6\log\left(\frac{a}{b}+x\right)\log\left(\frac{c}{d}+x\right)\log(a+bx)+3\log^2\left(\frac{c}{d}+x\right)\log(a}{}$$

```
[In] Integrate[Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2/((a + b*x)
*(e + f*x)),x]
```

```
[Out] (-2*Log[a/b + x]^3 + 3*Log[a/b + x]^2*Log[a + b*x] - 6*Log[a/b + x]*Log[c/d
 + x]*Log[a + b*x] + 3*Log[c/d + x]^2*Log[a + b*x] + 6*Log[a/b + x]*Log[c/d
 + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] - 3*Log[c/d + x]^2*Log[(d*(a + b*x)
)/(-(b*c) + a*d)] + 3*Log[a/b + x]^2*Log[(b*(c + d*x))/(b*c - a*d)] - 3*Log
[a/b + x]^2*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))] + 6*Log[a/
b + x]*Log[a + b*x]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))] -
6*Log[c/d + x]*Log[a + b*x]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b
*x))] + 6*Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[((b*e - a*f)*(
c + d*x))/((d*e - c*f)*(a + b*x))] + 3*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Lo
g[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2 + 3*Log[a + b*x]*Log[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2 - 3*Log[a/b + x]^2*Log[e
+ f*x] + 6*Log[a/b + x]*Log[c/d + x]*Log[e + f*x] - 3*Log[c/d + x]^2*Log[e
+ f*x] - 6*Log[a/b + x]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))
]*Log[e + f*x] + 6*Log[c/d + x]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a
 + b*x))]*Log[e + f*x] - 3*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*
x))]^2*Log[e + f*x] + 3*Log[a/b + x]^2*Log[(b*(e + f*x))/(b*e - a*f)] - 6*L
og[a/b + x]*Log[(f*(c + d*x))/(-(d*e) + c*f)]*Log[(b*(e + f*x))/(b*e - a*f)
] + 3*Log[(f*(c + d*x))/(-(d*e) + c*f)]^2*Log[(b*(e + f*x))/(b*e - a*f)] +
6*Log[a/b + x]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[(b*
(e + f*x))/(b*e - a*f)] - 6*Log[(f*(c + d*x))/(-(d*e) + c*f)]*Log[((b*e - a
*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[(b*(e + f*x))/(b*e - a*f)] + 3*
Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2*Log[(b*(e + f*x))/(b
*e - a*f)] - 6*Log[a/b + x]*Log[c/d + x]*Log[(d*(e + f*x))/(d*e - c*f)] + 3
*Log[c/d + x]^2*Log[(d*(e + f*x))/(d*e - c*f)] + 6*Log[a/b + x]*Log[(f*(c +
 d*x))/(-(d*e) + c*f)]*Log[(d*(e + f*x))/(d*e - c*f)] - 3*Log[(f*(c + d*x))
/(-(d*e) + c*f)]^2*Log[(d*(e + f*x))/(d*e - c*f)] - 6*Log[c/d + x]*Log[((b*
e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[(d*(e + f*x))/(d*e - c*f)]
 + 6*Log[(f*(c + d*x))/(-(d*e) + c*f)]*Log[((b*e - a*f)*(c + d*x))/((d*e -
c*f)*(a + b*x))]*Log[(d*(e + f*x))/(d*e - c*f)] - 3*Log[((b*e - a*f)*(c + d
*x))/((d*e - c*f)*(a + b*x))]^2*Log[((-(b*c) + a*d)*(e + f*x))/((d*e - c*f)
*(a + b*x))] + 6*Log[a/b + x]*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 6*
(Log[a/b + x] + Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])*PolyL
og[2, (b*(c + d*x))/(b*c - a*d)] + 6*Log[((b*e - a*f)*(c + d*x))/((d*e - c*
f)*(a + b*x))]*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] - 6*Log[((b*e - a*f)
*(c + d*x))/((d*e - c*f)*(a + b*x))]*PolyLog[2, ((b*e - a*f)*(c + d*x))/((d
*e - c*f)*(a + b*x))] - 6*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)] - 6*Poly
Log[3, (b*(c + d*x))/(b*c - a*d)] - 6*PolyLog[3, (b*(c + d*x))/(d*(a + b*x)
)] + 6*PolyLog[3, ((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/(3*b*e
- 3*a*f)
```

Maple [A] (verified)

Time = 5.45 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.66

method	result
derivativedivides	$\frac{\ln\left(-\frac{(af-be)(ad-cb)}{b(cf-de)(bx+a)}+\frac{d(af-be)}{(cf-de)b}\right)^2\ln\left(\frac{(af-be)(ad-cb)}{b(cf-de)(bx+a)}-\frac{d(af-be)}{(cf-de)b}+1\right)+2\ln\left(-\frac{(af-be)(ad-cb)}{b(cf-de)(bx+a)}+\frac{d(af-be)}{(cf-de)b}\right)\mathrm{Li}_2\left(-\frac{(af-be)}{b(cf-de}\right.}{af-be}$
default	$\frac{\ln\left(-\frac{(af-be)(ad-cb)}{b(cf-de)(bx+a)}+\frac{d(af-be)}{(cf-de)b}\right)^2\ln\left(\frac{(af-be)(ad-cb)}{b(cf-de)(bx+a)}-\frac{d(af-be)}{(cf-de)b}+1\right)+2\ln\left(-\frac{(af-be)(ad-cb)}{b(cf-de)(bx+a)}+\frac{d(af-be)}{(cf-de)b}\right)\mathrm{Li}_2\left(-\frac{(af-be)}{b(cf-de}\right.}{af-be}$
risch	$\frac{\ln\left(-\frac{(af-be)(ad-cb)}{b(cf-de)(bx+a)}+\frac{d(af-be)}{(cf-de)b}\right)^2\ln\left(\frac{(af-be)(ad-cb)}{b(cf-de)(bx+a)}-\frac{d(af-be)}{(cf-de)b}+1\right)}{af-be}+\frac{2\ln\left(-\frac{(af-be)(ad-cb)}{b(cf-de)(bx+a)}+\frac{d(af-be)}{(cf-de)b}\right)\mathrm{Li}_2\left(-\frac{(af-}{b(cf}\right.}{af-be}$

```
[In] int(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(b*x+a)/(f*x+e),x,method=_R
ETURNVERBOSE)
```

```
[Out] 1/(a*f-b*e)*(ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b
*e)/b)^2*ln((a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)-d/(c*f-d*e)*(a*f-b*e)/b
+1)+2*ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)*
polylog(2,-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)
-2*polylog(3,-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)
/b))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.29

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{(a+bx)(e+fx)}\,dx = -\frac{\log\left(\frac{bce-acf+(bde-adf)x}{ade-acf+(bde-bcf)x}\right)^2\log\left(-\frac{(bc-ad)fx+(bc-ad)e}{ade-acf+(bde-bcf)x}\right)+2\,\mathrm{Li}_2\left(\frac{(bc-ad)fx+(bc-ad)e}{ade-acf+(bde-bcf)x}+1\right)\log\left(\frac{bce-acf+(bde-adf)x}{ade-acf+(bde-bcf)x}\right.}{be-af}$$

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(b*x+a)/(f*x+e),x, a
lgorithm="fricas")
```

```
[Out] -(log((b*c*e - a*c*f + (b*d*e - a*d*f)*x)/(a*d*e - a*c*f + (b*d*e - b*c*f)*
x))^2*log(-((b*c - a*d)*f*x + (b*c - a*d)*e)/(a*d*e - a*c*f + (b*d*e - b*c*
f)*x)) + 2*dilog(((b*c - a*d)*f*x + (b*c - a*d)*e)/(a*d*e - a*c*f + (b*d*e
- b*c*f)*x) + 1)*log((b*c*e - a*c*f + (b*d*e - a*d*f)*x)/(a*d*e - a*c*f + (
b*d*e - b*c*f)*x)) - 2*polylog(3, (b*c*e - a*c*f + (b*d*e - a*d*f)*x)/(a*d*
e - a*c*f + (b*d*e - b*c*f)*x)))/(b*e - a*f)
```

Sympy [F]

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{(a+bx)(e+fx)}\,dx = \int \frac{\log\left(-\frac{acf}{-acf+ade-bcfx+bdex}-\frac{adfx}{-acf+ade-bcfx+bdex}+\frac{bce}{-acf+ade-bcfx+bdex}+\frac{bdex}{-acf+ade-bcfx+bdex}\right)^2}{(a+bx)\,(e+fx)}\,dx$$

```
[In] integrate(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))**2/(b*x+a)/(f*x+e),x)
```

```
[Out] Integral(log(-a*c*f/(-a*c*f + a*d*e - b*c*f*x + b*d*e*x) - a*d*f*x/(-a*c*f
+ a*d*e - b*c*f*x + b*d*e*x) + b*c*e/(-a*c*f + a*d*e - b*c*f*x + b*d*e*x) +
 b*d*e*x/(-a*c*f + a*d*e - b*c*f*x + b*d*e*x))**2/((a + b*x)*(e + f*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{(a+bx)(e+fx)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(b*x+a)/(f*x+e),x, a
lgorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Memory limit reached. Please ju
mp to an outer pointer, quit program and enlarge thememory limits before ex
ecuting the program again.
```

Giac [F]

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{(a+bx)(e+fx)}\,dx = \int \frac{\log\left(\frac{(be-af)(dx+c)}{(de-cf)(bx+a)}\right)^2}{(bx+a)(fx+e)}\,dx$$

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(b*x+a)/(f*x+e),x, a
lgorithm="giac")
```

```
[Out] integrate(log((b*e - a*f)*(d*x + c)/((d*e - c*f)*(b*x + a)))^2/((b*x + a)*(
f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{(a+bx)(e+fx)}\,dx = \int \frac{\ln\left(\frac{(a\,f-b\,e)\,(c+d\,x)}{(c\,f-d\,e)\,(a+b\,x)}\right)^2}{(e+f\,x)\,(a+b\,x)}\,dx$$

```
[In] int(log(((a*f - b*e)*(c + d*x))/((c*f - d*e)*(a + b*x)))^2/((e + f*x)*(a +
b*x)),x)
```

```
[Out] int(log(((a*f - b*e)*(c + d*x))/((c*f - d*e)*(a + b*x)))^2/((e + f*x)*(a +
b*x)), x)
```

3.107 $\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{e+fx} dx$

Optimal result 816
Rubi [A] (verified) 817
Mathematica [B] (verified) 819
Maple [B] (verified) 820
Fricas [F] 821
Sympy [F] 821
Maxima [F(-2)] 822
Giac [F] 822
Mupad [F(-1)] 822

Optimal result

Integrand size = 42, antiderivative size = 322

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{e+fx} dx = -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{f} + \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(1-\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{f} - \frac{2\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{f} + \frac{2\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\operatorname{PolyLog}\left(2,\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{f} + \frac{2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{f} - \frac{2\operatorname{PolyLog}\left(3,\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{f}$$

```
[Out] -ln((a*d-b*c)/d/(b*x+a))*ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/f+ln((
-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2*ln(1-(-a*f+b*e)*(d*x+c)/(-c*f+d*e)/
(b*x+a))/f-2*ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*polylog(2,b*(d*x+c)/
d/(b*x+a))/f+2*ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*polylog(2,(-a*f+b*
e)*(d*x+c)/(-c*f+d*e)/(b*x+a))/f+2*polylog(3,b*(d*x+c)/d/(b*x+a))/f-2*polyl
og(3,(-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))/f
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2554, 2404, 2354, 2421, 6724}

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{e+fx}\,dx = -\frac{2\,\text{PolyLog}\left(3,\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{f} - \frac{2\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{f} + \frac{2\,\text{PolyLog}\left(2,\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{f} - \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log^2\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{f} + \frac{\log\left(1-\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)\log^2\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{f} + \frac{2\,\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{f}$$

```
[In] Int[Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2/(e + f*x),x]
```

```
[Out] -((Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*
f)*(a + b*x))]^2)/f) + (Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))
]^2*Log[1 - ((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/f - (2*Log[((
b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*PolyLog[2, (b*(c + d*x))/(d*
(a + b*x))])/f + (2*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Po
lyLog[2, ((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/f + (2*PolyLog[3
, (b*(c + d*x))/(d*(a + b*x))])/f - (2*PolyLog[3, ((b*e - a*f)*(c + d*x))/(
(d*e - c*f)*(a + b*x))])/f
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2554

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Su
bst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m +
2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n
}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m] &
& IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = (-bc+ad)\text{Subst}\left(\int \frac{\log^2\left(\frac{(be-af)x}{de-cf}\right)}{(d-bx)(de-cf-(be-af)x)}\,dx, x, \frac{c+dx}{a+bx}\right)$$

$$= (-bc+ad)\text{Subst}\left(\int \left(\frac{b\log^2\left(\frac{(be-af)x}{de-cf}\right)}{(bc-ad)f(-d+bx)} + \frac{(be-af)\log^2\left(\frac{(be-af)x}{de-cf}\right)}{(bc-ad)f(de-cf-(be-af)x)}\right)\,dx, x, \frac{c+dx}{a+bx}\right)$$

$$= -\frac{b\text{Subst}\left(\int \frac{\log^2\left(\frac{(be-af)x}{de-cf}\right)}{-d+bx}\,dx, x, \frac{c+dx}{a+bx}\right)}{f} + \frac{((-bc+ad)(be-af))\text{Subst}\left(\int \frac{\log^2\left(\frac{(be-af)x}{de-cf}\right)}{de-cf+(-be+af)x}\,dx, x, \frac{c+dx}{a+bx}\right)}{(bc-ad)f}$$

$$= -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{f} + \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(1-\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{f} + \frac{2\text{Subst}\left(\int \frac{\log\left(\frac{(be-af)x}{de-cf}\right)\log\left(1-\frac{bx}{d}\right)}{x}dx, x, \frac{c+dx}{a+bx}\right)}{f} - \frac{2\text{Subst}\left(\int \frac{\log\left(\frac{(be-af)x}{de-cf}\right)\log\left(1+\frac{(-be+af)x}{de-cf}\right)}{x}dx, x, \frac{c+dx}{a+bx}\right)}{f}$$

$$= -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{f} + \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(1-\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{f} - \frac{2\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{f} + \frac{2\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\text{Li}_2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{f} + \frac{2\text{Subst}\left(\int \frac{\text{Li}_2\left(\frac{bx}{d}\right)}{x}dx, x, \frac{c+dx}{a+bx}\right)}{f} - \frac{2\text{Subst}\left(\int \frac{\text{Li}_2\left(-\frac{(-be+af)x}{de-cf}\right)}{x}dx, x, \frac{c+dx}{a+bx}\right)}{f}$$

$$= -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{f} + \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(1-\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{f} - \frac{2\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{f} + \frac{2\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\text{Li}_2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{f} + \frac{2\text{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{f} - \frac{2\text{Li}_3\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{f}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1080 vs. $2(322) = 644$.

Time = 0.40 (sec) , antiderivative size = 1080, normalized size of antiderivative = 3.35

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{e+fx}\,dx$$
$$= \frac{-\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right) + \log^2\left(\frac{a}{b}+x\right)\log(e+fx) - 2\log\left(\frac{a}{b}+x\right)\log\left(\frac{c}{d}+x\right)\log(e+fx) + \text{l}}{}$$

```
[In] Integrate[Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2/(e + f*x),
x]

[Out] (-(Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*
f)*(a + b*x))]^2) + Log[a/b + x]^2*Log[e + f*x] - 2*Log[a/b + x]*Log[c/d +
```

```
x]*Log[e + f*x] + Log[c/d + x]^2*Log[e + f*x] + 2*Log[a/b + x]*Log[((b*e -
a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[e + f*x] - 2*Log[c/d + x]*Log[
((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[e + f*x] + Log[((b*e -
 a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2*Log[e + f*x] - Log[a/b + x]^2*L
og[(b*(e + f*x))/(b*e - a*f)] + 2*Log[a/b + x]*Log[(f*(c + d*x))/(-(d*e) +
c*f)]*Log[(b*(e + f*x))/(b*e - a*f)] - Log[(f*(c + d*x))/(-(d*e) + c*f)]^2*
Log[(b*(e + f*x))/(b*e - a*f)] - 2*Log[a/b + x]*Log[((b*e - a*f)*(c + d*x))
/((d*e - c*f)*(a + b*x))]*Log[(b*(e + f*x))/(b*e - a*f)] + 2*Log[(f*(c + d*
x))/(-(d*e) + c*f)]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Lo
g[(b*(e + f*x))/(b*e - a*f)] - Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a
+ b*x))]^2*Log[(b*(e + f*x))/(b*e - a*f)] + 2*Log[a/b + x]*Log[c/d + x]*Log
[(d*(e + f*x))/(d*e - c*f)] - Log[c/d + x]^2*Log[(d*(e + f*x))/(d*e - c*f)]
 - 2*Log[a/b + x]*Log[(f*(c + d*x))/(-(d*e) + c*f)]*Log[(d*(e + f*x))/(d*e
- c*f)] + Log[(f*(c + d*x))/(-(d*e) + c*f)]^2*Log[(d*(e + f*x))/(d*e - c*f)
] + 2*Log[c/d + x]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log
[(d*(e + f*x))/(d*e - c*f)] - 2*Log[(f*(c + d*x))/(-(d*e) + c*f)]*Log[((b*e
 - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[(d*(e + f*x))/(d*e - c*f)]
+ Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2*Log[((-(b*c) + a*d
)*(e + f*x))/((d*e - c*f)*(a + b*x))] - 2*Log[((b*e - a*f)*(c + d*x))/((d*e
 - c*f)*(a + b*x))]*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] + 2*Log[((b*e -
 a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*PolyLog[2, ((b*e - a*f)*(c + d*x)
)/((d*e - c*f)*(a + b*x))] + 2*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))] - 2*
PolyLog[3, ((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/f
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 812 vs. $2(321) = 642$.

Time = 3.18 (sec) , antiderivative size = 813, normalized size of antiderivative = 2.52

method	result
derivativedivides	$(af-be)(ad-cb)\left(\frac{\ln\left(-\frac{(af-be)(ad-cb)}{b(cf-de)(bx+a)}+\frac{d(af-be)}{(cf-de)b}\right)^2\ln\left(\frac{(af-be)(ad-cb)}{b(cf-de)(bx+a)}-\frac{d(af-be)}{(cf-de)b}+1\right)+2\ln\left(-\frac{(af-be)(ad-cb}{b(cf-de)(bx+a}\right.}{(af-be)f(a}\right.$
default	$(af-be)(ad-cb)\left(\frac{\ln\left(-\frac{(af-be)(ad-cb)}{b(cf-de)(bx+a)}+\frac{d(af-be)}{(cf-de)b}\right)^2\ln\left(\frac{(af-be)(ad-cb)}{b(cf-de)(bx+a)}-\frac{d(af-be)}{(cf-de)b}+1\right)+2\ln\left(-\frac{(af-be)(ad-cb}{b(cf-de)(bx+a}\right.}{(af-be)f(a}\right.$
risch	Expression too large to display

```
[In] int(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(f*x+e),x,method=_RETURNVER
BOSE)
```

```
[Out] (a*f-b*e)*(a*d-b*c)*(1/(a*f-b*e)/f/(a*d-b*c)*(ln(-(a*f-b*e)*(a*d-b*c)/b/(c*
```

```
f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)^2*ln((a*f-b*e)*(a*d-b*c)/b/(c*f-d*e
)/(b*x+a)-d/(c*f-d*e)*(a*f-b*e)/b+1)+2*ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/
(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)*polylog(2,-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)
/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)-2*polylog(3,-(a*f-b*e)*(a*d-b*c)/b/(c*f-d
*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b))-b*(c*f-d*e)/(b*c*f-b*d*e)/(a*f-b*e)/f
/(a*d-b*c)*(ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*
e)/b)^2*ln(1+(b*c*f-b*d*e)/(-a*d*f+b*d*e)*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)
/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b))+2*ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b
*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)*polylog(2,-(b*c*f-b*d*e)/(-a*d*f+b*d*e)*(-(a
*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b))-2*polylog(3
,-(b*c*f-b*d*e)/(-a*d*f+b*d*e)*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/
(c*f-d*e)*(a*f-b*e)/b))))
```

Fricas [F]

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{e+fx}\,dx = \int \frac{\log\left(\frac{(be-af)(dx+c)}{(de-cf)(bx+a)}\right)^2}{fx+e}\,dx$$

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(f*x+e),x, algorithm
="fricas")
```

```
[Out] integral(log((b*c*e - a*c*f + (b*d*e - a*d*f)*x)/(a*d*e - a*c*f + (b*d*e -
b*c*f)*x))^2/(f*x + e), x)
```

Sympy [F]

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{e+fx}\,dx$$
$$= \int \frac{\log\left(-\frac{acf}{-acf+ade-bcfx+bdex} - \frac{adfx}{-acf+ade-bcfx+bdex} + \frac{bce}{-acf+ade-bcfx+bdex} + \frac{bdex}{-acf+ade-bcfx+bdex}\right)^2}{e+fx}\,dx$$

```
[In] integrate(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))**2/(f*x+e),x)
```

```
[Out] Integral(log(-a*c*f/(-a*c*f + a*d*e - b*c*f*x + b*d*e*x) - a*d*f*x/(-a*c*f
+ a*d*e - b*c*f*x + b*d*e*x) + b*c*e/(-a*c*f + a*d*e - b*c*f*x + b*d*e*x) +
 b*d*e*x/(-a*c*f + a*d*e - b*c*f*x + b*d*e*x))**2/(e + f*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{e+fx}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(f*x+e),x, algorithm
="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Memory limit reached. Please ju
mp to an outer pointer, quit program and enlarge thememory limits before ex
ecuting the program again.
```

Giac [F]

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{e+fx}\,dx = \int \frac{\log\left(\frac{(be-af)(dx+c)}{(de-cf)(bx+a)}\right)^2}{fx+e}\,dx$$

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(f*x+e),x, algorithm
="giac")
```

```
[Out] integrate(log((b*e - a*f)*(d*x + c)/((d*e - c*f)*(b*x + a)))^2/(f*x + e), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{e+fx}\,dx = \int \frac{\ln\left(\frac{(a\,f-b\,e)\,(c+d\,x)}{(c\,f-d\,e)\,(a+b\,x)}\right)^2}{e+f\,x}\,dx$$

```
[In] int(log(((a*f - b*e)*(c + d*x))/((c*f - d*e)*(a + b*x)))^2/(e + f*x),x)
```

```
[Out] int(log(((a*f - b*e)*(c + d*x))/((c*f - d*e)*(a + b*x)))^2/(e + f*x), x)
```

3.108 $\int \frac{\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+fx)}{be-af}\right)}{(a+bx)(c+dx)}\,dx$

Optimal result 823
Rubi [A] (verified) 824
Mathematica [B] (verified) 827
Maple [F] 828
Fricas [F] 829
Sympy [F(-1)] 829
Maxima [F(-2)] 829
Giac [F] 830
Mupad [F(-1)] 830

Optimal result

Integrand size = 65, antiderivative size = 433

$$\int \frac{\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+fx)}{be-af}\right)}{(a+bx)(c+dx)}\,dx = -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{2(bc-ad)} - \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+fx)}{be-af}\right)}{2(bc-ad)} + \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(1-\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{2(bc-ad)} - \frac{\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{bc-ad} + \frac{\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\operatorname{PolyLog}\left(2,\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{bc-ad} + \frac{\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{bc-ad} - \frac{\operatorname{PolyLog}\left(3,\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{bc-ad}$$

```
[Out] -1/2*ln((a*d-b*c)/d/(b*x+a))*ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(-
a*d+b*c)-1/2*ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2*ln(b*(f*x+e)/(-a*f
+b*e))/(-a*d+b*c)+1/2*ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2*ln(1-(-a*
f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))/(-a*d+b*c)-ln((-a*f+b*e)*(d*x+c)/(-c*f+d
*e)/(b*x+a))*polylog(2,b*(d*x+c)/d/(b*x+a))/(-a*d+b*c)+ln((-a*f+b*e)*(d*x+c
)/(-c*f+d*e)/(b*x+a))*polylog(2,(-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))/(-a*
d+b*c)+polylog(3,b*(d*x+c)/d/(b*x+a))/(-a*d+b*c)-polylog(3,(-a*f+b*e)*(d*x+
c)/(-c*f+d*e)/(b*x+a))/(-a*d+b*c)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.092, Rules used = {2589, 2554, 2404, 2354, 2421, 6724}

$$\int \frac{\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+fx)}{be-af}\right)}{(a+bx)(c+dx)}\,dx = -\frac{\text{PolyLog}\left(3,\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{bc-ad} - \frac{\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{bc-ad} + \frac{\text{PolyLog}\left(2,\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{bc-ad} - \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log^2\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{2(bc-ad)} - \frac{\log\left(\frac{b(e+fx)}{be-af}\right)\log^2\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{2(bc-ad)} + \frac{\log\left(1-\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)\log^2\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{2(bc-ad)} + \frac{\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{bc-ad}$$

```
[In] Int[(Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[(b*(e + f*x))
/(b*e - a*f)])/((a + b*x)*(c + d*x)),x]
```

```
[Out] -1/2*(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[((b*e - a*f)*(c + d*x))/((d*e -
 c*f)*(a + b*x))]^2)/(b*c - a*d) - (Log[((b*e - a*f)*(c + d*x))/((d*e - c*f
)*(a + b*x))]^2*Log[(b*(e + f*x))/(b*e - a*f)])/(2*(b*c - a*d)) + (Log[((b*
e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2*Log[1 - ((b*e - a*f)*(c + d*
x))/((d*e - c*f)*(a + b*x))])/(2*(b*c - a*d)) - (Log[((b*e - a*f)*(c + d*x)
)/((d*e - c*f)*(a + b*x))]*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(b*c -
a*d) + (Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*PolyLog[2, ((b
*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/(b*c - a*d) + PolyLog[3, (b*
(c + d*x))/(d*(a + b*x))]/(b*c - a*d) - PolyLog[3, ((b*e - a*f)*(c + d*x))/
((d*e - c*f)*(a + b*x))]/(b*c - a*d)
```

<u>Rule 2354</u>

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2554

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Su
bst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m +
2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n
}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m] &
& IGtQ[p, 0]
```

Rule 2589

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*Log[(i_.)*((j_.)*((g_.) + (h_.)*(x_))^(t_.))^(u_.)]*(v_), x_S
ymbol] :> With[{k = Simplify[v*(a + b*x)*(c + d*x)]}, Simp[k*Log[i*(j*(g +
h*x)^t)^u]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s + 1)/(p*r*(s + 1)*(b*c
- a*d))), x] - Dist[k*h*t*(u/(p*r*(s + 1)*(b*c - a*d))), Int[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^(s + 1)/(g + h*x), x], x] /; FreeQ[k, x]] /; FreeQ[{
a, b, c, d, e, f, g, h, i, j, p, q, r, s, t, u}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[p + q, 0] && NeQ[s, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = -\frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+fx)}{be-af}\right)}{2(bc-ad)} + \frac{f\int\frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{e+fx}\,dx}{2(bc-ad)}$$

$$= -\frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+fx)}{be-af}\right)}{2(bc-ad)} - \frac{1}{2}f\text{Subst}\left(\int \frac{\log^2\left(\frac{(be-af)x}{de-cf}\right)}{(d-bx)(de-cf-(be-af)x)}\,dx, x, \frac{c+dx}{a+bx}\right)$$

$$= -\frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+fx)}{be-af}\right)}{2(bc-ad)} - \frac{1}{2}f\text{Subst}\left(\int \left(\frac{b\log^2\left(\frac{(be-af)x}{de-cf}\right)}{(bc-ad)f(-d+bx)} + \frac{(be-af)\log^2\left(\frac{(be-af)x}{de-cf}\right)}{(bc-ad)f(de-cf-(be-af)x)}\right)dx, x, \frac{c+dx}{a+bx}\right)$$

$$= -\frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+fx)}{be-af}\right)}{2(bc-ad)} - \frac{b\text{Subst}\left(\int \frac{\log^2\left(\frac{(be-af)x}{de-cf}\right)}{-d+bx}\,dx, x, \frac{c+dx}{a+bx}\right)}{2(bc-ad)} - \frac{(be-af)\text{Subst}\left(\int \frac{\log^2\left(\frac{(be-af)x}{de-cf}\right)}{de-cf+(-be+af)x}\,dx, x, \frac{c+dx}{a+bx}\right)}{2(bc-ad)}$$

$$= -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{2(bc-ad)} - \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+fx)}{be-af}\right)}{2(bc-ad)} + \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(1-\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{2(bc-ad)} + \frac{\text{Subst}\left(\int \frac{\log\left(\frac{(be-af)x}{de-cf}\right)\log\left(1-\frac{bx}{d}\right)}{x}\,dx, x, \frac{c+dx}{a+bx}\right)}{bc-ad} - \frac{\text{Subst}\left(\int \frac{\log\left(\frac{(be-af)x}{de-cf}\right)\log\left(1+\frac{(-be+af)x}{de-cf}\right)}{x}\,dx, x, \frac{c+dx}{a+bx}\right)}{bc-ad}$$

$$= -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{2(bc-ad)} - \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+fx)}{be-af}\right)}{2(bc-ad)} + \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(1-\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{2(bc-ad)} - \frac{\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{bc-ad} + \frac{\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\text{Li}_2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{bc-ad} + \frac{\text{Subst}\left(\int \frac{\text{Li}_2\left(\frac{bx}{d}\right)}{x}\,dx, x, \frac{c+dx}{a+bx}\right)}{bc-ad} - \frac{\text{Subst}\left(\int \frac{\text{Li}_2\left(-\frac{(-be+af)x}{de-cf}\right)}{x}\,dx, x, \frac{c+dx}{a+bx}\right)}{bc-ad}$$

$$= -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{2(bc-ad)} - \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+fx)}{be-af}\right)}{2(bc-ad)}$$
$$+ \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(1-\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{2(bc-ad)} - \frac{\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{bc-ad}$$
$$+ \frac{\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\text{Li}_2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{bc-ad} + \frac{\text{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{bc-ad} - \frac{\text{Li}_3\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{bc-ad}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1855 vs. $2(433) = 866$.

Time = 0.44 (sec) , antiderivative size = 1855, normalized size of antiderivative = 4.28

$$\int \frac{\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+fx)}{be-af}\right)}{(a+bx)(c+dx)}\,dx$$
$$= \frac{2\log\left(\frac{c}{d}+x\right)\log\left(\frac{e}{f}+x\right)\log\left(\frac{d(a+bx)}{-bc+ad}\right)+2\log\left(\frac{a}{b}+x\right)\log\left(\frac{e}{f}+x\right)\log\left(\frac{b(c+dx)}{bc-ad}\right)-2(\log(a+bx)-\log(c}{}$$

```
[In] Integrate[(Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[(b*(e +
 f*x))/(b*e - a*f)])/((a + b*x)*(c + d*x)),x]
```

```
[Out] (2*Log[c/d + x]*Log[e/f + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 2*Log[a/b
+ x]*Log[e/f + x]*Log[(b*(c + d*x))/(b*c - a*d)] - 2*(Log[a + b*x] - Log[c
+ d*x])*(Log[a/b + x] - Log[c/d + x] + Log[((b*e - a*f)*(c + d*x))/((d*e -
c*f)*(a + b*x))])*(Log[e/f + x] - Log[(b*(e + f*x))/(b*e - a*f)]) + (Log[(d
*(a + b*x))/(-(b*c) + a*d)] - Log[(f*(a + b*x))/(-(b*e) + a*f)])*Log[(b*(e
+ f*x))/(b*e - a*f)]*(-2*Log[c/d + x] + Log[(b*(e + f*x))/(b*e - a*f)]) + L
og[a/b + x]^2*(-Log[e/f + x] + Log[(b*(e + f*x))/(b*e - a*f)]) + (Log[(b*(c
 + d*x))/(b*c - a*d)] - Log[(f*(c + d*x))/(-(d*e) + c*f)])*Log[(d*(e + f*x)
)/(d*e - c*f)]*(-2*Log[a/b + x] + Log[(d*(e + f*x))/(d*e - c*f)]) + Log[c/d
 + x]^2*(-Log[e/f + x] + Log[(d*(e + f*x))/(d*e - c*f)]) + 2*(-Log[(b*(c +
d*x))/(b*c - a*d)] + Log[(f*(c + d*x))/(-(d*e) + c*f)])*Log[(d*(e + f*x))/(
d*e - c*f)]*Log[((-(b*c) + a*d)*(e + f*x))/((d*e - c*f)*(a + b*x))] + (Log[
(-(b*e) + a*f)/(f*(a + b*x))] + Log[(b*(c + d*x))/(b*c - a*d)] - Log[((b*e
- a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])*Log[((-(b*c) + a*d)*(e + f*x))/
((d*e - c*f)*(a + b*x))]^2 + 2*(-Log[(d*(a + b*x))/(-(b*c) + a*d)] + Log[(f
*(a + b*x))/(-(b*e) + a*f)])*Log[(b*(e + f*x))/(b*e - a*f)]*Log[((b*c - a*d
)*(e + f*x))/((b*e - a*f)*(c + d*x))] + (Log[(d*(a + b*x))/(-(b*c) + a*d)]
+ Log[(-(d*e) + c*f)/(f*(c + d*x))] - Log[((d*e - c*f)*(a + b*x))/((b*e - a
*f)*(c + d*x))])*Log[((b*c - a*d)*(e + f*x))/((b*e - a*f)*(c + d*x))]^2 + 2
*(Log[e/f + x] - Log[((-(b*c) + a*d)*(e + f*x))/((d*e - c*f)*(a + b*x))])*P
olyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 2*Log[a/b + x]*PolyLog[2, (f*(a +
```

```
b*x))/(-(b*e) + a*f)] + 2*(Log[e/f + x] - Log[((b*c - a*d)*(e + f*x))/((b*
e - a*f)*(c + d*x))])*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] + (Log[e/f + x]
 - Log[(b*(e + f*x))/(b*e - a*f)])*(Log[a/b + x]^2 + Log[c/d + x]^2 - 2*(Lo
g[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)] + PolyLog[2, (d*(a + b*x))/(-(b*c
) + a*d)]) - 2*(Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + PolyLog[2,
 (b*(c + d*x))/(b*c - a*d)])) + 2*Log[c/d + x]*PolyLog[2, (f*(c + d*x))/(-(
d*e) + c*f)] + 2*(Log[c/d + x] + Log[((b*c - a*d)*(e + f*x))/((b*e - a*f)*(
c + d*x))])*PolyLog[2, (b*(e + f*x))/(b*e - a*f)] + 2*(Log[a/b + x] - Log[c
/d + x] + Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])*(Log[e/f +
x]*(Log[(f*(a + b*x))/(-(b*e) + a*f)] - Log[(f*(c + d*x))/(-(d*e) + c*f)])
+ PolyLog[2, (b*(e + f*x))/(b*e - a*f)] - PolyLog[2, (d*(e + f*x))/(d*e - c
*f)]) + 2*(Log[a/b + x] + Log[((-(b*c) + a*d)*(e + f*x))/((d*e - c*f)*(a +
b*x))])*PolyLog[2, (d*(e + f*x))/(d*e - c*f)] + 2*Log[((-(b*c) + a*d)*(e +
f*x))/((d*e - c*f)*(a + b*x))]*(PolyLog[2, (b*(e + f*x))/(f*(a + b*x))] - P
olyLog[2, -(((b*c - a*d)*(e + f*x))/((d*e - c*f)*(a + b*x)))]) + 2*Log[((b*
c - a*d)*(e + f*x))/((b*e - a*f)*(c + d*x))]*(PolyLog[2, (d*(e + f*x))/(f*(
c + d*x))] - PolyLog[2, ((b*c - a*d)*(e + f*x))/((b*e - a*f)*(c + d*x))]) -
 2*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)] - 2*PolyLog[3, (f*(a + b*x))/(-
(b*e) + a*f)] - 2*PolyLog[3, (b*(c + d*x))/(b*c - a*d)] - 2*PolyLog[3, (f*(
c + d*x))/(-(d*e) + c*f)] - 2*PolyLog[3, (b*(e + f*x))/(b*e - a*f)] - 2*Pol
yLog[3, (d*(e + f*x))/(d*e - c*f)] - 2*PolyLog[3, (b*(e + f*x))/(f*(a + b*x
))] + 2*PolyLog[3, -(((b*c - a*d)*(e + f*x))/((d*e - c*f)*(a + b*x)))] - 2*
PolyLog[3, (d*(e + f*x))/(f*(c + d*x))] + 2*PolyLog[3, ((b*c - a*d)*(e + f*
x))/((b*e - a*f)*(c + d*x))])/(2*(b*c - a*d))
```

Maple [F]

$$\int \frac{\ln\left(\frac{(-af+be)(dx+c)}{(-cf+de)(bx+a)}\right)\ln\left(\frac{b(fx+e)}{-af+be}\right)}{(bx+a)(dx+c)}dx$$

```
[In] int(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*ln(b*(f*x+e)/(-a*f+b*e))/(b*x
+a)/(d*x+c),x)
```

```
[Out] int(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*ln(b*(f*x+e)/(-a*f+b*e))/(b*x
+a)/(d*x+c),x)
```

Fricas [F]

$$\int \frac{\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+fx)}{be-af}\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\log\left(\frac{(fx+e)b}{be-af}\right)\log\left(\frac{(be-af)(dx+c)}{(de-cf)(bx+a)}\right)}{(bx+a)(dx+c)}\,dx$$

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*log(b*(f*x+e)/(-a*f+b*
e))/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(log((b*c*e - a*c*f + (b*d*e - a*d*f)*x)/(a*d*e - a*c*f + (b*d*e -
b*c*f)*x))*log((b*f*x + b*e)/(b*e - a*f))/(b*d*x^2 + a*c + (b*c + a*d)*x),
x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+fx)}{be-af}\right)}{(a+bx)(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*ln(b*(f*x+e)/(-a*f+b*e)
)/(b*x+a)/(d*x+c),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+fx)}{be-af}\right)}{(a+bx)(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*log(b*(f*x+e)/(-a*f+b*
e))/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Memory limit reached. Please ju
mp to an outer pointer, quit program and enlarge thememory limits before ex
ecuting the program again.
```

Giac [F]

$$\int \frac{\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+fx)}{be-af}\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\log\left(\frac{(fx+e)b}{be-af}\right)\log\left(\frac{(be-af)(dx+c)}{(de-cf)(bx+a)}\right)}{(bx+a)(dx+c)}\,dx$$

[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*log(b*(f*x+e)/(-a*f+b*e))/(b*x+a)/(d*x+c),x, algorithm="giac")

[Out] integrate(log((f*x + e)*b/(b*e - a*f))*log((b*e - a*f)*(d*x + c)/((d*e - c*f)*(b*x + a)))/((b*x + a)*(d*x + c)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+fx)}{be-af}\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\ln\left(-\frac{b\,(e+f\,x)}{a\,f-b\,e}\right)\,\ln\left(\frac{(a\,f-b\,e)\,(c+d\,x)}{(c\,f-d\,e)\,(a+b\,x)}\right)}{(a+b\,x)\,(c+d\,x)}\,dx$$

[In] int((log(-(b*(e + f*x))/(a*f - b*e))*log(((a*f - b*e)*(c + d*x))/((c*f - d*e)*(a + b*x))))/((a + b*x)*(c + d*x)),x)

[Out] int((log(-(b*(e + f*x))/(a*f - b*e))*log(((a*f - b*e)*(c + d*x))/((c*f - d*e)*(a + b*x))))/((a + b*x)*(c + d*x)), x)

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions 831

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                  end if
              else #result contains complex but optimal is not
                  if debug then
                          print("result contains complex but optimal is not");
                  fi;
                  return "C","Result contains complex when optimal does not.";
              fi;
          else # result do not contain complex
               # this assumes optimal do not as well. No check is needed here.
              if debug then
                     print("result do not contain complex, this assumes optimal do not as well")
              fi;
              if leaf_count_result<=2*leaf_count_optimal then
                  if debug then
                      print("leaf_count_result<=2*leaf_count_optimal");
                  fi;
                  return "A"," ";
              else
                  if debug then
                      print("leaf_count_result>2*leaf_count_optimal");
                  fi;
                  return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
              fi;
          fi;
      else #ExpnType(result) > ExpnType(optimal)
          if debug then
              print("ExpnType(result) > ExpnType(optimal)");
          fi;
          return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
      fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#           issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

  #debug=False
  if debug:
     print ("Enter is_atom, expn=",expn)

  if not hasattr(expn, 'parent'):
    return False


  #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
  try:
    if expn.parent() is SR:
      return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
      return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
      return expn in expn.parent().base_ring() or expn in expn.parent().gens()

    return False

  except AttributeError as error:
    print("Exception,AttributeError in  is_atom")
    print ("cought exception" , type(error).__name__ )
    return False


def expnType(expn):

  if debug:
    print (">>>>>Enter expnType, expn=", expn)
    print (">>>>>is_atom(expn)=", is_atom(expn))

  if is_atom(expn):
    return 1
  elif type(expn)==list:   #isinstance(expn,list):
    return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
  elif  is_sqrt(expn):
    if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
      return 1
    else:
      return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
  elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
    if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
      return expnType(expn.operands()[0])   #expnType(expn.args[0])
    elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
      if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)



    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```